A MACROECONOMIC MODEL WITH A FINANCIAL SECTOR MARKUS BRUNNERMEIER & YULIY SANNIKOY

Princeton University

... so far

- Go to the (debt) limits
 - KM: Limit is exogenous go to the limit
 - BruPed: Limit depends on future volatility
- "Safety cushion" self-insurance
 - Bewley/Aiyagari: aggregate variables are deterministic
 - Krusell & Smith: add aggregate risk no amplification (inv. is reversible)
 ...: add amplification in 3 period models
- BruSan10
 - Financial instability + Amplification + Persistence of shocks
 - Non-linear liquidity spirals adverse feedback loops
 - Go beyond log-linearization
 - Endogenous risk
 - "Volatility paradox"
 - Asset pricing implications
 - Fat tails
 - Endogenous correlation structure

- Bernanke & Gertler (1989), Carlstrom & Fuerst (1997)
 - Perfect (technological) liquidity, but persistence
 - Bad shocks erode net worth, cut back on investments, leading to low productivity & low net worth of in the next period



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- Brunnermeier & Pedersen (2009), Geanakoplos
 - Volatility interaction with margins/haircuts (leverage) debt constraint

Preview of results

- Full equilibrium dynamics + volatility dynamics
 - "Steady state" is endogenous depends on leverage, consumption etc.
 - Near "steady state"
 - (large) payouts balance profit making
 - intermediaries must be unconstrained and amplification is low
 - Below "steady state"
 - intermediaries constrained, try to preserve capital leading to high amplification and volatility —> precaution
- Crises episodes have significant endogenous risk, correlated asset prices, larger spreads and risk premia
- SDF is driven by constraint & $c \ge 0$
- "Volatility paradox"
- Securitization and hedging of idiosyncratic risks can lead to higher leverage, and greater systemic risk

... with volatility dynamics + precaution

 Unstable dynamics away from steady state due to (nonlinear) liquidity spirals



Model : Technology

Experts

Output: $y_t = ak_t$

- Consumption rate c_t
- Investment rate ι_t

•
$$\frac{dk_t}{k_t} = \underbrace{(\Phi(\iota_t) - \delta)}_{=g} dt + \sigma dZ_t$$

 $a \ge \underline{a}, \delta \le \underline{\delta}$ Output: $y_t = \underline{a}k_t$

Consumption rate <u>c</u>t

• Investment rate
$$\underline{\iota}_t$$

= $(\Phi(\underline{\iota}_t) - \underline{\delta})dt + \sigma dZ_t$

Model : Preferences Experts

Output: $y_t = ak_t$

- Consumption rate c_t
- Investment rate ι_t

•
$$\frac{dk_t}{k_t} = \underbrace{(\Phi(\iota_t) - \delta)}_{=g} dt + \sigma dZ_t$$

•
$$U = E_0 \left[\int_0^\infty e^{-\rho t} dc_t \right]$$

• $dc_t \ge 0$

 $a \geq \underline{a}, \delta \leq \underline{\delta}$ Output: $y_t = \underline{a}k_t$

- Consumption rate <u>c</u>t
- Investment rate $\underline{\iota}_t$ = $(\Phi(\underline{\iota}_t) - \underline{\delta})dt + \sigma dZ_t$

$$\rho \ge r \quad \bigcup = \mathsf{E}_{o} \left[\int_{0}^{\infty} e^{-rt} d\underline{c}_{t} \right]$$
$$\quad dc_{t} \in \mathbb{R}$$

Model : Financial Frictions

Experts

Output: $y_t = ak_t$

- Consumption rate c_t
- Investment rate ι_t

•
$$\frac{dk_t}{k_t} = \underbrace{(\Phi(\iota_t) - \delta)}_{=g} dt + \sigma dZ_t$$

•
$$U = E_o \left[\int_0^\infty e^{-\rho t} dc_t \right]$$

• $dc_t \ge 0$

Can issue only risk-free debt
 + solvency constraint

 $a \ge \underline{a}, \delta \le \underline{\delta}$ Output: $y_t = \underline{a}k_t$

Consumption rate <u>c</u>t

• Investment rate
$$\underline{\iota}_t$$

= $(\Phi(\underline{\iota}_t) - \underline{\delta})dt + \sigma dZ_t$

•
$$U = E_o \left[\int_0^\infty e^{-rt} d\underline{c}_t \right]$$

• $dc_t \in \mathbb{R}$

 $\rho \geq r$

Financially unconstraint

Model : Market for Physical Capital Experts

Output: $y_t = ak_t$

- Consumption rate c_t
- Investment rate l+

•
$$\frac{dk_t}{k_t} = \underbrace{(\Phi(\iota_t) - \delta)}_{=g} dt + \sigma dZ_t$$

