

# Optimal (Un)Conventional Monetary Policy

Andrey Alexandrov\* and Markus K. Brunnermeier†

February 27, 2026

## Abstract

We study the optimal joint interest rate and central bank balance sheet policies in a macro model with financial sector, sticky prices, aggregate and idiosyncratic risk. The optimal policy mix ensures an efficient path of aggregate output, as well as efficient distributions of consumption and risk across agents. With a fixed balance sheet composition, interest rate policy alone can close the output gap and implement the efficient path of the aggregates. It can also ensure its (near) efficient distribution, but only in a small vicinity around the stochastic steady state. Distributional efficiency away from the steady state requires active balance sheet management. The role of balance sheet policy is preparatory – it ensures that the effects of future shocks are distributed efficiently across agents, given the intended interest rate responses to those shocks.

---

\*Tor Vergata University of Rome.

†Princeton University.

We are grateful to Gaetano Bloise, Mikhail Golosov, Moritz Lenel, Sebastian Merkel, Jonathan Payne and Piotr Żoch (discussant), as well as seminar participants at Princeton and Tor Vergata University. Antonio Gentile provided valuable research assistance.

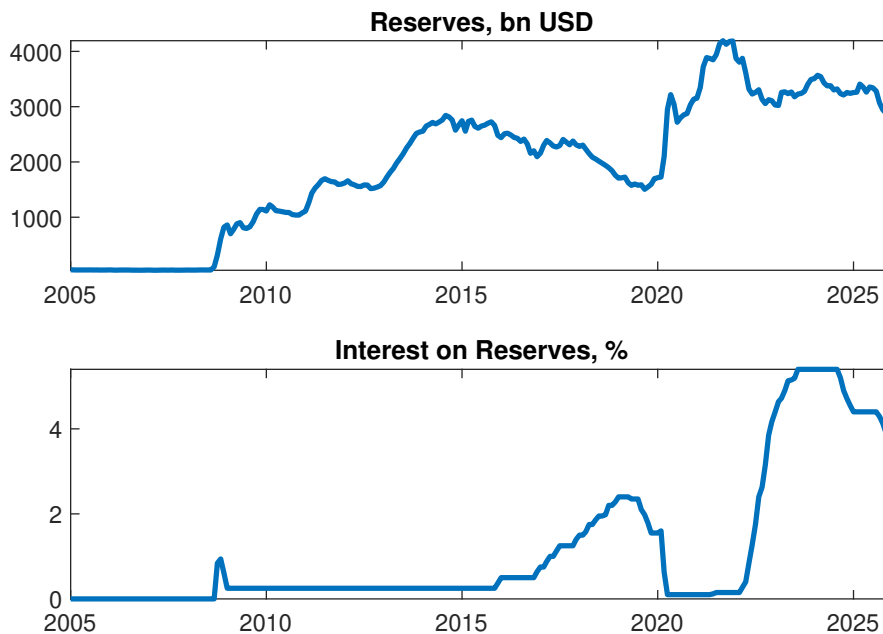
# 1 Introduction

Central banking has undergone significant transformations in the aftermath of the Global Financial Crisis (GFC). The crisis prompted central banks to adopt new experimental policies and expand their toolbox beyond traditional instruments. These developments have reshaped the approach to monetary policy, incorporating multiple policy instruments to achieve macroeconomic stability.

Modern central banking now relies on a combination of interest rate policies and central bank balance sheet management. Interest rate policies involve setting rates on both required and excess reserves. Simultaneously, balance sheet policies include active usage of quantitative easing (QE) and quantitative tightening (QT) in which central banks purchase or sell long-term government bonds in exchange for reserves. Being introduced as last-resort extraordinary measures, balance sheet policies have gradually become a routine instrument in the monetary policy toolbox.

The necessity to study all these policies jointly is motivated by the apparent interplay and interdependence of these policies observed in the data. Figure 1 shows the dynamics of the volume of reserves issued by the Federal Reserve (top panel) and the interest rate promised on these reserves (bottom panel). In the wake of the GFC, the Fed started to

Figure 1: Reserves and Interest Rates



actively expand its balance sheet by issuing reserves and purchasing various assets from private agents, with long-term government bonds among them (QE). This led to a large build-up of (excess) reserves and private banks' deposits. At the same time, the Fed introduced an interest rate on reserves to help steer the interbank rate, which was near

the zero lower bound at the time. Recent interest rate hikes *together* with previous central bank balance sheet expansions created a new liability for the central bank in the form of enormous interest rate payments, leading to concerns for the conduct of conventional interest rate policy. Such interaction between conventional and unconventional policies generates new challenges for central banking, and hence requires a new framework which allows for the study of the optimal size and composition of the central bank's balance sheet. To design effective policies, it is essential to understand the distinct role each monetary policy instrument plays, how they interact, and how they can be coordinated to maximize social welfare.

To analyze the interaction of interest rate and balance sheet policies, the following ingredients are at the center stage. First, there needs to be a financial sector which holds, among other assets, central bank reserves and issues deposits to households. Second, risk considerations and portfolio choice are essential because balance sheet policies redistribute risk. This includes the endogenous distribution of long-term bond holdings across different agents. Third, we include price rigidities to have the output-gap-management role of interest rate policy as in the mainstream monetary policy framework. Fourth, we allow monetary authority to conduct balance sheet management directly in response to aggregate shocks, whereas fiscal authority can only issue bonds and raise taxes gradually over time. This captures the real-world institutional constraints that make monetary policy more responsive to aggregate shocks, compared to fiscal policy which acts with a delay. Finally, unlike in most other papers studying balance sheet policies (Gertler and Karadi (2011), Karadi and Nakov (2021), Eren, Jackson, and Lombardo (2024), Nisticò and Seccareccia (2025)), we do not impose that central bank balance sheet expansions relax intermediaries' constraints. In addition, in our framework balance sheet management is fully anticipated, as in Haddad, Moreira, and Muir (2024).

We combine these features in a macro model with a financial sector, sticky prices, idiosyncratic and aggregate risk, building on the framework developed in Brunnermeier and Sannikov (2016), Merkel (2020) and Li and Merkel (2025). We characterize analytically the (constrained) efficient allocation in this model, as well as the optimal interest rate and balance sheet policy mix implementing it. The key monetary instruments are two interest rates: one on required and one on excess reserves, together with central bank balance sheet management. The two main inefficiencies that a social planner tries to correct stem from: (i) sticky prices leading to *aggregate* output and consumption gaps, (ii) pecuniary externalities leading to consumption and risk *allocation* inefficiencies. Interest rate policy alone is sufficient to address the former inefficiency, and even to ensure (near) optimal allocation of consumption across agents around the steady state. However, as aggregate shocks push the economy away from the steady state, optimal policy mix requires active balance sheet management.

We show that interest rate and balance sheet policies have unique roles, yet have to

be considered jointly as balance sheet management mediates the effects of the former and is preparatory. More specifically, the role of interest rate policy is to generate fluctuations in the price of long-term bonds. This in turn allows the central bank to steer aggregate consumption and net worth, as well as their distributions across agents, provided that agents have an efficient exposure to these fluctuations. The role of balance sheet policies is therefore to ensure that the economy is exposed to the efficient amount of risk generated by future interest rate movements. In this sense balance sheet management plays a preparatory role.

When addressing either one of the above mentioned inefficiencies, the size of the necessary interest rate movements depends on *prior* balance sheet policies. After a central bank balance sheet expansion, duration risk is taken off the private agents' balance sheets, requiring a more aggressive interest rate move in response to a subsequent shock. In this sense, there exists a significant substitutability between balance sheet and interest rate policies. Generous balance sheet policies have to be followed by aggressive interest rate responses to shocks. However, when addressing the two inefficiencies simultaneously, both balance sheet and interest rate policies are pinned down. We highlight that central bank's ability to set interest rates on required and excess reserves independently is important. The interest rate on excess reserves together with balance sheet management help to control immediate impact responses of both the output gap and net worth distribution across agents to an aggregate shock. The interest rate on required reserves is then used to manage the output gap along the subsequent transition path following the shock.

We note that the redistributive role of interest rate policy does not hinge on a particular fixed distribution of long-term bond holdings across agents. The reason is that besides the direct channel, through which interest rate cuts redistribute wealth towards long-term bond holders, another indirect channel is operative. An appreciation of long-term bonds increases total nominal wealth in the economy. Under sticky prices, this leads to a real wealth appreciation, pushing the price of all real assets up. Therefore, it additionally redistributes wealth towards agents that are levered in real assets, independently of the distribution of long-term bond holdings across agents. In fact, the endogeneity and responsiveness of this distribution to policy is key. Perfect long-term bond market segmentation removes the central bank's ability to shift the distribution of bonds across the two types of agents and impose the efficient risk exposure. However, a perfectly elastic long-term bond distribution allows the agents to hedge against aggregate risk and also precludes the optimal risk allocation. Therefore, optimal policy implementation requires an intermediate degree of bond market segmentation.

The rest of the paper is structured as follows. Section 2 sets up the model and discusses the equilibrium. Section 3 defines the welfare maximizing allocation. Section 4 illustrates the inefficiency of a laissez-faire economy, and section 5 describes welfare maximizing policies.

## 2 Model Setup

In this section we outline the theoretical framework, relegating the details and the model solution to appendix A. The model features households who hold productive capital and are exposed to time-varying idiosyncratic risk stemming from it, as well as intermediaries who possess a risk diversification technology. Prices are sticky and set by monopolistic firms that purchase the output good from households and add a variety to it, selling the resulting product to competitive final good producers. The government levies taxes and conducts both the interest rate and the balance sheet policies.

### 2.1 Capital

The only real asset in positive net supply in the economy is capital. When held by households,  $k_t$  units of capital produce  $ak_t$  units of a common input good per unit of time. Capital accumulation is determined by investment rate  $\iota_t$ , chosen by capital holders, and by exogenous idiosyncratic shocks  $d\tilde{Z}_t$  with volatility loading  $\tilde{\sigma}_t$ . Individual capital holdings follow:

$$\frac{dk_t}{k_t} = \underbrace{\left( \frac{1}{\phi} \log(1 + \phi \iota_t) - \delta \right)}_{g(\iota_t)} dt + \tilde{\sigma}_t d\tilde{Z}_t \quad (1)$$

where  $\delta$  is depreciation rate and  $g(\iota_t)$  is the expected growth rate of capital. Idiosyncratic volatility  $\tilde{\sigma}_t^2$  is a stationary stochastic process:

$$d\tilde{\sigma}_t^2 = \mu(\tilde{\sigma}_t^2)dt + \sigma(\tilde{\sigma}_t^2)dZ_t \quad (2)$$

with drift and diffusion terms depending on  $\tilde{\sigma}_t^2$ , and  $dZ_t$  denoting aggregate Brownian shocks to idiosyncratic volatility. These shocks drive the amount of idiosyncratic risk in the economy and are the only source of aggregate uncertainty in the model. Since idiosyncratic capital accumulation shocks net out in the cross-section, aggregate capital evolves as follows:<sup>1</sup>

$$\frac{dK_t}{K_t} = g(\iota_t)dt$$

Capital price in consumption units is denoted by  $q_t^K$  and is driven by aggregate shocks:

$$\frac{dq_t^K}{q_t^K} = \mu_t^{q^K} dt + \sigma_t^{q^K} dZ_t$$

where drift and volatility loading of capital price are endogenous and determined in equilibrium.

---

<sup>1</sup>Here we anticipate the result that all households choose the same investment rate despite heterogeneity in net worth levels. This is due to CRRA utility which greatly simplifies aggregation.

