Supplementary appendix to
Inverse Selection*

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February 2026

This note provides the details for the extensions described in Brunnermeier, Lamba, and Segura-Rodriguez [2026]. It is divided into four parts. The first part (Section 1) explores two extensions: First, we introduce competition into the model by allowing other "regular" insurers, who do not have access to the big data technology, to offer contracts. Second, the insurer is forced to put the statistical information (its big data advantage) in the public domain. The second part (Section 2) shows that the interiority assumption made in Section 2 of the main paper for the single peakedness of the profit function can be easily relaxed. In the third part (Section 3), we provide the formal results and proofs of behavioral extensions in the main text. Finally, the last part (Section 4) presents the proofs for the extensions described here in the note.

1 Two extensions

The high-level message of the extensions is that introducing competition or forcing the insurer to put its statistical model in the public domain both increase the payoff of the insuree (consumer surplus) and decrease the insurer's profit (producer surplus), but they have different aggregate impacts: introducing competition increases total surplus by increasing the total amount of insurance offered in the marketplace, and forcing the insurer to reveal its private information reduces total surplus by decreasing the total amount of insurance offered. The key channel differentiating the two is that in the former competition case, total information produced at the optimum goes down, and in the latter by design total information shared is maximal. Hence, information production is negatively correlated with total surplus in the two extensions, even though the consumer/insuree benefits in both.

1.1 Competition

Suppose there is an insurer who is informed about ρ, call it the big firm, and there is another insurer in the market that does not know this value and works with the prior F. Borrowing from the benchmark (in Section 3 of the main paper), the latter insurer is assumed to offer the Rothschild-Stiglitz contract evaluated at the expected value of ρ. The idea here is to introduce competition in a tractable way from regular firms who do not have the in-house expertise of big data.

To entice the insuree, the contract offered for each message sent by the big firm must provide the insuree a higher expected utility than the RS-contract offered by the other firm. Technically speaking, this is a strong IR requirement, and the rest of the problem remains the same as in Section 4 of the main paper.[1] The problem

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[1]Here, we assume that if indifferent, the insuree buys from the informed (or big data) insurer.

reads as follows:

$$\max_{r,C} \Pi \text{ s.t. } IC_{\theta_1}, IR^e_{\theta_1}$$

where r, C and IC_{θ_1} are as defined before, and $IR^e_{\theta_1}$ is modified so that the insuree's outside option is evaluated at the contract $c^e = \{p_e^{RS}(b), x_e^{RS}(b), p_e^{RS}(a), x_e^{RS}(a)\}$. Here $x_e^{RS}(b)$ and $x_e^{RS}(a)$ are defined in Proposition 1 of the main paper at $\rho = \rho^e = \mathbb{E}[\rho]$, and $p_e^{RS}(b)$ and $p_e^{RS}(a)$ are the corresponding prices that maximize the insurer's expected profits. So, $IR^e_{\theta_1}$ reads as follows:

$$IR^e_{\theta_1} : u_m(\theta_1; \theta_1) \geq u_e^{RS}(\theta_1) \text{ for } \theta_1 = a, b.$$

where $u_m(\theta_1;\theta_1)$ is defined in Equation (1) of the main paper for $\hat{\theta}_1 = \theta_1$, and $u_e^{RS}(\theta_1)$ is the rent of type θ_1 at the expected correlation in the RS model.

At a first pass, Figure 1 presents the mechanical calculation of profits of the insurer in the benchmark RS problem where ρ is common knowledge, but for the lower red curve, the IR constraint takes the stronger form described in this section. So following Section 4 of the main paper, the information design problem now is performed on this lower red curve as the full disclosure benchmark, instead of the upper blue curve. The same broad principles apply, and under a reasonable sufficient condition a direct comparison can be made of the "producer", "consumer" and "total" surplus, with and without competition. The sufficient condition bounds the rent the insuree can obtain by buying insurance from the competing firms that do not have access to big data.

The formal statement of the main result is as follows. As a matter of notation, let $\hat{\rho}^*$ be the peak point for the RS model where the IR is replaced with $IR^e_{\theta_1}$, so it is the peak of the lower red graph in Figure 1.

