Market Liquidity and Funding Liquidity

Markus K. Brunnermeier

Princeton, CEPR, NBER

Lasse Heje Pedersen

NYU, CEPR, NBER

Motivation

- Market liquidity
  - ease of trading an asset
  - asset-specific
- Funding liquidity
  - availability of funds
  - agent-specific
- these liquidity concepts are mutually reinforcing
  - funding liquidity to dealers, hedge funds, investment banks etc.
    $\Rightarrow$ enhances trading and market liquidity
  - market liquidity improves collateral value, i.e. lowers margins
    $\Rightarrow$ eases funding restriction

Stylized Facts on Market Liquidity

1. Sudden liquidity "dry-ups"
2. Correlated with volatility
   - cross section
   - time series
3. Flight to quality
4. Commonality of liquidity
   - within asset class (e.g. stocks)
   - across asset classes
5. Moves with the market

Outline

Leverage and Margins

- Financing a *long position* of $x_t^{j+} > 0$ shares at price $p_t^j = 100$:
  - Borrow 90 dollars per share;
  - Margin/haircut: $m_t^{j+} = 100 - 90 = 10$
  - Capital use: $10 x_t^{j+}$
- Financing a *short position* of $x_t^{j-} > 0$ shares:
  - Borrow securities, and lend collateral of 110 dollars per share
  - Short-sell securities at price of 100 dollars
  - Margin/haircut: $m_t^{j-} = 110 - 100 = 10$
  - Capital use: $10 x_t^{j-}$
- Margins/haircuts must be financed with capital:

$$\sum_j \left(x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-} \right) \leq W_t$$

where $x_t^j = x_t^{j+} - x_t^{j-}$

Capital

- Capital W_t:
  - Equity capital
    - LLP: NAV, subject to lock up
    - LLC: equity, reduced by assets that cannot be readily employed (e.g. goodwill, intangible assets, property)
  - Long-term unsecured debt
    - line of credit (material adverse change clause)
    - bonds/ loans: difficult to get for smaller securities firms
  - Short term debt: not counted
    - short-term loans, commercial paper, demand deposits

Cross-Margining

- Margins/haircuts must be financed with capital,

$$\sum_j \left(x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-} \right) \leq W_t, \qquad (1)$$

where $x_t^j = x_t^{j+} - x_t^{j-}$

- *Alternative:* perfect cross-margining
net out all offsetting risks, including diversification benefits, leading to a *portfolio* constraint:

$$M_t \left(x_t^1, \ldots, x_t^J \right) \leq W_t \qquad (2)$$

Regulatory Capital Requirements

- Basel Accord: banks
  - regulatory capital subject to constraint similar to (1)
  - alternatively, a bank can use its own model similar to (2)
- SEC Net Capital Rule: brokers
  - net capital = capital minus haircuts (compare to (1))
  - net capital must exceed a certain fraction of aggregate debt
- Regulation T: customers of brokers trading US equity
  - initial margin must be at least 50%

Model Setup

- Time: $t = 0, 1, 2, 3$
- J assets:
  - fundamental value $v_t^j = E_t[v^j]$ with final payoff v^j at $t = 3$
  - stochastic volatility with ARCH structure

$$\begin{aligned} v_t^j &= v_{t-1}^j + \Delta v_t^j = v_{t-1} + \sigma_t^j \varepsilon_t^j, \text{ where } \varepsilon_t^j \sim^{iid} \mathcal{N}(0,1) \\ \sigma_{t+1}^j &= \underline{\sigma}^j + \theta |\Delta v_t^j| \end{aligned}$$

- Market participants
  1. risk-averse customers
  2. speculators (dealers, hedge funds, ...)
  3. financiers (set margins speculators face)
- Competitive **stable** equilibria
- Let $\Lambda_t^j := p_t^j - v_t^j$ and $|\Lambda_t^j|$ be a measure of illiquidity

