

# Asset Pricing under Asymmetric Information Bubbles & Limits to Arbitrage

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# Overview

- All agents are rational
  - Bubbles under symmetric information
  - Bubbles under asymmetric information
- Interaction between rational arbitrageurs and behavioral traders - Limits to Arbitrage
  - Fundamental risk
  - Noise trader risk + Endogenous short horizons of arbs
  - Synchronization risk

# Historical Bubbles

- 1634-1637 Dutch Tulip Mania (Netherlands)
- 1719-1720 Mississippi Bubble (France)
- 1720 South Sea Bubble (England)
- 1990 Japan Bubble
- 1999 Internet/Technology Bubble

## A Technology Company

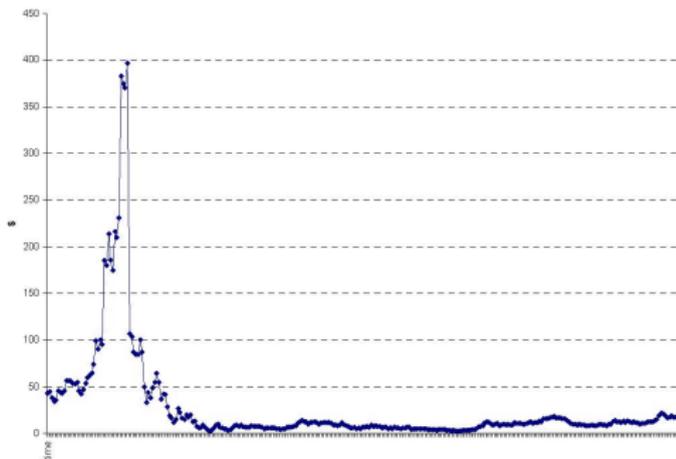
- Company X introduced a revolutionary wireless communication technology.
- It not only provided support for such a technology but also provided the informational content itself.
- It's IPO price was \$1.50 per share. Six years later it was traded at \$ 85.50 and in the seventh year it hit \$ 114.00.
- The P/E ratio got as high as 73.
- The company never paid dividends.

## The Story of RCA in 1920's

Company: Radio Corporate of America (RCA)

Technology: Radio

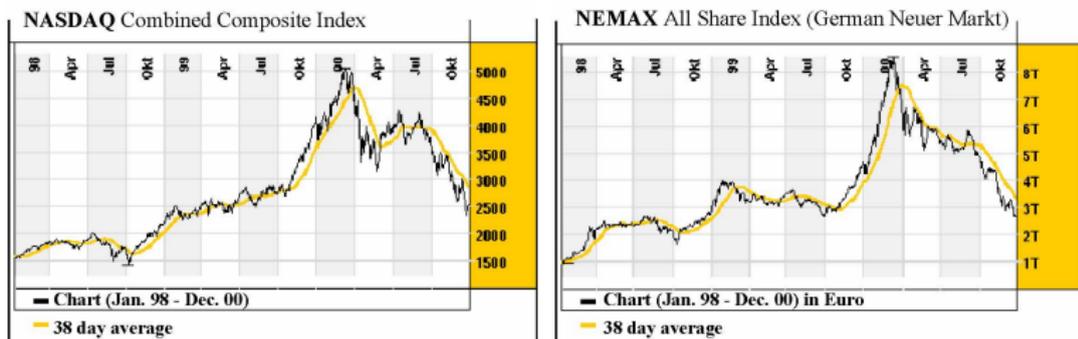
Years: 1920s



**Figure:** RCA's Stock Price from Dec 25 to Dec 50.

- RCA peaked at \$ 397 in Feb. 1929, down to \$ 2.62 in May 1932

## NASDAQ and “Neuer Markt”



Loss of ca. **60 %**  
from high of \$ 5,132

Loss of ca. **85 %**  
from high of Euro 8,583

**Figure:** NASDAQ and Neuer Markt during “Technology Bubble”.

## Bubbles under Symmetric Information

- Keynes' distinction between speculation and long-run investment:
  - Speculation: Buy "overvalued" in the hope to sell it to someone else at an even higher price
  - Investing: Buy and hold strategy
- Fundamental value: Was ist das?  
"highest WTP" if one forces agents to buy & hold the asset

no uncertainty:

uncertainty w/ risk-neutral agent:

uncertainty w/ risk-averse agents:

discounted value of dividends

*expected* discounted value

take expectations *w.r.t.* *EMM*

## Bubbles under Symmetric Information

- Problem of Keynes' buy and hold definition of fundamental value:
  - Retrade does also occur to dynamically complete the market (not only for speculation).
  - With retrade a different allocation can be achieved and hence the EMM is different.
  - Allow for retrade and take EMM which leads to highest fundamental value.

