The I Theory of Money

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Updates: http://scholar.princeton.edu/markus/files/i_theory_slides.pdf

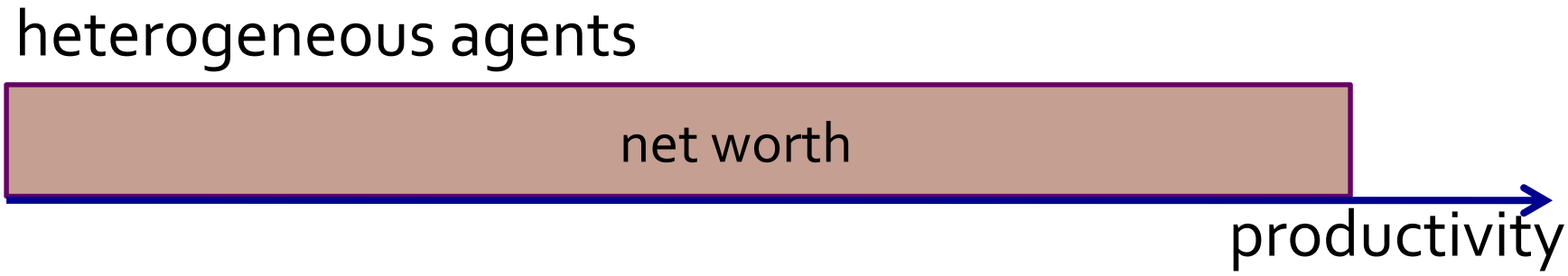
Motivation

- Unified framework to study financial and monetary stability
- **I**: **I**ntermediation (credit) - **I**nside money
- Value of money endogenous - store of value, liquidity
  - Samuelson, Bewley, Kiyotaki-Moore, …
- In downturns, intermediaries create less inside money
  - Value of **outside** (base) money goes up
  - Fisher (1933) **deflationary spiral** hits borrowers on liability side
  - **Endogenous** money multiplier = f(health of intermediary sector)
- Monetary policy (interest rates, open market operations)
  - Fills in demand for money when money multiplier contracts
  - Redistribution from/towards intermediary sector

Some Literature

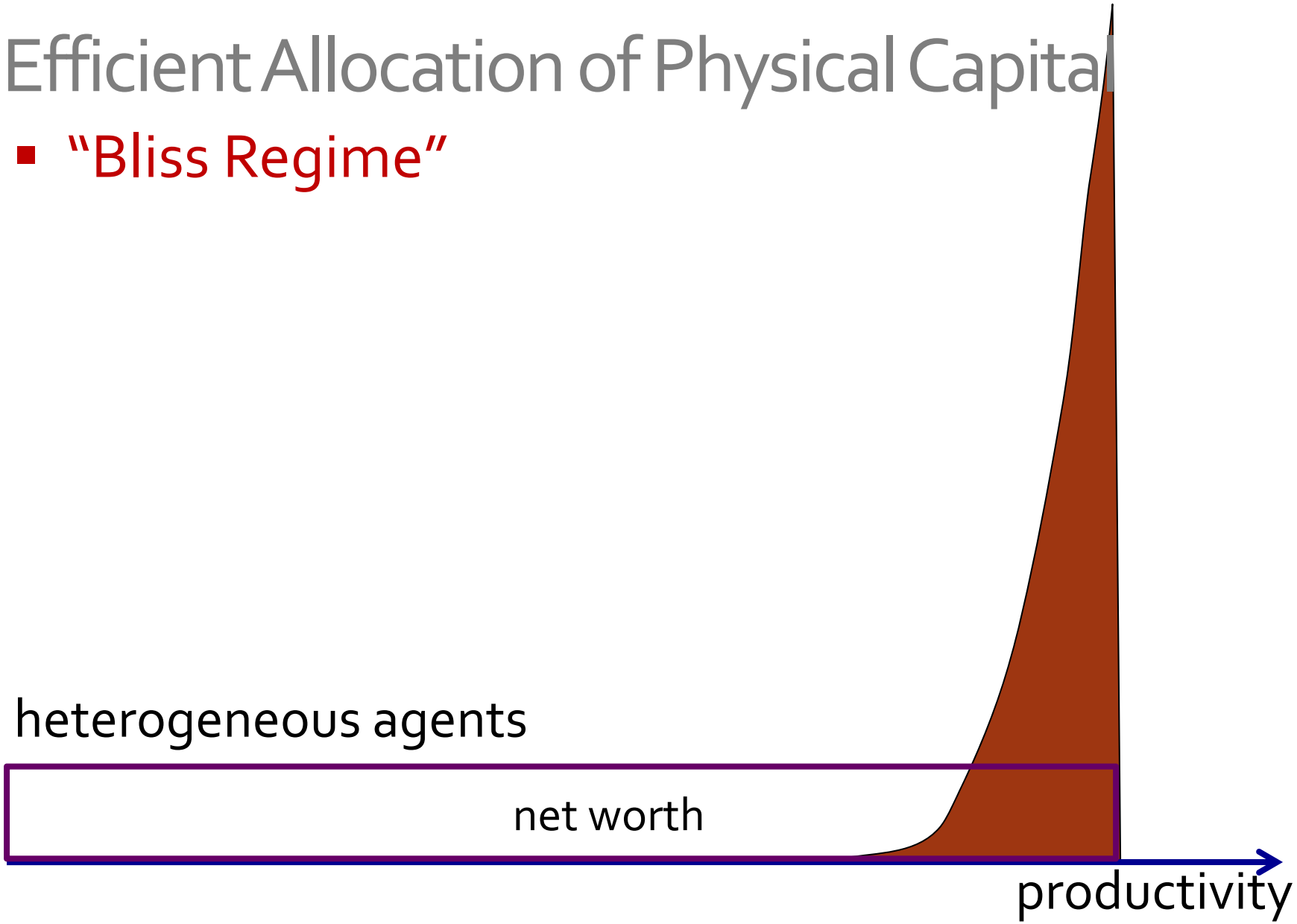
- Role of money
  - Unit of account
  - Medium of exchange
  - Store of value (Samuelson, Bewley, Scheinkman-Weiss, Kiyotaki-Moore)
- Without intermediaries
  - Inflation in downturns: less money needed since fewer transactions
- With intermediaries
  - Money view: (Friedman & Schwartz 1963)
    - "Moneyness" of bank liabilities decrease in downturns of intermediation
  - Credit view (demand/supply): (Tobin)
    BGG, KM, He & Krishnamurthy, BruSan10, Goodfriend 05, Curdia & Woodford 10, …
- Financial stability + monetary policy
  - Diamond & Rajan (2006), Stein (2012),

Outline of Modeling Ideas



Efficient Allocation of Physical Capital

- "Bliss Regime"

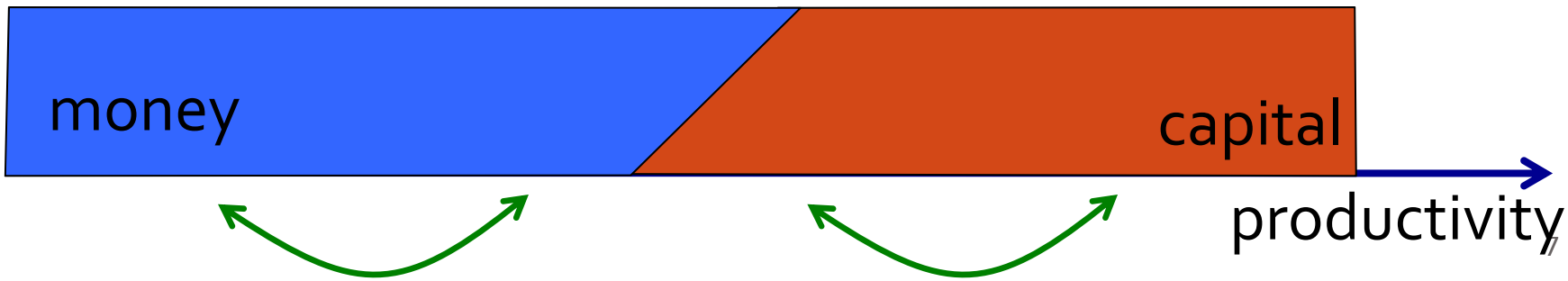


Allocation with Extreme Financial Constraint

- "Autarky Regime"

Switching Types and Money

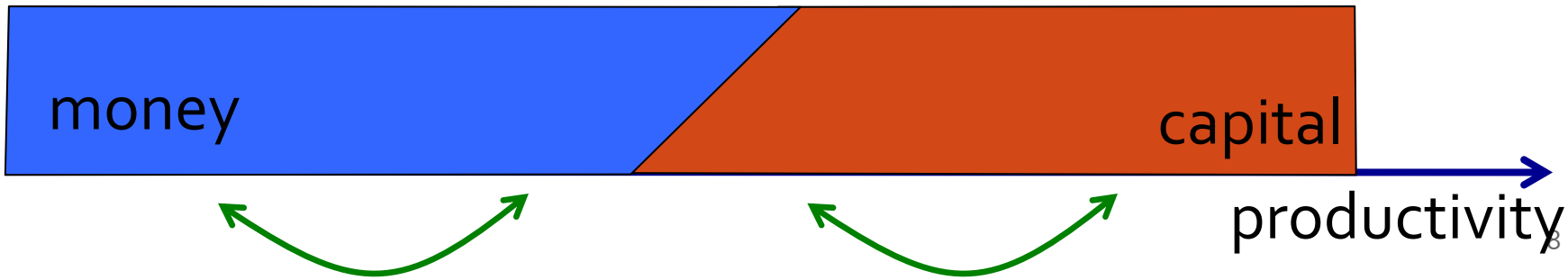
- "(Outside) Money Regime"
- Money (gold) intrinsically worthless, but …
- ∃ an equilibrium (coordination)
  - Agents store wealth in money while unproductive
  - Trade it for physical capital when become productive



Switching Types and Money

- "(Outside) Money Regime"

- Inefficiencies
  - Allocation (money has low return)
  - Underinvestment (marginal buyer is less productive
    ⇒ price of capital is low ⇒ capital production unattractive



Two Polar Regimes

Economy	Assets	Value of money	Price of capital
Frictions (severe)	No claims	high	low
Frictionless	Issue claims • Debt • Equity	low	high

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Two Polar Regimes with Intermediaries

Economy	Assets	Value of money	Price of capital		Intermediaries' capitalization
Frictions (severe)	No claims	high	low		defunct
Frictionless	Issue claims • Debt • Equity	low	high		perfect

- Role of intermediaries
  - Relax financing constraint by monitoring productive agents
  - Have to take on productive agent's equity risk (so that they have incentive to monitor)
  - Intermediation depends on their ability to absorb risk
    net worth of intermediaries

