Safe Assets: A Retrading Perspective

Markus K. Brunnermeier
Sebastian Merkel
Yuliy Sannikov

UCL 2023-10-06
Motivation

- What is a safe asset? What are its features?
  - Precautionary savings
  - Why is (US) government debt a safe asset?

- (Safe) asset pricing formula for incomplete market setting
  - What is service flow? How does it differ from
    - cash flow and convenience yield?
  - Flight-to-safety phenomenon (negative $\beta$)
  - Loss of safe-asset status / exorbitant privilege

- Is there a public debt valuation puzzle?
  - Valuation too large? Risk premium too low?

- Can government spend without taxation? How much?
  - Debt sustainability analysis and “Debt Laffer Curve”
Motivation

- Why is there debt valuation puzzle for US, Japan?
- Traditional FTPL equation:
  \[ \frac{B_t}{\rho_t} = E_t[PV_\xi (\text{primary surpluses})] + \text{Bubble} \]
- Persistently negative primary surpluses
- Negative surpluses in recessions: Asset Pricing (with SDF $\xi_t$)
Asset Pricing with Safe Assets

- Standard asset pricing: Buy and Hold Perspective

\[ price = \mathbb{E}[PV_\xi (\text{cash flow})] \]
Asset Pricing with Safe Assets

- Standard asset pricing with bubbles: Buy and Hold Perspective

\[ price = \mathbb{E}[PV_\xi (\text{cash flow})] + \text{bubble} \]

- Bubble if \( r < g \) (due to precautionary motive)
Asset Pricing with Safe Assets

- Standard asset pricing with bubbles: **Buy and Hold Perspective**

\[ \text{price} = \mathbb{E}[PV_\xi (\text{cash flow})] + \text{bubble} \]

- Bubble if \( r < g \) (due to precautionary motive)
- Effectively prices a buy-and-hold strategy
- But in incomplete markets, agents trade

- Alternative approach: **Dynamic Trading Perspective**

  - Value agents’ actual portfolio strategies, then aggregate
  - Discounts at higher effective rate \( r^{**} > g \)

\[ \text{price} = \mathbb{E}[PV^{**} (\text{cash flow})] + \mathbb{E}[PV^{**} (\text{service flow})] \]

Note: SDF \( \xi^{**} \) = “representative agent” discount rate \( \neq m \) (Reis 2022)
What’s is a Safe Asset Service Flow?

- Safe asset = good friend
  - Idiosyncratic risk: provides partial insurance through re-trading
What’s is a Safe Asset Service Flow?

- Safe asset = good friend
- Idiosyncratic risk: provides partial insurance through re-trading
What’s a Safe Asset? Exorbitant Privilege rises in Recessions

- Safe asset = good friend
  - Idiosyncratic risk: provides partial insurance through re-trading
  - Aggregate risk: appreciates in value in bad times (negative $\beta$)

In recessions:
Risk is higher
- Service flow is more valuable
- Cash flows are lower
  (depends on fiscal policy)
Safe Asset Definition

- **Equilibrium concept**
- **Good friend** (relative to own net worth return $d r_t^{n_i}$)
  - Idiosyncratic risk
  - Aggregate risk
- $Cov_t \left[ d \xi_t^i / \xi_t^i, d r_t^j - d r_t^{n_i} \right] > 0$
  - For agent $i$
    - with SDF $\xi_t^i$ is SDF of agent $i$ ($d \xi_t^i / \xi_t^i = -r_t^f dt - \varsigma_t^i d Z_t - \tilde{\varsigma}_t^i d \tilde{Z}_t^i$)
    - with net worth return $d r_t^{n_i}$
  - At time $t$ possible loss of safe asset status/exorbitant privilege

- **Re-tradeable:**
  - No asymmetric info – info insensitive
  - Service flow is derived from “dynamic re-trading”
Service Flow Term vs. Bubble Term

- **Service flow**
  - Partial insurance via retrading – (partially undo incomplete markets)
    - Bewley ... smooth consumption
    - BruSan ... retrade capital and safe asset + smooth consumption
  - Remaining (idiosyncratic) risk depresses cash flow return

- **Bubble**
  - \( \lim_{T \to \infty} E[\bar{\xi}_t P_t] > 0 \) if \( r_t^f + \text{risk premium} \leq g_t \) (on average)
    - \( r_t^f \) is depressed by precautionary savings (incl. uninsurable idiosyncratic risk)
  - **Transversality condition** holds for each individual, but not in aggregate
    - \( \neq \) complete markets
## Related Literature 1

- **FTPL literature:** Leeper, Sims, Woodford, Cochrane, ...