## 2.2 Households' problem

Households invest in capital, deposits and long-term bonds, and issue outside equity to offload idiosyncratic risk to intermediaries. They need liquid deposits to ensure smooth maintenance of capital, and the amount of deposits required depends on their velocity  $\nu_t$ :  $\nu_t D_t^H = q_t^K k_t$ , where  $D_t^H$  is the real value of held deposits. Higher velocity allows households to hold fewer deposits but comes at a cost  $\mathfrak{t}(\nu_t)$  that reduces return on capital, specified below. The cost itself is indexed by time as it is relative to the average cost in the economy, implying that in equilibrium the actual incurred cost is zero. This introduces a wedge in households' decisions without affecting the resource constraint. Formally, given average velocity  $\bar{\nu}_t$ , transaction cost for an individual choice of  $\nu_t$  is given by:

$$\mathfrak{t}(\nu_t) \equiv \mathfrak{t}(\nu_t) - \mathfrak{t}(\bar{\nu}_t)$$

where  $\mathfrak{t}(\cdot)$  is an increasing convex function. In addition, households choose investment rate  $\iota_t$  and capital utilization rate  $\nu_t$ , and face a corresponding dis-utility denoted by an increasing and convex function  $b(\nu_t)$ . With Poisson intensity  $\lambda^H$  a household switches type and becomes an intermediary, keeping their wealth. Denote by  $\theta_t^{D,H}$ ,  $\theta_t^{L,H}$ ,  $\theta_t^K$ ,  $\theta_t^{x,H}$  households' portfolio weights on deposits, long-term bonds, capital and outside equity, respectively, and by  $\rho$  – households' discount rate. Then, for a given initial wealth level  $n_0^H$ , they maximize the following objective:

$$V^H(n_0^H) = \max_{c_t^H, \nu_t, \iota_t, \nu_t, \theta_t^{D,H}, \theta_t^{L,H}, \theta_t^K, \theta_t^{x,H}} \mathbb{E} \left[ \int_0^T e^{-\rho t} (\log(c_t^H) - b(\nu_t)) dt + e^{-\rho T} V^I(n_T^H) \right]$$

subject to net worth evolution:

$$\frac{dn_t^H}{n_t^H} = -\frac{c_t^H}{n_t^H} dt + \theta_t^{D,H} dr_t^D + \theta_t^{L,H} dr_t^L + \theta_t^K dr_t^K(\nu_t, \iota_t, \nu_t) + \theta_t^{x,H} dr_t^{x,H} + \tau_t^H dt$$

and portfolio constraints:

$$\begin{aligned} 1 &= \theta_t^{D,H} + \theta_t^{L,H} + \theta_t^K + \theta_t^{x,H} \\ \nu_t \theta_t^{D,H} &= \theta_t^K \end{aligned}$$

as well as a solvency constraint  $n_t^H \geq 0$ , where  $T$  is the random type-switching time and  $\tau_t^H$  is the households' (per-period) wealth tax or transfer. Return on capital is given by:

$$\begin{aligned} dr_t^K(\nu_t, \iota_t, \nu_t) &= \left[ \frac{p_t a \nu_t - \iota_t - \tau_t^K + \mathfrak{d}_t}{q_t^K} - \mathfrak{t}(\nu_t) \right] dt + \frac{d(q_t^K k_t)}{q_t^K k_t} \\ &= \left[ \frac{p_t a \nu_t - \iota_t - \tau_t^K + \mathfrak{d}_t}{q_t^K} - \mathfrak{t}(\nu_t) + \mu_t^{q^K} + g(\nu_t) \right] dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t \quad (3) \end{aligned}$$

where  $av_t$  is the total output per unit of capital that is sold to monopolistic firms at an equilibrium price  $p_t$ ,  $\mathfrak{d}_t$  is a dividend transfer from those firms,<sup>2</sup> and  $\tau_t^K$  is capital tax. The above equation highlights that capital return is reduced by the transaction cost  $\mathfrak{t}_t(\nu_t)$  and is subject to idiosyncratic shocks. With some abuse of notation, returns on deposits, long-term bonds and outside equity, respectively, are given by:

$$dr_t^D = (i_t^D - \pi_t) dt \quad (4)$$

$$dr_t^L = \left( \frac{i^L}{P_t^L} + \mu_t^{PL} - \pi_t \right) dt + \sigma_t^{PL} dZ_t \quad (5)$$

$$dr_t^{x,H} = r_t^x dt + \sigma_t^{qK} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t \quad (6)$$

Real expected return on deposits is given by the difference between interest rate  $i_t^D$ , determined in equilibrium, and inflation rate  $\pi_t$ . Deposits are safe in real terms since the aggregate price level is sticky and does not respond to aggregate shocks. Perpetual long-term bonds pay a fixed nominal coupon  $i^L$  and are traded at an equilibrium nominal price  $P_t^L$  that evolves according to:

$$\frac{dP_t^L}{P_t^L} = \mu_t^{PL} dt + \sigma_t^{PL} dZ_t$$

Since nominal price of long-term bonds is stochastic, their return is not safe in either nominal or real terms, in contrast to deposits. Finally, outside equity issued by households inherits both the aggregate and idiosyncratic risk of their capital holdings, allowing them to offload it one-for-one. The expected return on outside equity  $r_t^x$  is determined endogenously in equilibrium.

## 2.3 Intermediaries' problem

Intermediaries invest in households' outside equity, central bank reserves and long-term bonds, and issue deposits. They face a required reserves constraint, demanding that their portfolio share of reserves  $\theta_t^R$  must be greater or equal than some threshold  $\underline{\theta}_t^R$ , set by the central bank. Analogously to households, they face type-switching Poisson risk and become households with intensity  $\lambda^I$ . Denote by  $\theta_t^{D,I}$ ,  $\theta_t^{L,I}$ ,  $\theta_t^{x,I}$  intermediaries' portfolio shares of deposits, long-term bonds and outside equity, respectively. Then, given some initial wealth level  $n_0^I$ , their objective is to maximize:

$$V^I(n_0^I) = \max_{c_t^I, \theta_t^{D,I}, \theta_t^{L,I}, \theta_t^{x,I}, \theta_t^R} \mathbb{E} \left[ \int_0^T e^{-\rho t} \log(c_t^I) dt + e^{-\rho T} V^H(n_T^I) \right]$$

---

<sup>2</sup>We assume that all physical production activity is associated with capital holding. As a result, capital return is not distorted by the presence of monopolistic producers in equilibrium.

subject to wealth evolution:

$$\frac{dn_t^I}{n_t^I} = -\frac{c_t^I}{n_t^I} dt + \theta_t^{L,I} dr_t^L + \theta_t^{D,I} dr_t^D + \theta_t^{x,I} dr_t^{x,I} + \theta_t^{\mathcal{R}} dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) + \tau_t^I$$

portfolio constraints:

$$\begin{aligned} 1 &= \theta_t^{D,I} + \theta_t^{L,I} + \theta_t^{\mathcal{R}} + \theta_t^{x,I} \\ \theta_t^{\mathcal{R}} &\geq \underline{\theta}_t^{\mathcal{R}} \end{aligned}$$

and a solvency constraint  $n_t^I \geq 0$ . Return on reserves depends on the amount of excess reserves held by intermediaries, as excess and mandatory reserves may pay different interest:

$$dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) = \left[ \frac{\underline{\theta}_t^{\mathcal{R}} \underline{i}_t + (\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}}) i_t}{\theta_t^{\mathcal{R}}} - \pi_t \right] dt \quad (7)$$

where  $\underline{i}_t$  is the interest rate on mandatory reserves and  $i_t$  is the interest rate on excess reserves. We denote by  $i_t^a$  the equilibrium average interest rate on reserves:

$$i_t^a = \frac{\underline{\theta}_t^{\mathcal{R}} \underline{i}_t + (\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}}) i_t}{\theta_t^{\mathcal{R}}} \quad (8)$$

Note that the marginal interest rate on reserves is given by:

$$i_t^m = i_t + \lambda_t^{\mathcal{R}} \quad (9)$$

where  $\lambda_t^{\mathcal{R}}$  is the Lagrange multiplier associated with the reserve requirement constraint. Returns on deposits and long-term bonds are the same as for the households and are given by (4) and (5). Return on outside equity, however, differs in two ways:

$$dr_t^{x,I} = (r_t^x + \tau_t^x) dt + \sigma_t^{q^K} dZ_t + \varphi \tilde{\sigma}_t d\tilde{Z}_t$$

First, intermediaries may receive an intermediation subsidy  $\tau_t^x$ , which augments their expected return beyond the rate paid by households. Second, when taking on idiosyncratic risk, intermediaries diversify away a fraction  $1 - \varphi$  of it and are only subject to the remaining share  $\varphi \in (0, 1)$ .<sup>3</sup> The diversification technology of intermediaries naturally applies to idiosyncratic risk only, exposing them in full to the capital price risk carried by outside equity claims.

---

<sup>3</sup>Note that intermediaries do not fully diversify idiosyncratic risk away ( $\varphi \neq 0$ ). As an example, such partial diversification occurs if each intermediary allocates a fraction  $1 - \varphi$  of their total outside equity holdings to a fully diversified portfolio containing outside equity claims issued by all households, and the remaining fraction  $\varphi$  – to a particular individual household, thus inheriting idiosyncratic risk of that household in full.

## 2.4 Monopolistic firms

We use the standard setting with monopolistically competitive intermediate goods producers and perfectly competitive final good producers, same as in Li and Merkel (2025) for a continuous-time reference. The final consumption good  $Y_t$  is a CES bundle of varieties  $Y_t^j$  with  $j \in [0, 1]$ :

$$Y_t = \left( \int_0^1 (Y_t^j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where  $\varepsilon > 1$  denotes the elasticity of substitution. Monopolistic firms purchase the common input good from households and produce a differentiated variety with linear technology  $Y_t^j = y_t^j$ . These firms sell their output to the final good producers at price  $P_t^j$ , which they can only adjust smoothly ( $dP_t^j = \pi_t^j P_t^j dt$ ) and at a flow cost  $\frac{\kappa}{2} (\pi_t^j)^2 Y_t dt$ , à la Rotemberg (1982). We consider a symmetric equilibrium with  $P_t^j = \mathcal{P}_t$  denoting the aggregate price level and  $\pi_t^j = \pi_t$  denoting its growth rate. Since this setup is standard, we provide the details in appendix A.3 and directly state the resulting New Keynesian Phillips Curve (NKPC):

$$\frac{\mathbb{E}[d\pi_t]}{dt} = \left( r_t^{f,H} - \frac{\mathbb{E}[dY_t]}{Y_t dt} + \varsigma_t^{C,H} \sigma_t^Y \right) \pi_t - \frac{\varepsilon}{\kappa} \left( p_t(1 - \tau_t) - \frac{\varepsilon - 1}{\varepsilon} \right) \quad (10)$$

where  $r_t^{f,H}$  and  $\varsigma_t^{C,H}$  are the shadow risk-free rate and price of aggregate risk of a representative household, respectively,  $Y_t = av_t K_t$  is total final good output with geometric volatility loading  $\sigma_t^Y$  on aggregate shocks  $dZ_t$ , and  $\tau_t$  is the usual subsidy addressing firms' market power. We can rewrite the NKPC in integral form:

$$\pi_t = \frac{\varepsilon}{\kappa Y_t} \mathbb{E}_t \int_t^\infty e^{-\int_t^s r_\tau^{f,H} d\tau} Y_s (m_s - m^f) ds$$

where  $r_t^f = r_t^{f,H} + \varsigma_t^{C,H} \sigma_t^Y$  is the appropriate risk-adjusted discount rate,  $m_s = p_s(1 - \tau_s)$  is the marginal cost and  $m^f = \frac{\varepsilon-1}{\varepsilon}$  is the flex-price marginal cost. The usual interpretation is that firms raise prices ( $\pi_t > 0$ ) whenever their future expected marginal costs  $m_s$  are above flex-price marginal costs  $m^f$ , or alternatively – whenever future expected markups  $1/m_s$  are below the flex-price markup  $\varepsilon/(\varepsilon - 1)$ .<sup>4</sup> Note that in the symmetric equilibrium, firms profits are  $(1 - p_t - \frac{\kappa}{2} \pi_t^2) Y_t$ . Firms transfer these profits to households, together with adjustment costs that they have paid, such that  $\mathfrak{d}_t = av_t - ap_t v_t$  and adjustment costs do not enter the resource constraint.