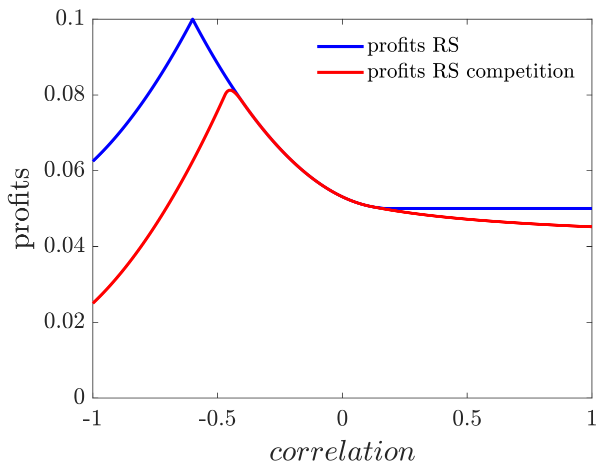

Figure 1: profit as a function of ρ with competition

Proposition A 1. *Define $\hat{\rho}^{**}$ such that $\mathbb{E}[\rho \mid \rho < \hat{\rho}^{**}] = \hat{\rho}^*$, and let I_0 be the insuree's information rent at the RS contract at ρ^e. Suppose $\rho^e > \rho^*$ and $\frac{2}{3}\left(\mu_{\hat{\rho}^{**}}(b) - \mu_{\hat{\rho}^{**}}(a)\right) > I_0$. Then we obtain the following results:*

- *The insurer discloses less information when there is competition. With competition, there is a pooling region in which the insurer sends the message $r(\rho) = \hat{\rho}^*$. This pooling region is a superset of the pooling region without competition.*
- *The insurer's ex-ante expected profit is lower with competition. The ex-ante expected profits in the pooling region without competition, $[\underline{\rho}, \rho^{**}]$, are strictly smaller with competition than without competition.*

- *Ex-post total surplus, which depends only on coverages and is larger when coverages are close to full-insurance, is, for each correlation, weakly larger with competition than without it. Total surplus is the same in the pooling regions with and without competition. Total surplus is weakly larger with competition than without competition in the full disclosure region with competition.*
- *Ex-ante expected consumer surplus is larger with competition than without it. If $I_0 > \mu_{\hat{\rho}^*}(b) - \mu_{\rho^*}(b)$, ex-post consumer surplus is point-wise strictly larger with competition than without competition for correlations that belong to the pooling region without competition. Consumer surplus is always weakly larger for correlations in the full disclosure region with competition.*

An analogous result follows if $\mathbb{E}[\rho] < \rho^*$.

1.2 A regulation for transparency

Given that the insurer wants to mislead the insuree, the government or a regulator might want to create a policy that forces the insurer to reveal all information it has. In our environment, this is equivalent to the insurer disclosing the correlation ρ. If the insurer is forced to reveal ρ, the environment is akin to Rothschild-Stiglitz environment, since now only the insuree has private information. Therefore, the insurer will implement the optimal Rothschild-Stiglitz menu. How does this policy affect the outcomes?

As we pointed out before in Proposition 2 of the main text, information and contract design lead the insurer to divide the correlation space into two regions: one in which there is complete pooling at the correlation ρ^* and another in which the insurer fully discloses its information. In the complete pooling area, the insurer offers the RS optimal menu at the correlation ρ^*. In this contract, the insurer does not offer any information rent to the insuree, since at that correlation, the insuree's information is completely uninformative about which θ_1 type has more likelihood of suffering a loss. Therefore, for those correlations, the insuree benefits from any policy that forces the insurer to reveal more information. Meanwhile, in the full revelation area, the insurer offers the optimal RS contract, so that the insuree is not affected by the government intervention. Therefore, the insuree is strictly better off for some correlations, and he is never worse off. Moreover, both total surplus and the insurer's profit decrease strictly from the intervention in comparison to the standard information design problem.