Customers

- 3 different types of customers $k \in \{0, 1, 2\}$
- CARA utility function: $u(W_3^k) = -\exp\{-\gamma W_3^k\}$
- endowment shock $\mathbf{z}^k$ in $t = 3$ s.t. $\sum_{k=0}^{2} \mathbf{z}^k = 0$
- become aware of $t = 3$-endowment shocks $\mathbf{z}^k$
  - *simultaneously* at $t = 0$ [with prob. $(1 - a)$]
  - *sequentially* at $t = k \in \{0, 1, 2\}$ [with "small" prob. $a < \bar{a}$]
- wealth dynamics: $W_{t+1}^k = W_t^k + \left(\mathbf{p}_{t+1} - \mathbf{p}_t\right)' \left(\mathbf{y}_t^k + \mathbf{z}^k\right)$
- customer k's demand

$$y_t^{j,k} = \frac{v_1^j - p_1^j}{\gamma(\sigma_{t+1}^j)^2} - z^{j,k} \quad \text{for } t = 1, 2$$

Speculators/Dealers

- risk-neutral
- wealth dynamics: $W_{t+1} = W_t + \left(\mathbf{p}_{t+1} - \mathbf{p}_t\right)' \mathbf{x}_t + \eta_{t+1}$
- margin constraint: $\sum_j \left(x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-}\right) \leq W_t$
- speculators' demand for $J = 1$

$$x_t^i = \begin{cases} W_t/m_t^+ & \text{if} \quad p_t < v_t \\ -W_t/m_t^- & \text{if} \quad p_t > v_t \\ \in \left[-W_t/m_t^-, W_t/m_t^+\right] & \text{if} \quad p_t = v_t \end{cases} \quad \text{for } t = 1, 2$$

$$x_0^i = \ldots$$

Financiers - Margin setting

- Margins are set based on Value-at-Risk (VaR)

$$\pi = Pr(-\Delta p^j_{t+1} > m^{j+}_t \,|\, \mathcal{F}^f_t)$$

- **Informed financiers** $(v_t \in \mathcal{F}^f_t)$:

$$\pi = Pr(-\Delta v^j_2 - \underbrace{\Lambda^j_2}_{=0} + \Lambda^j_1 > m^{j+}_1) = 1 - \Phi\left(\frac{m^{j+}_1 - \Lambda^j_1}{\sigma^j_2}\right)$$

$$\begin{aligned} m^{j+}_1 &= \Phi^{-1}(1-\pi)\,\sigma^j_2 + \Lambda^j_1 &&= \bar{\sigma}^j + \bar{\theta}|\Delta v^j_1| + \Lambda^j_1 \\ m^{j-}_1 &= \ldots &&= \bar{\sigma}^j + \bar{\theta}|\Delta v^j_1| - \Lambda^j_1 \end{aligned}$$

- **Uninformed financiers** (for $a \to 0$):

$$m^{j+}_1 = \Phi^{-1}(1-\pi)\,\sigma_2 = \bar{\sigma}^j + \bar{\theta}|\Delta p_1| = m^{j-}_1$$

Financiers - Margin setting

- Margins are set based on Value-at-Risk (VaR)

$$\pi = Pr(-\Delta p^j_{t+1} > m^{j+}_t \,|\, \mathcal{F}^i_t)$$

- **Informed financiers** ⇒ stabilizing margins

$$\pi = Pr(-\Delta v^j_2 - \underbrace{\Lambda^j_2}_{=0} + \Lambda^j_1 > m^{j+}_1) = 1 - \Phi\left(\frac{m^{j+}_1 - \Lambda^j_1}{\sigma^j_2}\right)$$

$$\begin{aligned} m^{j+}_1 &= \bar{\sigma}^j + \bar{\theta}|\Delta v^j_1| + \Lambda^j_1 \\ m^{j-}_1 &= \bar{\sigma}^j + \bar{\theta}|\Delta v^j_1| - \Lambda^j_1 \end{aligned}$$

- **Uninformed financiers** (for $a \to 0$) ⇒ destab. margins?