<sup>4</sup>See also equation (21) in Kaplan, Moll, and Violante (2018). The main difference to our setting is the absence of a risk premium term ( $\varsigma_t^{C,H} \sigma_t^Y$ ) due to the absence of aggregate risk in their setting.

## 2.5 Government

We consider a consolidated government, in which monetary and fiscal sides coordinate their policies. Its budget constraint is given by:

$$d\mathcal{R}_t + P_t^L dL_t + \mathcal{P}_t \tau_t^K K_t dt + \sigma_t^{PL} \sigma_t^L P_t^L L_t dt = i_t^a \mathcal{R}_t dt + i_t^L L_t dt \quad (11)$$

where  $\mathcal{R}_t$  is the stock of reserves issued by the central bank and  $L_t$  are the private holdings of long-term government bonds (total stock net of central bank holdings), evolving according to:

$$\begin{aligned} \frac{d\mathcal{R}_t}{\mathcal{R}_t} &= \mu_t^{\mathcal{R}} dt + \sigma_t^{\mathcal{R}} dZ_t \\ \frac{dL_t}{L_t} &= \mu_t^L dt + \sigma_t^L dZ_t \end{aligned}$$

where drifts and volatility loadings are chosen by the government. The budget constraint shows that government expenses consist of interest rate payments on reserves and coupon payments on long-term bonds, which can be financed by issuance of reserves or long-term bonds, as well as through capital taxation. The last term on the LHS is due to expected gains/losses from stochastic central bank long-term bond trades.<sup>5</sup> The remaining taxes and transfers are assumed to be self-financed and we refer the reader to appendix A.4 for further details.

The budget constraint in equation (11) masks an important assumption of our setup that we now state explicitly. We allow the central bank to directly respond to aggregate shocks when engaging in balance sheet management, whereas treasury's bond issuance and taxes need to be pre-announced and therefore anticipated by private agents before they go into effect. Formally, the central bank can purchase or sell long-term bonds directly loading on aggregate  $dZ_t$  innovations, whereas treasury's ability to issue long-term bonds or raise taxes are limited to drift terms only. This becomes apparent when stating the drift and diffusion terms separately:

$$\mu_t^{\mathcal{R}} \mathcal{R}_t + P_t^L \mu_t^L L_t + \mathcal{P}_t \tau_t^K K_t + \sigma_t^{PL} \sigma_t^L P_t^L L_t = i_t^a \mathcal{R}_t + i_t^L L_t \quad (12)$$

$$\sigma_t^{\mathcal{R}} \mathcal{R}_t + P_t^L \sigma_t^L L_t = 0 \quad (13)$$

The last equation states that whenever the central bank engages in balance sheet management in response to shocks, any long-term bond purchases or sales are directly balanced by issuance or absorption of reserve balances. The capital tax rate (as well all other self-financed taxes and subsidies) can jump upon shock arrivals, but these changes can only

---

<sup>5</sup>If the central bank purchases long-term bonds whenever their price goes down and sells them whenever their price goes up ( $\sigma_t^{PL}$  and  $\sigma_t^L$  are of the same sign), then in expectation it makes a gain, reflected in the drift component. See Appendix G for more details.

be reflected in the drift terms. This assumption is motivated by the factual differences in institutional constraints on the Federal Reserve and the Treasury. Whereas the Treasury can only issue bonds at auction dates set in advance and has little flexibility in adjusting the volume of issuance, the Fed can engage in asset purchases or sales on a daily basis and with greater flexibility. The model captures this difference in policy ‘responsiveness’ between the Fed and the Treasury by allowing the former to trade long-term bonds directly in response to stochastic innovations, and by restricting the corresponding ability of the latter to raise taxes or issue bonds instantaneously upon shock arrival.

We denote by  $\mathcal{B}_t = \mathcal{R}_t + P_t^L L_t$  the total nominal value of government debt (nominal wealth), by  $s_t$  the surplus-to-debt ratio:

$$s_t = \frac{\mathcal{P}_t \tau_t^K K_t}{\mathcal{B}_t} \quad (14)$$

and by  $\vartheta_t^L$  the share of long-term bonds in total nominal wealth:

$$\vartheta_t^L = \frac{P_t^L L_t}{\mathcal{B}_t} \quad (15)$$

While conventional monetary policy controls the short-term rates on reserves, balance sheet policy determines  $\vartheta_t^L$  – the composition of reserves and long-term bonds held by private agents. Given the treasury’s freedom to set capital tax  $\tau_t^K$  and central bank’s freedom to adjust the quantity composition of reserves and long-term bonds in the hands of private agents, we will treat  $s_t$  and  $\vartheta_t^L$  as policy variables. Then, taking into account government budget constraint (11), total nominal wealth  $\mathcal{B}_t$  evolves according to:

$$\frac{d\mathcal{B}_t}{\mathcal{B}_t} = \underbrace{\left[ (1 - \vartheta_t^L) i_t^a + \vartheta_t^L \frac{i_t^L}{P_t^L} - s_t + \vartheta_t^L \mu_t^{PL} \right]}_{\mu_t^{\mathcal{B}}} dt + \underbrace{\vartheta_t^L \sigma_t^{PL}}_{\sigma_t^{\mathcal{B}}} dZ_t \quad (16)$$

which shows that government debt increases because of issuance needed to finance interest rate payments not covered by tax revenues, and in addition because of changes in nominal bond prices.

## 2.6 Aggregation and Distributions

Let  $N_t$  be the total wealth in the economy, consisting of real wealth represented by aggregate value of capital  $q_t^K K_t$  and the real value of nominal wealth  $\mathcal{B}_t/\mathcal{P}_t$ :

$$N_t = q_t^K K_t + \frac{\mathcal{B}_t}{\mathcal{P}_t} \quad (17)$$

We denote by  $\vartheta_t$  the share of nominal assets in total wealth:

$$\vartheta_t = \frac{\mathcal{B}_t}{\mathcal{P}_t N_t} \quad (18)$$

Alternatively, one can decompose total wealth  $N_t$  by splitting it into wealth held by households and intermediaries, denoted by  $N_t^H$  and  $N_t^I$ , respectively:

$$N_t = N_t^H + N_t^I \quad (19)$$

and denoting by  $\eta_t$  the intermediaries' wealth share:

$$\eta_t = \frac{N_t^I}{N_t} \quad (20)$$

Analogously, we let  $\eta_t^L$  be the share of long-term bonds  $L_t$  held by intermediaries:

$$\eta_t^L = \frac{\theta_t^{L,I} N_t^I}{(P_t^L L_t) / \mathcal{P}_t} \quad (21)$$

Finally, we denote by  $\chi_t$  the share of capital risk that is offloaded by households and held by intermediaries. It can be stated as total value of outside equity relative to the total value of capital:

$$\chi_t = \frac{\theta_t^{x,I} N_t^I}{q_t^K K_t} \quad (22)$$

This concludes the model setup and we can proceed with defining the equilibrium.

## 2.7 Equilibrium

Our setup features three state variables and three forward-looking equations that pin down the rest of the economy.<sup>6</sup> We have some freedom in choosing which variables enter the state space and which forward-looking variables become part of the equilibrium definition. We make this choice to ensure a direct link between the equilibrium formulation and the constrained efficient allocation that we discuss later. In the following, we state the equilibrium definition and discuss some of the key equations. For more details, we refer the reader to appendix A.6.

**Definition 1.** *The Markovian equilibrium consists of state variables  $S \equiv \{\tilde{\sigma}^2, \eta, v\}$  with*

---

<sup>6</sup>Formally, aggregate capital stock  $K_t$  is a fourth state variable. However, it is only required to determine the dynamics of gross aggregate variables, but not the distributions or variables normalized by aggregate capital.

corresponding laws of motion

$$\begin{aligned} d\tilde{\sigma}_t^2 &= \mu(\tilde{\sigma}_t^2)dt + \sigma(\tilde{\sigma}_t^2)dZ_t \\ \frac{d\eta_t}{\eta_t} &= \mu_t^\eta dt + \sigma_t^\eta dZ_t \\ \frac{dv_t}{v_t} &= \mu_t^v dt + \sigma_t^v dZ_t, \end{aligned}$$

policy variables  $i^a(S)$ ,  $i^m(S)$ ,  $\vartheta^L(S)$ ,  $s(S)$ ,  $\tau^I(S)$ ,  $\tau^H(S)$ ,  $\tau^x(S)$ ,  $\tau(S)$  satisfying government budget constraints, and mappings  $P^L(S)$ ,  $\vartheta(S)$ ,  $\pi(S)$  satisfying agents' optimality conditions and market clearing.

The state space includes one exogenous state variable (idiosyncratic risk  $\tilde{\sigma}_t^2$ ) and two endogenous ones – net worth share of intermediaries  $\eta_t$  and utilization rate  $v_t$ . Their laws of motion can be derived from the wealth dynamics of households and intermediaries, and from the goods market clearing condition, and are provided in appendix A.6. The volatility loadings of the two endogenous state variables will be key for understanding the roles and interactions of interest rate and balance sheet policies. First, consider the diffusion term of intermediaries' wealth share  $\eta_t$ :

$$\eta_t \sigma_t^\eta = (\chi_t - \eta_t)(1 - \vartheta_t) \sigma_t^{q^K} + (\eta_t^L - \eta_t) \vartheta_t \vartheta_t^L \sigma_t^{P^L} \quad (23)$$

It shows that intermediaries' wealth share response to aggregate shocks depends on capital and nominal bond price responses ( $\sigma_t^{q^K}$  and  $\sigma_t^{P^L}$ ) weighted by their shares in total wealth ( $1 - \vartheta_t$  and  $\vartheta_t \vartheta_t^L$ ) and scaled by intermediaries' relative exposures to these assets. In particular, if  $\chi_t > \eta_t$ , then intermediaries' capital risk share is larger than their wealth share, which makes them effectively more exposed to capital risk than households. In this case, sudden capital price increases redistribute wealth away from households towards intermediaries, and sudden capital price decreases redistribute wealth in the opposite direction. The same logic applies to long-term bonds: if their bond share is larger than their wealth share, they become relatively more exposed to bond price risk than households. Second, consider the goods market clearing condition and the diffusion term of the utilization rate:

$$av_t K_t = \rho \left( q_t^K K_t + \frac{\mathcal{B}_t}{\mathcal{P}_t} \right) + \iota_t K_t \quad (24)$$

$$v_t \sigma_t^v = \frac{q_t^K}{a\phi(1 - \vartheta_t)} \left( (1 + \rho\phi)(1 - \vartheta_t) \sigma_t^{q^K} + \rho\phi \vartheta_t \vartheta_t^L \sigma_t^{P^L} \right) \quad (25)$$

The first line shows that total output is divided between consumption, which is a constant fraction  $\rho$  of total net worth because of log-utility, and investment. It then naturally follows from the second line that utilization's response to aggregate shocks is determined

by the weighted average between capital and bond price responses that drive aggregate net worth volatility. Note that the weighting is not symmetric because capital price also affects investment rate in equilibrium via Tobin's  $q$  relation:

$$\nu_t = \frac{q_t^K - 1}{\phi} \quad (26)$$

and therefore it affects aggregate output both through consumption and investment.

