

Asset Pricing under Asymmetric Information

Share Auctions

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A Classification of Market Microstructure Models

- simultaneous submission of demand schedules
 - competitive rational expectation models
 - **strategic share auctions**
- sequential move models
 - screening models in which the market maker submits a supply schedule first
 - static
 - ◇ uniform price setting
 - ◇ limit order book analysis
 - dynamic sequential trade models with multiple trading rounds
 - strategic market order models where the market maker sets prices ex-post

Auctions - Overview

- Unit demand versus divisible good (share) auctions
- Signal structure:
 - common value
 - private value (liquidity, non-common priors)
 - affiliated values
- Auction Formats:
 - Open-outcry auctions: English auctions (ascending-bid, progressive), Dutch auctions (descending-bid)
 - Sealed-bid auctions: First-price auction, second-price auction
 - Share auctions: uniform-price (Dutch) auction, discriminatory price auction

Results in (Unit Demand) Auction Theory

- A Refresher -

- 1 “Strategic equivalence” between Dutch auction and first price sealed-bid auction
(English auction is more informative than second-price auction.)
- 2 Second-price auction: Bidding your own private value is a (weakly) dominant strategy (Groves Mechanism)
- 3 Revenue Equivalence Theorem (RET)

2nd-Price Auction: Private Value

- Model Setup
 - Private value: v^i
 - Highest others' bid: $B_{\max}^{-i} = \max_{j \neq i} \{b^1, \dots, b^j, \dots, b^I\}$
- *Claim:* Bidding own value v^i is (weakly) dominant strategy
- Proof (note similarity to Groves Mechanism):
 - Overbid, i.e. $b^i > v^i$:
 - If $B_{\max}^{-i} \geq b^i$, he wouldn't have won anyway.
 - If $B_{\max}^{-i} \leq v^i$, he wins the object whether he bids b^i or v^i .
 - If $v^i < B_{\max}^{-i} < b^i$, he wins and gets negative utility instead of 0 utility.
 - Underbid, i.e. $b^i < v^i$:
 - If $b^i < B_{\max}^{-i} < v^i$, he loses instead of $u(v^i - B_{\max}^{-i}) > 0$.

Revenue Equivalence Theorem

- *Claim:* Any auction mechanism with risk-neutral bidders leads to the same *expected* revenue if
 - ① mechanism also assigns the good to the bidder with the highest signal
 - ② bidder with the lowest feasible signal receives zero surplus
 - ③ $v \in [\underline{V}, \overline{V}]$ from common, strictly increasing, atomless distribution
 - ④ private value OR pure common value with independent signals S^i with $v = f(S^1, \dots, S^I)$.
- Proof (Sketch):
Taken from book p. 185

Proof of RET

- Suppose the expected payoff $U^i(v^i)$ if $S^i = v^i$.
- If v^i -bidder mimics a $(v^i + \Delta v)$ -bidder,
 - payoff = payoff of a $(v^i + \Delta v)$ -bidder with the difference, that he values it Δv less than $(v^i + \Delta v)$ -bidder, if he wins
 - prob of winning: $P(v^i + \Delta)$ if he mimics the $(v^i + \Delta v)$ -bidder.
 - in any mechanism bidder should have no incentive to mimic somebody else, i.e.

$$U(v^i) \geq U(v^i + \Delta v) - \Delta v \Pr(v^i + \Delta v)$$

- $(v^i + \Delta v)$ -bidder should not want to mimic v^i -bidder, i.e.

$$U(v^i + \Delta v) \geq U(v^i) + \Delta v \Pr(v^i)$$

- Combining both inequalities leads to

$$\Pr(v^i) \leq \frac{U^i(v^i + \Delta v) - U^i(v^i)}{\Delta v} \leq \Pr(v^i + \Delta v)$$

Affiliated Values - Milgrom & Weber (1982)