## Bubbles under Symmetric Information

- with stochastic discount factor  $m_t$  (or pricing kernel  $m_t^*$ ) the price of an asset is given by

$$m_t p_t = E_t [m_{t+1} (p_{t+1} + d_{t+1})]$$

where  $m_{t+1}$  is related to MRS (divided by prob. of state)

- Alternatively one can also write pricing equation in terms of the equivalent martingale measure

$$p_t = E_t^{\hat{Q}} \left[ \frac{1}{1 + r_t^f} (p_{t+1} + d_{t+1}) \right]$$

- Securities with Finite Maturity
  - Reiterate pricing equation
  - Backwards induction rules out bubbles

## Bubbles under Symmetric Information

- Securities with Infinite Maturity

- Backwards induction argument fails since there is no well defined final period
- “Lack of market clearing at  $t = \infty$ ”
- Split the price in a fundamental component  $p_t^f$  and a bubble component  $b_t$ .
- By pricing equation, we get the following expectational difference equation

$$b_t = E_t^{\hat{Q}} \left[ \frac{1}{1 + r_t^f} b_{t+1} \right]$$

- Example 1: deterministic bubble  
⇒ has to grow at the risk-free rate
- Example 2 (Blanchard & Watson 1982): (risk-neutral investors)
  - bubble bursts in each period with prob.  $(1 - \pi)$ , persists with prob.  $\pi$
  - ⇒ bubble has to grow by a factor  $\frac{1+r_t^f}{\pi}$  (if it doesn't burst)

## Bubbles under Symmetric Information

- How can we rule out bubbles?
  - Negative bubbles (Blanchard & Watson 1982, Diba & Grossman 1988)
    - For  $b_t < 0$  difference equation implies that  $p_t$  will become negative.
    - Free disposal rules out negative prices.
  - Positive bubbles on assets with positive net supply if  $g < r$  (Brock, Scheinkman, Tirole 85, Santos & Woodford 97)
    - Argument: (*bubbles would outgrow the economy if  $r > g$* )
    - At any point in time  $t + \tau$ , the aggregate wealth of the economy contains bubble component  $b_\tau$ .
    - NPV <sub>$t$</sub>  of aggregate wealth  $W_{t+\tau}$  does not converge to zero as  $\tau \rightarrow \infty$
    - If aggregate consumption <sub>$t+\tau$</sub>  is bounded or grows at a rate  $g < r$ , NPV <sub>$t+\tau$</sub>  ( $C_{t+\tau}$ )  $\rightarrow 0$  as  $\tau \rightarrow \infty$ .
    - Household wealth exceeds PV of  $C$  for all  $t + \tau$  sufficiently far in the future.
    - This is inconsistent with optimization since household would consume part of wealth.

## Bubbles under Symmetric Information

- 5 Counter Examples (Santos and Woodford (1997)):
  - Example 1: fiat money (=bubble) in OLG models
    - allows (better) intergenerational transfers
    - without bubble households want to save more and hence MRS “implicit  $r$ ”  $< g$   
(can lead to overaccumulation of private capital and hence, dynamic inefficiency (see also Abel et al. (1989)))
  - Example 2: ...
  - *Common theme*: Pure existence of a bubble enlarges the trading space.  
leads to different allocation and EMM

## Bubbles under Asymmetric Information

- “Dynamic Knowledge Operator”

$$\mathcal{K}_t^i(E) = \left\{ \omega \in \Omega^{dynamic} : \mathcal{P}_i^t(\omega) \subseteq E \right\}$$

- Expected Bubbles versus Strong Bubbles
  - **expected bubble:**  
 $p_t >$  every agents marginal valuation at a date state  $(t, \omega)$
  - **strong bubble:** (arbitrage?)  
 $p_t >$  all agents know that no possible dividend realization can justify this price.

