Modern Macro, Money, and International Finance

Eco 529

Lecture 11: Cash vs. Cashless Economy – The I Theory of Money with Heterogenous Agents

Markus K. Brunnermeier
Princeton University
Key Takeaways

- Real vs. Nominal Debt/Cashless vs. Cash
  - Inflation risk can improve risk sharing
- Intertemporal unit of account
  - State-contingent Monetary Policy if $\sigma^B \neq 0$
- Equivalence of capital vs. risk allocation setting ($\kappa$ vs. $\chi$)
- Liquidity and Disinflationary Spiral

Policy
- Fiscal Policy
- Monetary Policy
  - Stealth recapitalization of intermediaries
  - Macroprudential Policy

Technical Takeaways
- Two sector money models
The big Roadmap: Towards the I Theory of Money

- One sector model with idio risk - “The I Theory without I”
  - (steady state focus)
  - Store of value
    - Insurance role of money within sector
  - Money as bubble or not
  - Fiscal Theory of the Price Level
  - Medium of Exchange Role ⇒ SDF-Liquidity multiplier ⇒ Money bubble

- 2 sector/type model with money and idio risk
  - Generic Solution procedure (compared to earlier lectures)
  - Equivalence btw experts producers and intermediaries
  - Real debt vs. nominal debt/money
    - Implicit insurance role of money across sectors
  - I Theory

- Welfare analysis
- Optimal Monetary Policy and Macroprudential Policy
- International Monetary Model
The 4 Roles of Money

- **Unit of account**
  - Intratemporal: Numeraire bounded rationality/price stickiness
  - Intertemporal: Debt contract incomplete markets

- **Store of value**
  - “I Theory of Money without I”
    Less risky than other “capital” – no idiosyncratic risk
  - Fiscal theory of the price level

- **Medium of exchange**
  - Overcome double-coincidence of wants problem

- **Record keeping device – money is memory**
  - Virtual ledger
Safe Assets $\supseteq$ (Narrow) Money

- **Asset Price** = $E[PV(\text{cash flows})] + E[PV(\text{service flows})]$  
  - dividends/interest

- **Service flows/convenience yield**
  1. **Collateral**: relax constraints (Lagrange multiplier)
  2. **Safe asset**: [good friend analogy]
     - When one needs funds, one can sell at stable price … since others buy
     - Partial insurance through **retrading** - market liquidity!
  3. **Money (narrow)**: relax double-coincidence of wants
     - Higher Asset Price = lower expected return

- **Problem**: safe asset + money status might burst like a bubble
  - Multiple equilibria: [safe asset tautology]
Models on Money as Store of Value

<table>
<thead>
<tr>
<th>Friction</th>
<th>OLG</th>
<th>Incomplete Markets + idiosyncratic risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk</td>
<td>deterministic</td>
<td>endowment risk&lt;br&gt;borrowing constraint</td>
</tr>
<tr>
<td></td>
<td></td>
<td>return risk&lt;br&gt;Risk tied up with Individual capital</td>
</tr>
<tr>
<td>Only money</td>
<td>Samuelson</td>
<td>Bewley</td>
</tr>
<tr>
<td>With capital</td>
<td>Diamond</td>
<td>Aiyagari</td>
</tr>
</tbody>
</table>

“| Theory without I”<br>Brunnermeier-Sannikov (AER PP 2016)
<table>
<thead>
<tr>
<th>(New) Keynesian Demand Management</th>
<th></th>
<th>I Theory of Money Risk (Premium) Management</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stimulate aggregate consumption</td>
<td></td>
<td>Alleviate balance sheet constraints</td>
</tr>
<tr>
<td>Woodford (2003)</td>
<td>Tobin (1982), HANK</td>
<td>BruSan</td>
</tr>
<tr>
<td>Price stickiness &amp; ZLB</td>
<td>Both</td>
<td>Financial frictions</td>
</tr>
<tr>
<td>Perfect capital markets</td>
<td>Financial frictions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Incomplete markets</td>
<td></td>
</tr>
<tr>
<td>Representative Agent</td>
<td>Heterogeneous Agents</td>
<td></td>
</tr>
<tr>
<td>Cut $i$</td>
<td>Cut $i$</td>
<td>Cut $i$ or QE</td>
</tr>
<tr>
<td>Reduces $r$ due to price stickiness</td>
<td>Changes bond prices</td>
<td>Changes asset prices</td>
</tr>
<tr>
<td>Consumption $c$ rises</td>
<td>Redistributes from low MPC to high MPC consumers</td>
<td>Ex-post: Redistributes to balance sheet impaired sector</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Price of Risk Dynamics</td>
</tr>
</tbody>
</table>

Price of Risk Dynamics
“Money and Banking” (in macro-finance)

- Money: store of value/safe asset/Gov. bond
- Banking: “diversifier”
  holds risky assets, issues inside money

Watch “Money and Banking”
YouTube Video Channel: “markus.economicus”
https://www.youtube.com/channel/UCV8DKGTKWjuyk/jkJOsRY70A/videos?pbjreload=10
“Money and Banking” (in macro-finance)

- **Money** store of value/safe asset/Gov. bond
- **Banking** “diversifier” holds risky assets, issues inside money

**Amplification/endogenous risk dynamics**
- Value of capital declines due to fire-sales **Liquidity spiral**
  - Flight to safety
- Value of money rises **Disinflation spiral** a la Fisher
  - Demand for money rises – less idiosyncratic risk is diversified
  - Supply for inside money declines – less creation by intermediaries
    - Endogenous money multiplier = f(capitalization of critical sector)

Paradox of Prudence
- Paradox of Thrift (in risk terms)

**Monetary Policy** (redistributive)
Roadmap

- Intro
- Equivalence btw experts producers and intermediaries
- Real vs. Nominal Debt
- I Theory of Money
- Policy
Intermediaries

- **Frictions:**
  - Household cannot diversify idio risk
  - Limited risky claims issuance
Equivalence

- $a^e = a^h$
- $\tilde{\sigma}^e < \tilde{\sigma}^h$

Equivalence

- $\sigma_t^A$
Equivalence

- Why equivalence btw. Intermediaries $\chi$-risk allocation model and experts $\kappa$-capital allocation model?

Poll 13: Why are both models equivalent?
- a) Since $a^e = a^h$.
- b) Intermediary sector doesn’t produce any output
- c) Risk $\chi$ and capital allocation $\kappa$ are fundamentally different.