•
$$U = E_o \left[\int_0^\infty e^{-\rho t} dc_t \right]$$

• $dc_t \geq 0$

Can issue only risk-free debt + solvency constraint

Less productive HH $a \ge a, \delta \le \delta$

Output:
$$y_t = \underline{a}k_t$$

- Consumption rate c_t
 - Investment rate l_t $= (\Phi(\iota_t) - \delta)dt + \sigma dZ_t$

$$\rho \ge r \quad \quad \mathsf{U} = \mathsf{E}_{\mathsf{o}} [\int_{0}^{\infty} e^{-rt} \, d\underline{c}_{t}]$$

• $dc_t \in \mathbb{R}$

- Financially unconstraint
- Liquid markets for capital k_t with endogenous price process for capital $dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$ 16

First Best – No Frictions

- Experts
 - Manage capital forever
 - Issue equity to less productive HH
 - Consume entire net worth at t = 0
- Price of capital

$$\overline{q} = \max_{\iota} \frac{a-\iota}{r-\Phi(\iota)+\delta}$$

- Earns a required return = r

Definition of Equilibrium

- An equilibrium consists of functions that for each history of macro shocks $\{Z_s, s \in [0, t]\}$ specify
 - *q_t* the price of capital
 - *k_t*, *k_t* capital holdings
 - $dc_t \ge 0, dc_t$ consumption of representative expert and households
 - $\iota_t, \underline{\iota}_t$ rate of internal investment, per unit of capital
 - r the risk-free rate
- such that
 - intermediaries and households maximize their utility, taking prices q_t as given and
 - markets for capital and consumption goods clear

Solution steps

- 1. Equilibrium conditions
 - Agents' optimization
 - Return from holding capital
 - Internal investment
 - Household's optimal portfolio choice
 - Experts optimal choice
 - Portfolio
 - Consumption
 - Market clearing conditions
- 2. Law of motion of state variable (wealth distribution)
- 3. Express in ODEs of state variable

Step 1: Equilibrium Conditions

- Return on Capital
 - $dr_t^k = dividend yield + capital gains rate$
- For experts:

For less productive households

$$d\underline{r}_t^k = \frac{\underline{a} - \underline{\iota}_t}{q_t} dt + \frac{d(\underline{k}_t q_t)}{\underline{k}_t q_t}$$

1. Capital Gains Rate $d(k_t q_t)/k_t q_t$

Capital

• $dk_t = (\Phi(\iota_t) - \delta)k_t dt + \sigma k_t dZ_t$ "cash flow news"

Price

•
$$dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$$
 "SDF news"

• $k_t q_t$ value dynamics

1. Capital Gains Rate $d(k_t q_t)/k_t q_t$

Capital

• $dk_t = (\Phi(\iota_t) - \delta)k_t dt + \sigma k_t dZ_t$ exogenous risk

Price

• $dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$ endogenous risk

• $k_t q_t$ value dynamics

$$d(k_t q_t) = (\Phi(\iota_t) - \delta + \mu_t^q + \sigma \sigma_t^q)(k_t q_t)dt + (\sigma + \sigma_t^q)(k_t q_t)dZ_t$$

exogenous endogenous

Ito's Lemma product rule: $d(X_tY_t) = dX_tY_t + X_tdY_t + \sigma^X\sigma^Y dt$

risk

1. Optimization

- Internal investment 1.
- 2. External investment

 - Solvency constraint

 x_t Given price dynamics $dq_t/q_t = \mu_t^q dt + \sigma_t^q dZ_t$ Solvency constraint $n_t \ge 0$ $n_t \ge 0$

- When to consume? dc_t 3.
- Bellman equation w/ value function $\theta_t n_t$

static

optimization

1. Internal investment – marginal Tobin's q

- Static problem
- Choose investment rate ι that solves $\frac{\overline{\Delta} \ \overline{\Delta}}{max} \Phi(\iota) \iota/q_t$
- FOC: $\Phi'(\iota) = \frac{1}{q_t}$ (marginal Tobin's q)
- Hence, optimal investment is $\iota_t = \underline{\iota}_t = \iota(q_t)$
- Substitute in optimal investment rate

g

1. External Investment - Leverage

- Less productive HH
 - x_t fraction of net worth invested in capital

$$\frac{d\underline{n}_t}{\underline{n}_t} = \underline{x}_t d\underline{r}_t^k + (1 - \underline{x}_t)rdt - \frac{d\underline{c}_t}{\underline{n}_t}$$