<table>
<thead>
<tr>
<th>Friction</th>
<th>OLG</th>
<th>Incomplete Markets + idiosyncratic risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk</td>
<td>deterministic</td>
<td>endowment risk, borrowing constraint, return risk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Risk tied up with Individual capital</td>
</tr>
<tr>
<td>Only money</td>
<td>Samuelson</td>
<td>Bewley</td>
</tr>
<tr>
<td>With capital</td>
<td>Diamond</td>
<td>Aiyagari</td>
</tr>
</tbody>
</table>

“*I Theory without I*”
Brunnermeier-Sannikov (AER PP 2016)
Angeletos (2006)
Related Literature 2

- **Safe Asset:**
  - Gorton-Pennachi (1990), Dang et al. (2017), Caballero et al. (2016), ...
  - Brunnermeier et al. (2017) , ESBies,

- **Equity premium**
  - Constantinides-Duffie (1996) – imposes “aggregate” transversality condition

- **Public Debt Evaluation Puzzles:**
  - Jiang et al. (2020,2021)

- **Fiscal debt sustainability $r$ vs. $g$:**
  - Bassetto-Cui (2018), Reis (2020)
Roadmap

- Motivation
- Steady state model (closed form solution)
- Stochastic idiosyncratic risk model
- (Safe) asset pricing with SDF $\xi_t^{**}$
- Flight-to-Safety, $\beta < 0$, and Excess Volatility of capital
- Equity issuance, non-safe mutual fund with $\beta > 0$
- Debt Laffer Curve
- Price level determination with a bubble
  - Fiscal space to defend Exorbitant Privilege (crypto, ...)

Calibration
Exorbitant Privilege Model with a Bubble: Safe Asset Model

- Model overview:
  - Continuous time, infinite horizon, one consumption good
  - Continuum of agents
    - Operate capital with \textit{time-varying idiosyncratic risk, AK production technology}
    - Can trade \textit{capital, government bond, and diversified equity claims}
  - Government
    - Exogenous spending
    - Taxes output
    - Issues (nominal) bonds
  - Financial Frictions: \textit{incomplete markets}
    - Agents \textit{cannot fully insure idiosyncratic risk (must retain skin in the game)}
    - Aggregate risk: fluctuations in \textit{volatility of idiosyncratic risk (and capital productivity)}

- Calibrated to US data to match macro and asset pricing moments
Model with Capital + Safe Asset

- Each heterogenous citizen $\tilde{i} \in [0,1]$
  \[ E \left[ \int_0^\infty e^{-\rho t} \log c^\tilde{i}_t \, dt \right] \] 
  s.t. 
  \[ \frac{dn^\tilde{i}_t}{n^\tilde{i}_t} = -\frac{c^\tilde{i}_t}{n^\tilde{i}_t} \, dt + dr^B_t + (1 - \theta^\tilde{i}_t) \left( dr^K_t (l^\tilde{i}_t) - dr^B_t \right) \] & No Ponzi

- Each citizen operates physical capital $k^\tilde{i}_t$
  
  - Output (net investment) 
    \[ a_t k^\tilde{i}_t \, dt - l^\tilde{i}_t k^\tilde{i}_t \, dt \]
  
  - Output tax 
    \[ \tau_t a_t k^\tilde{i}_t \, dt \]
  
  - \[ \frac{dk^\tilde{i}_t}{k^\tilde{i}_t} = \left( \Phi(l^\tilde{i}_t) - \delta \right) dt + \tilde{\sigma}_t d\tilde{Z}^\tilde{i}_t + d\Delta_t^k \]
    