- Next: Contrast Real Debt with Nominal Debt/Money Model
  - solve generic model and highlight the differences
Roadmap

- Intro
- Equivalence btw experts producers and intermediaries
- Real vs. Nominal Debt
- I Theory of Money
- Policy
Model with Intermediary Sector

Intermediary sector

- Hold equity up to $\tilde{\chi} \leq 1$
- Diversify idio risk to $\varphi\tilde{\sigma}$
- Consumption rate: $c_t^I$
- $E_0[\int_0^\infty e^{-\rho t} \log c_t^I dt]$

- Friction: Can only issue debt
  - 2 Models:
    1. **Real** debt issuance only (and money has no value)
    2. **Nominal** debt issuance
- Bond/money supply $\frac{dB_t}{B_t} = (\tilde{\mu}_t^B + i_t)dt + \sigma_t^B dZ_t$
- seigniorage distribution as in previous lecture (no fiscal impact – per period balanced budget)

Household sector

- Output: $y_t^h = a^h k_t^h$
- Investment rate: $\iota_t^h$
- Consumption rate: $c_t^h$
- $E_0[\int_0^\infty e^{-\rho t} \log c_t^h dt]$
Solving MacroModels Step-by-Step

0. Postulate aggregates, price processes & obtain return processes

1. For given $C/N$-ratio and SDF processes for each $i$ finance block
   a. Real investment $\iota$ + Goods market clearing (static)
      - Toolbox 1: Martingale Approach, HJB vs. Stochastic Maximum Principle Approach
   b. Portfolio choice $\theta$ + Asset market clearing
      Asset allocation $\kappa$ & risk allocation $\chi$
      - Toolbox 2: “price-taking social planner approach” – Fisher separation theorem
   c. “Money evaluation equation” $\vartheta$
      - Toolbox 3: Change in numeraire to total wealth (including SDF)

2. Evolution of state variable $\eta$ (and $K$)

3. Value functions
   a. Value fcn. as fcn. of individual investment opportunities $\omega$
      - Special cases: log-utility, constant investment opportunities
   b. Separating value fcn. $V^i(n^i; \eta, K)$ into $v^i(\eta)u(K)(n^i/n^i)^{1-\gamma}$
   c. Derive $\rho = C/N$-ratio and $\zeta, \xi$ prices of risks

4. Numerical model solution
   a. Transform BSDE for separated value fcn. $v^i(\eta)$ into PDE
   b. Solve PDE via value function iteration

5. KFE: Stationary distribution, Fan charts
0. Postulate Aggregates and Processes

- $q_t^K K_t$ value of physical capital
- $q_t^B K_t$ value of nominal capital/outside money/gov. debt
  - $\varphi_t := B_t / q_t^B K_t$ price level (inverse of “value of money”)
- $N_t := (q_t^K + q_t^B) K_t$ is total wealth in the economy
- $\vartheta_t := \frac{q_t^B}{q_t^K + q_t^B}$ fraction of nominal wealth
0. Postulate Aggregates and Processes

- \( q_t^K K_t \) value of physical capital
- \( q_t^B K_t \) value of nominal capital/outside money/gov. debt
  - \( \varphi_t := B_t / q_t^K K_t \) price level (inverse of “value of money”)
- \( N_t := (q_t^K + q_t^B)K_t \) is total wealth in the economy
- \( \vartheta_t := \frac{q_t^B}{q_t^K + q_t^B} \) fraction of nominal wealth

0. Postulate in the \( N_t \)-numeraire!

- \( \vartheta \)-price process
  - \( d\vartheta_t / \vartheta_t = \mu_t^\vartheta dt + \sigma_t^\vartheta dZ_t \),
- SDF for each \( \tilde{i} \) agent
  - \( \frac{d\xi_t^{\tilde{i}}}{\xi_t^{\tilde{i}}} = -r_t^{\tilde{i}} dt - \zeta_t^{\tilde{i}} dZ_t - \tilde{\zeta}_t^{\tilde{i}} d\tilde{Z}_t^{\tilde{i}} \)
  - Change of notation (dropped “hat”) compared to previous lectures!
0. Postulate Aggregates and Processes

- \( q^K_t K_t \): value of physical capital
- \( q^B_t K_t \): value of nominal capital/outside money/gov. debt
  - \( \phi_t := B_t / q^K_t K_t \): price level (inverse of “value of money”)
- \( N_t := (q^K_t + q^B_t) K_t \): is total wealth in the economy
- \( \vartheta_t := \frac{q^B_t}{q^K_t + q^B_t} \): fraction of nominal wealth

0. Postulate in the \( N_t \)-numeraire!

- \( \vartheta \)-price process: \( d\vartheta_t / \vartheta_t = \mu_t^\vartheta dt + \sigma_t^\vartheta dZ_t \),
- SDF for each \( \tilde{i} \) agent: \( d\xi^\tilde{i}_t / \xi^\tilde{i}_t = -r^\tilde{i}_t dt - \zeta^\tilde{i}_t dZ_t - \tilde{\zeta}^\tilde{i}_t d\tilde{Z}_t \)
  - Change of notation (dropped “hat”) compared to previous lectures!

Poll 19: Why is the drift \(-r^\tilde{i}_t\) and not simply \(-r^f_t\)?

a) With only nominal debt a real risk-free rate might not be in asset span.

b) Negative drift of the SDF in \( N_t \)-numeraire is not risk-free rate.
1a. Optimal \( t + \) Goods Market

- Use optional real investment \( t \) and goods market clearing
- Same as in Lecture 10
- Price of physical capital
  \[
  q^K_t = (1 - \vartheta_t) \frac{1 + \phi a}{(1 - \vartheta_t) + \phi \rho}
  \]
- Price of nominal capital
  \[
  q^B_t = \vartheta_t \frac{1 + \phi a}{(1 - \vartheta_t) + \phi \rho}
  \]
- Optimal investment rate
  \[
  \iota_t = \frac{(1 - \vartheta_t)a - \rho}{(1 - \vartheta_t) + \phi \rho}
  \]
- Moneyless equilibrium with \( q^B_t = 0 \Rightarrow \vartheta_t = 0 \Rightarrow q^K_t = \frac{1 + \phi a}{1 + \phi \rho} \)
1b. Price-taking Planner’s Allocation

- \[
\max_{\{\kappa_t, \chi_t, \bar{\chi}_t\}} \ E_t[dr_t^N(\kappa_t)] - \zeta_t \sigma(\kappa_t, \chi_t) - \bar{\zeta}_t \bar{\sigma}(\kappa_t, \bar{\chi}_t)
\]

- In our model(s):
  - \(\kappa = 0\) (households manage all physical capital)
  - \(\bar{\chi}_t = \chi_t\)
  - \(E_t[dr_t^N(\kappa_t)] = 0\)

Poll 21: Why is \(E_t[dr_t^N(\kappa_t)] = 0\)?