The policy instruments directly include the average and marginal interest rates  $i_t^a$  and  $i_t^m$ , even though those are determined by the interaction of interest rates on excess and required reserves, reserve requirement constraint and the associated Lagrange multiplier, see equations (8) and (9). To simplify the analysis, we let reserve requirements be sufficiently low (but never zero) to ensure that the constraint is never binding and the marginal interest rate  $i_t^m$  is equal to the excess reserves interest rate  $i_t$ . As discussed above, we treat the surplus-to-debt ratio  $s_t$  and long-term bond share in nominal wealth  $\vartheta_t^L$  as a policy choices. The remaining tax rates are peripheral to our analysis and need to satisfy government budget constraints provided in appendix A.6.

We now turn to the discussion of forward-looking variables. The forward-looking equation for inflation  $\pi_t$  is the New Keynesian Phillips Curve (10), which is standard and discussed above in section 2.4. The forward-looking equation for the nominal price of a long-term bond  $P_t^L$  is given by:

$$\frac{\mathbb{E} [dP_t^L]}{P_t^L} = \mu_t^{P^L} = i_t^m - \frac{i_t^L}{P_t^L} + \zeta_t^{n,I} \sigma_t^{P^L} \quad (27)$$

where  $\zeta_t^{n,I}$  denotes intermediaries' price of aggregate risk, given in equilibrium by  $\zeta_t^{n,I} = \sigma_t^\eta - \sigma_t^\vartheta + \sigma_t^B$ . This is a standard pricing condition, according to which the price of a perpetual bond is determined by the path of future *marginal* short-term interest rates, adjusted by risk premium. The last forward-looking equation is for the nominal wealth share  $\vartheta_t$ :

$$\begin{aligned} \frac{\mathbb{E} [d\vartheta_t]}{\vartheta_t} = \mu_t^\vartheta = & \rho - s_t - (1 - \vartheta_t) \left( (1 - \vartheta_t^L) (i_t^m - i_t^a + \nu_t^2 \mathfrak{t}'(\nu_t)) + \tilde{\zeta}_t^{n,H} \tilde{\sigma}_t \right) \\ & - (1 - \eta_t) \theta_t^{D,H} \nu_t^2 \mathfrak{t}'(\nu_t) + \sigma_t^{1-\eta} \sigma_t^\vartheta \end{aligned} \quad (28)$$

where  $\nu_t^2 \mathfrak{t}'(\nu_t)$  is the convenience yield on deposits associated with transaction services, and  $\tilde{\zeta}_t^{n,H}$  is the households' price of idiosyncratic risk. The above equation is best interpreted

in its integral form:

$$\begin{aligned} \vartheta_t = & \mathbb{E}_t \int_t^\infty e^{-\rho(\tau-t)} (s_\tau + (1 - \vartheta_\tau) ((1 - \vartheta_\tau^L) (i_\tau^m - i_\tau^a + \nu_\tau^2 \mathfrak{t}'(\nu_\tau)) + \zeta_\tau^{n,H} \tilde{\sigma}_\tau)) \vartheta_\tau d\tau \\ & + \mathbb{E}_t \int_t^\infty e^{-\rho(\tau-t)} ((1 - \eta_\tau) \theta_\tau^{D,H} \nu_\tau^2 \mathfrak{t}'(\nu_\tau) - \sigma_\tau^{1-\eta} \sigma_\tau^\vartheta) \vartheta_\tau d\tau \end{aligned} \quad (29)$$

There are several factors that determine the equilibrium distribution of wealth across real and nominal assets. First, positive surplus-to-debt ratio  $s_t$  implies that capital is taxed which provides incentives to hold effectively subsidized nominal assets. Second, idiosyncratic risk  $\tilde{\sigma}_t$  generates flight-to-safety motives, increasing the value of nominal assets, free of this type of risk. Third, the difference between marginal and average interest rates  $i_t^m - i_t^a$  allows the central bank to tilt portfolios towards nominal assets independently from fiscal measures.<sup>7</sup> Fourth, convenience yield on deposits increases the value of nominal assets. It appears twice in the above expression – once in the first line in relation to the share of reserves  $1 - \vartheta_t^L$ , which provide the same return as deposits in equilibrium, and once in the second line in relation to the value of deposits held by households. Finally, the aggregate risk associated with  $\vartheta_t$  is priced and affects its value in equilibrium.

We conclude this section by mentioning one more equation that will be key going forward, namely the one linking the convenience yield on deposits to long-term bond price volatility:

$$\frac{\sigma_t^\eta}{1 - \eta_t} \sigma_t^{P^L} = -\frac{\sigma_t^{1-\eta}}{\eta_t} \sigma_t^{P^L} = \nu_t^2 \mathfrak{t}'(\nu_t) \quad (30)$$

The equation provides two insights. First, since  $\nu_t \geq 0$ , the long-term bond price  $P_t^L$  must positively co-move with intermediaries' wealth share  $\eta_t$  in equilibrium and negatively – with households' wealth share  $1 - \eta_t$ . This is so because households have a preference for deposits relative to long-term bonds that do not provide transaction services. They prefer to issue long-term bonds to finance larger deposit holdings. To prevent them from doing so indefinitely, long-term bonds need to be a (relative) hedge for households and a (relative) anti-hedge for intermediaries, counterbalancing the transaction cost benefits, associated with deposits. Second, it highlights the importance of transaction costs for preventing aggregate risk sharing in this economy. Without transaction costs ( $\mathfrak{t}'(\nu_t) = 0$ ), agents share the aggregate consumption risk perfectly ( $\sigma_t^\eta = \sigma_t^{1-\eta} = 0$ ), as long as long-term bond prices move in equilibrium ( $\sigma_t^{P^L} \neq 0$ ).

We now move to the constrained efficient economy that will serve as a benchmark for optimal policy determination.

---

<sup>7</sup>The central bank can increase the marginal interest rate while keeping the average constant. Such a combination does not require financing additional interest rate expenses via taxes or debt dilution, and therefore has zero direct fiscal consequences. However, it has similar portfolio-tilting implications as financing interest rate payments via capital taxes. Effectively, it reproduces the freedom enjoyed by the central bank in a standard New Keynesian model with reserves in zero net supply.

### 3 Constrained Efficiency

To provide a constrained efficient benchmark, we consider a relaxed optimal policy problem, in which the treasury is endowed with additional instruments and is capable of both (i) issuing long-term bonds instantly in response to aggregate shocks and (ii) levying type-specific wealth taxes instantly in response to aggregate shocks. Formally, this introduces volatility loadings on the aggregate Brownian process  $dZ_t$  for the tax liabilities of households and intermediaries, and relaxes the assumption that instantaneous long-term bond purchases must be financed with reserve issuance. Economically, when used jointly with all the other policy instruments, this allows the government to independently choose the paths of net worth distributions across agent types  $\eta_t$  and across assets  $\vartheta_t$ , idiosyncratic risk distribution  $\chi_t$ , and utilization and investment rates  $v_t$  and  $\iota_t$ , only subject to the goods market clearing condition (24) and the Tobin's q rule (26).<sup>8</sup> The government's objective is therefore equivalent to that of a planner who chooses allocations directly (instead of policy instruments), takes investment choice of agents (Tobin's q) as given, and can freely allocate consumption across sectors, but not within sectors. In the following, we analyze the problem of such utilitarian planner.

The planner maximizes welfare of all agents, attaching equal Pareto weights to all individuals. Denote each agent by  $\tilde{i} \in [0, 1]$  and each agent's type at time  $t$  by  $i_t(\tilde{i}) \in \{I, H\}$ . Then consumption (or equivalently wealth) share of agent  $\tilde{i}$ 's sector is  $\eta_t^{i_t(\tilde{i})} = N_t^{i_t(\tilde{i})}/N_t$ , whereas agent's share within the sector is  $\tilde{\eta}_t^{\tilde{i}} = n_t^{\tilde{i}}/N_t^{i_t(\tilde{i})}$ . The planner's objective is to maximize total welfare:

$$W_0 = \max_{\{\iota_t, v_t, \vartheta_t, \eta_t, \chi_t\}_{t=0}^{\infty}} \int_0^1 \left[ \mathbb{E} \int_0^{\infty} e^{-\rho t} \left( \log(\eta_t^{i_t(\tilde{i})} \tilde{\eta}_t^{\tilde{i}} c_t K_t) - b(v_t) \mathbb{1}_{i_t(\tilde{i})=H} \right) dt \right] d\tilde{i}$$

subject to goods market clearing condition and Tobin's q relation:

$$\begin{aligned} c_t &= av_t - \iota_t = \rho \frac{q_t^K}{1 - \vartheta_t} \\ q_t^K &= (1 + \phi \iota_t) \end{aligned}$$

as well as the laws of motion for individual wealth shares within sectors:

$$\frac{d\tilde{\eta}_t^{\tilde{i}}}{\tilde{\eta}_t^{\tilde{i}}} = \begin{cases} \left( \lambda^I - \lambda^H \frac{1-\eta_t}{\eta_t} \right) dt + (1 - \vartheta_t) \frac{\chi_t}{\eta_t} \varphi \tilde{\sigma}_t d\tilde{Z}_t + \left( \frac{\eta_t}{1-\eta_t} - 1 \right) d\tilde{J}_t^I, & \text{if } i_t(\tilde{i}) = I \\ \left( \lambda^H - \lambda^I \frac{\eta_t}{1-\eta_t} \right) dt + (1 - \vartheta_t) \frac{1-\chi_t}{1-\eta_t} \tilde{\sigma}_t d\tilde{Z}_t + \left( \frac{1-\eta_t}{\eta_t} - 1 \right) d\tilde{J}_t^H, & \text{if } i_t(\tilde{i}) = H \end{cases}$$

where  $d\tilde{J}_t^I$  and  $d\tilde{J}_t^H$  are Poisson innovations with type-switching intensities  $\lambda^I$  and  $\lambda^H$  respectively. Net worth shares of individual agents within their sectors have a drift com-

---

<sup>8</sup>We keep the Tobin's q relationship for the constrained planner because otherwise it is optimal to allocate all wealth in nominal assets ( $\vartheta_t = 1$ ) and remove exposure to idiosyncratic risk entirely, as this decision has no consequences for the economy's growth rate.

ponent because sector-level wealth is growing at a different rate than individual-level wealth due to a constant flow of agents switching their types and either leaving or entering the sector. It has an idiosyncratic Brownian component because of idiosyncratic risk in capital, and an idiosyncratic jump component because individual wealth shares changes discontinuously at the time of a type switch, since the sector-level wealth relative to which it is computed changes discontinuously (from  $N_t^I$  to  $N_t^H$  or vice versa). As we show in Appendix B, the planner effectively attaches Pareto weight  $\lambda = \frac{\lambda^H}{\lambda^I + \lambda^H}$  to intermediaries and  $1 - \lambda$  to households, with  $\lambda$  being the physical share of intermediaries. We also show that the solution to the dynamic problem from above is equivalent to the solution of a static problem, in which the planner chooses allocation  $\{\iota_t, v_t, \vartheta_t, \eta_t, \chi_t\}$  period by period, given the current realization of  $\tilde{\sigma}_t^2$ :

$$\begin{aligned} & \max_{\{\iota_t, v_t, \vartheta_t, \eta_t, \chi_t\}} \log(av_t - \iota_t) - (1 - \lambda)b(v_t) + \frac{1}{\rho} \left( \frac{1}{\phi} \log(1 + \phi \iota_t) - \delta \right) \\ & + \lambda \log(\eta_t) + (1 - \lambda) \log(1 - \eta_t) + \frac{1}{\rho} \left[ \lambda \left( \lambda^I - \lambda^H \frac{1 - \eta_t}{\eta_t} \right) + (1 - \lambda) \left( \lambda^H - \lambda^I \frac{\eta_t}{1 - \eta_t} \right) \right] \\ & - \frac{(1 - \vartheta_t)^2 \tilde{\sigma}_t^2}{2\rho} \left[ \lambda \frac{\chi_t^2}{\eta_t^2} \varphi^2 + (1 - \lambda) \frac{(1 - \chi_t)^2}{(1 - \eta_t)^2} \right] \quad \text{s.t.} \quad av_t - \iota_t = \rho \frac{1 + \phi \iota_t}{1 - \vartheta_t} \end{aligned}$$

The reason for the equivalence is that myopic log-utility allows splitting different components of individual consumption into a sum, and the fact that the planner is not bound by equilibrium laws of motion and can freely pick consumption share  $\eta_t$  and utilization  $v_t$ , period by period.