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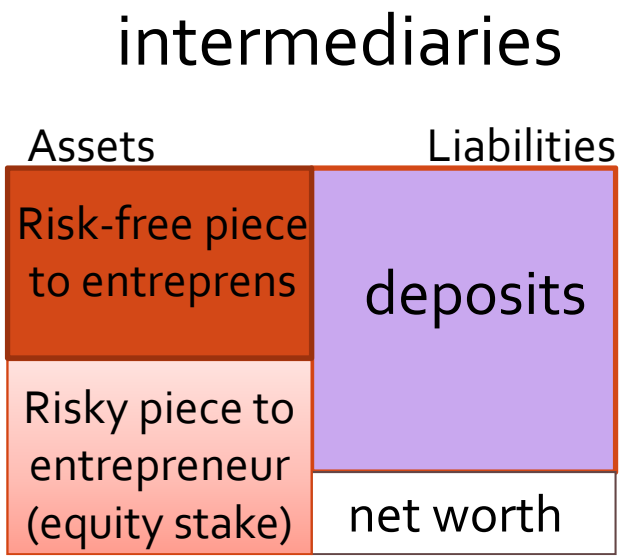
Intermediaries and Lending

- Intermediation is risky depends on banks' balance sheet
- Monitoring technology
  Diamond (1984)
  Homstrom-Tirole (1997)



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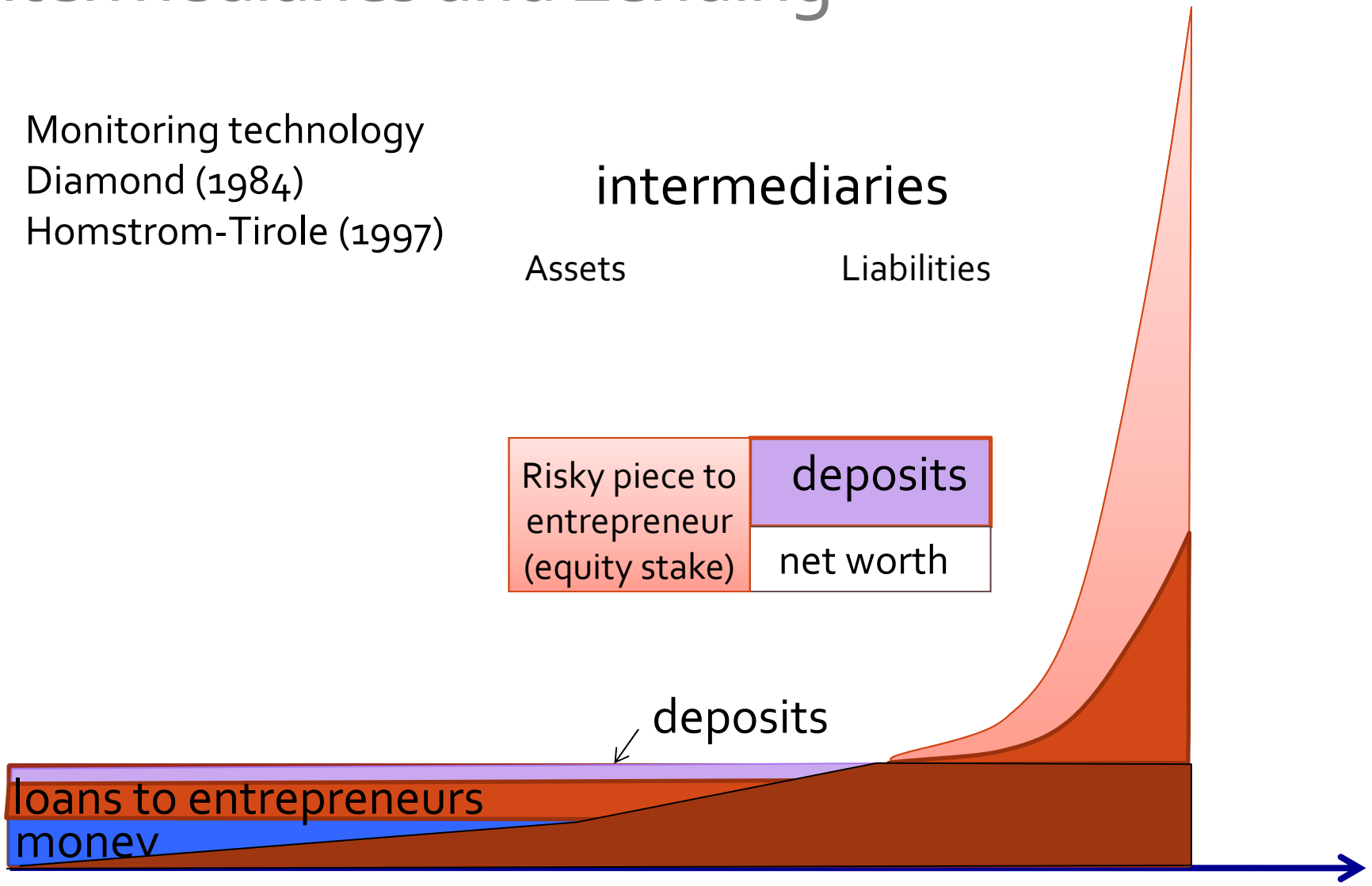

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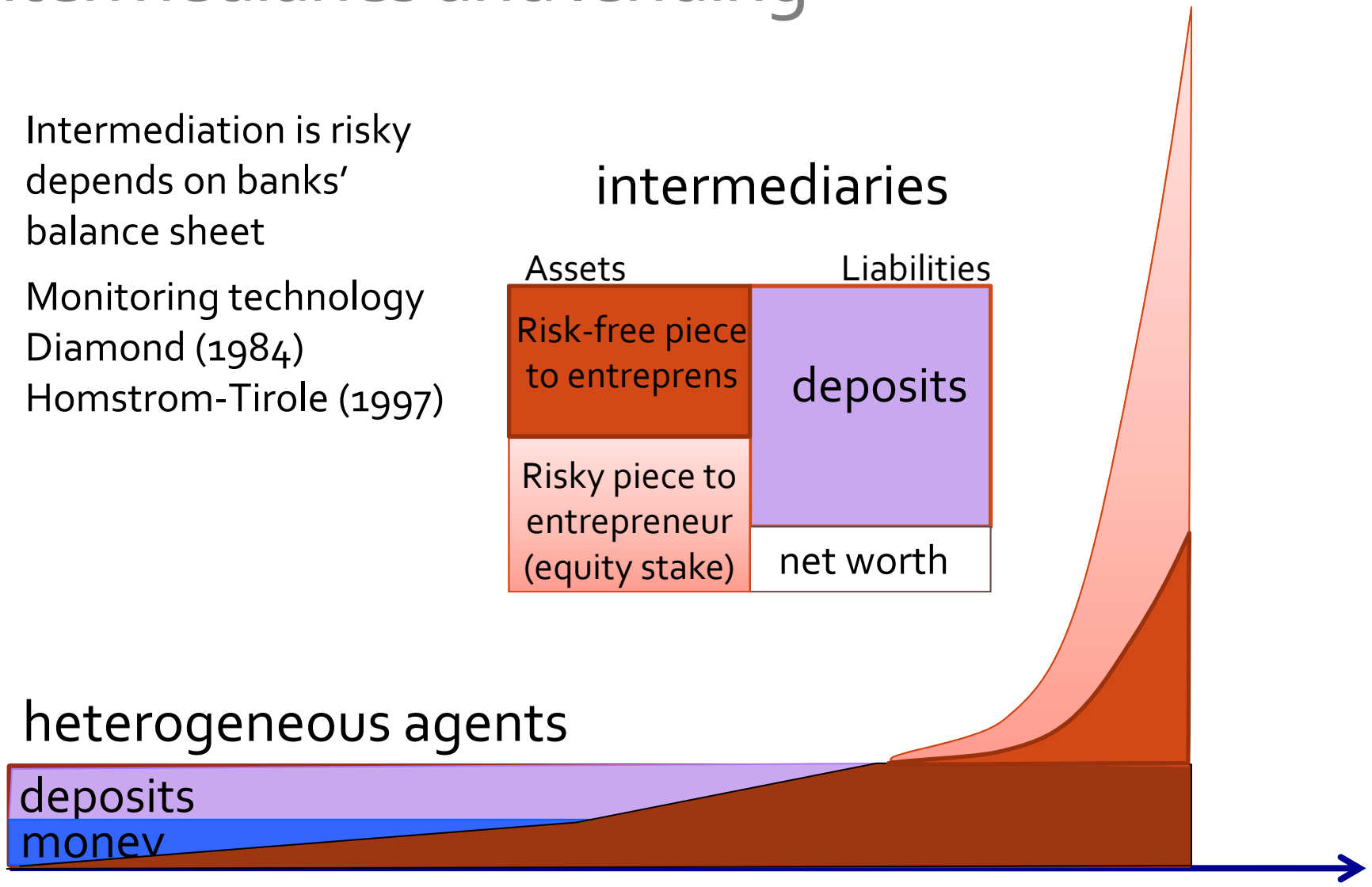
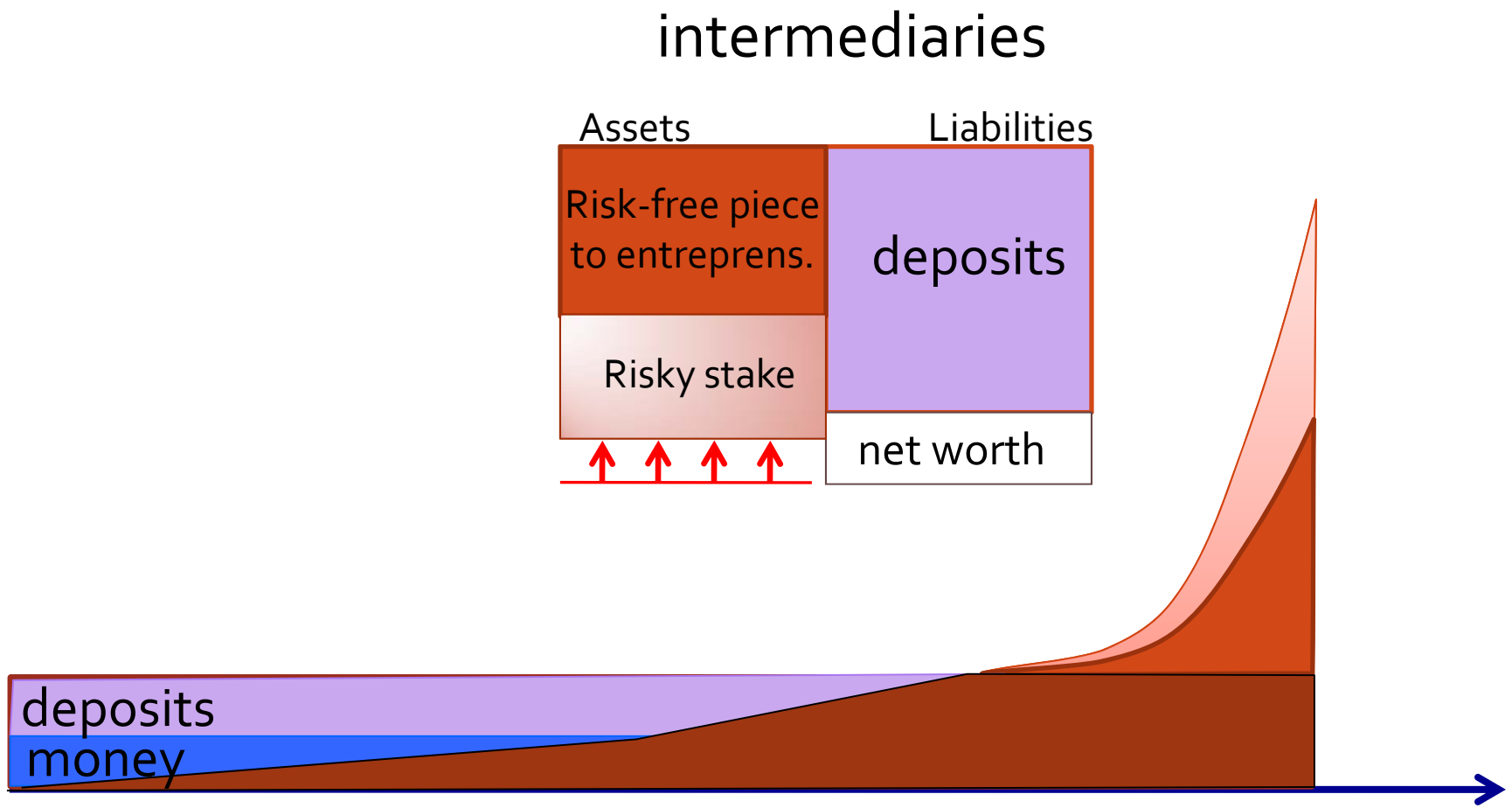