Consumption can be negative

Experts

x_t fraction of net worth invested in capital

$$\frac{dn_t}{n_t} = x_t dr_t^k + (1 - x_t)r dt - \frac{dc_t}{n_t}$$
 If x_t > 1 then expert uses leverage

Denote $d\zeta_t := dc_t/n_t$

1. External Investment and Consumption

- **1.** Households: risk free rate of r_t = households discount rate
 - Makes HH indifferent between consuming and saving, s.t. consumption market clears
 - Required return

$$\underbrace{\frac{\underline{a}-\iota(q_t)}{q_t} + \Phi(\iota(q_t)) - \underline{\delta} + \mu_t^q + \sigma\sigma_t^q}_{E_t[\underline{d\underline{r}_t^k}]/dt} \leq r \text{ with equality if capital>o}$$

- 2. Experts choose $\{x_t, d\zeta_t\}$ dynamic problem
 - Let future expected payoff under this strategy be

$$\theta_t n_t = E_t \left[\int_t^\infty e^{-\rho(s-t)} dc_s \right]$$

- Value function is proportional to n_t , since
 - Price takers
 - Consumption is proportional to their wealth

1. Solving dynamic optimization

- Let value of extra \$, (Note, $\theta_t 1 = \text{external funding premium})$ $d\theta_t = \mu_t^{\theta} \theta_t dt + \sigma_t^{\theta} \theta_t dZ_t$
- Use Ito's lemma to expand the Bellman equation $\rho \theta_t n_t dt = \max_{\substack{x_t \ge 0, d\zeta_t \ge 0}} n_t d\zeta_t + E[d(\theta_t n_t)]$
 - Consumption: $\theta_t \ge 1$, and $d\zeta_t > 0$ only when $\theta_t = 1$
 - Risk free: $r_{isk-free} + \mu_t^{\theta} = \rho_{isk-free} + E[change of invest-required return ment opportunities]}$
 - Capital: $\underbrace{\frac{a-\iota(q_t)}{q_t} + \Phi(\iota(q_t)) + \mu_t^q + \sigma \sigma_t^q - r}_{E[excess return of capital]} = \underbrace{-\sigma_t^\theta(\sigma + \sigma_t^q)}_{capital \ risk \ premium}$ with (in) equality if $x_t > (=)0$

1. Intuition – main forces at work

Investment

- Scale up
 - Scalable profitable investment opportunity
 - Higher leverage (borrow at r)
- Scale back
 - Precaution: don't exploit full (GE) debt capacity "dry powder"
 - Ultimately, stay away from fire-sales prices
 - Debt can't be rolled over if $d > k_t q$ (note, price is depressed)
 - Solvency constraint
- Consumption
 - Consume *early* and borrow $r < \rho$
 - Consume *late* to overcome investment frictions

aggregate leverage!

1. Aggregate Balance Sheets

Experts Liabilities Assets Assets Liabilities net worth N_t loans capital $\psi_t q_t K_t$ net worth capital debt $q_t K_t - N_t$ $(1-\psi_t)q_tK_t$

- Wealth distribution is summarized by $\eta_t = N_t/(q_tK_t)$ "experts' wealth share"
- In equilibrium everything (prices, capital allocation, investment) will be functions of η_t

Less productive HH

Step 2: Law of Motion of η

- $dN_t = \psi_t q_t K_t dr_t^k (\psi_t q_t K_t N) r dt dC_t$ • $dr_t^k = \cdots$, $\dots d(a_t K_t)$
- Recall $\frac{d(q_t K_t)}{q_t K_t} = \cdots$
 - Use Ito to derive $\frac{d(1/q_t K_t)}{1/q_t K_t} = \dots$
- Again Ito

•
$$d\eta_t =$$

 $(dN_t)\frac{1}{q_tK_t} + N_td\left(\frac{1}{q_tK_t}\right) + \psi_tq_tK_t\left(\sigma + \sigma_t^q\right)\frac{-1}{q_tK_t}\left(\sigma + \sigma_t^q\right)dt$
 $= \dots$

2. Law of Motion of η

$$\frac{d\eta_t}{\eta_t} = \mu_t^{\eta} dt + \sigma_t^{\eta} dZ_t - d\zeta_t$$

where

$$\sigma_t^{\eta} = \frac{\psi_t - \eta_t}{\eta_t} \left(\sigma + \sigma_t^{q} \right)$$
$$\mu_t^{\eta} = \sigma_t^{\eta} \left(\sigma + \sigma_t^{q} + \sigma_t^{\theta} \right) + \frac{a - \iota(q_t)}{q_t} + (1 - \psi_t) \left(\underline{\delta} - \delta \right)$$