- Affiliated Values - MLRP
- Model Setup

Bidder i 's signal: S^i

Highest of other bidders' signals: $S_{\max}^{-i} := \max_{j \neq i} \{S^j\}$

Define two-variable function:

$$V^i(x, y) = E[v^i | S^i = x, S_{\max}^{-i} = y]$$

- Optimal bidding strategy:
 - Second-price auction

$$b^i(x) = V^i(x, x)$$

- First-price auction: Solution to ODE

$$\frac{\partial b^i(x)}{\partial x} = [V^i(x, x) - b^i(x)] \frac{f_{S_{\max}^{-i}}(x|x)}{F_{S_{\max}^{-i}}(x|x)}$$

where f and F are the pdf and cdf of the conditional distribution of S_{\max}^{-i} , respectively.

Affiliated Values - Milgrom & Weber (1982)

- Revenue ranking with risk-neutral bidders:
 - English auction $>$ second-price auction $>$ first price auction
 - (Latter ranking might change with risk aversion Maskin & Riley 1984, Matthew 1983)
- END OF REFRESHER!

Share Auctions - Overview

- 1 Value v is commonly known
illustrate multiplicity problem, role of random supply
- 2 Random value v , but symmetric information
 - a) general demand function (no individual stock endowments)
 - b) linear equilibria (with individual endowments)
- 3 Random value v and asymmetric information
(CARA Gaussian setup)

Commonly Known Value v

— Illustration of Multiplicity Problem —

- Wilson (1979)
- Model Setup
 - I bidders/traders submit demand schedules
 - everybody knows value \bar{v}
 - non-random supply $X^{\text{sup}} = 1$ (normalization)
- *Benchmark*: unit demand auction $p^* = \bar{v}$
- *Share auctions*: Each bidder is a monopsonist who faces the *residual* supply curve.
- *Claim*: $p^* = \frac{\bar{v}}{2}$ is also an equilibrium if agents submit demand schedules $x(p) = \frac{1-2p/(I\bar{v})}{I-1}$.

Commonly Known Value v — Illustration of Multiplicity Problem —

Proof:

- Market clearing: $Ix(p^*) = 1 \Rightarrow p^* = \frac{\bar{v}}{2}$.
- Trader i 's residual supply curve:
 $X^{\text{sup}} - [(I - 1)x(p)] = 1 - [1 - 2p/(I\bar{v})] = \frac{2p}{I\bar{v}}$.
- Residual demand = residual supply: $x^i(p^*) = \frac{2p^*}{I\bar{v}}$.
- Trader i 's profit is $(\bar{v} - p)x^i(p) = (\bar{v} - p)\frac{2p}{I\bar{v}}$.
- By choosing $x^i(p)$, trader i effectively chooses the price p .
- Take FOC of $(\bar{v} - p)\frac{2p}{I\bar{v}}$ w.r.t. p : $(\bar{v})\frac{2}{I\bar{v}} - \frac{4p}{I\bar{v}} = 0$
- $\Rightarrow p^* = \frac{\bar{v}}{2}$ and $x^i = \frac{2(\bar{v}/2)}{I\bar{v}} = 1/I$.

Commonly Known Value v — Illustration of Multiplicity Problem —

- *Generalizations:* Any price $p^* \in [0, \bar{v})$ can be sustained in equilibrium if bidders simultaneously submit the following demand schedules:

$$x^i(p) = \frac{1}{I} [1 + \beta_p (p^* - p)], \text{ where } \beta_p = \frac{1}{(I-1)(\bar{v} - p^*)}$$

- Proof: Homework!

Commonly Known Value v

- Graphical Illustration for $I = 2$
- Each bidder is indifferent between any demand schedule as long as it goes through the optimal point.
- \Rightarrow multiple equilibria
- *Way out:* Introduce random supply $X^{\text{sup}} = u$

Value v is Random - No Private Info

- Model setup:
 - Value v is random - no private info
 - all traders have same utility function $U(\cdot)$
 - X^{sup} :
 - 1 deterministic/non-random X^{sup}
 \Rightarrow apply previous section and use certainty equivalence (Wilson)
 - 2 random supply $X^{\text{sup}} = u$

Value v is Random - No Private Info

- *Necessary Condition:* Any I bidder, symmetric strategy Nash equilibrium in continuously differentiable (downward sloping) demand functions with random supply $X^{\text{sup}} = u$ is characterized by

$$0 = E_v \left[U'((v - p)x(p)) \left[v - p + \frac{x(p)}{(I - 1) \partial x(p) / \partial p} \right] \right],$$

provided a equal tie breaking rule applies.