- a) Because capital is not reallocated, i.e. \(\kappa = 0\) all the time.
- b) In the \(N_t\)-numeraire return of total wealth \(dr_t^N = 0\).
1b. Price-taking Planner’s Allocation

- \[ \max_{\{\psi_t, x_t, \tilde{x}_t\}} E_t[dr_t^N(\kappa_t)] - \zeta_t \sigma(\kappa_t, x_t) - \tilde{\zeta}_t \tilde{\sigma}(\kappa_t, \tilde{x}_t) \]

- In our model(s):
  - \( \kappa = 0 \) (households manage all physical capital)
  - \( \tilde{x}_t = x_t \)
  - \( E_t[dr_t^N(\kappa_t)] = 0 \)
  - \( \sigma = (x_t \sigma_t^{xK}, (1 - x_t) \sigma_t^{xK}) \),
    - where \( \sigma_t^{xK} \) = Risk of the excess return of capital beyond benchmark asset
  - \( \tilde{\sigma} = (x_t \varphi \tilde{\sigma}, (1 - x_t) \tilde{\sigma}) \)
    - \( \varphi < 1 \)
1b. Price-taking Planner’s Allocation

- Minimize weighted average cost of financing

\[
\min_{\chi_t \leq \bar{\chi}} \left( \zeta^l_t \chi_t + \zeta^h_t (1 - \chi_t) \right) \sigma^{xK}_t + \left( \tilde{\zeta}^l_t \varphi \chi_t + \tilde{\zeta}^h_t (1 - \chi_t) \right) \tilde{\sigma}
\]

- FOC: (equality if \( \chi_t < \bar{\chi} \))

\[
\zeta^l_t \sigma^{xK}_t + \tilde{\zeta}^l_t \varphi \tilde{\sigma} \leq \zeta^h_t \sigma^{xK}_t + \tilde{\zeta}^h_t \tilde{\sigma}
\]

- **Real** debt model:
  \( \sigma^{xK}_t = \sigma + \underbrace{\sigma^{qK}_t}_{(recall \ q^K_t \ is \ constant)} \)

- **Nominal** debt model
  \( \sigma^{xK}_t = \left( -\sigma^g_t + \sigma^B_t \right)/(1 - \vartheta_t) \)
  - Risk of capital \( \sigma + \sigma^{qK}_t + \vartheta_t \sigma^B_t /(1 - \vartheta_t) - \sigma^N_t \) (in \( N_t \)-numeraire)
  - Risk of bond/money \( \sigma + \sigma^{qB}_t + \sigma^B_t - \sigma^N_t \) (in \( N_t \)-numeraire)
“Benchmark Asset Evaluation Equation”

- In $N_t$-numeraire $\eta^i_t$ takes on role of sector net worth $N^i_t$
- Return on individual agent’s net worth return (in $N_t$-numeraire)
  \[
  \frac{d\eta^i_t}{\eta^i_t} + \frac{d\tilde{\eta}^i_t}{\tilde{\eta}^i_t} + \rho dt = dt
  \]
  \[
  \text{sector share} \quad \text{within sector share}
  \]
- Martingale condition relative to benchmark asset is
  \[
  \mu^\eta_t + \rho - r^b_{t} = \zeta^i_t \left(\sigma^i_t - \sigma^b_{t}\right) + \tilde{\zeta}^i_t \tilde{\sigma}^i_t
  \]
- Take $\eta^i_t$-weighted sum (across 2 types $i = l, h$ here)
  \[
  \rho - r^b_{t} = \eta_t \zeta^l_t \left(\sigma^l_t - \sigma^b_{t}\right) + (1 - \eta_t) \zeta^h_t \left(-\frac{\sigma^l_t}{1 - \eta_t} \sigma^b_{t} - \sigma^b_{t}\right) + \eta_t \tilde{\zeta}^l_t \tilde{\sigma}^l_t + (1 - \eta_t) \tilde{\zeta}^h_t \tilde{\sigma}^h_t
  \]
- For log utility: $\zeta^l_t = \sigma^l_t, \zeta^h_t = -\frac{\sigma^l_t}{1 - \eta_t} \sigma^b_{t}$, $\tilde{\zeta}^l_t = \tilde{\sigma}^l_t$, $\tilde{\zeta}^h_t = \tilde{\sigma}^h_t$
  \[
  \rho - r^b_{t} = \eta_t \left(\sigma^p_t\right)^2 + (1 - \eta_t) \left(-\frac{\sigma^l_t}{1 - \eta_t} \sigma^b_{t}\right)^2 + \eta_t \left(\tilde{\sigma}^l_t\right)^2 + (1 - \eta_t) \left(\tilde{\sigma}^h_t\right)^2
  \]
“Benchmark Asset Evaluation Equation”

- **Real** debt = benchmark asset $bm$
  - Redundant equation for allocation just useful for deriving risk-free rate in $c$-numeraire $r_t^F$ (expressed in $N_t$-numeraire)

- **Nominal** debt/money = benchmark asset $bm$
  - Money evaluation equation (bubble)
  - Replace $r_t^{bm} = \mu_t^{\vartheta/B} := \mu_t^\vartheta - \mu_t^B - \sigma_t^B (\sigma_t^\vartheta - \sigma_t^B)$ (and $\sigma_t^{bm} = \sigma_t^\vartheta$)

$$\rho - \mu_t^{\vartheta/B} = \eta_t (\sigma_t^\eta)^2 + (1 - \eta_t) \left(-\eta_t \sigma_t^\eta\right)^2 + \eta_t \left(\tilde{\sigma}_t^\eta\right)^2 + (1 - \eta_t) \left(\tilde{\sigma}_t^\eta\right)^2$$

excess return = (required) “net worth weighted risk premium”

of $N_t$ (for holding risk in excess of money risk)
“Benchmark Asset Evaluation Equation” (FTPL Equation)

- **Nominal** debt/money = benchmark asset \( bm \)
  - Money evaluation equation
  - Replace \( r_t^{bm} = \mu_t^{\vartheta/B} = \mu_t^\vartheta - \mu_t^B - \sigma_t^B (\sigma_t^{\vartheta} - \sigma_t^B) \) (and \( \sigma_t^{bm} = \sigma_t^\vartheta \))

\[
\rho - \mu_t^{\vartheta/B} = \eta_t (\sigma_t^\eta)^2 + (1 - \eta_t) \left( -\frac{\eta_t}{1 - \eta_t} \sigma_t^\eta \right)^2 + \eta_t \left( \tilde{\sigma}_t^{\eta^i} \right)^2 + (1 - \eta_t) \left( \tilde{\sigma}_t^{\eta^h} \right)^2
\]