Negative Macro Shocks



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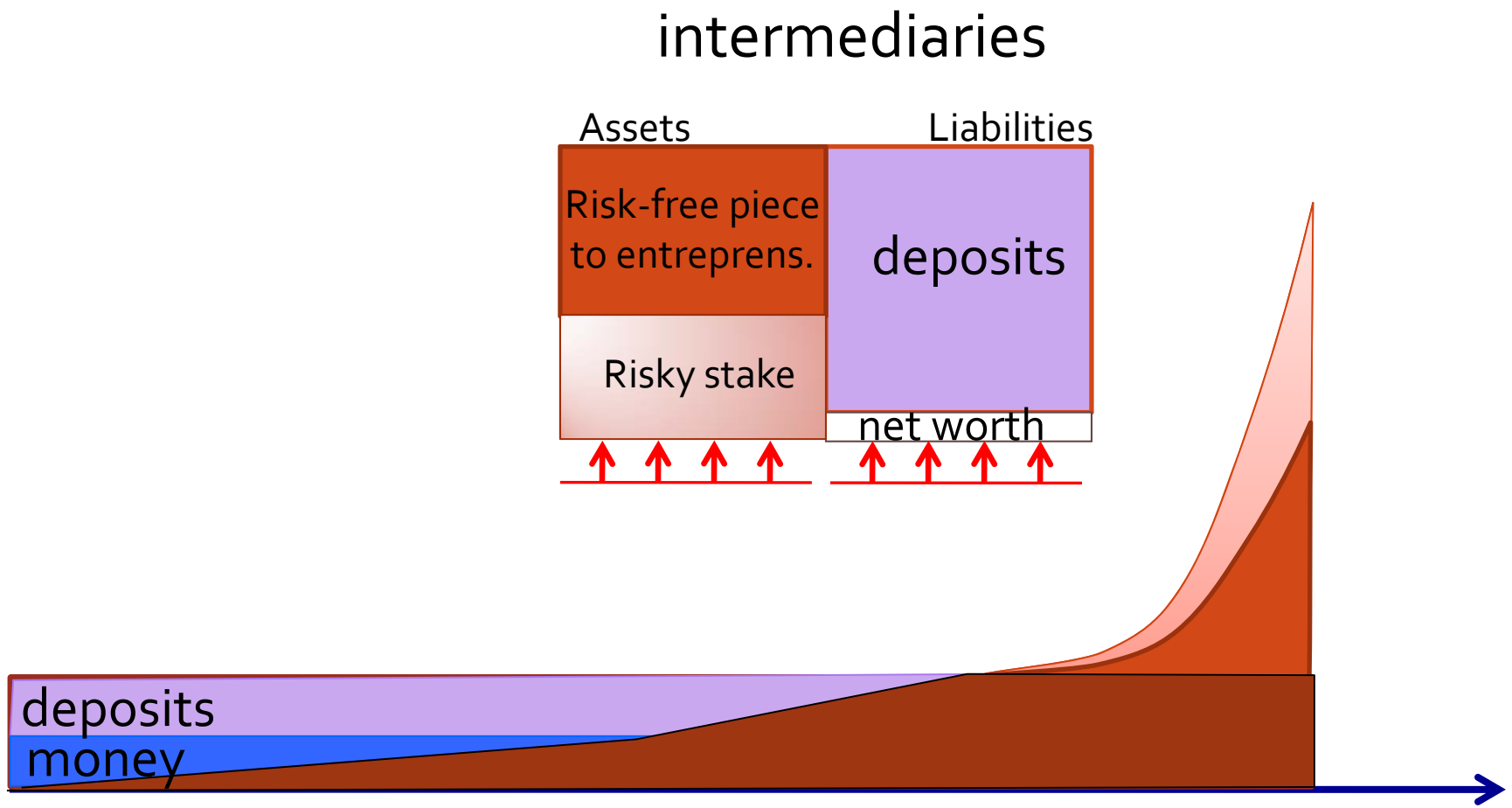

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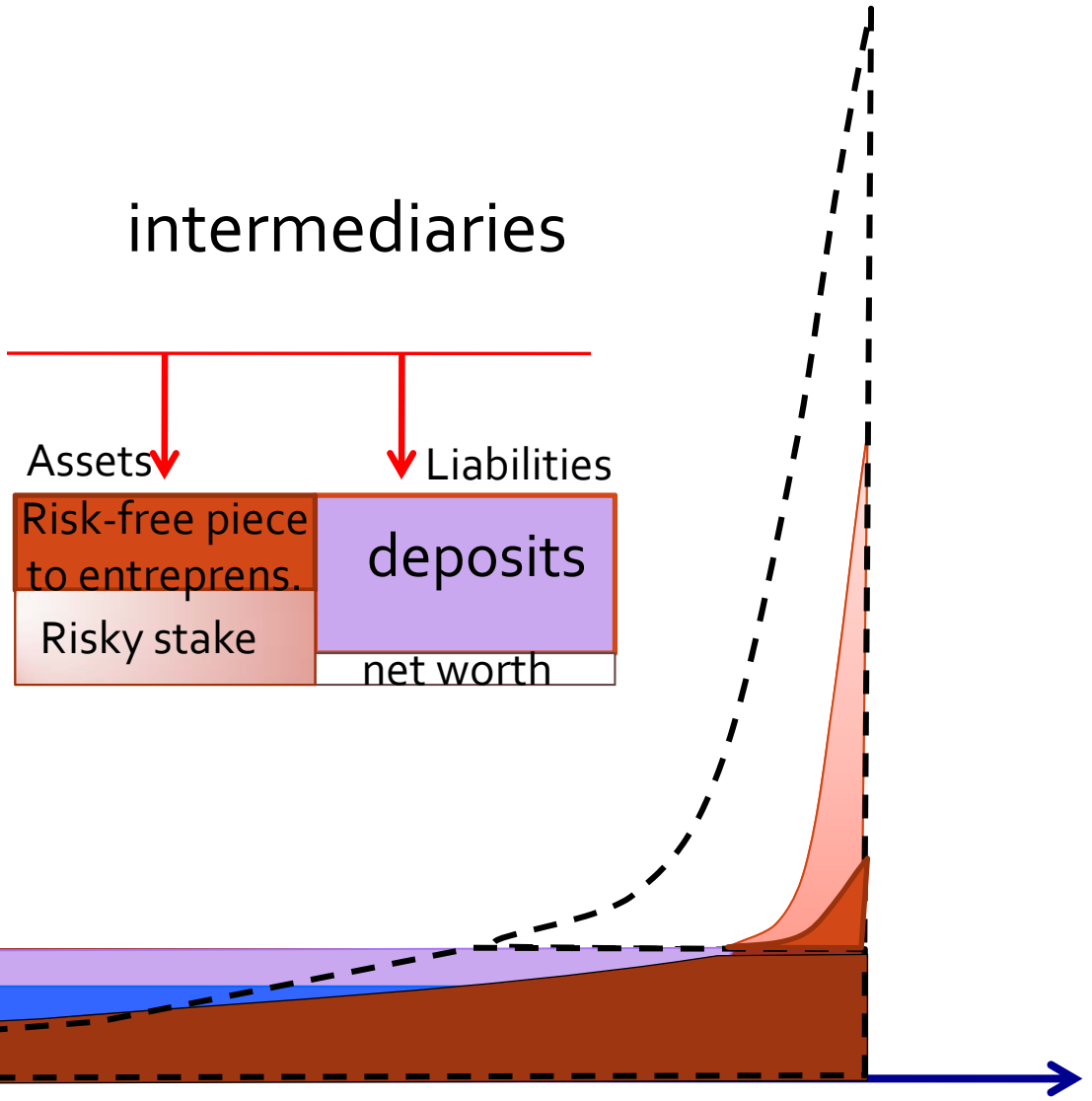
Shrinking Balance Sheets

- Intermediary net worth ↓
- **Capital:** fire sales, price q ↓
- **Money:**
  - Lending + deposits ↓
  - value of money p ↑
  - Multiplier ↓
- Allocation efficiency ↓

- Externality among banks!