Proof:

- Since $x^*(p)$ is invertible, all bidders can infer the random supply u from the equilibrium price p . In other words, each equilibrium price p' corresponds to a certain realization of the random supply u' . Bidders trade conditional to the equilibrium price by submitting demand schedules. Thus they implicitly condition their bid on the random supply u .

Value v is Random - No Private Info

- Every bidder i prefers his equilibrium strategy $x^{i,*}(p)$ to any other demand schedule $x^i(p) = x^{i,*}(p) + h^i(p)$. Let us focus on pointwise deviations at a single price p' , that is, for a certain realization u' of u . For a given aggregate supply u' , bidder i 's utility, is $E_v[U((v - p(x^i))x^i)]$.
- Deviating from $x_{p'}^{i,*}$ alters the equilibrium price p' . The marginal change in price for a given u' is given by totally differentiating the market clearing condition $x_{p'}^i + \sum_{-i \in \mathbb{I} \setminus i} x^{-i,*}(p) = u'$. That is, it is given by

$$\frac{dp}{dx^i} = - \frac{1}{\sum_{-i \in \mathbb{I} \setminus i} \partial x^{-i,*} / \partial p}$$

Value v is Random - No Private Info

- The optimal quantity $x_{p'}^{i,*}$ for trader i satisfies the first-order condition

$$E_v \left[U'(\cdot) \left(v - p + x_{p'}^{i,*} \frac{1}{\sum_{-i \in \mathbb{I} \setminus i} \partial x^{-i,*} / \partial p} \right) \right] = 0$$

for a given u' . This first-order condition has to hold for any realization u' of u , that is for any possible equilibrium price p' . For distributions of u that are continuous without bound, this differential equation has to be satisfied for all $p \in \mathbb{R}$. Therefore, the necessary condition is

$$E_v \left[U'(\cdot) \left(v - p + \frac{x^{i,*}(p)}{\sum_{-i \in \mathbb{I} \setminus i} \partial x^{-i,*} / \partial p} \right) \right] = 0.$$

- For a specific utility function $U(\cdot)$, explicit demand functions can be derived from this necessary condition.

Value v is random - No private info II

Special Cases I: Risk Neutrality

- For *risk neutral* bidders $U'(\cdot)$ is a constant.

$$p = E[v] + \underbrace{\left[\sum_{-i \in \mathbb{I} \setminus i} \frac{\partial x^{-i,*}}{\partial p} \right]^{-1}}_{\text{bid shading}} x^{i,*}(p).$$

- Imposing symmetry, $x(p) = (E[v] - p)^{\frac{1}{l-1}} k_0$, where $k_0 = p(0)$.

- inverse of it is $p(x) = E[v] - \underbrace{(1/k_0)^{(l-1)} (x)^{(l-1)}}_{\text{bid shading}}$.

- Note that equilibrium demand schedules are only linear for the two-bidder case.

Value v is random - No private info II Special Cases II: CARA utility

- $U(W) = -e^{-\rho W}$
- FOC:
$$\frac{\int e^{-\rho x^{i,*} v} v f(v) dv}{\int e^{-\rho x^{i,*} v} f(v) dv} - p + \underbrace{\left[\sum_{-i \in \mathbb{I} \setminus i} \frac{\partial x^{-i,*}}{\partial p} \right]^{-1}}_{\text{bid shading}} x^{i,*} = 0,$$
- where $f(v)$ is the density function of v .
- Homework: Check above FOC!
- Note: The integral is the derivative of the log of the moment generating function, $(\ln \Phi)'(-\rho x(p))$.