$$m^j_1 = \bar{\sigma}^j + \bar{\theta}|\Delta p_1|$$

1. Capital Constraint - Model Setup

2. Time-series Properties
   - Liquidity Dry-ups/ Fragility
   - Liquidity Spirals

3. Cross-Sectional Properties
   - Commonality of Market Liquidity
   - Flight to Quality

4. Risk of Liquidity Crisis
   - Skewness and Kurtosis

5. Related Literature

Liquidity Dry-ups/Fragility

Definition 1

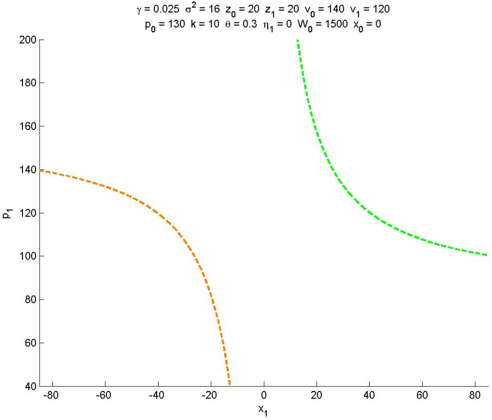
Liquidity is fragile if the price correspondence $p_t^*(\eta_1, v_t)$ is discontinuous in η_t or v_t.

Proposition 1

(i) With informed financiers, the market is fragile at time 1 if x_0 is large enough.
(ii) With uninformed financiers, the market is fragile at time 1 if x_0 large enough or if margins are increasing enough with illiquidity Λ_1. The latter happens if θ is large enough (i.e. ARCH effects are strong) and the financier's prior on a fundamental shock $(1-a)$ is large enough (i.e. $a < \bar{a}$).

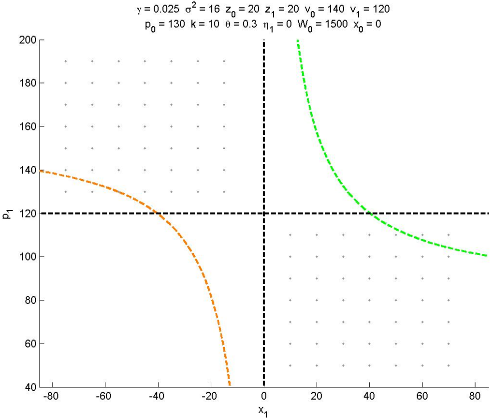
Example: Informed financier, ARCH & $x_0 = 0$ ($J = 1$)

Constraints: short: $\frac{W_1}{\bar{\sigma}+\bar{\theta}|\Delta v_1|-\Lambda_1}$ & long: $\frac{W_1}{\bar{\sigma}+\bar{\theta}|\Delta v_1|+\Lambda_1}$



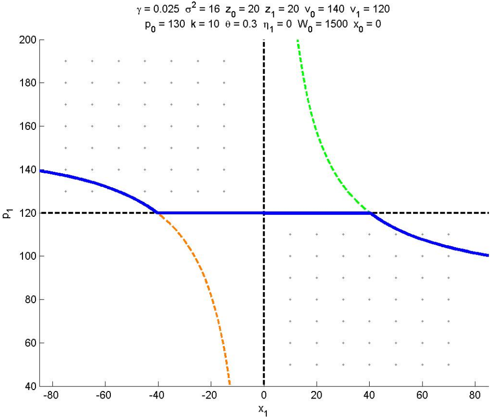
Example: Informed financier, ARCH & $x_0 = 0$

Short region ($p_1 > v_1$) & long region ($p_1 < v_1$)



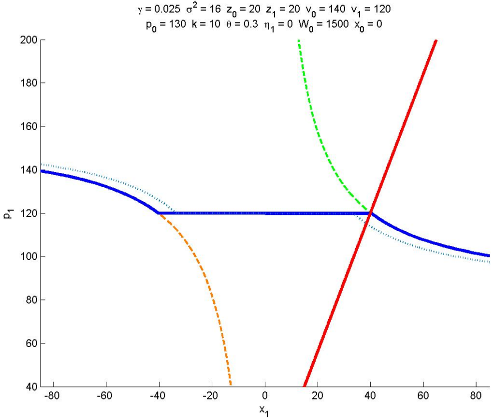
Example: Informed financier, ARCH & $x_0 = 0$

Speculators' demand



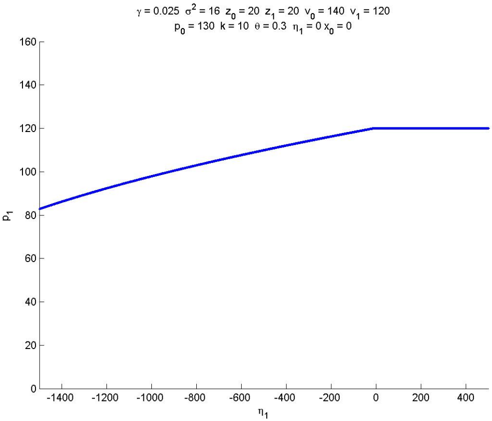
Example: Informed financier, ARCH & $x_0 = 0$