Step 3: Express as functions of η

- Use Ito's formula (extensively) to replace terms such as $\mu_t^{\theta}, \sigma_t^{q}, \dots$ with expressions $q', q'', \theta', \theta'' \rightarrow \text{ODEs}$
- Simplified example: Leland (1994). Value equity, E(V)
 - Firm's asset value follow $\frac{dV_t}{V_t} = rdt + \sigma dZ_t$ (state variable)
 - Debt coupon payment rate of C
 - Default when V_B is hit liquidation value αV_B with $\alpha \in (0,1)$
 - Postulate equity follows: $dE_t = \mu_t^E E_t dt + \sigma_t^E E_t dZ_t$ (q-measure)
 - $r = \mu^E C/E$, since any asset expected return under q is r.
 - Using Ito's lemma on E(V), $\mu_t^E E_t = rV_t E' + \frac{1}{2}\sigma^2 V_t^2 E''$

• So
$$r = \frac{rVE' + 1/2\sigma^2 V^2 E''}{E} - \frac{C}{E}$$
, boundaries $E(V_B) = 0$, $\lim_{V \to \infty} V - E(V) \to C/r_{33}$

Step 4: Numerical algorithm

- Algorithm 1: Compute $q''(\eta)$ and $\theta''(\eta)$ from $\eta, q(\eta), q'(\eta), \theta(\eta), \theta'(\eta)$
- Algorithm 2: solve system of ODE's numerically
 - Use Matlab ode45 solver



Drift and Volatility of η



36

Stationary Density



Endogenous Risk Through Amplification



Endogenous Risk Through Amplification



amplification coefficient

- Endogenous risk
 - zero near the steady state
 - large below steady state

$$\sigma_t^q = \frac{\frac{q'(\eta)}{q(\eta)}(\psi_t - \eta_t)\sigma}{1 - (\psi_t - \eta_t)\frac{q'(\eta)}{q(\eta)}}$$

Dynamics near and away from SS

- Intermediaries choose payouts endogenously
 - Exogenous exit rate in BGG/KM
 - Payouts occur when intermediaries are least constrained

 $q'(\eta^*)=0$

- Steady state: experts unconstrained
 - Bad shock leads to lower payout rather than lower capital demand

$$q'(\eta^*) = 0, \sigma_t^q(\eta^*) = 0$$

- Below steady state: experts constrained
 - Negative shock leads to lower demand
 - $q'(\eta)$ is high, strong amplification, $\sigma_t^q(\eta)$ is high

Note difference to BGG/KM

"Volatility Paradox" ... $\sigma(.05, .2, .5)$



Ext1: asset pricing (cross section)

- Capital: Correlation increases with σ^q
 - Extend model to many types *i* of capital

$$\frac{dk_t^i}{k_t^i} = \left(\Phi(\iota_t^i) - \delta\right)dt + \sigma dZ_t + \sigma' dZ_t^i$$

aggregate uncorrelated shock shock

- Experts hold diversified portfolios
 - Equilibrium looks as before, (all types of capital have same price) but
 - Volatility of $q_t k_t$ is $\sigma + \sigma' + \sigma^q$
 - Endogenous risk is perfectly correlated, exogenous risk not
 - For uncorrelated z^i and z^j correlation $(q_t^i k_t^i, q_t^j k_t^j)$ is $(\sigma + \sigma^q)/(\sigma + \sigma' + \sigma^q)$ which is increasing in σ^q

Ext1: asset pricing (cross section)

Outside equity:

- Negative sknewness
- **Excess volatility**
- Pricing kernel: e^{-rt}
 - Needs risk aversion!

Derivatives:

Volatility smirk

- (Bates 2000)
- More pronounced for index options (Driessen et al. 2009)

(in an extended version with outside equity)

Ext2: Idiosyncratic jump losses

$$dk_t^i = gk_t^i dt + \sigma k_t^i dZ_t + k_t^i dJ_t^i$$

- J_t^i is an idiosyncratic compensated Poisson loss process, loss distribution $F(y), y \in [-1,0]$ (per \$ of total assets) and intensity λ
- $q_t k_t^i$ drops below debt d_t , costly state verification
- Debt holders' loss rate $L(x) = \lambda \int_{-1}^{x} \left(\frac{1}{x} + y\right) dF(y)$

 $\frac{n_t}{k_t q_t} = \frac{1}{x_t}$

• Borrowing cost rate C(x)

• E.g.:
$$C(x) = \xi(x-1)$$

- BGG: verification costs
- KM: C(x) = 0 on $[0, \underline{x}]$ and ∞ otherwise
- Leverage bounded not only by precautionary motive, but also by the cost of borrowing