Value v is random - No private info II

Special Cases III: CARA-Gaussian setting

- in addition: $v \sim \mathcal{N}(\mu, \sigma_v^2)$
- Integral term simplifies to $E[v] - \rho x(p) \text{Var}[v]$
- $$p = \underbrace{E[v] - \rho \text{Var}[v]}_{\text{value of marginal unit}} x^{i,*}(p) + \underbrace{\frac{1}{\sum_{-i \in \mathbb{I} \setminus i} \frac{\partial x^{-i,*}}{\partial p}}}_{\text{bid shading}} x^{i,*}(p).$$
- Impose symmetry, $p(x) = E[v] - \rho \text{Var}[v] \frac{l-1}{l-2} x - k_1(x)^{l-1}$.
- Inverse for $k_1 = 0$, $x^i(p) = \frac{l-2}{l-1} \frac{E[v]-p}{\rho \text{Var}[v]}$
- This also illustrates that demand functions are only linear for $l \geq 3$ and for the constant $k_1 = 0$.

Double Auction View

- Model Setup
 - CARA-Gaussian setup
 - *Individual* endowment for each trader z^i
 - Aggregate random supply u . Total supply is $u + \sum_i z^i$. (only u is random)
 - Each trader's allocation is then $x^i = z^i + \Delta x^i(p^*)$
 - still symmetric information
- Focus on linear demand schedules:
- **Step 1:** Conjecture linear demand schedules $\Delta x^i = a^i - b^i p$ for all i (strategy profile)

Double Auction View

Residual supply is $u - \sum_{j \neq i} (a^j - b^j p) = \Delta x^i$

$$\Leftrightarrow p = \underbrace{\left(\sum_{j \neq i} a^j - u \right)}_{:= \tilde{p}_0} / \underbrace{\left(\sum_{j \neq i} b^j \right)}_{:= 1/\lambda^i} + 1/ \underbrace{\sum_{j \neq i} b^j}_{:= 1/\lambda^i} \Delta x^i$$

- **Step 2:** By conditioning on p , trader i can choose his demand for each realization of u (or \tilde{p}_0).
- **Step 3:** Best response
Trader i 's objective

$$(E[v] - \tilde{p}_0 - 1/\lambda^i \Delta x^i) \Delta x^i + E[v] z^i - \frac{1}{2} \rho^i \text{Var}[v] (z^i + \Delta x^i)^2$$

Double Auction View

Take FOC w.r.t. Δx^i

$$E[v] - \tilde{p}_0 - \frac{2}{\lambda^i} \Delta x^i - \rho^i \text{Var}[v] (z^i + \Delta x^i) = 0$$

SOC:

$$-\frac{2}{\lambda^i} - \rho^i \text{Var}[v] < 0 \iff \lambda^i \notin \left[-\frac{2}{\rho^i \text{Var}[v]}, 0 \right]$$

Best response is

$$\Delta x^i(\tilde{p}_0) = \frac{E[v] - \tilde{p}_0 - \rho^i \text{Var}[v] z^i}{2/\lambda^i + \rho^i \text{Var}[v]}$$

Double Auction View

In terms of price

$$\begin{aligned}\Delta x^i(p) &= \frac{\lambda^i \{ \eta^i \tau_v (E[v] - p) - z^i \}}{\eta^i \tau_v + \lambda^i} \\ &= \underbrace{\frac{\lambda^i \{ \eta^i \tau_v E[v] - z^i \}}{\eta^i \tau_v + \lambda^i}}_{:=a^i} - \underbrace{\frac{\lambda^i \eta^i \tau_v}{\eta^i \tau_v + \lambda^i}}_{:=b^i} p\end{aligned}$$

- **Step 4: Impose Rationality**

In symmetric equilibrium $b = b^i$, $\lambda = \lambda^i \forall i$. Hence,

$\sum_{j \neq i} b^j = \lambda^i$ becomes $(I - 1) b = \lambda$.