Add customers' supply



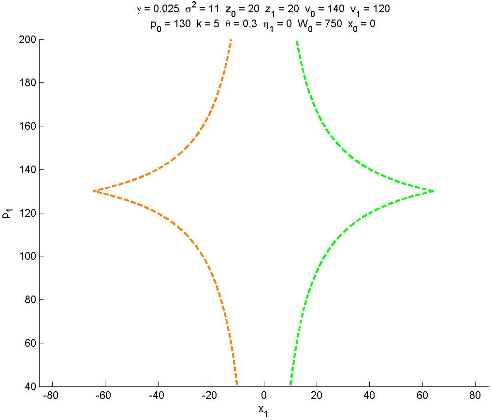
Example: Informed financier, ARCH & $x_0 = 0$

$\Rightarrow$ No fragility — "Cushioning effect of margins"



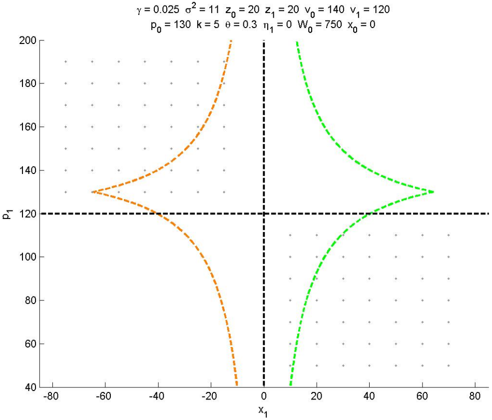
Example: Uninformed financier, ARCH & $x_0 = 0$

Constraints: short: $x_1 \geq -\frac{W_1}{\bar{\sigma}+\bar{\theta}|\Delta p_1|}$ & long: $x_1 \leq \frac{W_1}{\bar{\sigma}+\bar{\theta}|\Delta p_1|}$



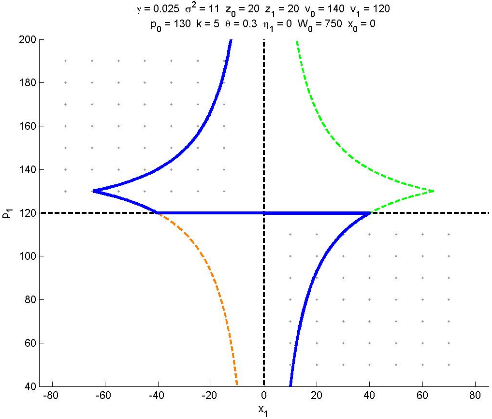
Example: Uninformed financier, ARCH & $x_0 = 0$

Short region ($p_1 > v_1$) & long region ($p_1 < v_1$)



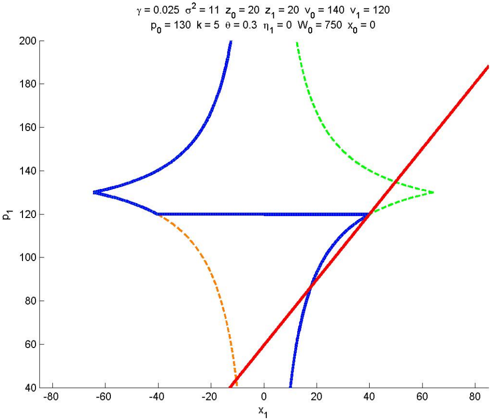
Example: Uninformed financier, ARCH & $x_0 = 0$

Speculators' demand



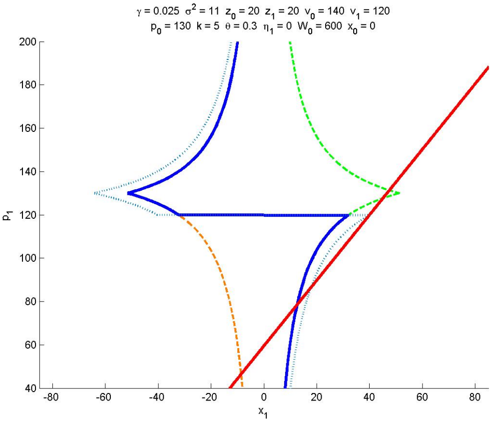
Example: Uninformed financier, ARCH & $x_0 = 0$