## Necessary Conditions for Bubbles

- Model setup in Allen, Morris & Postlewaite 1993:  
risky asset pays dividend  $d_T(\omega) : \Omega \mapsto \mathbb{R}_+$  at  $t = T$
- Necessary Conditions for Expected Bubbles
  - ① Initial allocation (interim) Pareto inefficient (Tirole 1982)
    - rational traders is not willing to buy “bubble asset” since some traders have realized their gains leaving a negative sum game for the buyers
  - ② Short-sale constraint strictly binds at some future time in some contingency for all  $i$ 
    - only don't sell to the position limit now, since shorting might be more profitable in the future

## Necessary Conditions for Bubbles

- Additional Necessary Conditions for Strong Bubbles
  - ① asymmetric information is necessary since traders must believe that the other traders do not know this fact.
  - ② Net trades of all traders cannot be CK (since CK of actions negates asymmetric info about events)  
⇒ no bubbles in economies with only two types of traders.
- Morris, Postelwaite & Shin (1995)-Model setup
  - now, all agents are risk-neutral
  - $p_T = d_T$  and  $p_t = \max_i E_t^i [p_{t+1} | \mathcal{P}_t^i]$  for all  $\omega \in \Omega$  and  $t = 1, \dots, T$ .
  - Let's focus on  $\omega$ , where  $d_T = 0$ ,  
 $E_T^{d_T=0} : \{\omega \in \Omega | d_T(\omega) = 0\}$

## Necessary Conditions for Bubbles

- Main Result:** Strong bubble can be ruled out at time  $t$  if
 
$$\mathcal{K}_t^G \mathcal{K}_{t+1}^G \cdots \mathcal{K}_{T-1}^G \left( E_T^{d_T=0} \right) = \{ \omega \in \Omega \mid p_t(\omega) = 0 \}$$
  - (That is, it is mutual knowledge in  $t$  that in period  $t + 1$  it will be mutual knowledge that ... in  $(T - 1)$  it will be mutual knowledge that  $d_T = 0$ .)
  - Sketch argument:
    - if it is mutual knowledge at  $T - 1$  that  $d_T = 0$ , then  $p_{T-1} = 0$ .
    - if it is mutual knowledge at  $T - 2$  that  $p_{T-1} = 0$ , then  $p_{T-2} = 0$ .
    - ...
    - Since knowledge can only improve over time. If it is at  $t$  already  $(T - t)$ -mutual knowledge that  $d_T = 0$ ,  $p_t = 0$ .

## Limits to Arbitrage - Overview

- Efficient Market Hypothesis - 3 levels of justifications
  - All traders are rational, since behavioral will not survive in the long-run (their wealth declines)
  - Behavioral trades cancel each other on average
  - Rational arbitrageurs correct all mispricing induced by behavioral traders.
- Fama/Friedman contra Keynes
  - “If there are many sophisticated traders in the market, they may cause these “bubbles” to burst before they really get under way.” (Fama 1965)
  - “It might have been supposed that competition between expert professionals, possessing judgment and knowledge beyond that of the average private investor, would correct the vagaries of the ignorant individual left to himself.” (Keynes 1936)

# Limits to Arbitrage - Overview

- Reasons for limits to arbitrage
  - Fundamental risk
  - Noise trader risk (DSSW 1990a, Shleifer & Vishny 1997)
  - Synchronization risk (Abreu & Brunnermeier 2002, 2003)
- Special case of market frictions (incl. liquidity)

## Noise Trader Risk

- *Idea:* Arbitrageurs do not fully correct the mispricing caused by noise traders due to
  - arbitrageurs short horizons
  - arbitrageurs risk aversion (face noise trader risk)
- Noise traders survive in the long-run (they are not driven out of the market.)