2 Relaxing assumption in Section 2 of the paper

We made the assumption that $\rho^* \in (\underline{\rho}, \overline{\rho})$. Recollect that ρ^* solves $\pi^{RS}(\rho^*) = \max_{\rho} \pi^{RS}$ or equivalently the equation $\mu_\rho(b) = \mu_\rho(a)$. This essentially means that the peak in profit in Figure 1a is attained within the feasible interval of correlation coefficients, $(\underline{\rho}, \overline{\rho})$. From the definition of μ_b and μ_a, it is easy to show that

$$\rho^* = \frac{q_1(1-q_1)\left[(1-q_2)(\mu_{bb} - \mu_{ab}) + q_2(\mu_{ba} - \mu_{aa})\right]}{\sigma\left[(1-q_1)(\mu_{aa} - \mu_{ab}) + q_1(\mu_{ba} - \mu_{bb})\right]}.$$

Notice that ρ^* is negative since the numerator is positive because we assume that $\mu_{bb} > \mu_{ab}$, $\mu_{ba} > \mu_{aa}$, and the denominator is negative, because we assume that $\mu_{ab} > \mu_{aa}$ and $\mu_{bb} > \mu_{ba}$. As $\sigma = \sqrt{q_1(1-q_1)}\sqrt{q_2(1-q_2)}$, whenever q_1 is close to 0 or 1, ρ^* is close to zero.

The assumption $\rho^* \in (\underline{\rho}, \overline{\rho})$ restricts the set of exogenous parameters that we consider. It does not affect any of the results substantially, but it is imposed to facilitate exposition. In particular, in Proposition 1, if

$\rho^* < \underline{\rho}$, type b's buyer would always be the high risk insuree, and if $\rho^* > \bar{\rho}$, type a's will always be the high risk insuree. Further, in Proposition 2, we will have another case to consider, either if $\rho^* < \underline{\rho}$ or $\rho^* > \bar{\rho}$, the insurer will use a full disclosure message rule, since the point at which she wants to pool buyers is not feasible.

3 Formal results for the behavioral models

In Section 5 of the paper, we presented two behavioral models: gullible and naive, and gave an overview of the main results. Here we present the formal statements and their proofs.

3.1 Gullible insuree

The insuree is gullible to the extent that he believes the correlation ρ told to him by the insurer. So, in addition to offering a contract, the insurer tells the insuree the correlation coefficient, and the insuree simply believes it; that is, the insuree is gullible. So, the insurer chooses r and C in tandem to create both maximal obfuscation and maximal price discrimination. Since the Bayes' consistency condition is not valid, technically the class of contract is given by $C = (c_{m,\rho})$ because the contract offered for the actual realization of ρ has no relation to the reported value m.

Proposition 1. *If the insuree is gullible,* $\exists\ \tilde{\rho} \in [\underline{\rho}, \overline{\rho}]$ *such that:*

1. *binary messages sent:* $\mathcal{M} = \{\underline{m}, \overline{m}\}$ *s.t.* $r(\rho) = \overline{m}$ *for* $\rho < \tilde{\rho}$ *and* $r(\rho) = \underline{m}$ *for* $\rho > \tilde{\rho}$,
2. *insuree's posterior is extreme:* $F_{\underline{m}} = \delta_{\underline{\rho}}$ *and* $F_{\overline{m}} = \delta_{\overline{\rho}}$ *where* δ_ρ *is Dirac measure on* ρ,
3. *profits are uniformly higher than benchmark:* $\pi(\rho) > \pi^{RS}(\rho)\ \forall\ \rho$ *almost surely,*
4. *coverages are generically separating and inefficient:* $x_\rho(H) \neq x_\rho(L)$ *and* $x_\rho \neq 1\ \forall\ \rho$ *a.s.*

Proof of Proposition 3. Since the insuree doesn't do Bayesian inference, the mapping r is independent of the contract offered at the optimum. The insurer's profit for a specific ρ is $\pi(\rho)$ given by:

$$\pi(\rho) = q_1 \left[p_{r(\rho)}(a) - \mu_\rho(a)x_{r(\rho)}(a)\right] + (1 - q_1)\left[p_{r(\rho)}(b) - \mu_\rho(b)x_{r(\rho)}(b)\right].$$

