Asset Pricing under Asymmetric Information Modeling Information & Solution Concepts

Markus K. Brunnermeier

Princeton University

August 22, 2007

References

Books:

Brunnermeier (2001), "Asset Pricing under Asym. Info."
Vives (2006), "Information and Learning in Markets"
O'Hara (1995), "Market Microstructure Theory"

Articles:

Biais et al. (JFM 2005), "Market Microstructure: A Survey"
many others - see syllabus

Some parts of these slides rely on Princeton lecture notes by Nöldeke (1993)

Two Interpretations of Asymmetric Information

- different information
- different interpretation of the same information (different background information)

Modeling information I

- State space Ω
  - state $\omega \in \Omega =$ full description of reality
    - fundamentals
    - signals
  - state space is common knowledge and fully agreed among agents

Modeling information II

- Partition
  - $(\omega_1, \omega_2, \omega_3), (\omega_4, \omega_5), (\omega_6, \omega_7, \omega_8)$
  - $\mathcal{P}_1^i, \mathcal{P}_2^i, \mathcal{P}_3^i$ (partition cells)
  - later more about 'knowledge operators' etc.
- Field (Sigma-Algebra) $\mathcal{F}^i$
- Probability measure/distribution P

Modeling information III

- Prior distribution
    - Common prior assumption (CPA) (Harsanyi doctrine)
        - any difference in beliefs is due to differences in info
        - has strong implications
    - Rational Expectations
        - $\text{prior}^i = \text{objective distribution } \forall i$
        - implies CPA
    - Non-common priors
        - Problem: almost everything goes
        - Way out: Optimal Expectations (structure model of endogenous priors)
- Updating/Signal Extraction

Modeling information III

- Updating (general)
  - Bayes' Rule

$$P^i\left(E_n|D\right)=\frac{P^i\left(D|E_n\right)P^i\left(E_n\right)}{P^i\left(D\right)},$$

  - if events $E_1, E_2, ..., E_N$ are a partition

$$P^i\left(E_n|D\right)=\frac{P^i\left(D|E_n\right)P^i\left(E_n\right)}{\sum_{n=1}^N P^i\left(D|E_n\right)P^i\left(E_n\right)},$$

Updating - Signal Extraction - general case

- Updating - Signal Extraction
  - $\omega = \{v, S\}$
  - desired property:
    signal realization S^H is always more favorable than S^L
  - formally: $G(v|S^H)$ FOSD $G(v|S^L)$
  - Milgrom (1981) shows that this is equivalent to $f_S(S|v)$ satisfies monotone likelihood ratio property (MLRP)
  - $f_S(S|v)/f_S(S|\bar{v})$ is increasing (decreasing) in S if $v > (<)\bar{v}$

$$\frac{f_S(S|v)}{f_S(S|v')} > \frac{f_S(S'|v)}{f_S(S'|v')} \forall v' > v \text{ and } S' > S.$$

  - another property:
    hazard rate $\frac{f_S(S|v)}{1-F(S|v)}$ is declining in v

Updating - Signal Extraction - Normal distributions

- updating normal variable X after receiving signal $S = s$

$$\begin{aligned} E[X|S=s] &= E[X] + \frac{Cov[X,S]}{Var[S]}(s - E[S]) \\ Var[X|S=s] &= Var[X] - \frac{Cov[X,S]^2}{Var[S]} \end{aligned}$$

- n multidimensional random variable $\left(\vec{X}, \vec{S}\right) \sim \mathcal{N}(\mu, \Sigma)$

$$\mu = \begin{bmatrix} \mu_X \\ \mu_S \end{bmatrix}_{n\times 1}; \Sigma = \begin{bmatrix} \Sigma_{X,X} & \Sigma_{X,S} \\ \Sigma_{S,X} & \Sigma_{S,S} \end{bmatrix}_{n\times n}.$$

- Projection Theorem $(X|S=s)$

$$\sim \mathcal{N}\left(\mu_X + \Sigma_{X,S}\Sigma_{S,S}^{-1}(s - \mu_S), \Sigma_{X,X} - \Sigma_{X,S}\Sigma_{S,S}^{-1}\Sigma_{S,X}\right)$$