## Noise Trader Risk - DSSW 1990a

- Model Setup of DSSW 1990a
  - OLG model
    - agents live for 2 periods
    - make portfolio decisions when they are young
  - 2 assets
    - safe asset  $s$       pays fixed real dividend  $r$   
perfect elastic supply  
numeraire, i.e.  $p_s = 1$
    - unsafe asset  $u$       pays fixed real dividend  $r$   
no elastic supply of  $X^{\text{sup}} = 1$   
price at  $t = p_t$
    - Fundamental value of  $s =$  Fundamental value of  $u$   
(perfect substitutes)
  - agents
    - mass of  $(1 - \mu)$  of arbitrageurs
    - mass of  $\mu$  of noise traders, who misperceive next period's price by  $\rho_t \sim \mathcal{N}(\rho_t^*, \sigma_\rho^2)$ , ( $\rho^*$  measures bullishness)
    - CARA utility function  $U(W) = -\exp\{-2\gamma W\}$  with certainty equivalent  $E[W] - \gamma \text{Var}[W]$ .

## Noise Trader Risk - DSSW 1990a

- Individual demand

- arbitrageur's

$$E[W] - \gamma \text{Var}[W] =$$

$$c_0 + x_t^a [r + E_t[p_{t+1}] - p_t(1+r)] - \gamma (x_t^a)^2 \text{Var}_t[p_{t+1}]$$

- noise traders

$$E[W] - \gamma \text{Var}[W] =$$

$$c_0 + x_t^n [r + E_t[p_{t+1}] + \rho_t - p_t(1+r)] - \gamma (x_t^n)^2 \text{Var}_t[p_{t+1}]$$

- Taking FOC

$$\text{arbitrageurs: } x_t^a = \frac{r + E_t[p_{t+1}] - (1+r)p_t}{2\gamma \text{Var}_t[p_{t+1}]}$$

$$\text{noise traders: } x_t^n = \frac{r + E_t[p_{t+1}] - (1+r)p_t}{2\gamma \text{Var}_t[p_{t+1}]} + \frac{\rho_t}{2\gamma \text{Var}_t[p_{t+1}]}$$

- Market Clearing

$$(1 - \mu) x_t^a + \mu x_t^n = 1$$

$$p_t = \frac{1}{1+r} [r + E_t[p_{t+1}] - 2\gamma \text{Var}_t[p_{t+1}] + \mu \rho_t]$$

## Noise Trader Risk - DSSW 1990a

Solve recursively,

$$p_{t+1} = \frac{1}{1+r} [r + E_{t+1} [p_{t+2}] - 2\gamma \text{Var}_{t+1} [p_{t+2}] + \mu \rho_{t+1}]$$

$$E_t [p_{t+1}] = \frac{1}{1+r} [r + E_t [p_{t+2}] - 2\gamma \text{Var}_t [p_{t+2}] + \mu \rho^*]$$

we will see later that  $\text{Var}_t [p_{t+\tau}]$  is a constant for all  $\tau$ .

Solve first order difference equation

$$p_t = 1 + \frac{\mu (\rho_t - \rho^*)}{1+r} + \frac{\mu \rho^*}{r} - \frac{2\gamma}{r} \text{Var}_t [p_{t+1}]$$

Note that  $\rho_t$  is the only random variable. Hence,

$$\text{Var}_t [p_{t+1}] = \text{Var} [p_{t+1}] = \frac{\mu^2 \sigma_\rho^2}{(1+r)^2}$$

$$p_t = 1 + \frac{\mu (\rho_t - \rho^*)}{1+r} + \frac{\mu \rho^*}{r} - \frac{(2\gamma) \mu^2 \sigma_\rho^2}{r(1+r)}$$

## Noise Trader Risk - DSSW 1990a

$$p_t = 1 + \frac{\mu(\rho_t - \rho^*)}{1+r} + \frac{\mu\rho^*}{r} - \frac{(2\gamma)\mu^2\sigma_\rho^2}{r(1+r)}$$

where

- $1$  = fundamental value
- $\frac{\mu(\rho_t - \rho^*)}{1+r}$  = deviation due to current misperception of noise traders
- $\frac{\mu\rho^*}{r}$  = average misperception of noise traders
- $-\frac{(2\gamma)\mu^2\sigma_\rho^2}{r(1+r)}$  = arbitrageurs' risk-premium
- Homework:
  - ① Check limiting cases
    - ①  $\gamma \rightarrow 0$
    - ②  $\sigma_\rho^2 \rightarrow 0$
  - ② Check whether there is also a fundamental equilibrium, where  $p_t = 1$  for all  $t$  (no risk  $\Rightarrow$  arbitrageurs buy everything)