- Replacing λ

$$b = \frac{(I - 1) b \eta \tau_v}{\eta \tau_v + (I - 1) b} = \frac{I - 2}{I - 1} \eta \tau_v \Rightarrow \lambda = (I - 2) \eta \tau_v$$

Double Auction View

- Note that only for $I \geq 3$ a symmetric equilibrium exists.
and

$$a^i = \frac{I-2}{I-1} \eta \tau_v E[v] - \frac{I-2}{I-1} z^i$$

- Put everything together

$$x^i(p) = z^i + \Delta x^i = \frac{I-2}{I-1} \frac{E[v] - p}{\rho \text{Var}[v]} + \frac{1}{I-1} z^i$$

Difference of Strategic Outcome to Competitive REE

① Trading

- traders are less aggressive
- endowments matter for holdings
 - Why? Price “moves against you”

② Excess “equilibrium” payoff

$$E[Q] = \rho \text{Var}[v] \left[\frac{1}{I} \sum_i z^i + \frac{I-1}{I-2} \frac{u}{I} \right]$$

- For $u = 0$, same as in competitive case. (Check homework)
- For $u > 0$, abnormally high - cost for liquidity (noise)
traders (sell when price is low)
- For $u < 0$, abnormally low - cost for liquidity (noise)
traders (buy when price is high)

③ As $I \rightarrow \infty$, convergence to competitive REE with sym. info

Value v is Random & Private Info Kyle (1989)

- Kyle (1989)
(similar to Hellwig 1980 setting, all traders receive signal $S^i = v + \varepsilon^i$)
- Simpler Model Setup (here):
 - CARA Gaussian setup
 - Signal structure (line in Grossman-Stiglitz 1980)
 - M uninformed traders
 - N informed traders, who observe same signal S
- Focus on linear demand functions only

Value v is Random & Private Info Kyle (1989)

- Step 1:** Conjecture symmetric, linear demand schedules
 for uninformed: $\Delta x^{un} = a^{un} - b^{un} p$
 for informed: $\Delta x^{in} = a^{in} - b^{in} p + c^{in} \Delta S$

Define

price impace (slope)

$$\lambda = Nb^{in} + Mb^{un}$$

'residual slope for informed'

$$\lambda^{in} = (N - 1) b^{in} + Mb^{un}$$

'residual slope for uninformed'

$$\lambda^{un} = Nb^{in} + (M - 1) b^{un}$$

intercept

$$A = Na^{in} + Ma^{un}$$

Equilibrium price is

$$p = \frac{1}{\lambda} (A - u + Nc^{in} \Delta S)$$

► Informed traders

- Step 2:** no info inference
- Step 3:** Best response same as before, just replace mean and variance, by conditional mean and variance

Value v is Random & Private Info Kyle (1989)

Best response (as a function of price) is

$$\begin{aligned}
 \Delta x^{in}(p) &= \frac{\lambda^{in} \{ \eta^{in} \tau_{v|S} (E[v|S] - p) - z^{in} \}}{\eta^{in} \tau_{v|S} + \lambda^{in}} \\
 &= \frac{\lambda^{in} \left\{ \eta^{in} \tau_{v|S} \left(E[v] + \frac{\tau_{\varepsilon}}{\tau_{v|S}} \Delta S - p \right) - z^{in} \right\}}{\eta^{in} \tau_{v|S} + \lambda^{in}} \\
 &= \underbrace{\frac{\lambda^{in} \{ \eta^{in} \tau_{v|S} E[v] - z^{in} \}}{\eta^{in} \tau_{v|S} + \lambda^{in}}}_{:=a^{in}} - \underbrace{\frac{\lambda^{in} \eta^{in} \tau_{v|S}}{\eta^{in} \tau_{v|S} + \lambda^{in}} p}_{:=b^{in}} \\
 &\quad + \underbrace{\frac{\lambda^{in} \eta^{in} \tau_{\varepsilon}}{\eta^{in} \tau_{v|S} + \lambda^{in}} \Delta S}_{:=c^{in}}
 \end{aligned}$$

SOC $\lambda^{in} \notin [-2\eta\tau_{v|S}, 0] \Rightarrow b^{in} > 0.$

Value v is Random & Private Info Kyle (1989)

- **Step 4:** Impose Rationality
(For $M = I$ is sym. info case.)