And due to the disentangling of the inference problem from the contractual problem, the insurer can solve the optimization independently for each value of ρ:

$$\max_{c,\hat{\mu}} \pi(\rho) \text{ subject to}$$

$$\begin{aligned}
&\hat{\mu}(\theta_1)x(\theta_1) - \tfrac{\eta}{2}(1 - x(\theta_1))^2 - p(\theta_1) \geqslant \hat{\mu}(\theta_1)x(\theta_1') - \tfrac{\eta}{2}(1 - x(\theta_1'))^2 - p(\theta_1') \ \ \forall\theta_1, \theta_1' \in \{b, a\} \quad IC_{\theta_1-\theta_1'}\\
&\hat{\mu}(\theta_1)x(\theta_1) - \tfrac{\eta}{2}(1 - x(\theta_1))^2 - p(\theta_1) \geqslant -\tfrac{\eta}{2} \quad \forall\theta_1 \in \{b, a\} \quad IR_{\theta_1}
\end{aligned}$$

where $c = \{p(\theta_1), x(\theta_1)\}_{\theta_1=b,a}$ is the contract and $\hat{\mu}$ is the belief that the insurer generates for the insuree. Note that both the contract and the belief chosen by the insurer depend on ρ, but since ρ is fixed for the optimization problem, we have simplified notation here for the rest of the calculations.

The objective is independent of $\hat{\mu}(\theta_1)$, and the constraint set is linear in this variable. Thus, the solution in bang-bang in $\hat{\mu}(\theta_1)$. So, the insurer will report either extremes of the feasible set of correlations, $\underline{\rho}$ or $\overline{\rho}$. This implies only two messages are sent at the optimum, say $\underline{m}$ and $\overline{m}$, that generate buyer's posteriors $F_{\underline{m}} = \delta_{\underline{\rho}}$ and $F_{\overline{m}} = \delta_{\overline{\rho}}$.

Suppose that the seller sends the message $\overline{m}$. This message generates posterior beliefs $\overline{\mu}(b) = \mu_{\overline{\rho}}(b)$ and $\overline{\mu}(a) = \mu_{\overline{\rho}}(a)$, where $\overline{\mu}(b) > \overline{\mu}(a)$. Furthermore, for any ρ, we have $\overline{\mu}(b) > \mu_\rho(b)$ and $\overline{\mu}(a) < \mu_\rho(a)$. Now, using the first-order approach, following steps from the proof of Proposition **??**, it is straightforward to show that in an interior solution

$$x(b) = 1 + \frac{\overline{\mu}(b) - \mu_\rho(b)}{\eta} > 1 \text{ and } x(a) = 1 - \frac{(1-q_1)\overline{\mu}(b) + q_1\mu_\rho(a) - \overline{\mu}(a)}{q_1\eta} < 1,$$

and at a corner solution $x(a) = 0$ and $x(b)$ takes the same value. An analogous argument shows that when sending the message $\underline{m}$, we obtain $x(b) < 1$ and $x(a) > 1$.

As a final step, we need to argue that for low correlations the seller will send messages $\overline{m}$ and for high correlations the seller will send the message $\underline{m}$. Let $\overline{\pi}(\rho)$ be the profits the seller obtains after sending message $\overline{m}$ when the actual correlation is ρ, and analogously define $\underline{\pi}(\rho)$. Plugging in we obtain that when the optimal contract is interior:

$$\frac{\partial\overline{\pi}(\rho)}{\partial\rho} - \frac{\partial\underline{\pi}(\rho)}{\partial\rho} = \frac{-\overline{\mu}(a) + (1-q_1)\overline{\mu}(b) + q_1\underline{\mu}(a)}{\eta}\frac{\partial\mu_\rho(a)}{\partial\rho} + \frac{\underline{\mu}(b) - (1-q_1)\overline{\mu}(b) - q_1\underline{\mu}(a)}{\eta}\frac{\partial\mu_\rho(b)}{\partial\rho}$$
$$< 0,$$

since $\frac{\partial\mu_\rho(a)}{\partial\rho} < 0$, $\frac{\partial\mu_\rho(b)}{\partial\rho} > 0$, $\overline{\mu}(a) < \underline{\mu}(a)$, $\overline{\mu}(a) < \overline{\mu}(b)$, $\overline{\mu}(b) > \underline{\mu}(b)$, and $\underline{\mu}(b) < \underline{\mu}(a)$.