The first line of the planner's objective contains terms affecting production and capital accumulation, which jointly determine the path of *aggregate* output, *aggregate* consumption and *total* wealth. In the following we say that an economy satisfies *aggregate* efficiency if its paths of aggregate output and aggregate consumption coincide with the planner's solution.<sup>9</sup> Note that this requires that both utilization rate  $v_t$  and investment rate  $\iota_t$  are constrained efficient, which necessarily implies constrained efficiency of nominal asset share  $\vartheta_t$  from goods market clearing (24) and Tobin's q (26).

The second and third lines capture the welfare effects of consumption, risk and wealth *distribution*. The first two terms in the second line reflect the direct welfare effect of allocating consumption in proportions  $\eta_t$  and  $1 - \eta_t$  across the two sectors. The third term captures welfare losses that are due to changes in individual consumption growth rates over the lifetime of an individual that occur because of type switching. Consumption smoothing motive calls for setting  $\eta_t = \lambda$  which maximizes the second line of the objective. The third line is the total welfare loss due to exposure to idiosyncratic risk, weighted across households and intermediaries. It depends on the portfolio share allocated to risky capital  $(1 - \vartheta_t)$  and how the risk is distributed ( $\chi_t$ ) relative to consumption shares ( $\eta_t$ ).

---

<sup>9</sup>Under log-utility, the path of aggregate consumption maps one-to-one to the path of total wealth.

In the following we say that an economy satisfies *distributional* efficiency if the paths of consumption and risk distributions across agents, and wealth distribution across assets all coincide with the planner's solution. Note that this requires that  $\eta_t$ ,  $\chi_t$  and  $\vartheta_t$  are all constrained efficient.

In the following we discuss the main properties of the planner's allocation.

**Proposition 1.** *Let  $\varphi \in (0, 1)$ ,  $\lambda \in (0, 1)$  and satisfy assumption (A1),  $\lambda^I > 0$  and  $\lambda^H > 0$  sufficiently small. Then there exists a unique solution to the planner's problem for  $\eta > \lambda$  and the constrained efficient allocation  $\{v^*(\tilde{\sigma}_t^2), \iota^*(\tilde{\sigma}_t^2), \vartheta^*(\tilde{\sigma}_t^2), \eta^*(\tilde{\sigma}_t^2), \chi^*(\tilde{\sigma}_t^2)\}$  has the following properties:*

- *Intermediaries are disproportionately exposed to risk:  $\chi^*(\tilde{\sigma}_t^2) > \eta^*(\tilde{\sigma}_t^2)$*
- *Capital utilization  $v^*(\tilde{\sigma}_t^2)$  is constant in  $\tilde{\sigma}_t^2$ .*
- *Intermediaries' wealth and risk shares  $\eta^*(\tilde{\sigma}_t^2)$  and  $\chi^*(\tilde{\sigma}_t^2)$  are increasing in  $\tilde{\sigma}_t^2$ .*
- *Nominal wealth share  $\vartheta^*(\tilde{\sigma}_t^2)$  is increasing and investment rate  $\iota^*(\tilde{\sigma}_t^2)$  is decreasing in  $\tilde{\sigma}_t^2$ .*

Assumption (A1) requires:

$$6\lambda(1-\lambda)(1-\varphi^2)(1-\lambda+\lambda\varphi^2) - (1-2\lambda)\varphi^2 \geq 0 \quad (\text{A1})$$

and restricts the admissible parameter space as depicted on Figure 6 in appendix B.1.

In general, the planner allocates a higher risk share to intermediaries, relative to their net worth share ( $\chi_t^* > \eta_t^*$ ), as these agents are able to diversify part of the idiosyncratic risk away.<sup>10</sup> Following an increase in idiosyncratic risk, the planner pushes more of this risk towards intermediaries ( $\chi_t^*$  goes up) but compensates them for it by increasing their wealth share  $\eta_t^*$ . At the same time, planner uses an alternative tool to counteract the rise in idiosyncratic risk – reallocates wealth towards nominal assets ( $\vartheta_t^*$  goes up). Finally, since idiosyncratic risk comes from capital, planner scales down on investment  $\iota_t^*$  but keeps utilization  $v_t^*$  constant as it does not interact with risk.

In the next section we compare the constrained efficient allocation with the one arising in competitive equilibrium in the absence of monetary or fiscal interventions, and discuss the frictions that need to be addressed by means of optimal policy.

## 4 Laissez-faire

Suppose that both monetary and fiscal policies are muted, meaning that all tax rates and government surplus are set to zero, the marginal and the average interest rates are

<sup>10</sup>Here and in the following, we interchangeably use two forms of notation for variables corresponding to planner's allocation, e.g.  $\chi_t^* \equiv \chi^*(\tilde{\sigma}_t^2)$ .

constant and equal to each other, the quantities of reserves and long-term bonds in the hands of private agents are constant. Since the marginal interest rate does not move, the nominal price of long-term bonds is constant:  $\sigma_t^{P^L} = 0$ . Rewrite the goods market clearing condition (24), dividing through by aggregate capital  $K_t$  and substituting capital price  $q_t^K$  from the Tobin's q relation (26) :

$$av_t = \rho + (1 + \rho\phi)\iota_t + \rho \frac{\mathcal{R}_t + P_t^L L_t}{\mathcal{P}_t K_t}$$

and consider a shock increasing idiosyncratic risk  $\tilde{\sigma}_t^2$ . Aggregate efficiency requires disinvestment (a decrease in  $\iota_t$ ) and constant utilization  $v_t$ . However, in the competitive equilibrium, the scaled real value of nominal assets  $(\mathcal{R}_t + P_t^L L_t)/\mathcal{P}_t K_t$  does not respond to shocks on impact. This is so because (i) prices are sticky, (ii) aggregate capital stock is unaffected by the change in idiosyncratic risk, and (iii) both quantities and prices of nominal assets are constant over time. Therefore, the only way to keep utilization at the constant efficient level is by inefficiently preventing capital disinvestment. Symmetrically, efficient disinvestment necessarily causes an inefficient decline in utilization. As a result, aggregate efficiency requires active monetary policy that can generate appropriate fluctuations in nominal wealth  $\mathcal{R}_t + P_t^L L_t$  to ensure efficient aggregate responses to shocks.

Consider now intermediaries' balance sheet and the volatility loading of their net worth share on aggregate shocks. Given that the price of long-term bonds is constant, equation (23) reduces to:

$$\eta_t \sigma_t^\eta = (\eta_t - \chi_t) \sigma_t^\vartheta \tag{31}$$

Recall that distributional efficiency requires that intermediaries hold more risk relative to their net worth share, such that  $\eta_t - \chi_t$  must be negative. At the same time, an increase in idiosyncratic risk must lead to a rebalancing of wealth towards nominal assets ( $\sigma_t^\vartheta > 0$ ) and towards intermediaries ( $\sigma_t^\eta > 0$ ). In a laissez-faire competitive equilibrium this is infeasible, since the LHS of equation (31) needs to be strictly positive, whereas its RHS – strictly negative. This is so because the distribution of wealth across assets ( $\vartheta_t$ ) is tightly linked to the distribution of wealth across agents ( $\eta_t$ ). In particular, flight to safety inevitably redistributes wealth away from intermediaries, as they are the ones levered in claims on real capital and short in nominal deposits. We stress that this inefficiency arises not because of incompleteness of the market for aggregate risk, but because of the tight link between aggregate and idiosyncratic risk distributions.<sup>11</sup> Therefore, the role of monetary policy is to counteract this force and redistribute wealth towards intermediaries despite portfolio rebalancing that affects them adversely. This again requires generating

---

<sup>11</sup>The agents can share aggregate risk perfectly and achieve  $\sigma_t^\eta = 0$ , but this requires a particular distribution of idiosyncratic risk, namely that  $\chi_t = \eta_t$ .

appropriate fluctuations in nominal wealth  $\mathcal{R}_t + P_t^L L_t$  to ensure efficient redistribution following shocks.

## 5 Optimal Policy

We now turn to optimal policy characterization, using the instruments available in the market equilibrium. In contrast to the constrained efficient planner, the government in our model can not issue long-term bonds or levy taxes instantly in response to aggregate shocks. This limitation leads to a necessity of a joint conduct of the interest rate and balance sheet policies.

We first study interest rate policy in isolation and its limitations. In this exercise we keep the balance sheet policy ‘passive’, meaning that the central bank maintains a constant  $\vartheta_t^L$ . We then introduce an ‘active’ balance sheet policy, allowing the central bank to optimally vary  $\vartheta_t^L$  and discuss the optimal policy mix.

### 5.1 Passive balance sheet management

Suppose that balance sheet management is passive, i.e.  $\vartheta_t^L = \vartheta^L \in (0, 1)$ , but all the other policy instruments can be set dynamically. As we show below, the government can achieve aggregate efficiency – the optimal path of aggregate consumption and output – by means of an appropriate conventional interest rate policy, but the welfare-maximizing allocation that takes distributional aspects into account can not be achieved.

Aggregate efficiency requires that both utilization rate  $v_t$  and investment rate  $\iota_t$  follow the efficient paths of  $v_t^*$  and  $\iota_t^*$ . However, goods market clearing implies that nominal wealth share  $\vartheta_t$  also follows the efficient path in this case, and it is therefore equivalent to focus on implementing efficient  $v_t$  and  $\vartheta_t$ , achieving efficient  $\iota_t$  as a by-product. Since nominal wealth share  $\vartheta_t$  is a forward-looking variable, it suffices to ensure that the expected change of  $\vartheta_t$  along the equilibrium path is efficient:  $\mu_t^\vartheta = \mu_t^{\vartheta,*}$ . From equation (28) it is apparent that this can be achieved by appropriately setting the surplus-to-debt rate  $s_t$  along the equilibrium path and does not require monetary policy intervention. Utilization  $v_t$ , in turn, is a state variable and therefore efficiency requires that  $v_t$  follows the efficient law of motion. In particular, it must be constant:  $\mu_t^v = \sigma_t^v = 0$ . It can be shown that the efficient drift can be achieved by an appropriate average interest rate ( $i_t^a$ ) policy. The most challenging task is ensuring that utilization responds to aggregate shocks efficiently and its volatility loading is zero. Rearranging (25), we obtain that optimal policy must provide:

$$\vartheta^L \sigma_t^{PL} = \frac{(1 + \rho\phi)}{(1 - \vartheta_t^* + \rho\phi)} \sigma_t^{\vartheta,*} \quad (32)$$

The next proposition states that this remaining condition can be satisfied under any passive balance sheet policy by an appropriate marginal interest policy  $i_t^m$ .