- Integrate

\[
\vartheta_t = E_t \left[ \int_t^\infty e^{-\rho(s-t)} \left( \eta_s \left( \sigma_s^{\eta^i} \right)^2 + (1 - \eta_s) \left( \sigma_s^{\eta^h} \right)^2 + \eta_s \left( \tilde{\sigma}_s^{\eta^i} \right)^2 + (1 - \eta_s) \left( \tilde{\sigma}_s^{\eta^h} \right)^2 - \mu_s^B - \sigma_s^B (\sigma_s^{\vartheta} - \sigma_s^B) \right) - \vartheta_s \, ds \right]
\]

Because \( \mu_t^\vartheta \) is the “geometric drift”
2. \( \eta \)-Evolution: Drift \( \mu_t^\eta \) (in \( N_t \)-numeraire)

- Take difference from two earlier equations

\[
\mu_t^\eta + \rho - r_t^{bm} = \zeta_t^l (\sigma_t^\eta - \sigma_t^{bm}) + \tilde{\eta}^l_t \\
\rho - r_t^{bm} = \eta_t \zeta_t^l (\sigma_t^\eta - \sigma_t^{bm}) + (1 - \eta_t) \zeta_t^l + \eta_t \tilde{\eta}^l_t + (1 - \eta_t) \tilde{\eta}^h_t
\]

\[
\mu_t^\eta = (1 - \eta_t) \left[ \zeta_t^l (\sigma_t^\eta - \sigma_t^{bm}) - \zeta_t^h \left( -\frac{\eta_t}{1 - \eta_t} \sigma_t^\eta - \sigma_t^{bm} \right) + \tilde{\eta}^l_t \tilde{\eta}_t^l - \tilde{\eta}^h_t \tilde{\eta}_t^h \right]
\]

- **Real** Debt
  - \( \sigma_t^{bm} = -\sigma_t^N = -\sigma \) (Recall \( \sigma_t^q = 0 \))

- **Nominal** Debt/Money
  - \( \sigma_t^{bm} = \sigma_t^q - \sigma^B \)
2. $\eta$-Evolution: $\eta$-Aggregate Risk

- $\sigma_{t}^{\eta} = \sigma_{t}^{r_{bm}} + (1 - \theta_{t}^{I}) \left( \sigma_{t}^{r_{K}} - \sigma_{t}^{r_{bm}} \right)$
  - Where portfolio share $1 - \theta_{t}^{I} = \frac{\chi_{t}}{\eta_{t}} (1 - \vartheta_{t})$

- **Real Debt**
  - Note $\sigma_{t}^{r_{K}} = 0$ given $N_{t} = q_{t}^{K} K_{t}$-numeraire
  - $\sigma_{t}^{\eta} = \frac{\chi_{t} - \eta_{t}}{\eta_{t}} \sigma$ (recall $\vartheta_{t} = 0$)

- No amplification since $q^{K}$ is constant

- Imperfect risk-sharing for $\chi_{t} \neq \eta_{t}$
Inflation Risk allows Perfect Risk Sharing

- **Nominal Debt**
  - Note \( \sigma^r_t = \sigma_t^{1-\vartheta} = -\frac{\vartheta_t}{1-\vartheta_t} \sigma_t^\vartheta \)
  - \( \sigma_t^\eta = \sigma_t^\vartheta - \sigma^B + \frac{\chi_t}{\eta_t} (1 - \vartheta_t) \left( -\frac{\vartheta_t}{1-\vartheta_t} \sigma_t^\vartheta - \sigma_t^\vartheta + \sigma^B \right) \)
  - Use \( \sigma_t^\vartheta = \frac{\vartheta_t'(\eta_t)}{\vartheta(\eta_t)} \eta_t \sigma_t^\eta \) and solve for \( \eta_t \sigma_t^\eta \) yields
    \[
    \eta_t \sigma_t^\eta = \frac{(\chi_t - \eta_t) \sigma_t^B}{1 - \frac{\chi_t - \eta_t}{\eta_t} \left( -\frac{\vartheta_t'(\eta_t)}{\vartheta(\eta_t)} \eta_t \right)}
    \]

- Intermediaries’ balance sheet perfectly hedges agg. risk for \( \sigma^B = 0! \)

- Proposition: Aggregate risk is perfectly shared for \( \sigma^B = 0! \)
  - Via inflation risk
  - Stable inflation (targeting) would ruin risk-sharing
    - Example: Brexit uncertainty. Use inflation reaction to share risks within UK
2. Within Type $\tilde{\eta}$-Risk

- Within intermediary sector
  \[
  \tilde{\sigma}_t^{\tilde{\eta}_I} = (1 - \theta_t^I) \varphi \tilde{\sigma} = \frac{\chi_t}{\eta_t} (1 - \vartheta_t) \varphi \tilde{\sigma}
  \]

- Within household sector
  \[
  \tilde{\sigma}_t^{\tilde{\eta}_H} = (1 - \theta_t^H) \tilde{\sigma} = \frac{1 - \chi_t}{1 - \eta_t} (1 - \vartheta_t) \tilde{\sigma}
  \]
Solving for $\chi_t$

- Recall planner condition: (equality if $\chi_t < \bar{\chi}$)
  \[
  \zeta_t^l \sigma_t^{xK} + \tilde{\zeta}_t^l \phi \bar{\sigma} \leq \zeta_t^h \sigma_t^{xK} + \tilde{\zeta}_t^h \bar{\sigma}
  \]

### Price of Risks

<table>
<thead>
<tr>
<th></th>
<th>Real Debt</th>
<th>Nominal Debt with $\sigma^B = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta_t^l = \sigma_t^n$</td>
<td>$\frac{\chi_t - \eta_t}{\eta_t}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\zeta_t^h = \frac{-\eta_t}{1 - \eta_t} \sigma_t^n$</td>
<td>$\frac{\chi_t - \eta_t}{1 - \eta_t}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\tilde{\zeta}_t^l = \frac{\chi_t}{\eta_t} (1 - \vartheta_t) \phi \bar{\sigma}$</td>
<td>$\frac{\chi_t}{\eta_t} \phi \bar{\sigma}$</td>
<td>$\frac{\chi_t}{\eta_t} (1 - \vartheta_t) \phi \bar{\sigma}$</td>
</tr>
<tr>
<td>$\tilde{\zeta}_t^h = \frac{1 - \chi_t}{1 - \eta_t} (1 - \vartheta_t) \bar{\sigma}$</td>
<td>$\frac{1 - \chi_t}{1 - \eta_t} \bar{\sigma}$</td>
<td>$\frac{1 - \chi_t}{1 - \eta_t} (1 - \vartheta_t) \bar{\sigma}$</td>
</tr>
</tbody>
</table>
Solving for $\chi_t$