Thus, if for a correlation $r(\rho) = \underline{m}$, then for all $\rho' > \rho$ the seller sends the same message, $r(\rho') = \underline{m}$. Analogously if $r(\rho) = \overline{m}$, then for all $\rho' < \rho$, $r(\rho') = \overline{m}$. This and our characterization above show that there is a $\tilde{\rho} \in [\underline{\rho}, \bar{\rho}]$ such that $r(\rho) = \overline{m}$ for $\rho < \tilde{\rho}$ and $r(\rho) = \underline{m}$ for $\rho > \tilde{\rho}$. □

3.2 Naive insuree

The agent ignores the signals offered by the contract about the correlation coefficient, so that $F_m = F\ \forall m \in \mathcal{M}$. Thus, in this situation, the role of r is moot. The insurer designs the contract as a function of ρ with the knowledge that the insuree will evaluate his payoffs using the prior F.

Proposition 2. *If the insuree is naive (and thus sticks to the prior):*

1. *profits are higher in expectation than without information:* $\mathbb{E}(\pi(\rho)) > \pi^{RS}(\mathbb{E}(\rho))$,
2. *coverages features both pooling and separation,*
3. *coverages are generically inefficient:* $x_\rho(\theta_1) \neq 1\ \forall\ \rho$ *a.s.,*

Proof of Proposition 4. Here the mapping r is redundant since the insuree does not update his prior. Since the IC and IR constraints are linear in beliefs, what matters for the insuree is the expected probability of loss as evaluated through the prior F, call these $\mu^e(b)$ and $\mu^e(a)$. The insurer offers a contract $c_\rho = \{p_\rho(\theta_1), x_\rho(\theta_1)\}_{\theta_1=b,a}$ to maximize $\pi(\rho)$ subject to appropriate notions of IC and IR—same as in the proof of Proposition **??** with $\mu_\rho(\theta_1)$ replaced with $\mu^e(\theta_1)$. Suppose $\mu^e_b > \mu^e_a$.

Following steps as before, we obtain that at an interior solution:

$$x_\rho(b) = 1 + \frac{\mu^e(b) - \mu_\rho(b)}{\eta} \text{ and } x_\rho(a) = 1 - \frac{(1-q_1)\mu^e(b) + q_1\mu_\rho(a) - \mu^e(a)}{q_1\eta}.$$

Notice that $x_\rho(b)$ is decreasing in ρ and $x_\rho(a)$ is increasing in ρ. Then there are two "corner" solutions: one in which $x_\rho(a) = 0$ and $x_\rho(b) = 1 + \frac{\mu^e(b)-\mu_\rho(b)}{\eta}$, and another one in which $x_\rho(a) = x_\rho(b) = 1 + \mu^e(a) -$

$\frac{q_1\mu_\rho(a)+(1-q_1)\mu_\rho(b)}{\eta}$. This happens when the two allocations listed above violate $x_\rho(a) = 0$ and $x_\rho(b) \leqslant x_\rho(a)$, respectively.

This proves part 2 that coverages can be pooling and separating and part 3 that coverages are generically not equal to 1. So, we are now only left to show part 1, that is, $\mathbb{E}(\pi(\rho)) > \pi^{RS}(\mathbb{E}(\rho))$. We show that the profit generated by contract above is convex in ρ. Since at correlation $\mathbb{E}(\rho)$ we have $\pi(\mathbb{E}(\rho)) = \pi^{RS}(\mathbb{E}(\rho))$, Jensen's inequality implies that $\mathbb{E}(\pi(\rho)) > \pi^{RS}(\mathbb{E}(\rho))$.

In an interior contract, we have that

$$\frac{\partial^2\pi}{\partial\rho^2} = \frac{q_1}{\eta}\left(\frac{\partial\mu_a}{\partial\rho}\right)^2 + \frac{1-q_1}{\eta}\left(\frac{\partial\mu_b}{\partial\rho}\right)^2 > 0,$$

for the corner solution in which $x_a(\rho) = 0$ we have that $\frac{\partial^2\pi}{\partial\rho^2} = 0$ and for the corner solution in which $x(a,\rho) = x(b,\rho)$ we have

$$\frac{\partial^2\pi}{\partial\rho^2} = \frac{1}{\eta}\left(q_1\frac{\partial\mu_a}{\partial\rho} + (1-q_1)\frac{\partial\mu_b}{\partial\rho}\right)^2 > 0.$$

Therefore, the profit function is convex and we obtain the inequality as desired.