    - $d\tilde{Z}^\tilde{i}_t$ idiosyncratic Brownian

- Financial Friction: Incomplete markets: no $d\tilde{Z}^\tilde{i}_t$ claims

- Aggregate risk:
  \[ \tilde{\sigma}_t, a_t, g_t \] exogenous process by aggregate Brownian $dZ_t$
  
  - E.g. Heston model: 
    \[ d\tilde{\sigma}^2_t = -\psi \left( \tilde{\sigma}^2_t - (\tilde{\sigma}^0)^2 \right) dt - \sigma\tilde{\sigma}_t dZ_t \]

  - $a_t = a(\tilde{\sigma}_t), g_t = g(\tilde{\sigma}_t)$
Government: Taxes, Bond/Money Supply, Gov. Budget

- **Policy Instruments** \( K_t := \int k_t \, d\tilde{\tau} \)
  - Government spending \( g_t K_t \) (with exogenous \( g_t \))
  - Proportional output tax \( \tau_t a_t K_t \)
  - Nominal government debt supply \( \frac{dB_t}{B_t} = \mu_t^B \, dt \)
  - Floating nominal interest rate \( i_t \) on outstanding bonds

- **Government budget constraint (BC)**

\[
\left( \mu_t^B - i_t \right) B_t + \varrho_t K_t \left( \tau_t a_t - g_t \right) = 0
\]

\( \bar{\mu}_t^B := \mu_t^B \)  
Not market clearing,  
Payment/redistribution to bond holders  
\( q_t^B \) clears bond market

- **Equilibrium selection:** No No-Ponzi constraint
Optimal Choices & Market Clearing

- **Optimal investment rate**
  \[ \Phi(\iota) = \frac{1}{\phi} \log(1 + \phi \iota) \]
  \[ \iota_t = \frac{1}{\phi} (q_t^K - 1) \]

- **Consumption**
  \[ \frac{c_t}{n_t} =: \rho \quad \Rightarrow C_t = \rho (q_t^K K_t + q_t^B B_t) = (a - \iota_t - g) K_t \]

- **Portfolio**
  - Solve for \( \theta_t \) later

  \[ 1 - \theta_t = \frac{q_t^K}{q_t^K + q_t^B} =: 1 - \vartheta_t \]
  \[ \vartheta = \text{fraction of wealth in nominal claims} \]

  Bond market clears by Walras law

- **Goods market**
  \[ C_t = \rho (q_t^K K_t + q_t^B B_t) = (a - \iota_t - g) K_t \]
### Equilibrium (before solving for portfolio choice)

<table>
<thead>
<tr>
<th>Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_t^B$ = $\vartheta_t \frac{1 + \phi \tilde{a}}{(1 - \vartheta_t) + \phi \tilde{p}_t}$</td>
</tr>
<tr>
<td>$q_t^K = (1 - \vartheta_t) \frac{1 + \phi \tilde{a}}{(1 - \vartheta_t) + \phi \tilde{p}_t}$</td>
</tr>
<tr>
<td>$\iota_t = \frac{(1 - \vartheta_t)\tilde{a} - \tilde{p}_t}{(1 - \vartheta_t) + \phi \tilde{p}_t}$</td>
</tr>
</tbody>
</table>

- Moneyless equilibrium with $q_t^B = 0 \Rightarrow \vartheta_t = 0$
- Next, determine portfolio choice.

\[
\tilde{a} = a - g
\]

For log utility
\[
\tilde{p}_t = \rho
\]
Portfolio choice $\theta \Rightarrow$ Evaluation Equation

- Asset pricing equation (martingale method):
  
  $\mathbb{E}_t \left[ \frac{dr^{K,i}_t}{dt} \right] = \frac{\tilde{a} - \iota_t}{q^K_t} + \frac{q^K_B}{q^K_t} \tilde{\mu}^B + \Phi(\iota_t) - \delta + \mu^q^K = r^f_t + \tilde{\zeta}_t \tilde{\sigma}$