- **Real debt**

  $$\chi_t = \min \left\{ \frac{\eta_t(\sigma^2 + \tilde{\sigma}^2)}{\sigma^2 + [(1 - \eta_t)\phi^2 + \eta_t]\tilde{\sigma}^2}, \bar{\chi} \right\}$$

- **Nominal debt**

  $$\chi_t = \min \left\{ \frac{\eta_t}{(1 - \eta_t)\phi^2 + \eta_t}, \bar{\chi} \right\}$$
<table>
<thead>
<tr>
<th></th>
<th>Real Debt</th>
<th>Nominal Debt with $\sigma^B = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_t$</td>
<td>$\min \left{ \frac{\eta_t (\sigma^2 + \bar{\sigma}^2)}{\sigma^2 + \left[ (1 - \eta_t) \varphi^2 + \eta_t \right] \bar{\sigma}^2}, \bar{\chi} \right}$</td>
<td>$\min \left{ \frac{\eta_t}{(1 - \eta_t) \varphi^2 + \eta_t}, \bar{\chi} \right}$</td>
</tr>
<tr>
<td>$\mu^\eta_t$</td>
<td>$\frac{\chi_t - \eta_t \chi_t - 2 \chi_t \eta_t + \eta_t^2}{\eta_t (1 - \eta_t)} \sigma^2 + (1 - \eta_t) \left( \frac{\chi_t}{\eta_t} \right)^2 \varphi^2 - \left( \frac{1 - \chi_t}{1 - \eta_t} \right)^2 \bar{\sigma}^2$</td>
<td>$(1 - \eta_t) (1 - \vartheta)^2 \left( \left( \frac{\chi_t}{\eta_t} \right)^2 \varphi^2 - \left( \frac{1 - \chi_t}{1 - \eta_t} \right)^2 \right) \bar{\sigma}^2$</td>
</tr>
<tr>
<td>$\sigma^\eta_t$</td>
<td>$\frac{\chi_t - \eta_t \sigma}{\eta_t}$</td>
<td>0</td>
</tr>
<tr>
<td>$q^K_t$</td>
<td>$\frac{1 + \phi a}{1 + \phi \rho}$</td>
<td>$(1 - \vartheta_t) \frac{1 + \phi a}{(1 - \vartheta_t) + \phi \rho}$</td>
</tr>
<tr>
<td>$q^B_t$</td>
<td>0</td>
<td>$\vartheta_t \frac{1 + \phi a}{(1 - \vartheta_t) + \phi \rho}$</td>
</tr>
<tr>
<td>$\vartheta_t$</td>
<td>0</td>
<td>$\rho - \mu^\vartheta_t + \mu^B_t$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= (1 - \vartheta_t)^2 \left( \eta_t \frac{\chi_t^2 \varphi^2}{\eta_t^2} - (1 - \eta_t) \left( \frac{1 - \chi_t}{1 - \eta_t} \right)^2 \right) \bar{\sigma}^2$</td>
</tr>
<tr>
<td>$\iota_t$</td>
<td>$\frac{a - \rho}{1 + \phi \rho}$</td>
<td>$\frac{(1 - \vartheta_t) a - \rho}{(1 - \vartheta_t) + \phi \rho}$</td>
</tr>
</tbody>
</table>
Example: Nominal Debt/Money with $\bar{\chi} = 1$

- $a = .15, \rho = .03, \sigma = .1, \phi = 2, \delta = .03, \tilde{\sigma}^e = .2, \tilde{\sigma}^h = .3, \varphi = ., \bar{\chi} = 1$

Blue: real debt model
Red: nominal model
Contrasting Real with Nominal Debt

- **Real** debt model:
  - Changes in $\eta$ are absorbed by risk-free rate moves
  - Aggregate risk
    - $\iota(\eta)$ and $q^K(\eta)$ are constant

- **Nominal** debt/money model
  - Inflation risk completes markets
  - Perfect aggregate risk sharing
    - Banks balance sheet is perfectly hedged!!!
  - Risk-free rate is high
  - $\iota(\eta)$ and $q^K(\eta)$ are functions of $\eta$
Example: Nominal Debt with Limit on Risk Offloading

- $\rho = .05, \kappa = 2, \tilde{\sigma} = .5, \phi = .4, \bar{\chi} = .8$
Combining Nominal & Real Debt

- Adding real debt to money model does not alter the equilibrium, since
  - Markets are complete w.r.t. to aggregate risk (perfect aggregate risk sharing)
  - Markets are incomplete w.r.t. to idiosyncratic risk only
  - Real debt is a redundant asset

- Note: Result relies on absence of price stickiness

- Both Settings: Real Debt and Money/Nominal Debt converge in the long-run to the “I Theory without I” steady state model of Lecture 10 if $\bar{\chi} = 1$. 

37
Claim: \( \mathcal{G}(\eta) \) and average idiosyncratic risk exposure, \( X(\eta) \), is minimized at the stochastic steady state of \( \eta \).

Intuition: at steady state both sectors earn same risk premia + idiosyncratic seems well spread out ... less desire to hold money to self-insure

With \( \sigma_t^B = 0 \ \forall t \)

\[
\sigma_t^\eta = 0, \text{ (perfect risk sharing with nominal debt)}
\]

\[
\mu_t^\eta = (\bar{\sigma}_t^l)^2 - \eta_t(\bar{\sigma}_t^l)^2 - (1 - \eta_t)(\bar{\sigma}_t^h)^2 = (1 - \eta_t)(1 - \vartheta_t)^2 \frac{\chi_t^2 \phi^2}{\eta_t^2} \left( \frac{1 - \chi_t^2}{1 - \eta_t^2} \right) \bar{\sigma}^2
\]

Money valuation equation

\[
\rho - \mu_t^{\vartheta/B} = (1 - \vartheta_t)^2 \left( \eta_t \frac{\chi_t^2 \phi^2}{\eta_t^2} - (1 - \eta_t) \frac{(1 - \chi_t)^2}{(1 - \eta_t)^2} \bar{\sigma}^2 \right) \frac{\eta_t (\bar{\sigma}_t^l)^2 + (1 - \eta_t)(\bar{\sigma}_t^h)^2}{\eta_t (\bar{\sigma}_t^l)^2 + (1 - \eta_t)(\bar{\sigma}_t^h)^2}
\]

where \( \chi_t = \min \left( \frac{\eta_t}{\eta_t + (1 - \eta_t)^2}, \bar{\chi} \right) \)
Cashless/Bondless Limit with Jump