An analogous argument shows the proposition for the case $\mu^e_b < \mu^e_a$. □

4 Proofs

This section presents the proof of Proposition A1 that was presented above.

Proof of Proposition A1. The proof is divided into 3 steps.

Step 1. We first characterize the insurer's optimal profits when ρ is common knowledge and there is competition, that is, the lower red curve in Figure 1.

Suppose $\mathbb{E}[\rho] > \rho^*$ and denote by I_0 type b insuree's information rent in the standard RS problem at correlation $\mathbb{E}[\rho]$. Recall that since $\mathbb{E}[\rho] > \rho^*$, type a's information rent at that correlation is zero. Given that both the insuree and the insurer know that the correlation is ρ, the insurer's problem is

$$\begin{aligned}
&max_{x_\rho,p_\rho} q_1(p_\rho(a) - \mu_\rho(a)x_\rho(a)) + (1-q_1)(p_\rho(b) - \mu_\rho(b)x_\rho(b))\\
&\quad \text{s.t. } \mu_\rho(b)x_\rho(b) - \frac{\eta}{2}(1-x_\rho(b))^2 - p_\rho(b) \geq -\frac{\eta}{2} + I_0\\
&\quad \mu_\rho(a)x_\rho(a) - \frac{\eta}{2}(1-x_\rho(a))^2 - p_\rho(a) \geq -\frac{\eta}{2}\\
&\quad \mu_\rho(b)x_\rho(b) - \frac{\eta}{2}(1-x_\rho(b))^2 - p_\rho(b) \geq \mu_\rho(b)x_\rho(a) - \frac{\eta}{2}(1-x_\rho(a))^2 - p_\rho(a)\\
&\quad \mu_\rho(a)x_\rho(a) - \frac{\eta}{2}(1-x_\rho(a))^2 - p_\rho(a) \geq \mu_\rho(a)x_\rho(b) - \frac{\eta}{2}(1-x_\rho(b))^2 - p_\rho(b)
\end{aligned}$$

This problem is analogous to the RS problem, with the exception that the RHS of type b's IR constraint has increased by I_0. This difference changes the set of restrictions that bind for each correlation.

Consider first a correlation $\rho < \rho^*$. In the RS problem, type b's IR constraint and type a's IC constraint are the only ones that bind. This is still true in this case because only the RHS of type b's IR constraint has increased. Therefore, the optimal coverages are the same and the only difference is that now the insurer has to decrease both prices by I_0. Therefore, in this region, the profit function is still increasing and convex.

If instead $\rho > \rho^*$, RS contract might not be feasible because, according to proposition 1 in the main text, type b's utility in RS contract is equal to

$$u_\rho(b) = \mu_\rho(b) - \mu_\rho(a) - \frac{1-q_1}{\eta q_1}(\mu_\rho(b) - \mu_\rho(a))^2 - \frac{\eta}{2},$$

and this problem requires that it is at least equal to $I_0 - \frac{\eta}{2}$, which is not when $\mu_\rho(b) - \mu_\rho(a)$ is small.

Since u_ρ is strictly concave, and by definition $u_{\mathbb{E}[\rho]}(b) = I_0 - \frac{\eta}{2}$, there exist two thresholds $\tilde{\rho}_1 \leq \tilde{\rho}_2$ such that $u_\rho(b) < I_0 - \frac{\eta}{2}$ for $\rho \in (\rho^*, \tilde{\rho}_1) \bigcup (\tilde{\rho}_2, \bar{\rho})$ and $u_\rho(b) > I_0 - \frac{\eta}{2}$ for $\rho \in (\tilde{\rho}_1, \tilde{\rho}_2)$. Then type b's constraint IR should bind only for correlations $\rho \in (\rho^*, \tilde{\rho}_1) \bigcup (\tilde{\rho}_2, \bar{\rho})$.