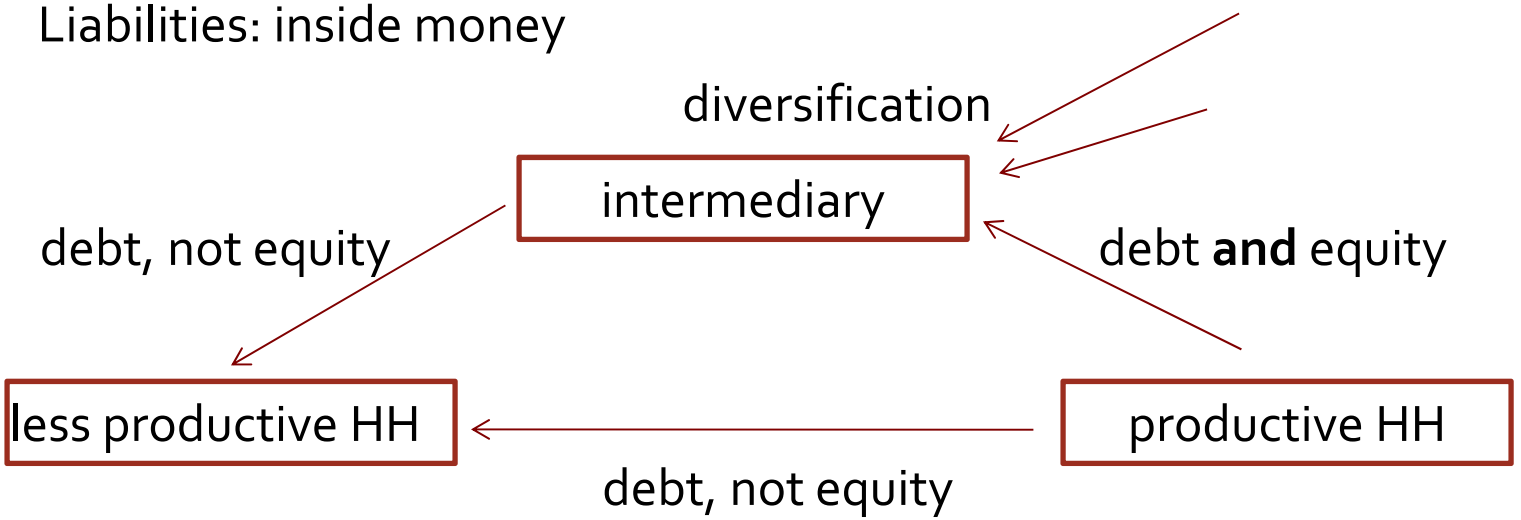
Overview

- **Passive** monetary policy: "Gold standard"
  - Quantity of outside money fixed
  - Interest rate zero
  - A negative macro shock hits intermediaries
    - Asset side: liquidity spiral ("skin in the game")
    - Liability side: deflationary spiral

- **Active** Monetary Policy
  - Introduce long-term bond
    - Short-term interest rate policy
    - Value of long-term bonds rises in downturns – substitute for reduction of inside money
    - Asset purchase and OMO
  - Redistributional effects

Formal Model: Key Frictions

- HH can borrow from other HH, cannot issue equity
  - Inefficient: risky projects cannot sustain high leverage
- … but HH can issue equity to intermediaries
- Intermediaries
  - Assets: diversified asset across households
  - Liabilities: inside money



The Model: Technology

consumption rate

Output: $y_t^{\omega} = a^{\omega} k_t^{\omega} = (c_t^{\omega} + i_t^{\omega}) k_t^{\omega}$

investment rate

Capital: $dk_t^{\omega} = (\Phi(i_t^{\omega}) - \delta^{\omega}) k_t dt + d\varepsilon_t^{\omega}$

$\Phi(0) = 0, \Phi' > 0, \Phi'' < 0$

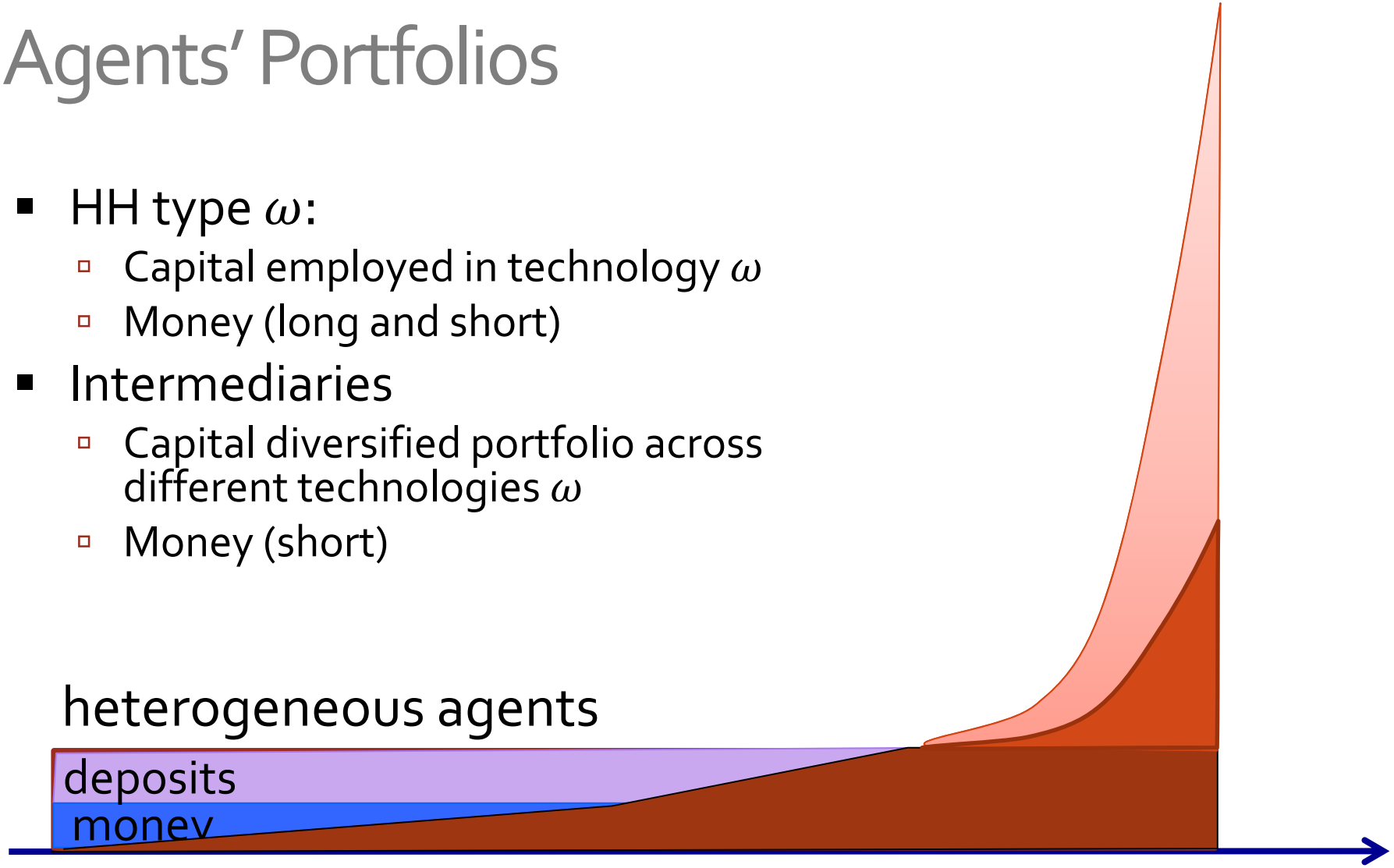
$Cov[\varepsilon_t^{\omega}, \varepsilon_t^{\omega\prime}]$

- heterogeneous agents

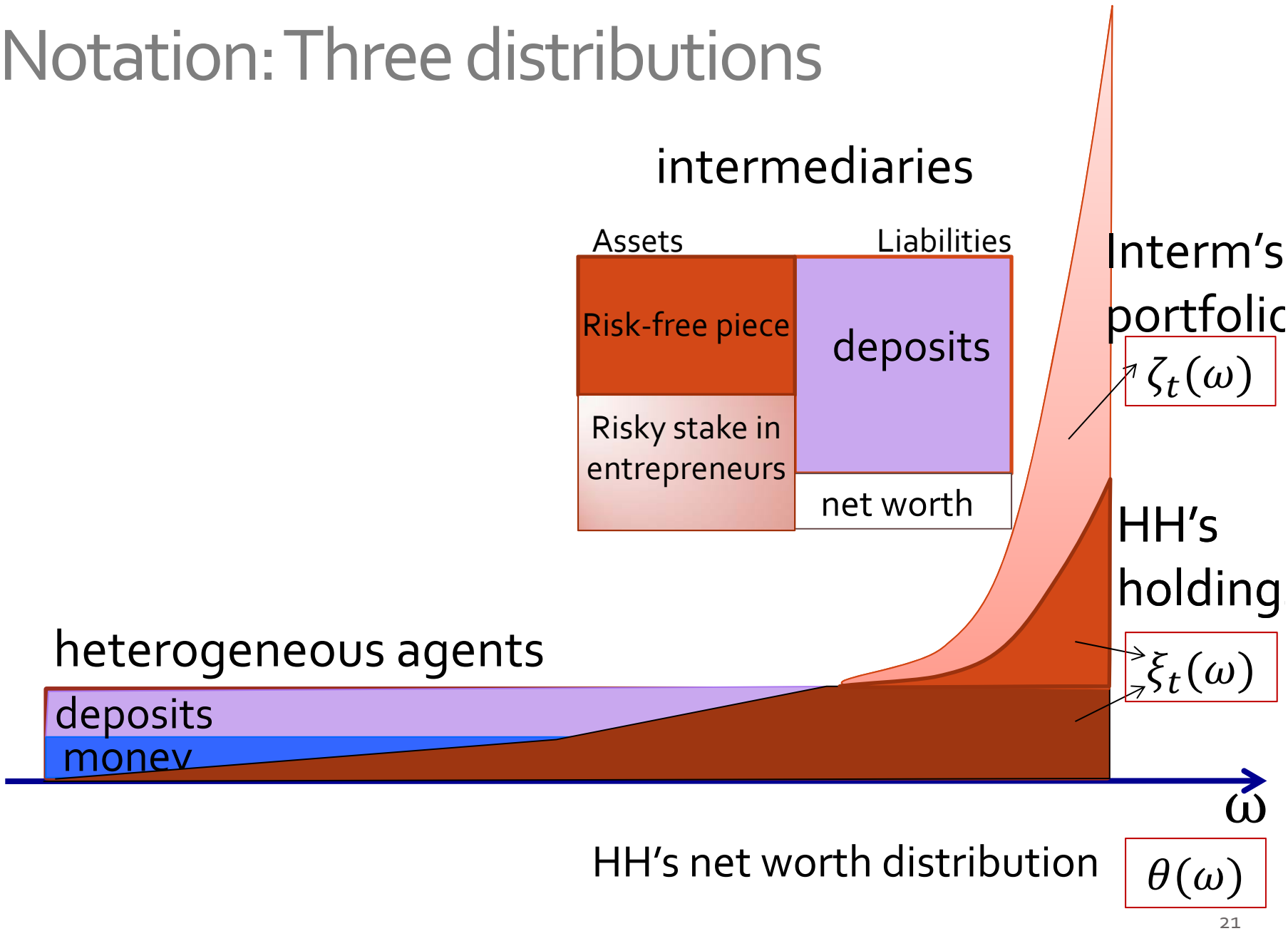
- Outside money (gold) is in fixed supply
- Contracting friction: contract on $q_t k_t$ but not on k_t