Rewrite $b^{in} = \frac{\lambda^{in}\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S} + \lambda^{in}}$ as $b^{in} \frac{\eta^{in}\tau_{v|S} + \lambda^{in}}{\eta^{in}\tau_{v|S}} = \lambda^{in}$ and notice

$$\lambda = \lambda^{in} + b^{in}$$

$$\lambda = b^{in} \frac{\eta^{in}\tau_{v|S} + \lambda^{in}}{\eta^{in}\tau_{v|S}} + b^{in} \text{ and}$$

$$\text{def for } \lambda^{un} : Mb^{un} = b^{in} \frac{\eta^{in}\tau_{v|S} + \lambda^{in}}{\eta^{in}\tau_{v|S}} - (N-1)b^{in}$$

Value v is Random & Private Info Kyle (1989)

► **Uninformed traders:**

- **Step 2:** Information Inference from

$$p = \frac{1}{\lambda} (A - u + Nc^{in}\Delta S)$$

$$E[v|p] = E[v] + \frac{\lambda}{Nc^{in}} \left(\frac{\phi\tau_{\epsilon}}{\tau_{v|p}} \right) \left[p - \frac{A}{\lambda} \right] \text{ and } \tau_{v|p} = \tau_v +$$

$$\begin{aligned} \text{where } \phi &= \frac{N^2 (c^{in})^2 \tau_u}{N^2 (c^{in})^2 \tau_u + \tau_{\epsilon}} \\ &= \frac{N^2 (b^{in})^2 \tau_u \tau_{\epsilon}}{N^2 (b^{in})^2 \tau_u \tau_{\epsilon} + (\tau_{v|S})^2} \text{ since } c^{in} = \frac{\tau_{\epsilon}}{\tau_{v|S}} b^{in} \end{aligned}$$

Value v is Random & Private Info Kyle (1989)

- **Step 3:** Best response $\Delta x^{un}(p) =$

$$= \frac{\lambda^{un} \left\{ \eta^{un} \tau_{v|p} (E[v|p] - p) - z^{un} \right\}}{\eta^{un} \tau_{v|p} + \lambda^{un}}$$

$$= \frac{\lambda^{un} \left\{ \eta^{un} \tau_{v|p} \left(E[v] - \frac{1}{Nc^{in}} \frac{\phi \tau_{\varepsilon}}{\tau_{v|p}} (A - \lambda p) - p \right) - z^{un} \right\}}{\eta^{un} \tau_{v|p} + \lambda^{un}}$$

$$= \frac{\lambda^{un} \left\{ \eta^{un} \tau_{v|p} \left(E[v] - \frac{1}{Nc^{in}} \frac{\phi \tau_{\varepsilon}}{\tau_{v|p}} A \right) - z^{un} \right\}}{\eta^{un} \tau_{v|p} + \lambda^{un}}$$

$:= a^{un}$

$$- \frac{\lambda^{un} \eta^{un} \tau_{v|p} \left(1 - \frac{\lambda}{Nc^{in}} \frac{\phi \tau_{\varepsilon}}{\tau_{v|p}} \right)}{\eta^{un} \tau_{v|p} + \lambda^{un}} p$$

$:= b^{un}$

Simplification I: Information Monoplist

- Since $Nc^{in} = \frac{\tau_\varepsilon}{\tau_v|S} b^{in}$,

$$b^{un} (\eta^{un} \tau_v|p + \lambda^{un}) = \lambda^{un} \eta^{un} (\tau_v|p - \frac{\lambda}{b^{in}} \phi \tau_v|S).$$

- Using $\lambda = b^{in} \frac{\eta^{in} \tau_v|S + \lambda^{in}}{\eta^{in} \tau_v|S} + b^{in}$,