## Do Noise Traders Die Out over Time (Evolutionary Argument)

- Relative Expected Returns

- Difference in returns

- $\Delta R_{n-a} = (x_t^n - x_t^a) [r + p_{t+1} - p_t (1 + r)]$

- Aside 1:  $(x_t^n - x_t^a) = \frac{\rho_t}{(2\gamma) \text{Var}_t[p_{t+1}]} = \frac{(1+r)^2 \rho_t}{(2\gamma) \mu^2 \sigma_\rho^2}$

(Note for  $\mu \rightarrow 0$ ,  $(x_t^n - x_t^a) \rightarrow \infty$ )

- Aside 2: By market clearing  $E[r + p_{t+1} - p_t (1 + r)] =$

$$(2\gamma) \text{Var}_t[p_{t+1}] - \mu \rho_t = \frac{(2\gamma) \mu^2 \sigma_\rho^2}{(1+r)^2} - \mu \rho_t$$

$$\Rightarrow E_t[\Delta R_{n-a}] = \rho_t - \frac{(1+r)^2 (\rho_t)^2}{(2\gamma) \mu^2 \sigma_\rho^2}$$

- Taking unconditional expectations

$$E[\Delta R_{n-a}] = \rho^* - \frac{(1+r)^2 \rho^* + (1+r)^2 \sigma_\rho^2}{(2\gamma) \mu^2 \sigma_\rho^2} > 0 \text{ only if } \rho^* > 0$$

## Do Noise Traders Survive over Time (Evolutionary Argument)

- Taking unconditional expectations

$$E[\Delta R_{n-a}] = \rho^* - \frac{(1+r)^2 \rho^* + (1+r)^2 \sigma_\rho^2}{(2\gamma) \mu^2 \sigma_\rho^2} > 0 \text{ only if } \rho^* > 0$$

- “Overoptimistic/bullish” traders hold riskier positions and have higher expected returns.
- In evolutionary process, they will have more off-springs and hence they won't die out.

## Myopia due to Liquidation Risk

- Why are professional arbitrageurs myopic?
- Model setup of Shleifer & Vishny (1997) [slightly modified]
  - Two assets
    - risk free bond and
    - risky stock with final value  $v$
  - Two types of fund managers:
    - Good fund managers know fundamental value  $v$
    - Bad fund managers have no additional information (just gamble with “other people’s money”).
  - Two trading rounds  $t = 1$  and  $2$  (in  $t = 3$   $v$  is paid out)
  - Individual investors
    - entrust their money  $F_1$  to a fund manager without knowing the fund managers’ skill level - “separation of brain and money”
    - can withdraw their funds in  $t = 2$
  - Noise traders submit random demand

## Myopia due to Liquidation Risk

- Price setting:
  - $P_3 = v$
  - $P_2$  is determined by aggregate demand of fund managers and liquidity/noise traders
- Focus on case where
  - ①  $P_1 < v$  (asset is undervalued)
  - ②  $P_2 < P_1$  asset price goes even further down in  $t_2$  due to
    - sell order by noise traders
    - sell order by other informed traders
- Performanced-based fund flows (see Chevalier & Ellison 1997)
  - If price drops, the probability increases that the fund manager is “bad”.
  - Individual investors withdraw their money at  $t = 2$ .
  - Shleifer & Vishny assume  $F_2 = F_1 - aD_1 \left(1 - \frac{P_2}{P_1}\right)$ , where  $D_1$  is the amount the fund manager invested in the stock.

## Myopia due to Liquidation Risk

- “Good” manager’s problem who has invested in risky asset
  - He has to liquidate his position at  $P_2 < P_1$  (exactly when mispricing is the largest!)  
That is, he makes losses, even though the asset was initially undervalued.
  - Due to this “early liquidation risk”, at  $t = 1$  a rational fund manager is reluctant to fully exploit arbitrage opportunities at  $t = 1$ .
  - Focus on short-run price movements  $\Rightarrow$  myopia of professional arbitrageurs

## Synchronization Risk

- “Bubbles and Crashes” (Abreu & Brunnermeier 2003)  
(for bubbles)
- “Synchronization Risk and Delayed Arbitrage” (Abreu & Brunnermeier 2002)  
(for any form of mispricing)
- see power point slides (file “08 slides Eco525.ppt”)