Agents' Portfolios

- HH type ω:
  - Capital employed in technology ω
  - Money (long and short)
- Intermediaries
  - Capital diversified portfolio across different technologies ω
  - Money (short)

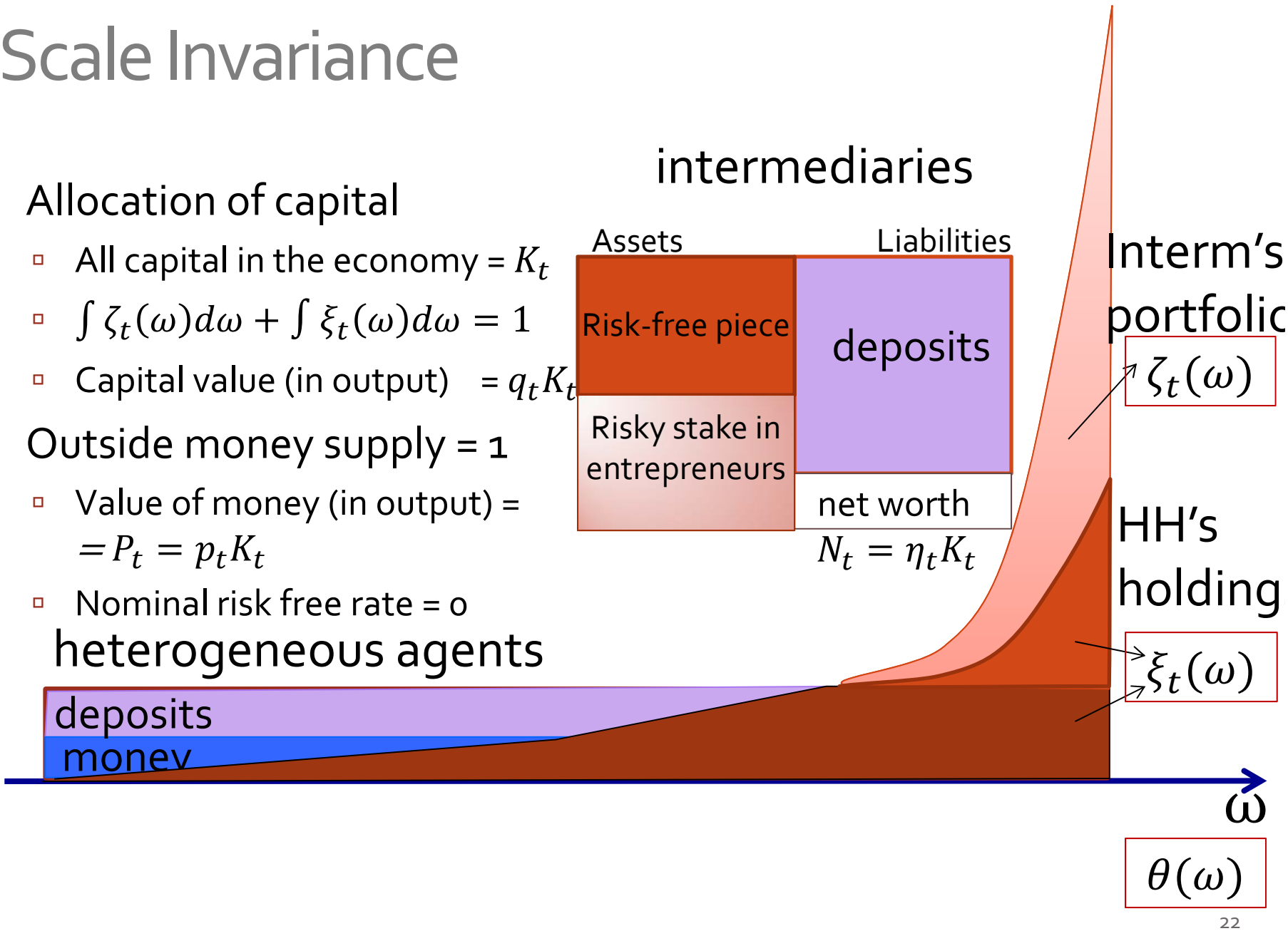


Notation: Three distributions



Scale Invariance

- Allocation of capital
  - All capital in the economy = K_t
  - $\int \zeta_t(\omega)d\omega + \int \xi_t(\omega)d\omega = 1$
  - Capital value (in output) $= q_t K_t$
- Outside money supply = 1
  - Value of money (in output) = $= P_t = p_t K_t$
  - Nominal risk free rate = 0



The Model: Preferences

- All agents have logarithmic utility with discount rate

$$E\left[\int_0^\infty e^{-\rho t}\log c_t\, dt\right]$$

- Retirement: intermediary gets utility boost, when it decides to become a household forever
- Implications of log utility:
  - Consumption $= \rho \times net\ worth$
  - Required return $= Cov[asset\ risk, net\ worth\ risk]$
  - Consumption is independent of investment opportunity
  - Asset demands are myopic
    (no Mertonian hedging demand, no precautionary motive)

Equilibrium Definition

- For each history of shocks $\{\{d\varepsilon_s^\omega\}_\omega, s \in [0,t]\}$
  - HH type ω max utility
    - Consumption
    - Investment
    - Allocation between technology ω, $\xi_t(\omega)$, and money
  - Intermediaries max utility
    - Consumption
    - Portfolio across technology ωs, $\zeta_t(\omega)$, and money
    - Retirement decision
  - Market clearing
    - Capital: Supply of K_t at price q_t
    - Money: Supply of 1 at the price $P_t = p_t K_t$
    - Output: numeraire

Derivation - Roadmap

- Individual choices
  - $c_t = \rho *$ net worth
  - i_t^{ω}
  - Required excess return = Cov [asset risk, net worth risk]
    - Postulate: $dq_t = \mu_t^q q_t dt + d\varepsilon^q$ and $dp_t = \mu_t^p p_t dt + d\varepsilon_t^p$
- Market clearing
  - Endogenously determines $\mu_t^q, d\varepsilon_t^q, \mu_t^p, d\varepsilon_t^p$
- Step 1: Derive equilibrium conditions (optimality + m-clearing)
- Step 2: Derive law of motion of η
  - Depends on postulated price processes q_t and p_t (fixed point)
- Step 3: $\mu_t^q, d\varepsilon_t^q, \mu_t^p, d\varepsilon_t^p$ as functions of η

Internal Investment Decision

$$dk_t^\omega = (\Phi(i_t^\omega) - \delta^\omega)dt + d\varepsilon_t^\omega$$

- Given the price of capital q_t, the optimal investment solves

$$\max_i \Phi(i)q_t - i \Rightarrow i^*(q_t)$$

- Determines for each HH ω
  - $c^\omega(q_t) = a^\omega - i^*(q_t)$
  - $g^\omega(q_t) = \Phi(i^*(q_t)) - \delta^\omega$

Return on Physical Capital

- Recall: $dk_t^\omega / k_t^\omega = (\Phi(\iota_t^\omega) - \delta^\omega)dt + d\varepsilon_t^\omega$
- Postulate: $dq_t = \mu_t^q q_t dt + q_t d\varepsilon_t^q \leftarrow$ endogenous

$$dR_t^\omega = \left(\frac{a^\omega - \iota^\omega}{q_t} + \Phi(\iota_t^\omega) - \delta^\omega + \mu_t^q + Cov\left[d\varepsilon_t^\omega, d\varepsilon_t^q\right] \right) dt + \left(d\varepsilon_t^\omega + d\varepsilon_t^q\right)$$

dividend yield

capital gains rate

risk (endogenous + exogenous)

maximized when $\Phi'(\iota_t^\omega) q_t = 1$. ι_t^ω increases in q_t, independent of ω

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Return on Money

- Convenient to normalize $P_t = p_t K_t$
  - In the long-run value of money is proportional to K_t
  - In the short run it fluctuates with shocks
- Postulate: $dp_t = \mu_t^p p_t dt + p_t d\varepsilon_t^p \leftarrow$ endogenous

$$\frac{dK_t}{K_t} = \underbrace{\int (\zeta(\omega) + \xi(\omega)) g^\omega(q_t) d\omega}_{\mu_t^K} + \underbrace{\int \zeta(\omega) + \xi(\omega)\, d\varepsilon_t^\omega}_{d\varepsilon_t^K}$$

a dollar invested in money earns return

$$dR_t^M = (\mu_t^K + \mu_t^p + Cov[d\varepsilon_t^K, d\varepsilon_t^p])dt + \underbrace{d\varepsilon_t^K + d\varepsilon_t^p}_{d\varepsilon_t^M}$$

Intermediaries' "Risk Balance Sheet"

Assets	Liabilities
$q_t K_t \int \zeta_t(\omega)\left(d\varepsilon_t^q + d\varepsilon_t^\omega\right)d\omega$	$\left(q_t K_t \int \zeta_t(\omega)d\omega - N_t\right)d\varepsilon_t^M$
	$N_t d\varepsilon_t^N$

$$dN_t = -\rho N_t dt + N_t dr_t^M + q_t K_t \int \zeta_t(\omega) Cov\left[d\varepsilon_t^q + d\varepsilon_t^\omega - d\varepsilon_t^M, d\varepsilon_t^N\right]d\omega\, dt + q_t K_t \int \zeta_t(\omega)\left(d\varepsilon_t^q + d\varepsilon_t^\omega - d\varepsilon_t^M\right)d\omega$$

$$d\eta_t = d(N_t/K_t) = \cdots$$

Equilibrium Conditions

1. Market clearing for capital goods and bonds

$$\int \zeta_t(\omega)d\omega + \int \xi_t(\omega)d\omega = 1$$

2. Market clearing for output:

$$\int (\zeta_t(\omega) + \xi(\omega))a^{\omega}(q_t)d\omega - \iota_t = \rho(q_t + p_t)$$

3. Valuation of capital ω -- return = Cov(risk, net worth risk)
   - Intermediaries

$$E[dR_t^{\omega} - dR_t^M] \leq Cov\left[d\varepsilon_t^q + d\varepsilon_t^M, d\varepsilon_t^N\right] \qquad (= if\ \zeta_t(\omega) > 0)$$