Add customers' supply — two stable equilibria



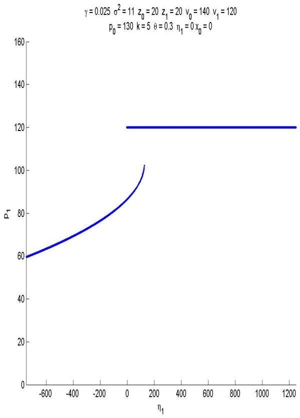
Example: Uninformed financier, ARCH & $x_0 = 0$

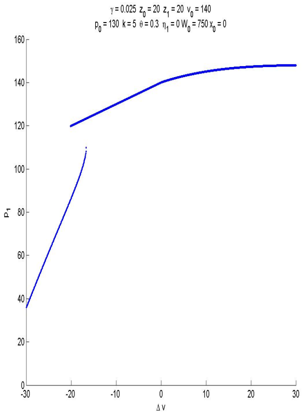
Add customers' supply — fragility for $\eta_1 = -150$



Example: Uninformed financier, ARCH & $x_0 = 0$

Example: fragility due to destabilizing margins

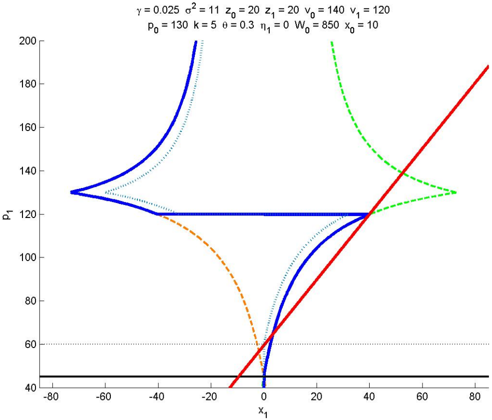




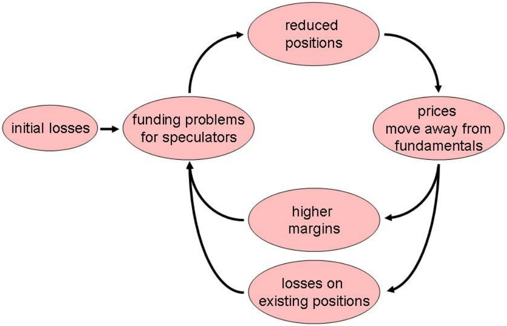
p_1 as correspondence of η_1 $\quad$ p_1 as correspondence of Δv_1

Example: Uninformed financier, ARCH & $x_0 = 10 > 0$

Leveraged x_0-position — 'tilted star' & bankruptcy line



Liquidity Spirals



Liquidity Spirals

Proposition 2

In a stable illiquid equilibrium with $Z_1 > 0$, $x_1 > 0$, and

$$\frac{\partial p_1}{\partial \eta_1} = \frac{1}{\frac{2}{\gamma(\sigma_2)^2} m_1^+ + \frac{\partial m_1^+}{\partial p_1} x_1 - x_0}.$$

A **margin spiral** arises if $\frac{\partial m_1^+}{\partial p_1} < 0$, which can happen if finaniers are uninformed and a is small.
A **loss spiral** arises if speculators' previous position is in the opposite direction as the demand pressure $x_0 Z_1 > 0$.

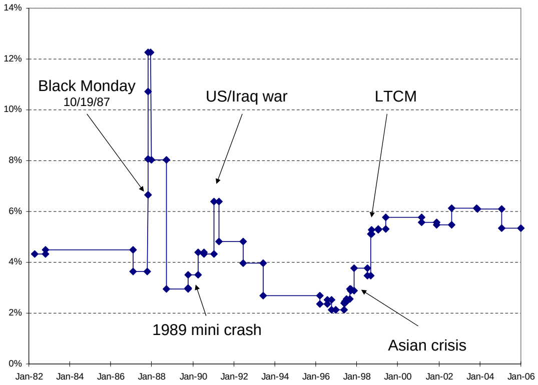
$$\frac{1}{k-l} = \frac{1}{k} + \frac{l}{k^2} + \frac{l^2}{k^3} + \ldots$$