**Proposition 2.**

*Optimal paths of aggregate output and consumption can be achieved under any passive balance sheet policy  $\vartheta^L \in (0, 1)$ . In particular, for any  $\vartheta^L \in (0, 1)$  there exists a corresponding marginal interest rate policy  $i_t^m$  ensuring (32) holds.*

The proposition follows directly from (32) and the fact that marginal interest rate policy  $i_t^m$  can generate arbitrary bond price volatility  $\sigma_t^{PL}$ , see (27). The necessity of monetary policy interventions is standard with sticky prices, but operates in a different way in models with government debt in positive net supply. An increase in idiosyncratic risk leads to portfolio rebalancing towards safe nominal assets. Under sticky prices and absent an appropriate interest rate policy, nominal wealth  $\mathcal{B}_t/\mathcal{P}_t$  can not expand, meaning that capital price  $q_t^K$  must drop to satisfy portfolio rebalancing. As a consequence, this leads to a drop in total wealth and consumption demand on impact and generates a recession. By cutting the marginal interest rate  $i_t^m$  and boosting the price of long-term bonds, the central bank can compensate for the stickiness of the nominal price level and stabilize wealth and consumption. The role of the marginal interest rate policy is to introduce sufficient fluctuations in the bond price in response to aggregate shocks ( $\sigma_t^{PL}$ ), conditional on the passive balance sheet management policy  $\vartheta^L$ .<sup>12</sup>

A direct corollary of Proposition 2 is that a larger central bank balance sheet (a smaller  $\vartheta^L$ ) requires higher bond price volatility going forward. Central bank balance sheet expansion reduces the sensitivity of the economy to subsequent interest rate movements, which needs to be compensated by a larger magnitude of these movements. We highlight that such multiplicity in the mix of optimal interest rate and balance sheet policies is due to the central bank's ability to set the marginal and average interest rates independently. If we restrict  $i_t^m = i_t^a$ , then the optimal path of the single interest rate set to ensure  $\mu^v = 0$  would pin down bond price volatility  $\sigma_t^{PL}$ , which may or may not be sufficient to ensure the efficient response of output on impact for a given arbitrary  $\vartheta^L$ .

Can the government achieve efficient allocation of consumption and risk, and therefore implement the planner's allocation with the remaining instruments? Such policies would aim to additionally achieve distributional efficiency, namely the efficient paths of wealth and idiosyncratic risk distributions  $\eta_t$  and  $\chi_t$ .<sup>13</sup> While risk allocation  $\chi_t$  can be corrected by a corresponding subsidy and the drift of intermediaries' wealth share  $\mu_t^\eta$  – by a wealth tax, achieving the efficient responsiveness of wealth distribution to aggregate shocks ( $\sigma_t^\eta$ ) is challenging. Rearranging (23), we obtain that the following equation must

---

<sup>12</sup>A similar outcome can be achieved with tradable lump-sum taxes, as discussed in Li and Merkel (2025).

<sup>13</sup>Note that the efficient path of wealth distribution across real and nominal assets  $\vartheta_t$  is already implemented under aggregate efficiency.

hold:

$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta,*} + (\chi_t^* - \eta_t^* + \vartheta_t^* (\eta_t^L - \chi_t^*)) \vartheta_t^L \sigma_t^{PL} \quad (33)$$

This is where passive balance sheet policy falls short. To see that, recall that aggregate efficiency determines the required nominal net worth volatility  $\vartheta_t^L \sigma_t^{PL}$  as given by (32). Together with (33) this pins down the implied distribution of long-term bonds across agent types  $\eta_t^L$ . Consider now the deposit convenience yield expression (30) and transaction constraint  $\nu_t \theta_t^{D,H} = \theta_t^K$  stated with equilibrium variables:

$$\frac{\sigma_t^{\eta,*}}{1 - \eta_t^*} \vartheta_t^L \sigma_t^{PL} = \vartheta_t^L \nu_t^2 \mathfrak{t}'(\nu_t) \quad (34)$$

$$\nu_t [\chi_t^* - \eta_t^* + \vartheta_t^* (1 - \chi_t^*) - (1 - \eta_t^L) \vartheta_t^L \vartheta_t^*] = 1 - \vartheta_t^* \quad (35)$$

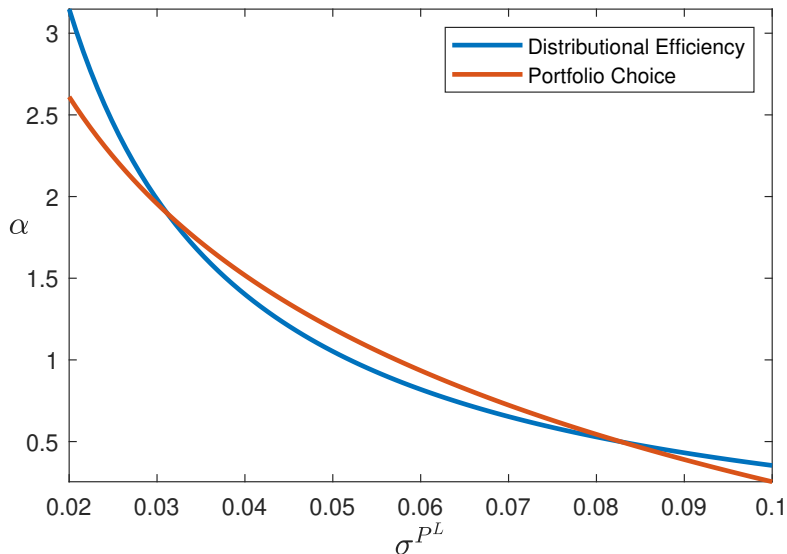
Given the planner's allocation and the required  $\vartheta_t^L \sigma_t^{PL}$  and  $\eta_t^L$ , this is a system of two equations with two unknowns ( $\nu_t$  and  $\vartheta_t^L$ ) that needs to be satisfied at all times  $t$ , i.e. in all states of  $\tilde{\sigma}_t^2$ . With a passive balance sheet policy and therefore a time-invariant  $\vartheta_t^L$ , a solution is generically impossible. For example, the government can choose a balance sheet composition that would ensure efficient dynamics following small shocks around the stochastic steady state. However, as idiosyncratic risk wanders away from the steady state, competitive equilibrium departs further and further from the efficient allocation. This calls for active balance sheet management.

## 5.2 Active balance sheet management

We now allow the central bank to pursue an active balance sheet policy, i.e. to set  $\vartheta_t^L$  as a function of the exogenous state  $\tilde{\sigma}_t^2$ . Since we have already discussed in detail the implementation of the efficient path of aggregate consumption, we now focus on the implementation of its efficient distribution across agents, and only then consider both objectives jointly. In particular, the central bank must set its interest rate and balance sheet policies to ensure that intermediaries' net worth share  $\eta_t$  has an efficient loading on aggregate shocks  $\sigma_t^\eta$  as in (33), at the same time respecting the equilibrium deposit convenience yield (34) and transaction constraint (35). The challenge in this case is the endogeneity of long-term bond holdings distribution  $\eta_t^L$  – the share of long-term bonds held by the intermediaries. The central bank needs to account for the fact that by increasing bond price volatility  $\sigma_t^{PL}$ , it subjects long-term bond holders to larger duration risk and stimulates portfolio reallocation. Since in the efficient equilibrium long-term bonds are a (relative) anti-hedge for the intermediaries ( $\sigma_t^{\eta,*}$  and  $\sigma_t^{PL}$  are of the same sign), higher  $\sigma_t^{PL}$  would lead to a smaller share of long-term bonds held by intermediaries and therefore requires a larger total nominal wealth volatility  $\sigma_t^B$  to ensure efficient redistribution. For

a particular  $\vartheta_t^L$ , we represent this trade-off graphically in Figure 2. On the x-axis we show

Figure 2: Allocation Efficiency



long-term bond price volatility  $\sigma_t^{PL}$ , determined by the marginal interest rate policy  $i_t^m$ . On the y-axis we put intermediaries' share in total long-term bond holdings  $\eta_t^L$ . The red line is intermediaries' portfolio choice – as long-term bonds become riskier, intermediaries scale down on their holdings. The blue line represents intermediaries' long-term bonds exposure, required for the efficient reallocation according to (33) – larger  $\sigma_t^{PL}$  implies larger nominal wealth volatility  $\sigma_t^B$  and allows for smaller intermediaries' share in long-term bond holdings  $\eta_t^L$ . In this example, there are two optimal bond price volatilities  $\sigma_t^{PL}$  for a given  $\vartheta_t^L$ , corresponding to the two intersections. A moderate interest rate policy generates low bond price volatility and incentivizes intermediaries to hold large amounts of long-term bonds. An aggressive interest rate policy leads to high bond price volatility and therefore small long-term bond holdings for the intermediaries. Note that such multiplicity does not apply for any balance sheet policy  $\vartheta_t^L$ , and in fact there could be no solutions for some of these policies. However, the next proposition establishes that there always exists a combination of interest rate and balance sheet policies that delivers the efficient consumption and risk allocation:

**Proposition 3.** *There exists a joint balance sheet and marginal interest rate policy such that (33) is satisfied in the competitive equilibrium.*

We relegate the proof to Appendix C. We highlight that our result does not require long-term bond market segmentation and goes through even if intermediaries are not holding any long-term bonds. Indeed, suppose that  $\eta_t^L = 0$ . In that case, countercyclical

interest rate policy can still implement the efficient allocation by ensuring:

$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta,*} + (\chi_t^* - \eta_t^* - \vartheta_t^* \chi_t^*) \vartheta_t^L \sigma_t^{PL} \quad (36)$$

as long as

$$\chi_t^* - \eta_t^* - \chi_t^* \vartheta_t^* > 0$$

In a competitive equilibrium with  $\eta_t^L = 0$ , the above condition implies that intermediaries issue more deposits than the amount of reserves they hold, which should hold for any reasonable calibration. Despite the fact that households hold all long-term bonds, long-term bond appreciation can still lead to a wealth transfer towards intermediaries. This is because long-term bonds affect the wealth share in two ways: a direct and an indirect way. To see that, rewrite the volatility of  $\eta$  as follows:

$$\eta_t \sigma_t^\eta = (\eta_t - \chi_t) \sigma_t^\vartheta + (\chi_t - \eta_t) (1 - \vartheta_t) \underbrace{\sigma_t^B}_{\vartheta_t^L \sigma_t^{PL}} + (\eta_t^L - \eta_t) \vartheta_t \vartheta_t^L \sigma_t^{PL} \quad (37)$$

The last term on the RHS is the direct effect of bond revaluations, discussed before. The second term on the RHS is the indirect effect, which is independent of intermediaries' long-term bond holdings. The key to understanding this effect are the real wealth effects of nominal wealth changes under sticky prices. Using the definition of  $\vartheta_t$ , we can write:

$$q_t^K = \frac{\mathcal{R}_t + P_t^L L_t}{\mathcal{P}_t K_t} \frac{1 - \vartheta_t}{\vartheta_t}$$

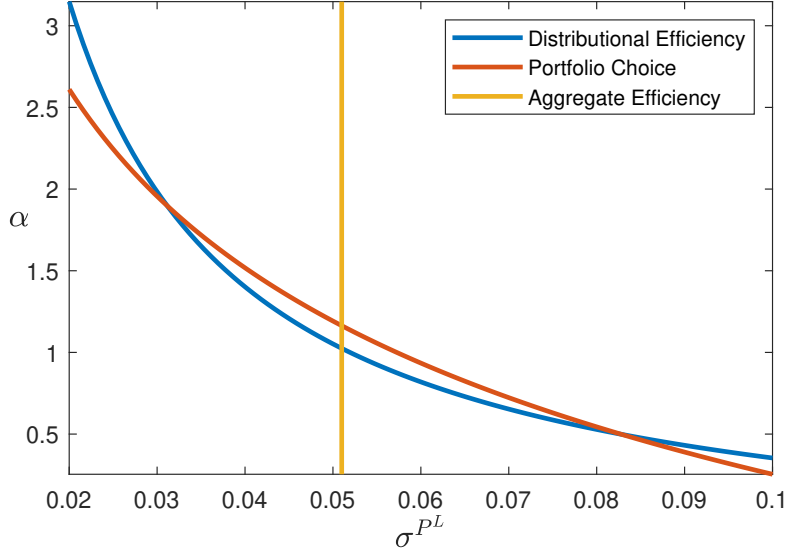
For a given  $\vartheta_t$ , an appreciation of long-term bond price  $P_t^L$  leads to an increase in total nominal wealth  $\mathcal{B}_t = \mathcal{R}_t + P_t^L L_t$  and, with a sticky price level  $\mathcal{P}_t$ , to an appreciation in total real wealth and hence the capital price  $q_t^K$ . As discussed above,  $\chi_t - \eta_t$  is the intermediaries' *relative* exposure to capital price risk. If this positive, an appreciation of capital price  $q_t^K$  redistributes wealth towards intermediaries, independently of the distribution of long-term bond holdings across the two agent types. We note that this indirect effect is equally operative under flexible prices. The only difference is that under flexible prices nominal wealth appreciation leads to a jump in the price level  $\mathcal{P}_t$ , which devalues intermediaries' deposit liabilities and redistributes wealth towards them, again precisely because intermediaries are levered in capital ( $\chi_t > \eta_t$ ).