Now suppose that none of the IC constraints bind. Then $x_a(\rho)$ and $x_\rho(b)$ would be both equal to 1 and $p_\rho(b) = \mu_\rho(b) + \frac{\eta}{2} - I_0$ and $p_\rho(a) = \mu_\rho(a) - \frac{\eta}{2}$. Comparing both prices allows us to find which IC constraint should bind: If $p_\rho(b) > p_\rho(a)$, i.e., $\mu_\rho(b) - \mu_\rho(a) > I_0$, type b's IC constraint should bind and in the opposite case type a's IC constraint should bind. We denote by $\hat{\rho}^*$ the unique correlation for which $\mu_{\hat{\rho}^*}(b) - \mu_{\hat{\rho}^*}(a) = I_0$, i.e., for $\rho < \hat{\rho}^*$ type a's IC constraint binds and for $\rho > \hat{\rho}^*$ type b's IC constraint binds.

For correlations $\rho > \hat{\rho}^*$ either type b's IC and IR constraints and type a's IR constraint bind or only type b's IC constraint and type a's IR constraint bind. In the first case, the optimal coverages are $x_\rho(b) = 1$ and $x_\rho(a) = \frac{I_0}{\mu_\rho(b) - \mu_\rho(a)}$ and optimal prices are equal to $p_\rho(b) = \mu_\rho(b) + \frac{\eta}{2} - I_0$ and $p_\rho(a) = \frac{\mu_\rho(a) I_0}{\mu_\rho(b) - \mu_\rho(a)} + \frac{\eta}{2} - \frac{\eta}{2}\left(1 - \frac{I_0}{\mu_\rho(b) - \mu_\rho(a)}\right)^2$. Therefore, for these correlations, profit is equal to

$$\frac{\eta}{2} - (1-q_1)I_0 - q_1 \frac{\eta}{2}\left(1 - \frac{I_0}{\mu_\rho(b) - \mu_\rho(a)}\right)^2$$

. This function is clearly decreasing since $I_0 < \mu_\rho(b) - \mu_\rho(a)$ and $\mu_\rho(b) - \mu_\rho(a)$ is increasing in ρ. Further, this profit function is strictly convex iff $\mu_\rho(b) - \mu_\rho(a) > \frac{3}{2}I_0$, this is, this profit function is locally concave close to $\hat{\rho}^*$, as we had proved before that close to $\hat{\rho}^*$ type b's IR constraint binds. In the second case, optimal coverages and prices are exactly the same as in the RS problem. Therefore, the profit function in this set is strictly decreasing and strictly convex.

Finally, we need to characterize the optimal contract for correlations in the set $(\rho^*, \hat{\rho}^*)$. In this set, type b's IR constraint and type a's IC constraint always bind. Further, type a's IR constraint might bind. If it binds, $x_\rho(b) = \frac{I_0}{\mu_\rho(b) - \mu_\rho(a)}$, $x_\rho(a) = 1$, $p_\rho(b) = \frac{\mu_\rho(b) I_0}{\mu_\rho(b) - \mu_\rho(a)} + \frac{\eta}{2} - I_0 - \frac{\eta}{2}\left(1 - \frac{I_0}{\mu_\rho(b) - \mu_\rho(a)}\right)^2$ and $p_\rho(a) = \frac{\eta}{2} + \mu_\rho(a)$. If it does not bind, the optimal coverages would be $x_\rho(b) = 1 + \frac{q_1}{(1-q_1)\eta}(\mu_\rho(b) - \mu_\rho(a))$ and $x_\rho(a) = 1$ and prices would be $p_\rho(b) = \frac{\eta}{2} - I_0 + \mu_\rho(b) x_\rho(b) - \frac{\eta}{2}\left(1 - x_\rho(b)\right)^2$ and $p_\rho(a) = \frac{\eta}{2} - I_0 + \mu_\rho(a) + x_\rho(b)(\mu_\rho(b) - \mu_\rho(a))$. Type a's IR constraint binds for correlations close to $\hat{\rho}^*$ since type a's utility $I_0 - \frac{\eta}{2} - \mu_\rho(b) + \mu_\rho(a) - \frac{q_1}{(1-q_1)\eta}(\mu_\rho(b) - \mu_\rho(a))^2$, which is less than $-\frac{\eta}{2}$ for correlations close to $\hat{\rho}^*$.