- Removing cash/nominal gov. bonds (comparative static)
  - $B > 0$ vs. $B = 0$
    - Price flexibility $\Rightarrow$ Neutrality of money
  - Discontinuity at $\lim_{B \to 0}$

- Remark:
  - Different from Woodford (2003) – medium of exchange role of money
    - CIA becomes relevant for fewer and fewer goods

- Inflation on nominal claims (bond/cash)
  - Change $\mu^B$ and subsidize capital
  - Continuous process
## Theory of Money

- **Aim:** intermediary sector is not perfectly hedged
- **Idiosyncratic risk** that HH have to bear is time-varying
- **Needed:** Intermediaries’ aggregate risk ≠ aggregate risk of economy
- **One way to model:** 2 technologies $a$ and $b$

<table>
<thead>
<tr>
<th>Technology</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital share</td>
<td>$1 - \tilde{\kappa}$</td>
<td>$\tilde{\kappa}$</td>
</tr>
<tr>
<td>Risk</td>
<td>$\frac{dk_t}{k_t} = (\cdot)dt + \sigma^a dZ_t + \tilde{\sigma} d\tilde{Z}_t$</td>
<td>$\frac{dk_t}{k_t} = (\cdot)dt + \sigma^b dZ_t + \tilde{\sigma} d\tilde{Z}_t$</td>
</tr>
<tr>
<td>Intermediaries</td>
<td>No</td>
<td>Yes, reduce $\tilde{\sigma}$ to $\varphi \tilde{\sigma}$</td>
</tr>
<tr>
<td>Excess risk</td>
<td>$\sigma_t^a x K^a$, $\sigma_t^b x K^b$</td>
<td>$\sigma_t^b - \sigma^a - \frac{\sigma^\vartheta - \sigma^B}{1 - \vartheta}$</td>
</tr>
</tbody>
</table>

\[
\sigma_t^a x K^a - \sigma_t^b x K^b = -\tilde{\kappa} (\sigma^b - \sigma^a) - \frac{\sigma^\vartheta - \sigma^B}{1 - \vartheta} \]

\[
(1 - \tilde{\kappa}) (\sigma_t^b - \sigma^a) - \frac{\sigma^\vartheta - \sigma^B}{1 - \vartheta} \]

\[
\sigma := \frac{\sigma^\vartheta - \sigma^B}{1 - \vartheta} \]
I Theory: Balance Sheets

- Frictions:
  - Household cannot diversify idio risk
  - Limited risky claims issuance
  - Only nominal deposits
Overview Slide that Explains the Role of Each Model Ingredient

- $\mathcal{X}$ -- avoid degenerated distribution (households dying out)
- $\varphi$
  - if $\varphi = 1$ intermediaries would die out,
  - if $\varphi = 0$ don’t earn risk premium (except for aggregate risk)
- $\sigma^b > \sigma^a$ -- avoid perfect hedging for intermediaries
  - (except $\sigma^B \neq 0$ -- for example risk-free asset is in zero net supply)
    (like AER paper/handbook chapter)
- Fraction $\kappa$ of $K$ has aggregate risk of $\sigma = \sigma^b - \sigma^a$,
  rest has risk of zero (it’s exogenous) (allocation does not determine total risk in aggregate economy)
  (To keep it clean (taste choice): price-taking planner’s choice is less involved)

- ...
1b. Price-taking Planner’s Allocation

- Minimize weighted average cost of financing

\[ \min_{\chi_t \leq \chi} (1 - \bar{\kappa})\zeta_t^h \sigma_t^x K^a + \left( \zeta_t^l \chi_t + \zeta_t^h (\bar{\kappa} - \chi_t) \right) \sigma_t^x K^b + \left( \zeta_t^l \varphi \chi_t + \zeta_t^h (1 - \chi_t) \right) \tilde{\sigma} \]

- FOC: (equality if \( \chi_t < \bar{\chi} \))

\[ \zeta_t^l \sigma_t^x K^b + \zeta_t^l \varphi \tilde{\sigma} \leq \zeta_t^h \sigma_t^x K^b + \zeta_t^h \tilde{\sigma} \]

\[ \sigma_t^x K^b = (1 - \bar{\kappa})\sigma - \frac{\sigma^\vartheta - \sigma^B}{1 - \vartheta} \]

- Price of risk with log-utility in total wealth numeraire:

\[ \sigma_t^\eta \left( (1 - \bar{\kappa})\sigma - \frac{\sigma^\vartheta - \sigma^B}{1 - \vartheta} \right) + \left[ (1 - \vartheta) \frac{\chi_t}{\eta_t} \varphi \tilde{\sigma} \right] \varphi \tilde{\sigma} \leq \frac{-\eta_t \sigma_t^\eta}{1 - \eta_t} \left( (1 - \bar{\kappa})\sigma - \frac{\sigma^\vartheta - \sigma^B}{1 - \vartheta} \right) + \left[ (1 - \vartheta) \frac{(1 - \chi_t)}{(1 - \eta_t)} \tilde{\sigma} \right] \tilde{\sigma} \]
1c. Money Evaluation + 2. $\eta$-Drift

- As before in money/nominal debt model

- Money evaluation

$$\rho - \mu_t^{\vartheta/B} = \eta_t \left( (\sigma_t^\eta)^2 + (\tilde{\sigma}_t^\eta)^2 \right) + (1 - \eta_t) \left( \left( \frac{\eta_t \sigma_t^\eta}{1 - \eta_t} \right)^2 + \left( \tilde{\sigma}_t^\eta \right)^2 \right)$$

- $\eta$-drift

$$\mu_t^\eta = (1 - \eta_t) \left( (\sigma_t^\eta)^2 + (\tilde{\sigma}_t^\eta)^2 - \left( \frac{\eta_t \sigma_t^\eta}{1 - \eta_t} \right)^2 - \left( \tilde{\sigma}_t^\eta \right)^2 \right) - \sigma_t^\eta \frac{\sigma_t^{\vartheta/B}}{\sigma_t^{\vartheta} - \sigma^B}$$
\[ \eta - \text{Volatility and Amplification} \]

\[ \sigma_t^\eta = \sigma_t^{rB} + (1 - \theta_t^l)\sigma_t^{xK^b} \]
- Where portfolio share \( 1 - \theta_t^l = \frac{x_t}{\eta_t}(1 - \vartheta) \)

\[ \sigma_t^\eta = \sigma_t^\vartheta - \sigma^B + \frac{\chi_t(1 - \vartheta_t)}{\eta_t} \left( (1 - \bar{\kappa})\sigma - \frac{\sigma_t^\vartheta - \sigma^B}{1 - \vartheta_t} \right) \]

\[ \Rightarrow \eta_t\sigma_t^\eta = \frac{(1 - \vartheta_t)\chi_t(1 - \bar{\kappa})\sigma + (\chi_t - \eta_t)\sigma^B}{1 - \frac{\chi_t - \eta_t}{\eta_t} \left( \frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)} \right)} \]
- Note that \( \frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)} = (1 - \vartheta_t) \left( \frac{q^{K'}(\eta_t)\eta_t}{q^K(\eta_t)} + \frac{-q^{B'}(\eta_t)\eta_t}{q^B(\eta_t)} \right) \)

\[ \eta_t\sigma_t^\eta = (1 - \vartheta_t)\chi_t(1 - \bar{\kappa})\sigma \text{ if } \sigma^B = \sigma^\vartheta \]