  $\mathbb{E}_t [dr^{B}_t] = -\tilde{\mu} + \Phi(\iota_t) - \delta + \mu^q_B = r^f_t$

  $\mathbb{E}_t \left[ \frac{dr^{K,i}_t}{dt} \right] - \mathbb{E}_t \left[ \frac{dr^{B}_t}{dt} \right] = \frac{\tilde{a} - \iota_t}{q^K_t} + \frac{1}{1 - \theta_t} \tilde{\mu}^B + \mu^q^K - \mu^q_B = \tilde{\zeta}_t \tilde{\sigma}$

- Goods market clearing: $\tilde{\rho}(q^B_t + q^K_t) = (\tilde{a} - \iota_t) \Rightarrow \frac{\tilde{a} - \iota_t}{q^K_t} = \frac{-\mu_t^\theta / (1 - \theta_t)}{\tilde{\rho}}$

- Price of idiosyncratic risk: $\tilde{\zeta}_t = \gamma \tilde{\sigma}^n_t = \gamma (1 - \theta_t) \gamma \tilde{\sigma}$

- Capital market clearing: $1 - \theta_t = 1 - \vartheta_t$

- FTPL/Money Valuation Equation:
  
  $\mu^\theta_t = \rho + \tilde{\mu}^B - (1 - \theta_t)^2 \tilde{\sigma}^2$

  In steady state $\mu^\theta_t = 0$: $\sqrt{\rho + \tilde{\mu}^B / \tilde{\sigma}}$

Recall: $\vartheta = \text{fraction of wealth in nominal claims}$

Price of risk

Agent $i$'s SDF, $\xi^i_t / \xi^i_t = -r^f_t dt - \tilde{\zeta}_t d\tilde{Z}_t - \xi^i_t d\tilde{Z}_t$
Two Stationary Equilibria – in closed form

<table>
<thead>
<tr>
<th>Non-Monetary</th>
<th>Monetary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0^B = 0$</td>
<td>$\frac{B_0}{\delta_0} / K_t = q^B = \frac{(\tilde{\sigma} - \sqrt{\rho + \tilde{\mu}^B}) (1 + \phi \tilde{a})}{\sqrt{\rho + \tilde{\mu}^B + \phi \tilde{\sigma} \rho}}$</td>
</tr>
<tr>
<td>$q_0^K = \frac{1 + \phi \tilde{a}}{1 + \phi \rho}$</td>
<td>$q^K = \frac{\sqrt{\rho + \tilde{\mu}^B} (1 + \phi \tilde{a})}{\sqrt{\rho + \tilde{\mu}^B + \phi \tilde{\sigma} \rho}}$</td>
</tr>
<tr>
<td>$\lambda = \frac{\tilde{a} - \rho}{1 + \phi \rho_0}$</td>
<td>$\lambda = \frac{\tilde{a} \sqrt{\rho + \tilde{\mu}^B - \tilde{\sigma} \rho}}{\sqrt{\rho + \tilde{\mu}^B + \phi \tilde{\sigma} \tilde{\rho}}}$</td>
</tr>
</tbody>
</table>

- $\rho$ time preference rate
- $\phi$ adjustment cost for investment rate
- $\tilde{\mu}_t^B = \mu_t^B - i_t$ bond issuance rate beyond interest rate
- $\tilde{a} = a - g$ part of TFP not spend on gov.
Remarks for steady state case:

- **Real risk-free rate**
  \[ r_f = \Phi(\mu_B^\beta) - \delta - \mu^B \]

  - \( \mu_B^B = 0 \Rightarrow s = 0 \) no primary surplus (no cash payoff for bond)
  - \( q_B^B K = \frac{B}{P} > 0 \) bond trades at a **bubble** due to service flow

  - \( \mu_B^B > 0 \Rightarrow s < 0 \) primary deficit (constant fraction of GDP)
    - As long as \( q_B^B > 0 \) “mine the bubble”