   - HH ω

$$E[dR_t^{\omega} - dR_t^M] \leq Cov\left[d\varepsilon_t^q + d\varepsilon_t^M, d\varepsilon_t^{N(\omega)}\right] \quad (= if\ \xi_t(\omega) > 0)$$

net worth risk of HH ω, $d\varepsilon_t^{N(\omega)}$, depends on $\xi_t(\omega)$ and its net worth

Dynamics with One State Variable η

- N_t denotes aggregate net worth of intermediaries
  - Depends on portfolio $\zeta_t(\omega)$, returns and retirement
- $q_t K_t + P_t - N_t$ is the aggregate net worth of HH
  - Allocation depends on returns, switching types
  - Assume HH types switch very fast, so distribution over types $\theta(\omega)$ is invariant
- + scale invariance in K_t

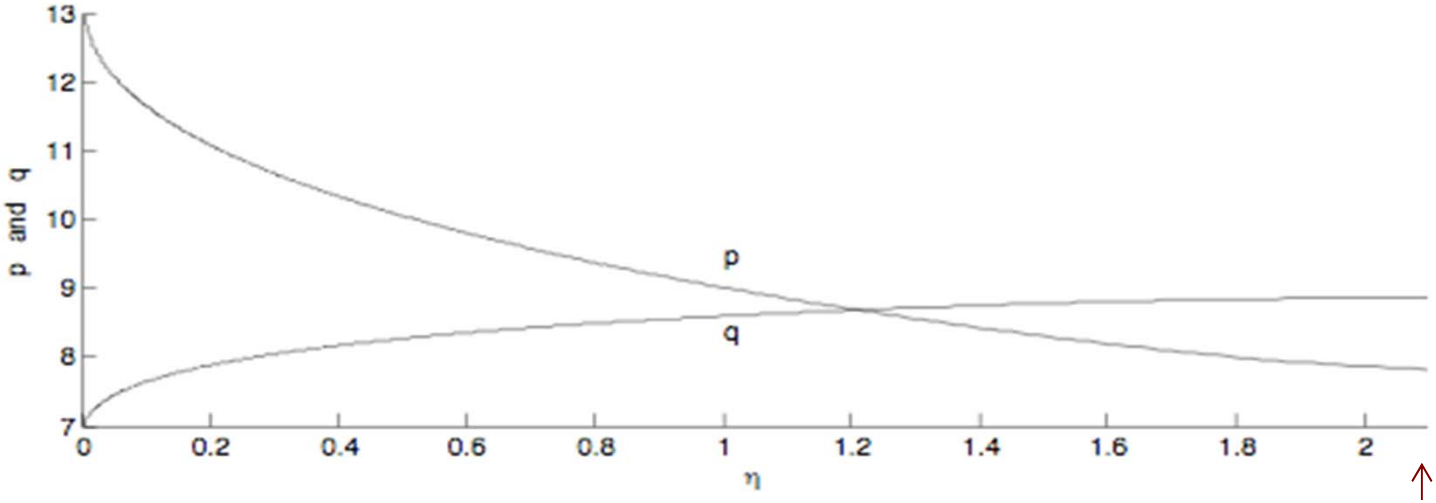
- Wealth distribution is characterized by a single state variable $\eta_t = N_t/K_t$

Example

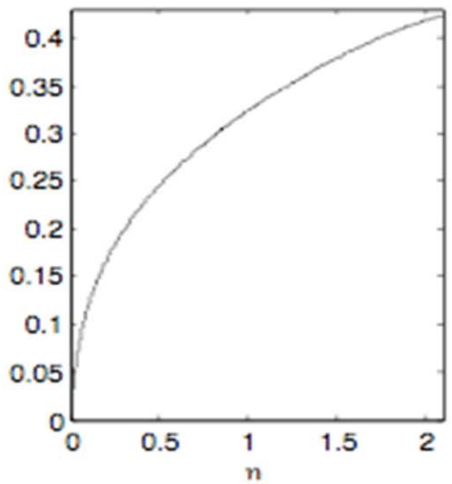
- Three household types ω only
  - Low: very bad technology, hold money
    own 65% of HH wealth
  - Medium: risk-free technology,
    prefer to hold capital over money
    own 35% of HH worth
  - High: risky production – low net worth
    no net worth

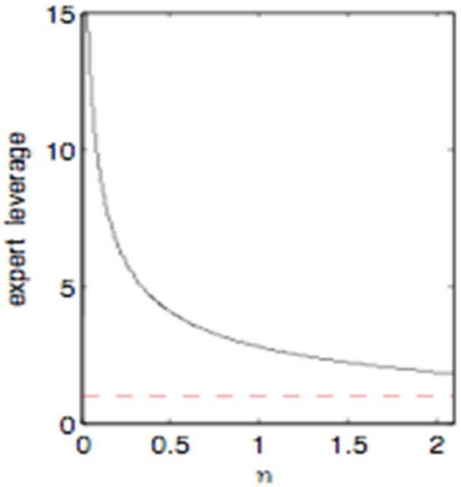
- Intermediaries choose to invest only in high ω due to monitoring cost

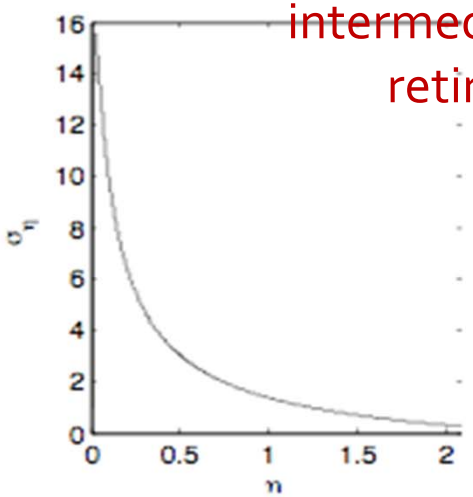
Example



some intermediaries retire







allocation to the most productive technology

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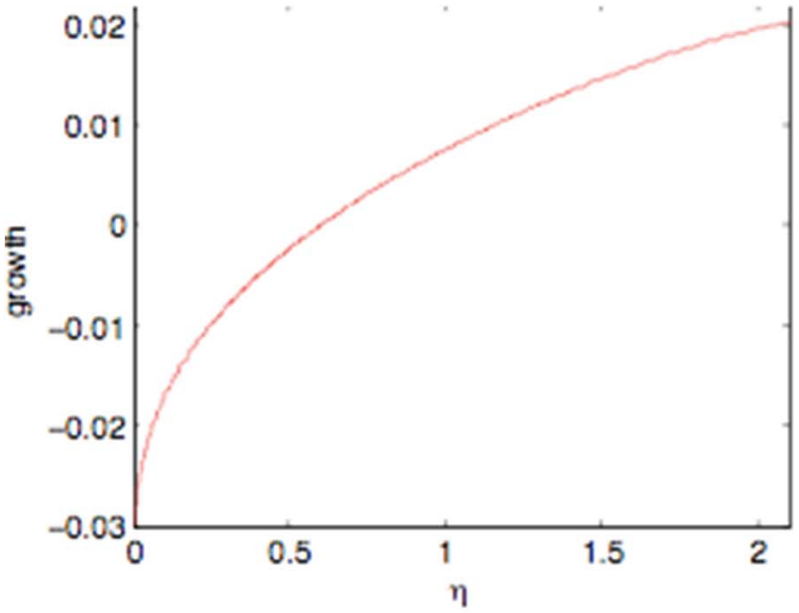
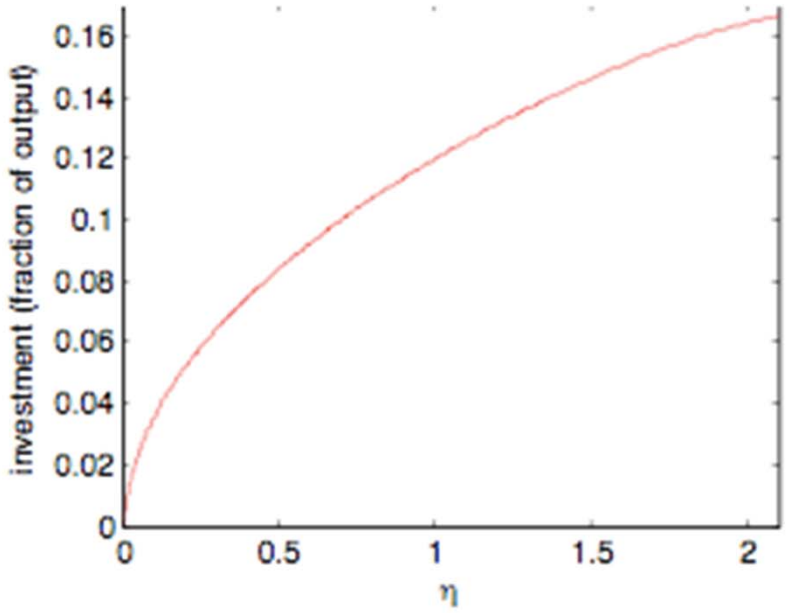
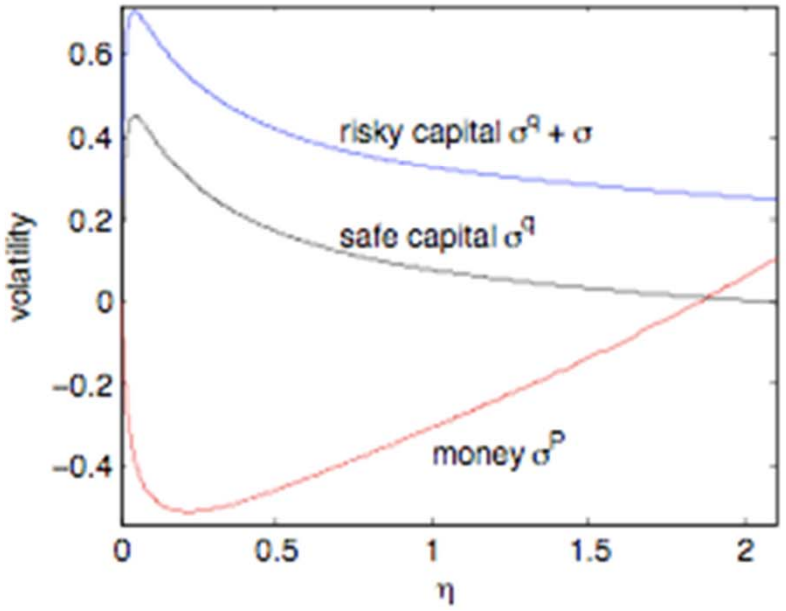
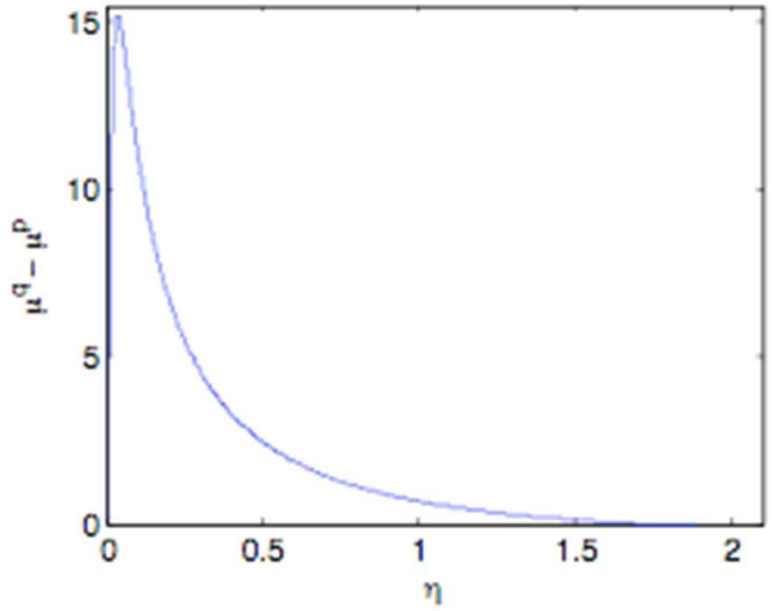
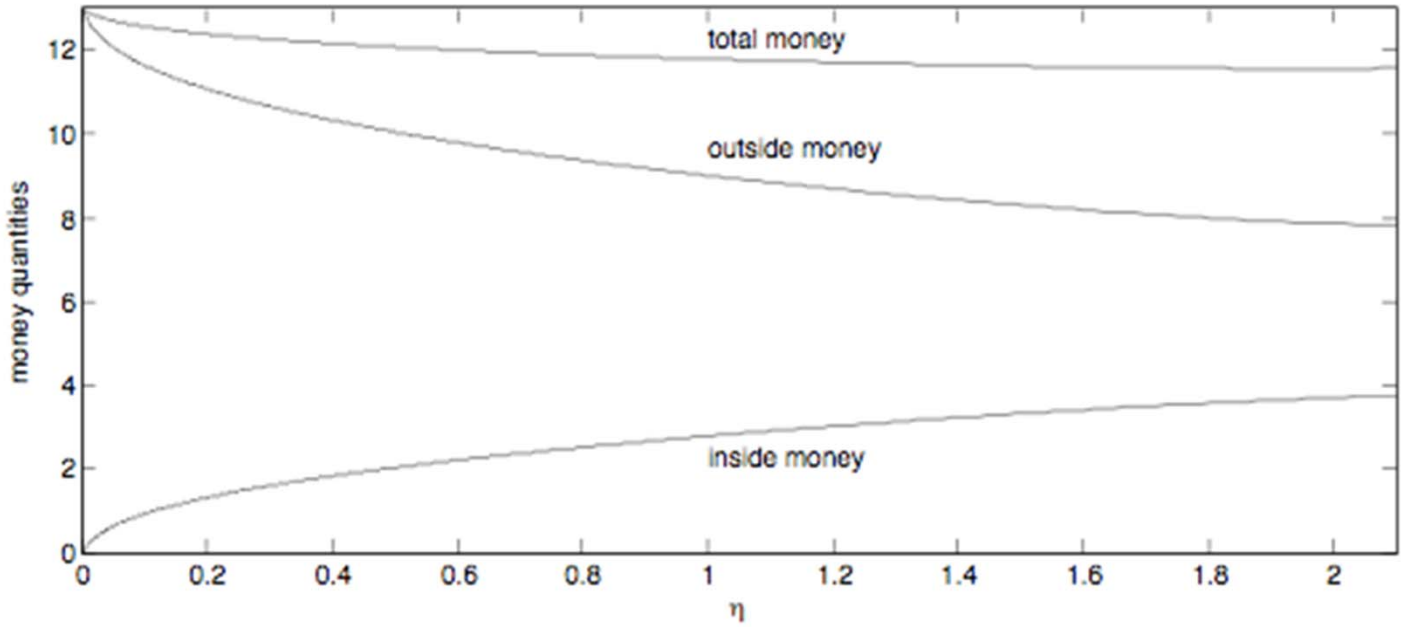
$\alpha = 1, r = 5\%, \delta^H = 0, \delta^M = 3\%, \sigma = 25\%, \theta^L = .65, \theta^M = 35\%, \theta^H = 0\%, \Phi(i) = (0.02i)^{1/2}$