Policy removes endog. amplification

Liquidity Spiral  Disinflationary Spiral
- **Frictions:**
  - Household cannot diversify idio risk
  - Limited risky claims issuance
  - Only nominal deposits
Consequences of a Shock in 4 Steps

1. **Shock:** destruction of some capital
   - % loss in intermediaries net worth > % loss in assets
   - Leverage shoots up
   - Intermediaries %-loss > Household %-losses
     - $\eta$-derivative shifts losses to intermediaries

2. **Response:** shrink balance sheet / delever
   - For given prices no impact

3. Asset side: asset price $q^K$ shrinks
   - Further losses, leverage ↑, further deleveraging

4. **Disinflationary spiral**
   - Money supply declines
   - Value of money $q^B$ rises

4a. Liability side:
   - Money demand rises
   - HH face more idiosyncratic risk (can’t diversify)
Policy

- Fiscal policy

- Monetary policy without fiscal implications

- Macroprudential policy
Fiscal policy

- Includes monetary policy that has fiscal implications
- ...
Monetary Policy

- No fiscal implications, no seigniorage $\tau^{i,\tilde{i}} = 0 \ \forall i, \tilde{i}$
- Any seigniorage is paid out to government debt/money holders in form of interest
- Introducing interest rates on bond/reserves $i_t$.

\[
dr_t^B = i_t \, dt + \frac{d(1/P_t)}{1/P_t} = i_t \, dt + \frac{d(q_t^B K_t / B_t)}{q_t^B K_t / B_t}
= \left\{ i_t + \Phi(t_t) - \delta + \mu_t^q - \left[ \mu_t^B + (\sigma_t^q - \sigma_t^B) \sigma_t^B \right] \right\} \, dt + (\sigma_t^q - \sigma_t^B) \, dZ_t^{\tilde{q}}.
\]

To study monetary policy without fiscal implications, we let $\sigma_t^B = 0$, so

\[
dr_t^B = \left\{ i_t - \mu_t^B + \Phi(t_t) - \delta + \mu_t^q \right\} \, dt + \sigma_t^q \, dZ_t^{\tilde{q}}.
\]
Monetary Policy: Super-neutrality

- If interest paid on bond holdings is simply financed by issuing new bonds (issuing money), then money is
  - Neutral
  - Super-neutral

\[ \frac{dB_t}{B_t} = i_t dt \]

- Fisher equation

\[ dr_t^B = i_t dt - d\pi_t \]
Introducing Long-term Gov. Bond

- Introduce long-term (perpetual) bond
  - No default ... held by intermediaries in equilibrium

Value of long-term fixed $i$-bond is endogenous

$$dP_t^L / P_t^L = \mu_t^PL \, dt + \sigma_t^PL \, dZ_t$$
Redistributive MoPo: Ex-post perspective

- Adverse shock $\rightarrow$ value of risky claims drops
- Monetary policy
  - Interest rate cut $\Rightarrow$ long-term bond price
  - Asset purchase $\Rightarrow$ asset price
  - $\Rightarrow$ “stealth recapitalization” - redistributive
  - $\Rightarrow$ risk premia
- Liquidity & Deflationary Spirals are mitigated
Introducing long-term bonds

- Long-term bond
  - yields fixed coupon interest rate on face value $F(i,m)$
  - Matures at random time with arrival rate $1/m$
  - Nominal price of the bond $P_t^{B(i,m)}$
  - Nominal value of all bonds outstanding of a certain maturity
    \[ B_t^{(m)} = P_t^{B(i,m)} F(i,m) \]
  - Nominal value of all bonds $B_t = \sum_m B_t^{(m)}$

- Special bonds
  - Reserves: $B_t^{(0)}$ and note $P_t^{B(0)} = 1$
  - Consol bond: $B_t^{(\infty)}$
Debt evolution w/o fiscal implications

- $dB_t^{(0)} = i_t B_t^{(0)} dt$
  \[ + \sum_{(i,m)} \left[ \left( i + \frac{1}{m} \right) F_t^{(i,m)} dt - \frac{B_t^{(i,m)}}{F_t^{(i,m)}} (dF_t^{i,m} + \frac{1}{m} F_t^{(i,m)} dt) \right] \]

- Money $B_t^{(0)}$ is different since it pays floating interest rate

- If we have only reserves and consol bond, then
  \[ dB_t^{(0)} + \frac{B_t^{(i,\infty)}}{F_t^{(i,\infty)}} dF_t^{i,\infty} = i_t B_t^{(0)} dt + iF_t^{(i,\infty)} dt \]

New notation:
\[ B_t^{(0)} = M_t \]
\[ F_t^{(i,\infty)} = F_t^L \]
Define fraction of value of bonds that are not in short-term reserves

\[ \phi_t^L = \frac{p_t^L f_t^L}{B_t} \]

Let’s postulate the price of a single long-term consol bond

\[ \frac{dP_t^L}{P_t^L} = \mu_t^p dt + \sigma_t^p dZ_t \]

In the total net worth numeraire the

\[ E_t [d\sigma_t^L - d\sigma_t^M] = \sigma_t^{P\kappa} \sigma_t^n (\text{for now assuming that only intermediaries find it worthwhile to hold consul bonds}) \]

\[ \sigma_t^n = \cdots (\text{in net worth numeraire}) \quad (5.3) \]

\[ dr_t^L = dr_t^M + \sigma_t^{P\kappa} \sigma_t^n dt + \sigma_t^{P\kappa} dW_t \]
Return of total bond portfolio (in total net worth numeraire)

\[ dr^B_t = \mu^9_t \, dt + \sigma^9_t \, dZ_t \] (since no fiscal implications)

\[ dr^B_t = dr^M_t + \vartheta^L_t (dr^L_t - dr^M_t) \]

\[ dr^B_t = dr^M_t + \vartheta^L_t (\sigma_t^{PL} \sigma_t^\eta \, dt + \sigma_t^{PL} \, dZ_t) \]