  - \( \mu_B^B < 0 \Rightarrow s > 0 \) and \( r > g \) primary surplus (constant fraction of GDP)
    - \( q_B^B K_t = E_t[PV_{rf}(sK_t)] \) no bubble, but service flow
    - \( \frac{B_0}{\varphi_0} = \mathbb{E} \left[ \int_0^\infty e^{-r f t} sK_t dt \right] \)
Flight-to-Safety when idiosyncratic risk is $\tilde{\sigma}_t$ high $\Rightarrow$ negative $\beta$ for Gov. Bond

Gov. debt value rises in recessions

Capital price
Safe Asset – Service flow >> Cash flow

- Asset Price = \( E[\text{PV(} \text{primary surplus/cash flows})] + E[\text{PV(} \text{service flows})] \)
Debt Valuation (FTPL) – Two Perspectives

- **Buy and Hold Perspective:**
  \[ \frac{B_0}{\phi_0} = \lim_{T \to \infty} \left( E \left[ \int_0^T \xi_t s_t K_t dt \right] + E \left[ \xi_T \frac{B_T}{\phi_T} \right] \right) \]
  - Valuation of strategy that buys and holds a fixed fraction of outstanding debt

- Agent i’s SDF, \( \xi_t: \frac{d \xi_t}{\xi_t} = -r_t^d dt - \zeta_t dZ_t - \tilde{\zeta}_t^i d\tilde{Z}_t^i \), idiosyncratic consumption vol. \( \tilde{\sigma}_t^C \)
Debt Valuation (FTPL) – Two Perspectives

**Buy and Hold Perspective:**

\[ \frac{B_0}{\varphi_0} = \lim_{T \to \infty} \left( \mathbb{E} \left[ \int_0^T \xi_t s_t K_t dt \right] + \mathbb{E} \left[ \frac{\xi_t}{\varphi_T} B_T \right] \right) \]

- Valuation of strategy that buys and holds a fixed fraction of outstanding debt

**Dynamic Trading Perspective:**

\[ \eta_0 \frac{B_0}{\varphi_0} = \mathbb{E} \left[ \int_0^\infty \xi_t \eta_t^i s_t K_t dt \right] + \mathbb{E} \left[ \int_0^\infty \xi_t \eta_t^i (\tilde{\sigma}_t^c)^2 \frac{B_t}{\varphi_t} dt \right] \]

- Valuation of equilibrium cash flows from individual bond portfolios, incl. trading cash flows (aggregated over all agents \( i \) to obtain total value of debt)

- Agent \( i \)'s SDF, \( \xi_t: \frac{d \xi_t}{\xi_t} = -r_t^f dt - \xi_t dZ_t - \tilde{\xi}_t d\tilde{Z}_t \), idiosyncratic consumption vol. \( \tilde{\sigma}_t^c \)
Debt Valuation (FTPL) – Two Perspectives

- **Buy and Hold Perspective:**
  - \( \frac{B_0}{\varphi_0} = \lim_{T \to \infty} \left( \mathbb{E} \left[ \int_0^T \xi_t s_t K_t dt \right] + \mathbb{E} \left[ \xi_i \frac{B_T}{\varphi_T} \right] \right) \)
  - Valuation of strategy that buys and holds a fixed fraction of outstanding debt

- **Dynamic Trading Perspective:**
  - \( \frac{B_0}{\varphi_0} = \mathbb{E} \left[ \int_0^\infty \left( \int \xi_t \eta_t^i \right) s_t K_t dt \right] + \mathbb{E} \left[ \int_0^\infty \left( \int \xi_t \eta_t^i \right) \left( \tilde{\sigma}_t \right)^2 \frac{B_t}{\varphi_t} dt \right] \)
  - Valuation of equilibrium cash flows from individual bond portfolios, incl. trading cash flows (aggregated over all agents \( i \) to obtain total value of debt)

- Agent \( i \)'s SDF, \( \xi_t^i \): \( d \xi_t^i / \xi_t^i = -r^f_t dt - \xi_t^i dZ_t - \tilde{\xi}_t^i d\tilde{Z}_t^i \), idiosyncratic consumption vol. \( \tilde{\sigma}_t^i \)
“Aggregate Intertemporal Budget Constraint

\[ \frac{q^K_t K_t}{\text{total (net) wealth}} + \frac{q^B_t K_t}{\text{total (net) wealth}} = \mathbb{E}_t \left[ \int_t^{\infty} \int_{\xi_t^i}^{\xi_s^i} \int_{\eta_t^i}^{\eta_s^i} c_s ds \right] \quad (*) \]