In the first case, the profits are equal to

$$\frac{\eta}{2} - (1-q_1)I_0 - (1-q_1)\frac{\eta}{2}\left(1 - \frac{I_0}{\mu_\rho(b) - \mu_\rho(a)}\right)^2,$$

which is strictly increasing and concave since in this case $I_0 > \mu_\rho(b) - \mu_\rho(a)$. In the second case, the profits are equal to

$$\frac{\eta}{2} - I_0 + q_1(\mu_\rho(b) - \mu_\rho(a)) - \frac{\eta q_1^2}{2(1-q_1)}\left(\mu_\rho(b) - \mu_\rho(a)\right)^2,$$

which is clearly strictly increasing and strictly convex since $\mu_\rho(b) - \mu_\rho(a)$ is increasing in ρ.

In summary, with competition, the insurer's profit function has a unique global and local maximum at the correlation $\hat{\rho}^*$, it is strictly increasing to the left of this correlation and strictly decreasing to its right. Further, it is locally concave close to that correlation, and strictly convex for the rest of the support. A similar argument shows the analogous result for the case in which $\mathbb{E}[\rho] < \rho^*$.

Step 2. In this step, we use the argument in the proof of Proposition 2 to show that the insurer's optimal message function has the same structure as in the case without competition.

First, it is clear that the insurer wants to send the same message m for all correlations $\rho < \hat{\rho}^*$ and for some correlations $\rho > \hat{\rho}^*$ in order to make sure that once that insuree observes the message m, the insuree's posterior expected correlation is exactly equal to $\hat{\rho}^*$. As in the proof of Proposition 2 one can argue that to maximize the probability that is assigned to this message, we must have $r(\rho) = m$ iff $\rho < \hat{\rho}^{**}$ with $\hat{\rho}^{**}$ as the correlation defined to be the unique correlation for which $\mathbb{E}[\rho \mid \rho < \hat{\rho}^{**}] = \hat{\rho}^*$.

Now, by assumption $\mu_{\hat{\rho}^{**}}(b) - \mu_{\hat{\rho}^{**}}(a) > \frac{3}{2} I_0$. According to the first step, this condition guarantees that the profit function at any correlation $\rho > \hat{\rho}^{**}$ is strictly convex. Therefore, the insurer wants to disclose all information for any correlation $\rho > \hat{\rho}^{**}$.

Therefore, we conclude that the first bullet point in the statement is satisfied.

Step 3. Finally, we make some comparisons in terms of the insurer's profits, the insuree's information rent (or consumer surplus) and in terms of total surplus to see how efficiency is affected. We consider the case in $\mathbb{E}[\rho] > \rho^*$. The complementary case is analogous.

First, with competition, all correlations $\rho < \hat{\rho}^{**}$ are pooled at $\hat{\rho}^*$, and in this correlation we showed that both types' IR constraints bind and type b's IC constraint binds, so that $\pi^{comp}(\hat{\rho}^*) < \pi^{RS}(\hat{\rho}^*)$. Further, compared to the case without competition, the insurer is pooling the correlations for which it was pooling before at ρ^* and the correlations in the interval $[\rho^{**}, \hat{\rho}^{**}]$. Strict convexity of π^{RS} profit function implies that without competition the expected profit for those correlations smaller than $\hat{\rho}^{**}$ is larger than $\pi^{RS}(\hat{\rho}^*)$. Therefore, ex-ante expected profits for correlations $\rho < \hat{\rho}^{**}$ decreased after the insurer faces competition. Further, for correlations $\rho > \hat{\rho}^{**}$, both with competition and without it the insurer fully discloses information. However, with competition the insuree's outside option has increased, so that the insurer's profit has decreased point-wise for these correlations. In conclusion, insurer's ex-ante profits decreased with competition.

Second, the pooling region is larger with competition than without it, and in this region total surplus is maximized since both types of insurees obtain a contract which provides full insurance. For correlations, $\rho > \hat{\rho}^{**} > \hat{\rho}^*$, we show in Step 1, that with competition type b's coverage is equal to RS coverage and type a's coverage is either larger or equal to RS coverage, depending on whether type b's IR constraint binds or not. Therefore, in this region total surplus is weakly larger.

Third, these two arguments imply immediately that ex-ante expected consumer surplus is larger. □

References

M. K. Brunnermeier, R. Lamba, and C. Segura-Rodriguez. Inverse selection. Princeton University, Cornell University and Banco Central de Costa Rica, 2026.