AssetLiabilities $k_t q_t$ $d_t = k_t q_t - n_t$ n_t n_t

Ext2: Idiosyncratic losses

- Experts borrowing rate > r
 - Compensates for verification cost
- $d\eta_t$ = diffusion process (without jumps) because losses cancel out in aggregate
- Results:
 - Borrowing costs (even in downturn) make system more stable --- note difference to KM!
 - Non-degenerated deterministic steady state $x^0 = 1/\eta^0$
 - $\rho r = x^0(x^0 1)C'(x^0) + C(x^0)$
 - If C(x) large as $x \to \infty$, then experts cannot hold capital η close to zero

Ext2: Idiosyncratic losses



Borrowing costs (even in downturns) stabilize system

Ext3: Securitization

- Experts can contract on shocks Z_t and dJⁱ_t directly among each other, zero contracting costs
- In principle, good thing (avoid verification costs)
- Equilibrium
 - experts fully hedge idiosyncratic risks
 - experts hold their share (do not hedge) aggregate risk Z_{t} , market price of risk depends on $\sigma_t^{\theta}(\sigma + \sigma_t^q)$
 - with securitization experts lever up more (as a function of η_t) and bonus payments occur "sooner"
 - financial system becomes less stable
 - risk taking is endogenous (Arrow 1971, Obstfeld 1994)

Ext4: Policy measures

- Policy 1: Capital requirement (leverage constraint) $x \leq \bar{x}(\eta)$
 - Increases η^*
 - Small effect, e.g. $\bar{x}(\eta) = \bar{x}$ which binds on 70% in downturns increases η^* only by 2%
 - Depresses price, more misallocation inefficient
 - Overall, mostly inefficient
- Policy 2: Forced retained earnings until $\eta^* = 0.7$
 - Improves welfare
 - Price of q rises, θ non-continuous and risk premia negative around η^*
 - Less frequent, but more severe crisis, low speed of recovery



stabilizing effect

Ext4: Policy measures



Microfoundation: Contracting friction

 Focus on contracts in which agents is required to hold sufficient levered equity stake in projects



 The more risk entrepreneur wants to unload, the more they have to be monitored (by someone who takes on exposure)

Microfoundation of contracts (extra)

- Agency problem of entrepreneur
 - Increase capital depreciation rate, private benefit b per \$1 destroyed
 - Incentive constraint: entrepreneur equity stake $\geq b$
- Are these contracts optimal? No
 - Entrepreneur reward depends on $k_t q_t$, but q_t is determined by market why not hedge q_t to get a better performance?
 - Shocks to k_t are common across entrepreneurs, why not hedge those and get first best?
 - In practice markets aggregate information to determine $k_t q_t$, but hard to distinguish between shocks to k_t (cash flow news) and q_t (SDF news)
- Optimal contracts get first-best, but miss important phenomena
- Same as in Kiyotaki & Moore, BGG, He & Krishnamurthy

Interlinked balance sheets



Microfoundation of capital structures

- Assumption: value of assets $q_t k_t^i$ is contractable, k_t^i not
- Agency problem of entrepreneur
 - Can take projects w/ NPV<0, private benefit b(m) < 1 per \$ destroyed
 - *m* is amount of monitoring by intermediary
 - Incentive constraint: $\alpha^E \ge b(m)$, binds in equ. $\Rightarrow \alpha^{E(m)}$
- Agency problem of intermediary
 - Save monitoring cost c(m) per \$1 if shirking
 - Incentive constraint: $\alpha^I \ge c(m)$
- Solvency constraint: $n \ge 0$ (implied by IC constraints)
- Assume c(m) + b(m) is a constant for all m entrepreneurs' & intermediaries' net worth are substitutes
 - Special case: if entrepreneurs' net worth =0, then m s.t. b(m) = 0

Merging productive HH & Intermediaries



Conclusion

- Incorporate financial sector in macromodel
 - Higher growth
 - Exhibits instability
 - similar to existing models (BGG, KM) in term of persistence/amplification, but
 - non-linear liquidity spirals (away from steady state) lead to instability
- Risk taking is endogenous
 - "Volatility paradox:" Lower exogenous risk leads to greater leverage and may lead to higher endogenous risk
 - Correlation of assets increases in crisis
 - With idiosyncratic jumps: countercyclical credit spreads
 - Securitization helps share idiosyncratic risk, but leads to more endogenous risk taking and amplifies systemic risk
- Welfare: (Pecuniary) Externalities
 - excessive exposure to crises events

Thank you!