Special Signal Structures

- $\mathcal{N}$-Signals of form: $S_n = X + \varepsilon_n$
(Let X be a scalar and $\tau_y = \frac{1}{Var[y]}$),

$$E[X|s_1,...,s_N] = \mu_X + \frac{1}{\tau_X + \sum_{n=1}^N \tau_{\varepsilon_n}} \sum_{n=1}^N \tau_{\varepsilon_n}(s_n - \mu_X)$$

$$Var[X|s_1,...,s_N] = \frac{1}{\tau_X + \sum_{n=1}^N \tau_{\varepsilon_n}} = \frac{1}{\tau_{X|s_1,...,s_N}}$$

  - If, in addition, all ε_n i.i.d. then

$$E[X|s_1,...,s_N] = \mu_X + \underbrace{\frac{1}{\tau_X + N\tau_{\varepsilon_n}}}_{Var[X|s_1,...,s_N]} N\tau_{\varepsilon_n}\left(\sum_{n=1}^N \frac{1}{N}s_n - \mu_X\right),$$

where $\bar{s} := \sum_{n=1}^N \left(\frac{1}{N}\right) s_n$ is a *sufficient statistic*

Special Signal Structures

- $\mathcal{N}$-Signals of form: $X = S + \varepsilon$

$$\begin{aligned} E[X|S=s] &= s \\ Var[X|S=s] &= Var[\varepsilon] \end{aligned}$$

- Binary Signal: Updating with binary state space/signal
  - q = precision = prob$(X = H|S = S^H)$

- "Truncating signals": $v \in [\overline{S}, \underline{S}]$
  - v is Laplace (double exponentially) distributed or uniform
  - posterior is a truncated exponential or uniform

  (see e.g. Abreu & Brunnermeier 2002, 2003)

Solution/Equilibrium Concepts

- Rational Expectations Equilibrium
    - Competitive environment
    - agents take prices as given (price takers)
    - Rational Expectations (RE) $\Rightarrow$ CPA
    - *General Equilibrium Theory*
- Bayesian Nash Equilibrium
    - Strategic environment
    - agents take strategies of others as given
    - CPA (RE) is typically assumed
    - *Game Theory*
    - distinction between normal and extensive form games simultaneous move versus sequential move

The 5 Step Approach

	REE	**BNE** (sim. moves)
Step 1	Specify joint priors Conject. price mapping**s** $P : \{\mathcal{S}^1, ..., \mathcal{S}^I, u\} \rightarrow \mathbb{R}_+^{\mathbb{J}}$	Specify joint priors Conjecture strategy profile**s**
Step 2	Derive posteriors	Derive posteriors
Step 3	Derive individual demand	Derive best response
Step 4	Impose market clearing	
Step 5	Impose Rationality Equate undet. coeff.	Impose Rationality No-one deviates

A little more abstract

- REE
  Fixed Point of Mapping: $\mathcal{M}_P(P(\cdot)) \mapsto P(\cdot)$
- BNE (simultaneous moves)
  Fixed Point of Mapping:
  strategy profiles $\mapsto$ strategy profiles
- What's different for sequential move games?
  - late movers react to deviation
  - equilibrium might rely on 'strange' out of equilibrium moves
  - refinement: subgame perfection
- Extensive form move games with asymmetric information
  - Sequential equilibrium (agents act sequentially rational)
  - Perfect BNE (for certain games)
    - nature makes a move in the beginning (chooses type)
    - action of agents are observable

A Classification of Market Microstructure Models

- simultaneous submission of demand schedules
  - competitive rational expectation models
  - strategic share auctions
- sequential move models
  - screening models:
    (uninformed) market maker submits a supply schedule first
    - static
      - ◇ uniform price setting
      - ◇ limit order book analysis
    - dynamic sequential trade models with multiple trading rounds
  - signalling models:
    informed traders move first, market maker second