Lucas-type models: \( q^B = 0 \) (also \( C_t = Y_t \), no idiosyncratic risk)
- Value of equity (Lucas tree) = PV of consumption claim
- Volatility equity values require (low) volatile RHS of (*)

This model: even for constant RHS of (*), \( q^K_t K_t \) can be volatile due to flight to safety:
- increase in \( \sigma_t \) \( \Rightarrow \) Portfolio reallocation from capital to bonds, \( q^K_t K_t \downarrow, \frac{B_t}{\varphi_t} = q^K_t K_t \uparrow \),

Quantitatively relevant? Yes
- Excess return volatility
  - 2.9% in equivalent bondless model (\( s = 0 \) and no bubble)
  - 12.9% in our model
Service Flow Term, Convenience Yield, Ponzi Scheme

- **Service flow**
  - Convenience yield: relaxes collateral constraint or CIA constraint (money)
    - Traditional measure: BAA-US Treasury spread
  - Here: Partially completing markets through retriming
    - Low interest rate (cash flow) asset can be issued by everyone
      Hence, corporate-Treasury spread = 0

- **Ponzi scheme** is not feasibly for everyone
  **No Ponzi constraint** may be binding
  - Who can run a Ponzi scheme?
    - ... assigned by equilibrium selection
  - Likely to government, private entities are subject to solvency constraint
    - ... still there is a Debt Laffer Curve


Roadmap

- Motivation
- Steady state model (closed form solution)
- Stochastic idiosyncratic risk model
- (Safe) asset pricing with SDF $\xi^*$
- Flight-to-Safety, $\beta < 0$, and Excess Volatility of capital
- Equity issuance, non-safe mutual fund with $\beta > 0$
- Debt Laffer Curve
- Price level determination with a bubble
  - Fiscal space to defend Exorbitant Privilege (crypto, ...)

Calibration
Equity Markets (ETF)

- **Equity Market**
  - Each citizen $i$ can sell off a fraction $(1 - \bar{\chi})$ of capital risk to outside equity holders
  - Return $d'r_{t}^{E,i}$
    - Same risk as $d'r_{t}^{K,i}$
    - But $\mathbb{E}_{t} [d'r_{t}^{E,i}] < \mathbb{E}_{t} [d'r_{t}^{K,i}]$... due to insider premium
  - Prop.: Model equations as before but replace $\bar{\sigma}$ with $\bar{\chi}\bar{\sigma}$
Epstein-Zin preferences for calibration (EIS=1)

- Citizen \( \tilde{i} \) maximizes \( V_0^{\tilde{i}} \) where \( V_t^{\tilde{i}} \) is recursively defined by

\[
V_t^i = E_t \left[ \int_t^\infty (1 - \gamma) \rho V_s^i \left( \log(c_s^i) - \frac{1}{1 - \gamma} \log \left( (1 - \gamma) V_s^i \right) \right) ds \right]
\]

- Needed to generate realistic prices of risk (Sharpe ratio)
Numerical Illustration (Calibration)

- Exogenous processes:
  - **Recessions** feature high idiosyncratic risk and low consumption
  - $\tilde{\sigma}_t$: Heston (1993) model of stochastic volatility
    \[ d\tilde{\sigma}_t^2 = -\psi \left( \tilde{\sigma}_t^2 - (\tilde{\sigma}^0)^2 \right) dt - \sigma \tilde{\sigma}_t dZ_t \]
  - $a_t$: $a_t = a(\tilde{\sigma}_t)$
    \[ a_t(\tilde{\sigma}_t) = a^0 - \alpha a (\tilde{\sigma}_t - \tilde{\sigma}^0) \]
  - $\varrho_t = 0$

- Government (bubble-mining policy)
  \[ \tilde{\mu}_t^B = \tilde{\mu}_t^B,0 + \alpha^B (\tilde{\sigma}_t - \tilde{\sigma}^0) \]