RHS becomes $\lambda^{un} \eta^{un} \left(\tau_v|p - \left(\frac{\eta^{in} \tau_v|S + \lambda^{in}}{\eta^{in} \tau_v|S} + 1 \right) \phi \tau_v|S \right)$ or

$$\lambda^{un} \eta^{un} \left[(1 - \phi) \tau_v - \phi \eta^{in} \frac{(\tau_v|S)^2}{\eta^{in} \tau_v|S - b^{in}} \right].$$

Since we can write $\phi = \frac{N^2 (b^{in})^2 \tau_u \tau_\varepsilon}{N^2 (b^{in})^2 \tau_u \tau_\varepsilon + (\tau_v|S)^2}$, RHS is

$$\lambda^{un} \eta^{un} \left[\frac{\tau_v - \eta^{in} \frac{(b^{in})^2 \tau_u \tau_\varepsilon}{\eta^{in} \tau_v|S - b^{in}}}{N^2 (b^{in})^2 \tau_u \tau_\varepsilon + (\tau_v|S)^2} \right] (\tau_v|S)^2 =: \zeta (b^{in}).$$

Simplification I: Information Monoplist

- Using $Mb^{un} = b^{in} \frac{\eta^{in} \tau_{v|S} + \lambda^{in}}{\eta^{in} \tau_{v|S}} - (N - 1) b^{in}$, one can eliminate b^{un} and λ^{un} .

...

- Finally,

$$\begin{aligned} & \frac{1}{M} \frac{\eta^{in} \tau_{v|S}}{\eta^{in} \tau_{v|S} - b^{in}} \left[\eta^{un} \tau_{v|p} + \left(\frac{M-1}{M} \frac{\eta^{in} \tau_{v|S}}{\eta^{in} \tau_{v|S} - b^{in}} + 1 \right) \right] \\ = & \eta^{un} \left[\frac{M-1}{M} \frac{\eta^{in} \tau_{v|S}}{\eta^{in} \tau_{v|S} - b^{in}} + 1 \right] \zeta(b^{in}) \end{aligned}$$

Simplification II: Information Monopoliſt & Competitive Outsiders

- Taking the right limit:
 - As $M \rightarrow \infty$, $M\eta^{un} \rightarrow \bar{\eta}^{un}$, i.e. each individual uninformed trader becomes infinitely risk averse.
 - Why not just $M \rightarrow \infty$? uninformed trader would dominate and informed traders demand becomes relatively tiny.
- Above equation simplifies to (multiply by M and notice that $\eta^{un} \rightarrow 0$)

$$\begin{aligned} & \frac{\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S} - b^{in}} \left[\frac{\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S} - b^{in}} + 1 \right] b^{in} = \\ & = M\eta^{un} \left[\frac{\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S} - b^{in}} + 1 \right] \zeta(b^{in}) \end{aligned}$$

Simplification I: Information Monoplist

$$\frac{\eta^{in} \tau_{v|S}}{\eta^{in} \tau_{v|S} - b^{in}} b^{in} = \bar{\eta}^{un} \zeta (b^{in})$$

- Sub in $\zeta (b^{in})$ [check at home!]

$$\begin{aligned} \eta^{in} b^{in} \tau_{v|S} + \eta^{in} (b^{in})^2 \tau_u \tau_\varepsilon [b^{in} + \bar{\eta}^{un} \tau_{v|S}] &= \\ &= \bar{\eta}^{un} \tau_{v|S} \tau_v [\eta^{in} \tau_{v|S} - b^{in}] \end{aligned}$$

- Plot both sides and one can see that the unique real root to this cubic equation is in the acceptable (recall SOC) interval $(0, \eta^{in} \tau_{v|S})$.
- Let $\vartheta \in (0, 1)$, and $b^{in*} = \vartheta \eta^{in} \tau_{v|S}$.
- Using $Mb^{un} = b^{in} \frac{\eta^{in} \tau_{v|S} + \lambda^{in}}{\eta^{in} \tau_{v|S}} \rightarrow \frac{\vartheta}{1-\vartheta} \eta^{in} \tau_{v|S}$.