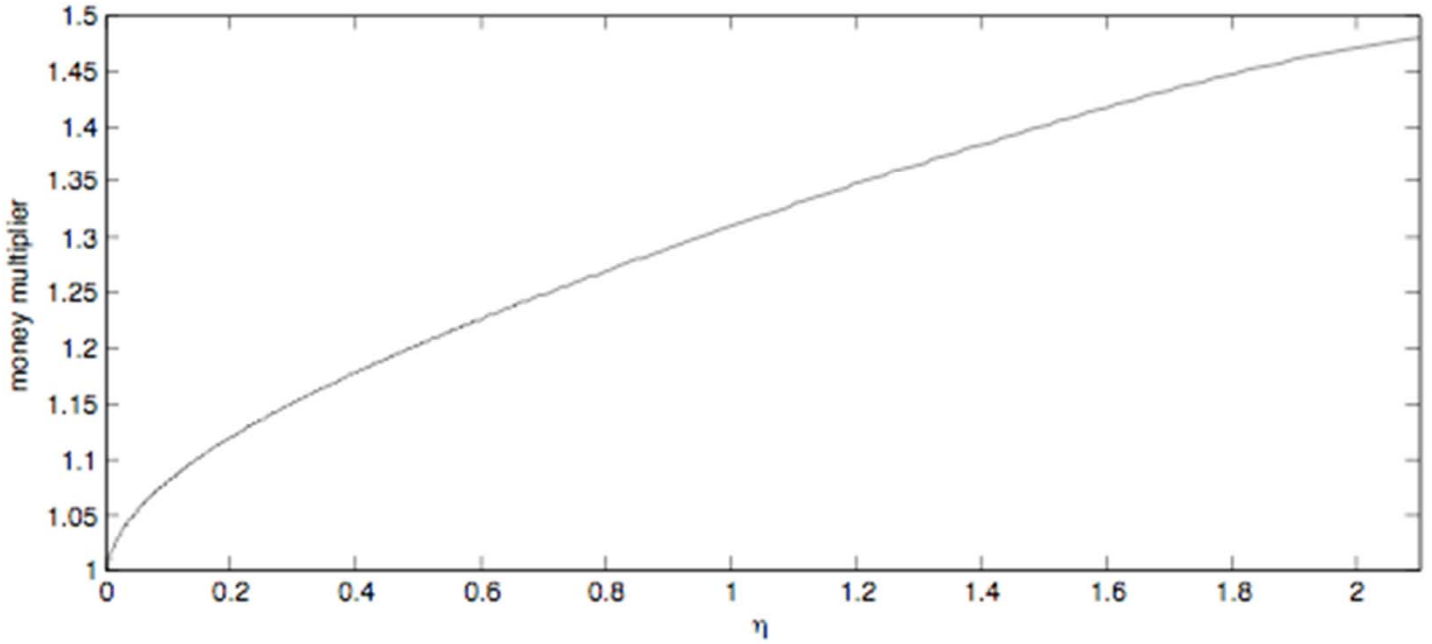

growth
0.02
0.01
0
−0.01
−0.02
−0.03
0
0.5
1
1.5
2
η


investment (fraction of output)
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.5
1
1.5
2
η


volatility
0.6
0.4
0.2
0
−0.2
−0.4
risky capital $\sigma^q + \sigma$
safe capital σ^q
money σ^p
0
0.5
1
1.5
2
η

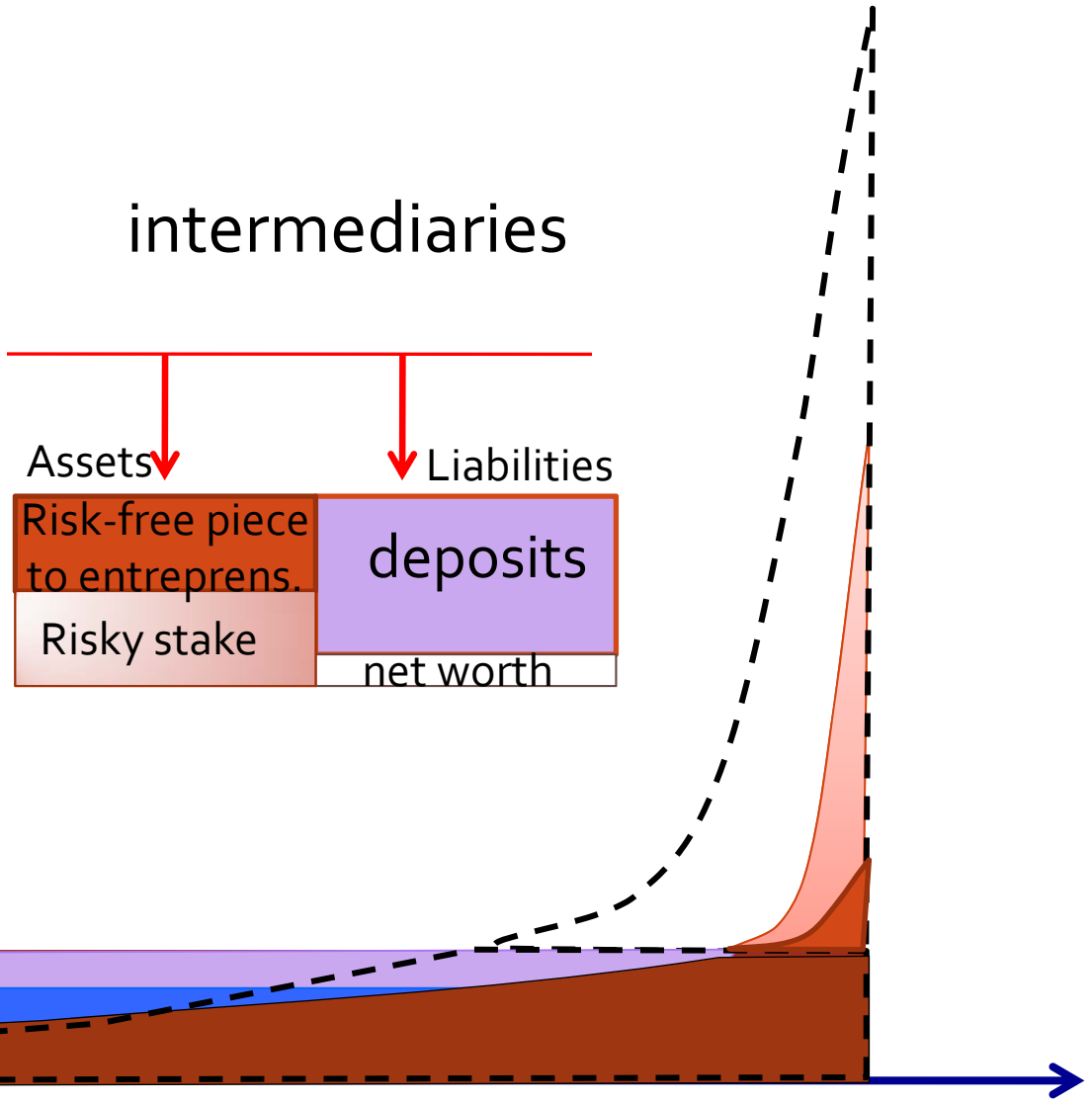

$\mu^q - \mu^p$
15
10
5
0
0
0.5
1
1.5
2
η


total money
outside money
inside money
money quantities
η
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


money multiplier
η
1
1.05
1.1
1.15
1.2
1.25
1.3
1.35
1.4
1.45
1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

After a negative shock

- Intermediary net worth ↓
- Balance sheets ↓
- Competition among banks ↓
- **Capital:** fire sales, price q ↓
- **Money:**
  - Lending + deposits ↓
  - value of money p ↑
  - Multiplier ↓
- Banks are hit on both sides of their balance sheet
- Allocation efficiency ↓
- Externality among banks!