Example: 1987 Crash

- Increased volatility caused banks to require more margin
- funding problems for marketmakers
  - failures at NYSE, Amex, OTC, trading firms, etc.
  - "thirteen [NYSE specialist] units had no buying power" because of their funding constraint (SEC (1988))
- $\Rightarrow$ mutually reinforcing
- Fed response:
  "calls were placed by high ranking officials of the FRBNY to senior management of the major NYC banks, indicating that ... they should encourage their Wall Street lending groups to use additional liquidity being supplied by the FRBNY to support the securities community"

Margin for S&P500 Futures

Margin requirement for CME members
as a fraction of the S&P500 index level

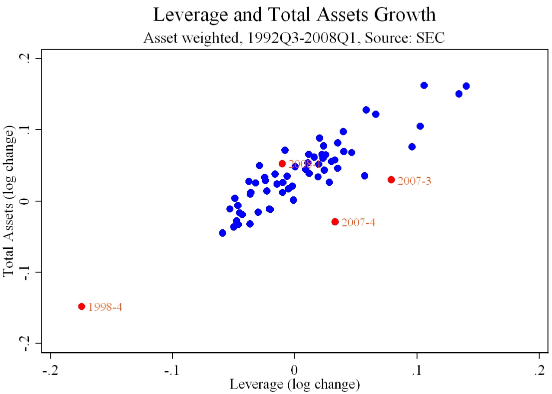


Example: 1998 Liquidity Crisis

- Salomon closed down proprietary trading
  - η-shock: less aggregate funding of trading in certain markets
- Russian default
  - Δv-shock: adverse fundamental shocks
- increased spreads & reduced market liquidity
- increased margins/haircuts & reduced funding liquidity

De-leveraging of I-Banks

esp. in Fall of 1998 — Source: Adrian-Shin (2008)



Multiple Assets - Speculators' Optimal Stra

Speculator maximizes *expected profit per capital use*

- expected profit $v_1^j - p_1^j = -\Lambda_1^j$ or $-(v_1^j - p_1^j) = \Lambda_1^j$
- capital use m_1^j

Shadow cost of capital, **funding liquidity**,

$$\phi_1 = 1 + \max\{\max_j \frac{v_1^j - p_1^j}{m_1^{j+}}, \max_j \frac{-(v_1^j - p_1^j)}{m_1^{j-}}\}$$

speculators

- invest only in securities with highest ratio $\frac{|\Lambda_1^j|}{m_1^j}$ (speculators determine price)
- do not invest in securities with lower ratio (customers determine price)

(If funding is abundant, $\phi_1 = 1$ and $\Lambda_1^j = 0\ \forall j$.)

Equilibrium

either

- funding is abundant, $\phi_1 = 1$, and market illiquidity $\Lambda_1^j = 0$ for all j;

or

- funding is tight, $\phi_1 > 1$, and

$$|\Lambda_1^j|(\phi_1) = \min\{\underbrace{(\phi_1 - 1)m_1^j}_{x_1^j \neq 0}, \underbrace{|\bar{\Lambda}_1^j(\mathbf{Z}_1, \cdot)|}_{x_1^j = 0}\}$$

Recall,

$$\Lambda_1^j = p_1^j - v_1^j$$

Commonality of Market Liquidity

Proposition 3

(iii) (Commonality of Market Liquidity) The market illiquidity $|\Lambda|$ of any two securities k and l comove,

$$Cov_0\left[|\Lambda_1^k|, |\Lambda_1^l|\right] \geq 0$$

and market illiquidity comoves with funding illiquidity, ϕ_1

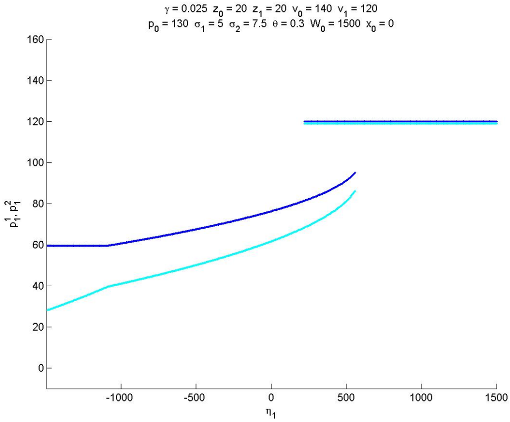
$$Cov_0\left[|\Lambda_1^k|, \phi_1\right] \geq 0$$

(iv) (Commonality of Fragility) Jumps in market liquidity occurs simultaneously for all assets for which speculators are marginal.