Remarks I

- 3 effects for informed monopolist
 - For given λ^{in} , price moves against informed trader \Rightarrow lower b^{in} .
 - informational effect
 - For given $\tau_{v|p}$: Uninformed trader make inferences from prices \Rightarrow their demand will react less strongly to increases in p . \Rightarrow residual supply curve is steeper \Rightarrow lower b^{in}
 - Increase in $b^{in} \Rightarrow \tau_{v|p}$ increases \Rightarrow makes uninformed more aggressive \Rightarrow lowers $\lambda^{in} \Rightarrow$ higher b^{in} .
- Comparative statics

...

Comparative Statics

① $Var [u] \nearrow \infty, \phi \searrow 0$ (price carries no info), $\bar{b}^{un} \rightarrow \bar{\eta}^{un} \tau_v,$
 $b^{in} \rightarrow \frac{\bar{\eta}^{un} \tau_v}{\bar{\eta}^{un} \tau_v + \eta^{in} \tau_v | S} \eta^{in} \tau_v | S$

same as in monopoly solution with competitive
 “Walrasian” outsiders (homework: check this!)

② As $Var [u] \searrow 0, (\tau_u \nearrow \infty), b^{in} \rightarrow 0$, from cubic equation.

- Actually, $(b^{in})^2 \tau_u \rightarrow \frac{\tau_v | S \tau_v}{\tau_\varepsilon}$. Hence, $\phi \rightarrow \frac{\tau_v}{2\tau_v + \tau_\varepsilon} < \frac{1}{2}$.
- Furthermore, $\bar{b}^{un} \rightarrow 0, \lambda^{in} \rightarrow 0, a^{in} \rightarrow 0, a^{un} \rightarrow 0$.
- Hence, NO TRADE EQUILIBRIUM, that is $\Delta x^i(p) \rightarrow 0$, even though initial endowments $\{z^i\}_{i \in I}$ are not well diversified.
- One needs noise to lubricate financial markets.
- NOTE: this result hinges on the unbounded support of normal distribution.

Does Asymmetric Information Without Noise Trading Lead to Market Break Down?

- 1 Limit $Var [u] = 0$ in above simplified Kyle (1989) setting
 \Rightarrow non-existence of an equilibrium
- 2 Bhattacharya & Spiegel (1991) setup:
as before, but (i) z^{in} is random and (ii) $Var [u] = 0$
 \Rightarrow non-existence of an equilibrium
[due to unbounded support of $X^{SUP} S$ (Noeldeke 1993,
Hellwig1993)]
- 3 Finite number of signals and CARA (Noeldeke 1992)
 \Rightarrow if initial allocation is inefficient a fully revealing trade
equilibrium always exists.
(with bounded support, allows construction starting from worst
possible type)
- 4 Finite number of signals and HARA & NIARA
(nonincreasing)
 \Rightarrow market may break down (in very specific circumstances)

Price Discrimination vs. Uniform Pricing

- total payment:
 - uniform prices: total payment is $p x^i(p)$.
 - discriminatory: total payment is $\int_0^{x^i(p)} p(q) dq$ (area below demand schedule $p(q)$).
- Discriminatory pricing eliminates equilibria with $p < \bar{v}$ (commonly known)
- Demand curves in mean variance setting (Viswanathan & Wang)

- uniform pricing:

$$p = E[v] - \rho \text{Var}[v] x^{i,*}(p) + \frac{1}{\sum_{-i \in \mathbb{I} \setminus i} \frac{\partial x^{-i,*}}{\partial p}} x^{i,*}(p).$$

- discriminatory pricing: (intercept & slope change)

$$p = E[v] - \rho \text{Var}[v] x^{i,*}(p) + \frac{1}{\sum_{-i \in \mathbb{I} \setminus i} \frac{\partial x^{-i,*}}{\partial p}} \frac{1}{H(u)}.$$

where $H(u) = \frac{g(u)}{1-G(u)}$ (hazard rate of random u .)