Observations

- As η goes down:
- Intermediaries take on less risk, competition decreases
- Price of capital q and investment, i(q), decrease
- Capital is allocated less efficiently
- Unproductive households hold less inside money (loans to intermediaries/entrepreneurs) and more outside fiat money
- Price of outside money goes up (deflation)
- Additional source of amplification in economy with money:
  - value of assets fall
  - value of liabilities increase (due to deflation)

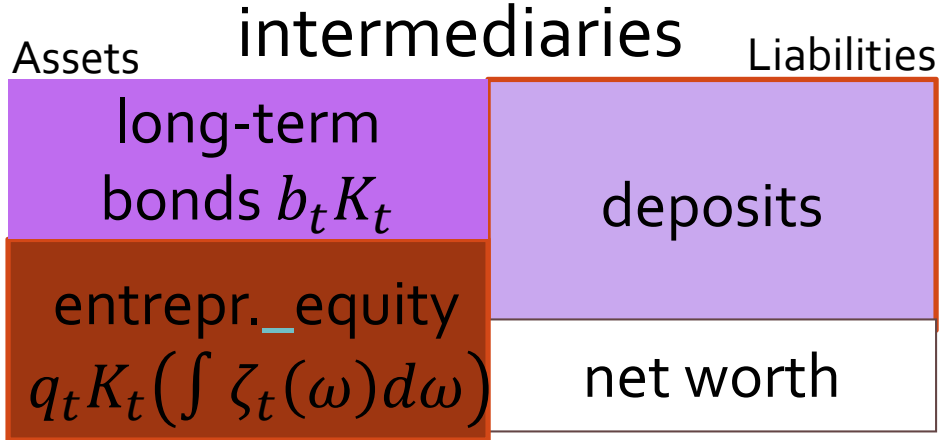
Monetary Policy

- So far, Gold Standard
  - outside money fixed,
  - pays no interest
  - no central bank

  - Introduce consul (perpetual) bond
    - pays interest rate in short-term (outside) money
- Monetary Policies
  - Short-term interest rate policy
    - Central bank accepts deposits & pays interest rate (by printing money)
      - E.g. short-term interest rate is lowered when η becomes small
    - Budget neutral policies (at any point in time)
  - Asset purchase program
    - Bond – open market operations (OMO/QE)

Money and Long-term Bond

- Policy instruments (functions of η_t)
  - Central bank pays interest $r_t \geq 0$ on money (by printing)
  - Sets total outstanding value $b_t K_t$ of perpetual bond
    - By changing interest r_t
    - Additional Quantitative Easing/Open market operations – to get around ZLB
- Endogenous market reaction
  - Price of long-term bond (in money, per unit coupon rate)
  - $dB_t = \mu_t^B B_t dt + B_t d\varepsilon_t^B$
  - q_t = price of capital
  - $p_t K_t$ = value of money



Extra steps

- Under Gold standard
  - Return on money: $\frac{d(p_t K_t)}{p_t K_t}$
- Now, $\frac{d(p_t K_t)}{p_t K_t}$ depends on OMO/QE also …
- To derive return on money and bonds use trick:
  1. Return on (bond – money) = return on bond in money – interest on money *Both are nominal – price of bond is all what matters*
  2. $\frac{d(p_t+b_t)K_t}{(p_t+b_t)K_t}$ = return on a portfolio of money and bonds *Like before*
  - system of two linear equations for returns on bonds & money

Disentangling Money and Bonds

- Given
  - flow of motion of η
  - Endogenous $p(\eta), q(\eta), B(\eta)$ and exogenous $r(\eta), b(\eta)$ functions and
  - Price of bond: $\frac{dB_t}{B_t} = \mu_t^B dt + d\varepsilon_t^B$ ($\frac{1}{B_t}$ is current yield)
- Figure out return on
  - money: $dr_t^M = \mu_t^M dt + d\varepsilon_t^M$
  - bonds: $dr_t^B = dr_t^M - r_t dt + (\frac{1}{B_t} + \mu_t^B + Cov[\varepsilon_t^B, \varepsilon_t^M])dt + d\varepsilon_t^B$
  - all monetary instruments: $\frac{d(p_t+b_t)K_t}{(p_t+b_t)K_t} = dr_t^M + \frac{b_t}{p_t+b_t}(dr_t^B - dr_t^M)$

$$= \left(\mu_t^p + \mu_t^b + \mu_t^K + Cov\left[\varepsilon_t^p + \varepsilon_t^b, \varepsilon_t^K\right]\right)dt + d\varepsilon_t^p + d\varepsilon_t^b + d\varepsilon_t^K$$

- Collecting shocks: $d\varepsilon_t^M + \frac{b_t}{p_t+b_t} d\varepsilon_t^B = d\varepsilon_t^p + d\varepsilon_t^b + d\varepsilon_t^K$

Equilibrium Conditions

1. Market clearing for capital goods and bonds

$$\int \zeta_t(\omega)d\omega + \int \xi_t(\omega)d\omega = 1, \qquad \zeta_t^B + \int \xi_t^B(\omega)d\omega = 1$$

2. Market clearing for output:

$$\int (\zeta_t(\omega) + \xi(\omega))c^{\omega}(q_t)d\omega = \rho(q_t + p_t + b_t)$$

3. Valuation of capital ω -- return = Cov(risk, net worth risk)

$$E[dr_t^{\omega} - dr_t^M] \leq Cov\left[d\varepsilon_t^q + d\varepsilon_t^M, d\varepsilon_t^N\right] \qquad (= if\ \zeta_t(\omega) > 0)$$

$$E[dr_t^{\omega} - dr_t^M] \leq Cov\left[d\varepsilon_t^q + d\varepsilon_t^M, d\varepsilon_t^{HH-N}\right] \quad (= if\ \xi_t(\omega) > 0)$$

4. Valuation of bonds

$$E[dr_t^B - dr_t^M] = Cov[d\varepsilon_t^B, d\varepsilon_t^N] \qquad \text{(assuming } \zeta_t^B > 0)$$

$$E[dr_t^B - dr_t^M] \leq Cov[d\varepsilon_t^B, d\varepsilon_t^{HH-N}] \quad \text{(= if } \xi_t^B(\omega) > 0)$$

Short-term interest rate policy

- Without long-maturity assets changes in short-term interest rate have no effect
  - Interest rate change equals instantaneous inflation change
- With bonds: of all monetary instruments, fraction $p_t/(p_t+b_t)$ is cash and $b_t/(p_t+b_t)$ are bonds
  - deflationary spiral is less pronounced because as η goes down, growing demand for money is absorbed by increase in value of long-term bonds
  - also, intermediaries hedge risks better by holding long-term bonds
  - however, intermediaries also have greater incentives to increase leverage/risk-taking ex-ante
- Effectiveness of monetary policy depend on maturity structure (duration) of government debt

Conclusion

- Unified macro model to analyze both
  - Financial stability
  - Monetary stability
    - Liquidity spirals
    - Fisher deflation spiral
- Capitalization of banking sector is key state variable
  - Price stickiness plays no role (unlike in New Keynesian models)
- Monetary policy rule
  - Affects money supply
  - Redistributional feature
  - Time inconsistency problem – "Greenspan put"

	New Keynesian	I-Theory
Key friction	Price stickiness & ZLB	Financial friction
Driver	Demand driven as firms are obliged to meet demand at sticky price	Misallocation of funds increases incentive problems and restrains firms/banks from exploiting their potential
Monetary policy • First order effects	Affect HH's intertemporal trade-off Nominal interest rate impact real interest rate due to price stickiness	Ex-post: redistributional effects between financial and non-financial sector Ex-ante: insurance effect leading to moral hazard in risk taking (bubbles) - Greenspan put -
• Second order effects	Redistributional between firms which could (not) adjust price	
Time consistency	Wage stickiness Price stickiness + monopolistic competition	Moral hazard

	New Keynesian	I-Theory
Risk build-up phase		Endogenous due to accommodating monetary policy
Net worth dynamics	zero profit no dynamics	dynamic
State variables	Many exogenous shocks Intermediation/friction shock	Endogenous intermediation shock
Monetary policy rule	Taylor rule (is approximately optimal only if difference in u' is well proxied by output gap) • spreads • credit aggregates (?)	Depends on signal quality and timeliness of various observables
Policy instrument	Short-term interest rate + expectations	Short-term interest rate **+ long-term bond** + expectations
Role of money	In utility function (no deflation spiral)	Storage Precautionary savings

	Monetarism	I-Theory
Focus	Price stability	Price and Financial stability
Theory	Quantity theory of money P*Y = v*M Transaction role of money	Distribution of wealth (liquidity, balance sheet) endogenous money multiplier
Monetary aggregates	Mo (Brunner, Meltzer) M1-2(Friedman,Schwartz) Inside and outside money are *perfect substitutes*	Outside money is only *imperfect substitute* for inside money (intermediation) Bank underwriting (*credit lines*) is substitute to bank deposits (difficult to measure M1-3 in a meaningful way)
Monetary policy	Constant growth of M2 (Friedman)	Recapitalize banks through monetary policy Switch off deflationary pressure

Intermediaries and lending

- Monitoring technology
  Diamond (1984)
  Homstrom-Tirole (1997)