- *Intuition:* Funding liquidity is the driving common factor.

Commonality and Flight to Quality

Two asset example: $\underline{\sigma}^2 = 7.5 > 5 = \underline{\sigma}^1$ (Hint: asset 2 = light blue curve)



Flight to Quality

Proposition 3, continued

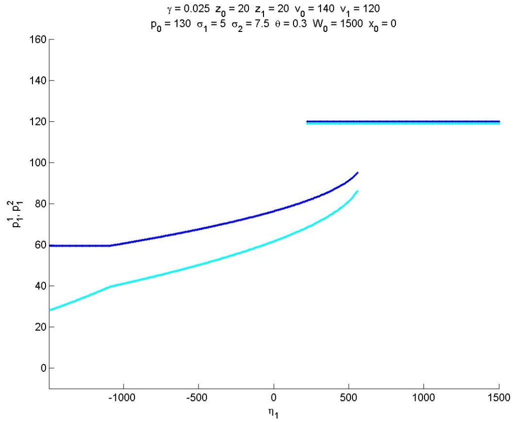
(i) (Quality=Liquidity) Assets with lower fundamental volatility have better market liquidity.
(ii) (Flight to Quality) The market liquidity differential between high- and low-fundamental-volatility securities is bigger when speculator funding is tight, that is, $\underline{\sigma}^l < \underline{\sigma}^k$ implies that $|\Lambda_1^k|$ increases more then $|\Lambda_1^l|$ with a negative funding shock,

$$\frac{\partial|\Lambda_1^l|}{\partial(-\eta_1)} \leq \frac{\partial|\Lambda_1^k|}{\partial(-\eta_1)},$$

$$Cov_0[|\Lambda_1^l|, \phi_1] \leq Cov_0[|\Lambda_1^k|, \phi_1].$$

Commonality and Flight to Quality

Tow asset example: $\underline{\sigma}^2 = 7.5 > 5 = \underline{\sigma}^1$ (Hint: asset 2 = light blue curve)



Risk of Liquidity Crisis - $t = 0$

1. pricing kernel depends on future funding liquidity, ϕ_{t+1}
2. funding liquidity risk can matter even before margin requirements actually bind
3. conditional skewness of price p_1 due to the funding constraint
4. margins m_0 and illiquidity Λ_0 can be positively related due to liquidity risk even if financiers are informed.

Risk of Liquidity Crisis - $t = 0$

- Pledgable capital interpretation of W_t
  - if $W_t < 0$, losses have to be covered with unpledgable capital
  - speculators' "utility" $\phi_1 W_1$ (also for $W_1 < 0$)
  - weakest assumption that curbs speculators' risk taking, since objective function linear.

1. Pricing kernel reflects funding liquidity (shadow cost) ϕ_{t+1}.

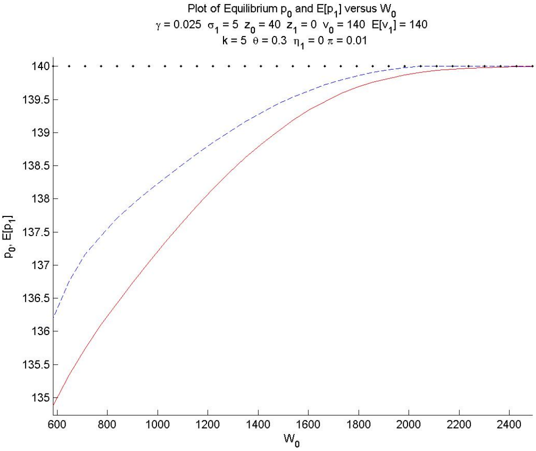
$$p_0 = E_0[\underbrace{\frac{\phi_1}{E_0[\phi_1]}}_{\text{kernel}} p_1], \text{ if } \phi_0 = 1 \text{ (unconstrained case)}.$$