### 5.3 Implementing the optimal allocation

Implementing the optimal allocation requires achieving both the efficient path of aggregate consumption, as well as its distribution across sectors. The former pins down the required nominal wealth volatility  $\sigma_t^B$  given by (32). This serves as an additional constraint for interest rate and balance sheet policy that deliver allocation efficiency. Graphically, it

adds a vertical line to Figure 2, requiring the intersection of all three: As before, the lines

Figure 3: Optimal policy



are depicted for a fixed balance sheet policy  $\vartheta_t^L$  and implementing the optimal allocation requires active management of  $\vartheta_t^L$ . Whether the solution exists (or is unique) depends on functional forms and parameter values. Suppose that a solution exists, then one can construct the bond price  $P_t^L$  as function of  $\tilde{\sigma}_t^2$ , e.g. by normalizing the value at the stochastic steady state to one and using the optimal  $\sigma_t^{PL}$  to compute the implied values on the entire state space. Given the bond price mapping  $P^L(\tilde{\sigma}_t^2)$ , one can uniquely identify the required marginal interest rate policy  $i_t^m$  that would ensure the required  $P_t^L$  along the equilibrium path from the equilibrium bond pricing condition:

$$\mu_t^{PL} = i_t^m - \frac{i_t^L}{P_t^L} + \sigma_t^{PL} \left( \sigma_t^{\eta,*} - \sigma_t^{\vartheta,*} + \sigma_t^{\mathcal{B}} \right)$$

The remaining steps require setting intermediation subsidy  $\tau_t^x$ , average interest rate  $i_t^a$ , surplus-to-debt ratio  $s_t$  and wealth taxes  $\tau_t^I$  and  $\tau_t^H$  to ensure that  $\chi_t$  and drifts of  $\eta_t$ ,  $\vartheta_t$ , and  $v_t$  are efficient on the equilibrium path.

If, however, there is no optimal solution, then constrained efficient allocation is not compatible with competitive equilibrium and the government faces a trade-off between aggregate and distributional efficiency. The next proposition establishes existence and uniqueness of an optimal policy mix under some conditions on the transaction cost function  $\mathfrak{t}(\nu)$ .

**Proposition 4.** *Suppose that transaction cost function  $\mathfrak{t}(\nu)$  is such that (i)  $\mathfrak{t}'(\nu) \cdot (\nu^2 - b_t \nu)$  has only one extremum on the  $(0, b_t)$  interval for all  $t$ , where:*

$$b_t = \frac{1 - \vartheta_t^*}{\chi_t^* - \eta_t^* + \vartheta_t^*(1 - \chi_t^*)} > 0$$

and (ii):

$$\mathfrak{t}' \left( \frac{1 - \vartheta_t^*}{\chi_t^* - \eta_t^* + \vartheta_t^* (\eta_t^{L,*} - \chi_t^*)} \right) \geq \left( \frac{\chi_t^* - \eta_t^* + \vartheta_t^* (\eta_t^{L,*} - \chi_t^*)}{1 - \vartheta_t^*} \right)^2 \frac{\sigma_t^{\eta,*}}{1 - \eta_t^*} \frac{(1 + \rho\phi)}{(1 - \vartheta_t^* + \rho\phi)} \sigma_t^{\vartheta,*} \quad (38)$$

where  $\eta_t^{L,*}$  satisfies:

$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta,*} + (\chi_t^* - \eta_t^* + \vartheta_t^* (\eta_t^{L,*} - \chi_t^*)) \frac{(1 + \rho\phi)}{(1 - \vartheta_t^* + \rho\phi)} \sigma_t^{\vartheta,*} \quad (39)$$

Then there exists a unique optimal policy mix  $\{\vartheta_t^L, i_t^m\}$ .

We provide the proof in Appendix D. Condition (i) is more technical and requires the transaction cost function  $\mathfrak{t}(\nu)$  to have a sufficiently ‘well-behaved’ derivative, meaning that the rate at which  $\mathfrak{t}'(\nu)$  increases with  $\nu$  does not vary ‘too much’. It is needed for uniqueness but is irrelevant for existence. As an example, it is easy to verify that power functions with derivatives of the form  $\mathfrak{t}'(\nu) = c\nu^{p-1}$  with  $c > 0$  and  $p \geq 2$  satisfy this condition. Condition (ii) is more economic and demands the transaction cost function to have a sufficiently large derivative in a relevant range, which is determined by the efficient planner’s allocation. In other words, transaction costs need to be sufficiently sensitive to changes in deposit velocity in the relevant region. Absent this sensitivity, the government can not impose the required distribution of long-term bonds across the two agent types  $\eta_t^{L,*}$  by varying the bond price sensitivity to aggregate shocks  $\sigma_t^{PL}$  and therefore can not achieve the efficient allocation.

This sensitivity can be linked to the degree of bond market segmentation. As one extreme scenario, consider the limiting case in which  $\mathfrak{t}'(\nu)$  converges to zero everywhere, meaning that deposits lose their special role for households and condition (ii) is violated. In such an economy, the degree of bond market segmentation is zero and households and intermediaries can perfectly hedge against aggregate risk by trading long-term bonds and deposits. As a result, the government is not capable of achieving the efficient allocation, which requires certain exposure to aggregate risk. The opposite – complete bond market segmentation – would yield the same result. In a model where only intermediaries (or households) can hold long-term bonds,  $\eta_t^L$  is fixed at one (or zero), which also prevents the central bank from manipulating the distribution of long-term bond holdings across the agents and ensuring efficient risk exposure.

## 5.4 Optimal policy over the cycle

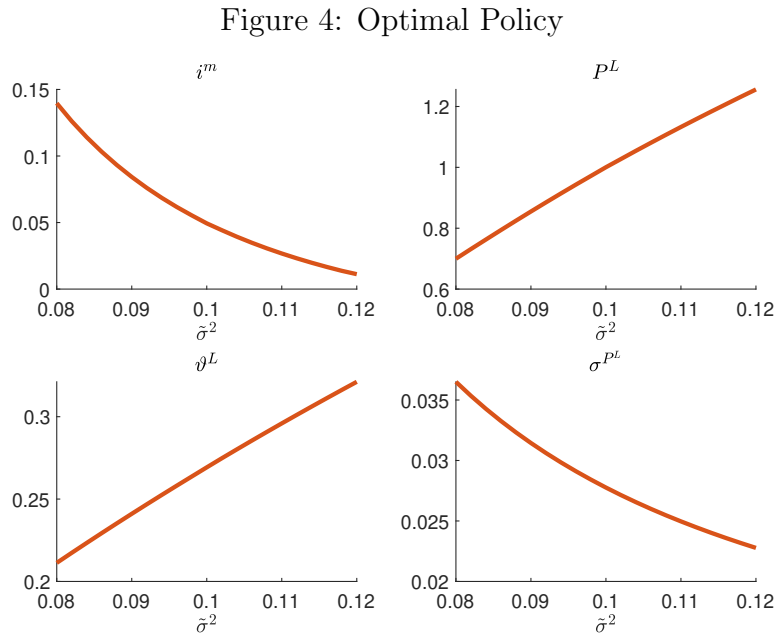
In previous sections we described the general implementation of the optimal policy mix, without providing specific details on the directions in which policy instruments must move

over the business cycle. While increases in idiosyncratic risk call for interest rate cuts,<sup>14</sup> the prescription for balance sheet policy  $\vartheta_t^L$  is ambiguous. We now consider a special case in which we can characterize the balance sheet policy as well.

**Proposition 5.** *Suppose that idiosyncratic risk  $\tilde{\sigma}_t^2$  has constant geometric volatility  $\sigma$ :  $\sigma(\tilde{\sigma}_t^2) = \sigma\tilde{\sigma}_t^2$ , and let  $\tilde{\sigma}_t^2$  be close to zero. Then, in response to an increase in idiosyncratic risk, the optimal policy response consists of:*

- a cut in the marginal interest rate  $i_t^m$
- a milder interest rate policy going forward (lower  $\sigma_t^{PL}$ )
- an increase in long-term bond share (higher  $\vartheta_t^L$ )

The proposition is illustrated on Figure 4. As idiosyncratic risk increases, the central



bank cuts the marginal interest rate to generate long-term bond price appreciation and efficient redistribution of wealth towards intermediaries, along the two channels discussed in the previous section. Following the initial cut, the interest rate policy going forward becomes milder, reducing the long-term bond price volatility. This is needed to prevent intermediaries from scaling down on their long-term bond holdings and to secure the efficient redistribution in response to future shocks. As the bond price sensitivity  $\sigma_t^{PL}$  goes down, a larger share of long-term bonds in the economy  $\vartheta_t^L$  is needed to preserve the efficient volatility of nominal wealth  $\sigma_t^B = \vartheta_t^L \sigma_t^{PL}$  required for the efficient path of aggregate output and consumption.

<sup>14</sup>This is evident from (32), where the LHS is positive by Proposition 1, meaning that  $\sigma_t^{PL}$  must be positive. This requires the central bank to cut the marginal interest rate when idiosyncratic risk goes up to ensure long-term bond price appreciation.

## 5.5 Varying aggregate risk

Finally, we perform a comparative statics exercise with respect to aggregate risk.

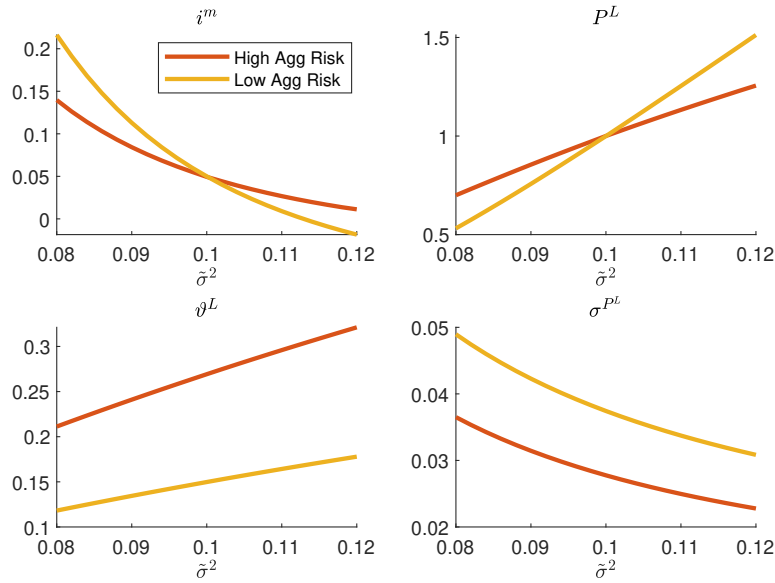
**Proposition 6.** *Suppose that idiosyncratic risk  $\tilde{\sigma}_t^2$  has constant geometric volatility  $\sigma$ :  $\sigma(\tilde{\sigma}_t^2) = \sigma\tilde{\sigma}_t^2$  and there exists a unique optimal policy mix implementing the efficient allocation. Denote by  $\frac{\partial \log \vartheta_t^L}{\partial \log \sigma}$  and  $\frac{\partial \log \sigma_t^{PL}}{\partial \log \sigma}$  the elasticities of the optimal balance sheet policy and bond volatility, respectively, with respect to aggregate risk  $\sigma$ . Then:*

$$\begin{cases} \frac{\partial \log \vartheta_t^L}{\partial \log \sigma} = 2, & \frac{\partial \log \sigma_t^{PL}}{\partial \log \sigma} = -1 & \text{if } \eta_t^{L,*} = 1 \\ \frac{\partial \log \vartheta_t^L}{\partial \log \sigma} > 2, & \frac{\partial \log \sigma_t^{PL}}{\partial \log \sigma} < -1 & \text{if } \eta_t^{L,*} > 1 \\ \frac{\partial \log \vartheta_t^L}{\partial \log \sigma} < 2, & \frac{\partial \log \sigma_t^{PL}}{\partial \log \sigma} > -1 & \text{if } \eta_t^{L,*} < 1 \end{cases}$$

where  $\eta_t^{L,*}$  is the share of long-term bonds held by the intermediaries, as implied by the optimal policy.