- Calibration to US data (1970-2019, period length is one year)
### Parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>value</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\sigma}^0$</td>
<td>$\tilde{\sigma}_t$ stoch. steady state</td>
<td>0.54</td>
<td>external calibration</td>
</tr>
<tr>
<td>$\psi$</td>
<td>$\tilde{\sigma}_t^2$ mean reversion</td>
<td>0.67</td>
<td>MLE targeting common idiosyncratic volatility (Herskovic et al. 2018)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$\tilde{\sigma}_t^2$ volatility</td>
<td>0.4</td>
<td>Heaton, Lucas (1996, 2000, 2001), Angeletos (2007) (range [0.2, 0.6])</td>
</tr>
<tr>
<td>$\bar{\chi}$</td>
<td>undiversifiable idio. risk</td>
<td>0.3</td>
<td></td>
</tr>
</tbody>
</table>

**calibration to match model moments**

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>value</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>risk aversion</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>time preference</td>
<td>0.138</td>
<td>chosen jointly to match (approximately)</td>
</tr>
<tr>
<td>$a^0$</td>
<td>$a_t$ stoch. steady state</td>
<td>0.63</td>
<td>- volatility of $Y$, $C$, $I$, $S/Y$</td>
</tr>
<tr>
<td>$g$</td>
<td>gov. expenditures</td>
<td>0.138</td>
<td>- average $C/Y$, $G/Y$, $S/Y$, $I/K$, $q^KK/Y$, $q^BK/Y$</td>
</tr>
<tr>
<td>$\tilde{\mu}_t^{B,0}$</td>
<td>$\tilde{\mu}_t^B$ stoch. steady state</td>
<td>0.0023</td>
<td>- mean equity premium</td>
</tr>
<tr>
<td>$\alpha^a$</td>
<td>$a_t$ slope</td>
<td>0.071</td>
<td>- equity Sharpe ratio</td>
</tr>
<tr>
<td>$\alpha^B$</td>
<td>$\tilde{\mu}_t^B$ slope</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>capital adjustment cost</td>
<td>8.5</td>
<td></td>
</tr>
</tbody>
</table>

**other parameters**

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>value</th>
<th>target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>0.055</td>
<td>economic growth rate (ultimately irrelevant for all results)</td>
</tr>
</tbody>
</table>


## Quantitative Model Fit

<table>
<thead>
<tr>
<th>Moment symbol</th>
<th>Description</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(Y)$</td>
<td>output volatility</td>
<td>1.3%</td>
<td>1.3%</td>
</tr>
<tr>
<td>$\sigma(C)/\sigma(Y)$</td>
<td>relative consumption volatility</td>
<td>0.63</td>
<td>0.64</td>
</tr>
<tr>
<td>$\sigma(S/Y)$</td>
<td>surplus volatility</td>
<td>1.1%</td>
<td>1.1%</td>
</tr>
<tr>
<td>$\mathbb{E}[S/Y]$</td>
<td>average surplus-output ratio</td>
<td>-0.0004</td>
<td>-0.0005</td>
</tr>
<tr>
<td>$\mathbb{E}[q^KK/Y]$</td>
<td>average capital-output ratio</td>
<td>3.48</td>
<td>3.73</td>
</tr>
<tr>
<td>$\mathbb{E}[q^BK/Y]$</td>
<td>average debt-output ratio</td>
<td>0.74</td>
<td>0.71</td>
</tr>
<tr>
<td>$\mathbb{E}[d\bar{r}^E - d\bar{r}^B]$</td>
<td>average (unlevered) equity premium</td>
<td>3.62%</td>
<td>3.40%</td>
</tr>
<tr>
<td>$\frac{\mathbb{E}[d\bar{r}^E - d\bar{r}^B]}{\sigma(d\bar{r}^E - d\bar{r}^B)}$</td>
<td>equity sharpe ratio</td>
<td>0.31</td>
<td>0.31</td>
</tr>
</tbody>
</table>
Fiscal Sustainability \textit{given} Exorbitant Privilege: Debt Laffer Curve