$$p_0 = E_0[\phi_1]E_0[p_1] + Cov_0[\frac{\phi_1}{E_0[\phi_1]}, p_1]$$

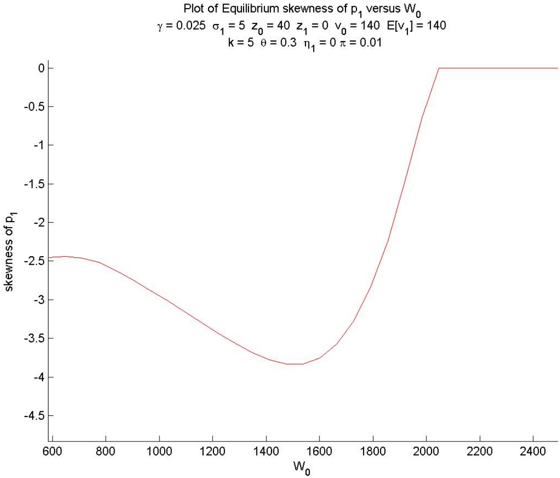
Market & Funding Liquidity

Brunnermeier & Pedersen

p_0 and $E_0[p_1]$

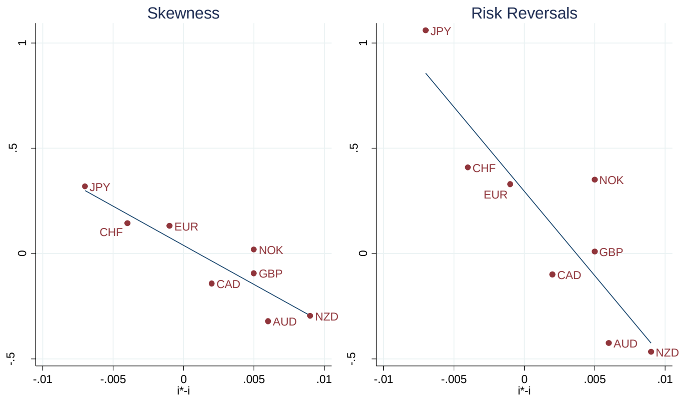


Conditional Skewness and Kurtosis



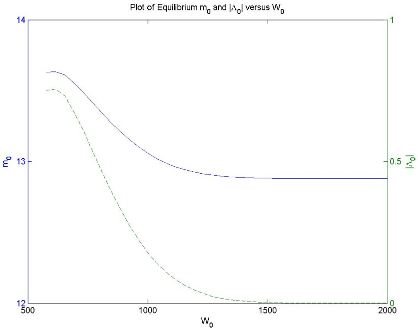
Conditional Skewness in FX

Brunnermeier, Nagel, Pedersen (NBER Macro Annual 2008)



Margins m_0 can increase with $|\Lambda_0|$

- in $t = 1$: margins, m_1, are only increasing **in** $|\Lambda_1|$ if
  - financiers are uninformed
  - fundamentals follow ARCH structure
- in $t = 0$: margins, m_0, can be increasing **with** $|\Lambda_0|$ even when financiers are informed.
  - decline in W_0 leads to
    - increase in $|\Lambda_0|$
    - increase in m_0 since p_1 is more volatile



Related Theoretical Literature

This Paper:	**Related Theoretical Literature:**
Cushioning Effect	Gromb-Vayanos (2002), Geanakopolos (2003)
Conditions for destabilizing margins	—
Fragility	Asym. info: Gennotte-Leland (1990)
Loss Spiral	Grossman (1988), Kiyotaki-Moore (1997), Shleifer-Vishny (1997), Xiong (2001), Gromb-Vayanos (2002), Morris-Shin (2004)
Margin Spiral	Vayanos (2004)
Flight to Quality	—
Commonality of Liquidity	Contagion: Allen-Gale(2000b), Kyle-Xiong(2001)

Paper links literatures on:
asset pricing, microstructure, limits of arb, corporate finance, macro, GE

Conclusion

1. Sudden liquidity "dry-ups"
   - fragility
   - liquidity spirals
   - due to destabilizing margins (financiers imperfectly informed + ARCH)
2. Market liquidity correlated with volatility:
   - volatile securities require more capital to finance
3. Flight to quality / flight to liquidity:
   - when capital is scarce, traders withdraw more from "capital intensive" high-margin securities
4. Commonality of liquidity:
   - these funding problems affect many securities
5. Market liquidity moves with the market
   - because funding conditions do