Return of a single coin (reserve unit/short-term bond)

\[ dr^M_t = (\mu^9_t - \vartheta^L_t \sigma_t^{PL} \sigma_t^\eta) \, dt + (\sigma^9_t - \vartheta^L_t \sigma_t^{PL}) \, dZ_t \]

\[ \vartheta^L_t \sigma_t^{PL} \] shows importance of long-term bond price variation

- the \( dZ \)-term is a “risk-transfer”.
- The \( dt \)-term shows that it also affects risk premia
\( \eta \)-Volatility and Amplification

\[
\sigma_t^\eta = \sigma_t^{rM} + (1 - \theta_t^{M,I} - \theta_t^{L,I})\sigma_t^{xK} + \theta_t^{L,I}(\sigma_t^{L} - \sigma_t^{rM})
\]

- Where portfolio share \( 1 - \theta_t^{M,I} - \theta_t^{L,I} = \frac{\chi_t}{\eta_t}(1 - \theta_t) \) and \( \theta_t^{L,I} = \theta_t^* \theta_t / \eta_t \)

\[
\sigma_t^\eta = \sigma_t^\vartheta - \vartheta_t^L \sigma_t^{PL} + \frac{\chi_t(1 - \vartheta_t)}{\eta_t} \left( (1 - \bar{\kappa})\sigma - \frac{\sigma_t^\vartheta}{1 - \vartheta_t} + \vartheta_t^L \sigma_t^{PL} \right) + \frac{\vartheta_t^L \vartheta_t}{\eta_t} \sigma_t^{PL}
\]

Collect \( \sigma_t^{PL} \)-terms

\[
\sigma_t^\eta = \sigma_t^\vartheta + \frac{\chi_t(1 - \vartheta_t)}{\eta_t} \left( (1 - \bar{\kappa})\sigma - \frac{\sigma_t^\vartheta}{1 - \vartheta_t} \right) + \frac{\chi_t(1 - \vartheta_t) + \vartheta_t - \eta_t \vartheta_t^L \sigma_t^{PL}}{\eta_t}
\]

Replace \( \sigma_t^\vartheta = \frac{\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)} \sigma_t^\eta \) and \( \sigma_t^{PL} = \frac{p_t^{L'}(\eta_t)\eta_t}{p_t^{L}(\eta_t)} \sigma_t^\eta \)

\[
\Rightarrow \eta_t \sigma_t^\eta = \frac{(1 - \vartheta_t)\chi_t(1 - \bar{\kappa})\sigma}{1 - \chi_t - \eta_t \left( \frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)} \right) + \vartheta_t^L \left( \frac{p_t^{L'}(\eta_t)\eta_t}{p_t^{L}(\eta_t)} \right) \chi_t(1 - \vartheta_t) + \vartheta_t - \eta_t}
\]

- Recall that \( \frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)} = (1 - \vartheta_t) \left( \frac{q_t^K(\eta_t)\eta_t}{q_t^K(\eta_t)} + \frac{-q_t^B(\eta_t)\eta_t}{q_t^B(\eta_t)} \right) \) ... and is the mitigation term due to policy
Derive $\mu_t^n$

- Same steps as before
Monetary Policy: Ex-post perspective

- **Money view**
  - **Friedman-Schwartz**
  - Restore money supply
    - Replace missing inside money with outside money
  - Aim: Reduce deflationary spiral
    - ... but banks extent less credit & diversify less idiosyncratic risk away
    - ... as households have to hold more idiosyncratic risk, money demand rises
    - Undershoots inflation target

- **Credit view**
  - **Tobin**
  - Restore credit
  - Aim: Switch off deflationary spiral & liquidity spiral

- **I Theory**: “Stealth” recapitalization of impaired sector
  - Interest policy and OMO affect asset prices
MoPo Benchmark 1: Removing endogenous Risk

- The policy that removes endogenous risk, $\sigma_t^B = \sigma_t^\vartheta$

- FOC gives (in closed form)

$$\chi_t = \min \left( \frac{\eta_t}{\eta_t + (1 - \eta_t)\phi^2 + (1 - \bar{\psi})^2 (\sigma^b)^2 / \bar{\sigma}^2}, \bar{\psi} \right)$$

- $\eta$-Evolution

  - $\sigma^\eta = (1 - \vartheta_t) \frac{\chi_t}{\eta_t} (1 - \bar{\psi}) \sigma^b$

  - $\eta_t \mu^\eta_t = \eta_t (1 - \eta_t)(1 - \vartheta_t)^2 \left( \frac{1-2\eta_t}{(1-\eta_t)^2} \frac{\chi_t^2}{\eta_t^2} (1 - \bar{\psi})^2 (\sigma^b)^2 + \frac{\chi_t^2 \phi^2 \bar{\sigma}^2}{\eta_t^2} - \frac{(1-\chi_t)^2 \phi^2 \bar{\sigma}^2}{\eta_t^2} \right)$

Closed form up to $\vartheta_t$ (which is choice of planner)
Numerical Example

- $\rho = 0.05$, $\phi = 2$, $\bar{\sigma} = 0.5$, $\varphi = 0.4$, $\bar{\chi} = 0.8$, $\sigma^a = 0$, $\sigma^b = 0.1$

Policy: $\mu^M = 0$, $\sigma^M = \sigma^G \eta$
Numerical Example

- \( \rho = .05, \phi = 2, \tilde{\sigma} = .5, \varphi = .4, \bar{\chi} = .8, \sigma^a = 0, \sigma^b = .1 \)

\( \vartheta \) falls (because low \( \eta \)-regions with high idiosyncratic risk are visited less frequently)

Policy: \( \mu^M = 0, \sigma^M = \sigma^{\vartheta \eta} \)

Volatility goes down/amplification removed

With policy, risk is lower, but recovery is faster

No amplification when \( \eta_t = \kappa_t = \bar{\kappa} \)
Optimal Policy

- Next lecture after we have covered welfare analysis
Recall

- Unified macro “Money and Banking” model to analyze
  - Financial stability - Liquidity spiral
  - Monetary stability - Fisher disinflation spiral

- Exogenous risk &
  - Sector specific
  - Idiosyncratic

- Endogenous risk
  - Time varying risk premia – flight to safety
  - Capitalization of intermediaries is key state variable

- Monetary policy rule
  - Risk transfer to undercapitalized critical sectors
  - Income/wealth effects are crucial instead of substitution effect
  - Reduces endogenous risk – better aggregate risk sharing
    - Self-defeating in equilibrium – excessive idiosyncratic risk taking
Flipped Classroom Experience

Series of 4 YouTube videos, each about 10 minutes
Thank you!

markus@princeton.edu