- Issue bonds at a faster rate $\ddot{\mu}^B$ (esp. in recessions)
  - $\Rightarrow$ tax precautionary self insurance $\Rightarrow$ tax rate
  - $\Rightarrow$ real value of bonds, $\frac{B}{\phi}$, $\Rightarrow$ “tax base”
- Less so in recession due to flight-to-safety

Sizeable revenue only if Gov. debt has negative $\beta$
Properties of US primary surpluses
- Average surplus \( \approx 0 \)
- Procyclical surplus (\( > 0 \) in booms, \( < 0 \) in recessions)

Two valuation puzzles from standard perspective: (Jiang, Lustig, van Nieuwerburgh, Xiaolan, 2019, 2020)

1. "Public Debt Valuation Puzzle"
   - Empirical: \( E[PV(\text{surpluses})] < 0 \), yet \( \frac{B}{\varphi} > 0 \)
   - Our model: bubble/service flow component overturns results

2. "Gov. Debt Risk Premium Puzzle"
   - Debt should be positive \( \beta \) asset, but market don’t price it this way
   - Our model: can be rationalized with countercyclical bubble/service flow
Roadmap

- Motivation
- Steady state model (closed form solution)
- Stochastic idiosyncratic risk model
- (Safe) asset pricing with SDF $\xi_t^{**}$
- Flight-to-Safety, $\beta < 0$, and Excess Volatility of capital
- Equity issuance, non-safe mutual fund with $\beta > 0$
- Debt Laffer Curve
- Price level determination with a bubble
  - Fiscal space to defend Exorbitant Privilege (crypto, ...)

Calibration
Why Does Gov. Safe Asset Survive in Presence of ETFs?

- Diversified stock portfolio is free of idiosyncratic risk
  - Trading in stocks (ETF) can also self-insure idiosyncratic risk
    - Good friend in idiosyncratically bad times

- But: poor hedge against aggregate risk, losses value in recessions
  - Positive $\beta$
    - Bad friend in aggregate bad times
  - Why positive $\beta$? (after all $r_f$ goes down in recessions, lowers discount rate)
    - Equity are claims to capital, but marginal capital holder is insider
    - Insider bears idiosyncratic risk, must be compensated
    - $\bar{\sigma}_t \uparrow \Rightarrow$ insider premium $E_t[dr_t^K] - E_t[dr_t^E] \uparrow \Rightarrow$ payouts to stockholders fall

- Share of inside equity relative to outside equity compensation increases with $\bar{\sigma}_t$
  - E.g. time of promise to issuance of new shares diluting outside equity holders
Exorbitant Privilege

1. Pay low real interest rate $r$ (cash flow) on safe asset

2. Run **Ponzi scheme**
   
   *Issue more bonds to fund primary deficit*
   
   - Dilute existing bond holdings - “mine the bubble”
   - Tax on “precautionary savings”/self-insurance
     
     ... but it is limited ⇒ Debt Laffer Curve

- Safe-asset status = exorbitant privilege is like a bubble (it can pop)
  
  - Jump to bad equilibrium
  - Safe-asset status can jump to foreign safe asset or crypto asset
Conclusion

- **Safe Asset** = good friend
  - **Individually:** allows self-insurance through retrading
  - **Aggregate:** appreciates in bad times (negative $\beta$)

- **Asset pricing with safe assets**
  - Service Flow term $>>$ convenience yield (BAA-Treasury spread)
  - Flight to Safety creates
    - Countercyclical safe asset valuation
    - Large stock market volatility

- **Exorbitant privilege:**
  - “Safe-asset status”:
    - low cash flow due to service flow (partially completing market via re-trading)
  - Extra space, but Debt Laffer Curve (≠ MMT)
  - Power to run Ponzi scheme
  - Debt sustainability analysis (off-equilibrium)

- **Fiscal space to ensure that bubble is attached to gov. bond (not on crypto)**

- **Remark:** Competing Safe Assets
  - Within country private bonds are partial safe assets
  - Across countries ⇒ Spillover of US Monetary Policy
Extra Slides