The proposition states that as long as optimal bond distribution  $\eta_t^{L,*}$  is close to one, the central bank needs to contract its balance sheet in response to an increase in aggregate risk, accompanied by a milder interest rate policy, ensuring lower bond price volatility. The intuition is that intermediaries choose lower long-term bond holdings as aggregate risk goes up, decreasing their exposure to long-term bond price risk and preventing efficient redistribution. As a countermeasure, the central bank moderates its interest risk policy and decreases the risk carried by long-term bonds. However, this calls for a larger amount of long-term bonds held by private agents to ensure efficient redistribution, which requires the central bank to increase private holdings of long-term bonds. These policy responses are similar to the ones following an increase in idiosyncratic risk (when it is close to zero) as presented in Proposition 5. Figure 5 shows optimal policy as a function of idiosyncratic risk  $\tilde{\sigma}^2$  for two economies – with low and high aggregate risk  $\sigma$ . In line with proposition 6, the economy with larger aggregate risk features a smaller central bank balance sheet (larger  $\vartheta_t^L$ ) and a more moderate marginal interest rate policy.

Figure 5: Optimal Policy



## References

- Brunnermeier, Markus K. and Yuliy Sannikov (2016). *The I theory of money*. Working Paper.
- Eren, Egemen, Timothy Jackson, and Giovanni Lombardo (2024). *The macroprudential role of central bank balance sheets*. Working Paper 1173.
- Gertler, Mark and Peter Karadi (2011). “A model of unconventional monetary policy”. In: *Journal of Monetary Economics* 58.1, pp. 17–34.
- Haddad, Valentin, Alan Moreira, and Tyler Muir (2024). *Asset Purchase Rules: How QE Transformed the Bond Market*. Working Paper.
- Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante (2018). “Monetary Policy According to HANK”. In: *American Economic Review* 108.3, 697–743.
- Karadi, Peter and Anton Nakov (2021). “Effectiveness and addictiveness of quantitative easing”. In: *Journal of Monetary Economics* 117, pp. 1096–1117.
- Li, Ziang and Sebastian Merkel (2025). *Flight to Safety and New Keynesian Demand Recessions*. Working Paper.
- Merkel, Sebastian (2020). *The Macro Implications of Narrow Banking: Financial Stability versus Growth*. Working Paper.
- Nisticò, Salvatore and Marialaura Seccareccia (2025). *Unconventional Policy and Idiosyncratic Risk*. Working Paper 7/2022.
- Rotemberg, Julio J. (1982). “Sticky Prices in the United States”. In: *Journal of Political Economy* 90.6, pp. 1187–1211.

# A Model Solution

In this section, as well as in the rest of the paper, we denote arithmetic drifts and volatility loadings with a subscript and geometric ones – with a superscript, such that for any variable  $x_t$ :

$$\begin{aligned} dx_t &= \mu_{x,t}dt + \sigma_{x,t}dZ_t \\ \frac{dx_t}{x_t} &= \mu_t^x dt + \sigma_t^x dZ_t \end{aligned}$$

## A.1 Households' problem

Households invest in capital, deposits and long-term bonds, and issue outside equity. They need liquid deposits to ensure smooth maintenance of capital, and the amount of deposits required depends on their velocity  $\nu_t$ :  $\nu_t D_t^H = q_t^K k_t$ , where  $D_t^H$  is the real value of held deposits. Higher velocity allows households to hold fewer deposits but comes at a cost  $\mathfrak{t}(\nu_t)$  that reduces return on capital. The cost itself is indexed by time as it is relative to the average cost in the economy, implying that in equilibrium the actual incurred cost is zero. This introduces a wedge in households' decisions without affecting the resource constraint. Formally, given average velocity  $\bar{\nu}_t$ , transaction cost for an individual choice of  $\nu_t$  is given by:

$$\mathfrak{t}(\nu_t) \equiv \mathfrak{t}(\nu_t) - \mathfrak{t}(\bar{\nu}_t)$$

where  $\mathfrak{t}(\cdot)$  is an increasing convex function. In addition, households face dis-utility of capital utilization, denoted by an increasing and convex function  $b(\nu_t)$ . With intensity  $\lambda^H$  a household becomes an intermediary, keeping their net worth. Denote by  $\theta_t^{D,H}, \theta_t^{L,H}, \theta_t^K, \theta_t^{x,H}$  households' portfolio weights on deposits, long-term bonds, capital and outside equity, respectively. They solve the following problem:

$$\begin{aligned} V_0^H &= \max_{c_t^H, \nu_t, \iota_t, \nu_t, \theta_t^{D,H}, \theta_t^{L,H}, \theta_t^K, \theta_t^{x,H}} \mathbb{E} \left[ \int_0^T e^{-\rho t} (\log(c_t^H) - b(\nu_t)) dt + e^{-\rho T} V_T^I \right] \quad \text{s.t.} \\ \frac{dn_t^H}{n_t^H} &= -\frac{c_t^H}{n_t^H} dt + \theta_t^{D,H} dr_t^D + \theta_t^{L,H} dr_t^L + \theta_t^K dr_t^K(\nu_t, \iota_t, \nu_t) + \theta_t^{x,H} dr_t^{x,H} + \tau_t^H dt \\ 1 &= \theta_t^{D,H} + \theta_t^{L,H} + \theta_t^K + \theta_t^{x,H} \\ \nu_t \theta_t^{D,H} &= \theta_t^K, \end{aligned}$$

where  $T$  is the random type switching time. Denoting by  $\chi_t$  the share of risk that households offload to intermediaries and substituting  $\theta_t^{x,H} = -\chi_t \theta_t^K$ ,  $\theta_t^K = \nu_t \theta_t^{D,H}$ , and

$\theta_t^{L,H} = 1 - \theta_t^{D,H} - \theta_t^K(1 - \chi_t)$  one can rewrite the problem as:

$$V_0^H = \max_{c_t^H, v_t, \iota_t, \nu_t, \theta_t^{D,H}, \chi_t} \mathbb{E} \left[ \int_0^T e^{-\rho t} (\log(c_t^H) - b(v_t)) dt + e^{-\rho T} V_T^I \right] \quad \text{s.t.}$$

$$\frac{dn_t^H}{n_t^H} = -\frac{c_t^H}{n_t^H} dt + dr_t^L + \theta_t^{D,H} (dr_t^D - dr_t^L + \nu_t(dr_t^K(v_t, \iota_t, \nu_t) - dr_t^L - \chi_t(dr_t^{x,H} - dr_t^L))) + \tau_t^H dt$$

Write returns in the following way:

$$\begin{aligned} dr_t^K(v_t, \iota_t, \nu_t) &= r_t^K(v_t, \iota_t, \nu_t)dt + \sigma_t^{r,K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t \\ dr_t^D &= r_t^D dt + \sigma_t^{r,D} dZ_t \\ dr_t^L &= r_t^L dt + \sigma_t^{r,L} dZ_t \\ dr_t^{x,H} &= r_t^x dt + \sigma_t^{r,K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t \end{aligned}$$

The state is  $n_t^H$  and let  $\xi_t^H$  be the co-state. Note that the random switch from household to intermediary type does not affect net worth  $n_t^H$  but only the value associated with it, and therefore only introduces an idiosyncratic jump risk component in the co-state  $\xi_t^H$ . Then the Hamiltonian and the co-state evolution for households are given by:

$$\begin{aligned} H_t^H &= e^{-\rho t} (\log(c_t^H) - b(v_t)) - \xi_t^H c_t^H \\ &+ \xi_t^H n_t^H \left[ r_t^L + \theta_t^{D,H} (r_t^D - r_t^L + \nu_t(r_t^K(v_t, \iota_t, \nu_t) - r_t^L - \chi_t(r_t^x - r_t^L))) + \tau_t^H \right] \\ &- \xi_t^H n_t^H \zeta_t^H \left[ \sigma_t^{r,L} + \theta_t^{D,H} (\sigma_t^{r,D} - \sigma_t^{r,L} + \nu_t(1 - \chi_t)(\sigma_t^{r,K} - \sigma_t^{r,L})) \right] \\ &- \xi_t^H n_t^H \tilde{\zeta}_t^H \nu_t \theta_t^{D,H} (1 - \chi_t) \tilde{\sigma}_t \\ d\xi_t^H &= -\frac{\partial H_t^H}{\partial n_t^H} dt - \zeta_t^{\xi,H} \xi_t^H dZ_t - \tilde{\zeta}_t^{\xi,H} \xi_t^H d\tilde{Z}_t + j_t^{\xi,H} \xi_t^H (dJ_t^H - \lambda^H dt) \end{aligned}$$

with  $j_t^{\xi,H} = \frac{\xi_t^I - \xi_t^H}{\xi_t^H}$ . The FOCs wrt  $c_t^H$ ,  $\iota_t$ ,  $v_t$ ,  $\nu_t$ ,  $\theta_t^{D,H}$  and  $\chi_t$  are:

$$\begin{aligned} \xi_t^H &= e^{-\rho t} \frac{1}{c_t^H} \\ \frac{\partial r_t^K(v_t, \iota_t, \nu_t)}{\partial \iota_t} &= 0 \\ e^{-\rho t} b'(v_t) &= \xi_t^H n_t^H \nu_t \theta_t^{D,H} \frac{\partial r_t^K(v_t, \iota_t, \nu_t)}{\partial \nu_t} \\ r_t^K(v_t, \iota_t, \nu_t) - r_t^L - \chi_t(r_t^x - r_t^L) + \nu_t \frac{\partial r_t^K(v_t, \iota_t, \nu_t)}{\partial \nu_t} &= \zeta_t^H (1 - \chi_t) (\sigma_t^{r,K} - \sigma_t^{r,L}) + \tilde{\zeta}_t^H (1 - \chi_t) \tilde{\sigma}_t \\ r_t^D - r_t^L + \nu_t (r_t^K(v_t, \iota_t, \nu_t) - r_t^L - \chi_t(r_t^x - r_t^L)) &= \zeta_t^H (\sigma_t^{r,D} - \sigma_t^{r,L} + \nu_t(1 - \chi_t)(\sigma_t^{r,K} - \sigma_t^{r,L})) \\ &+ \tilde{\zeta}_t^H \nu_t (1 - \chi_t) \tilde{\sigma}_t \\ r_t^x - r_t^L &= \zeta_t^H (\sigma_t^{r,K} - \sigma_t^{r,L}) + \tilde{\zeta}_t^H \tilde{\sigma}_t \end{aligned}$$

We use the result that  $c_t^H = \rho n_t^H$  and manipulate the last three equations to obtain:<sup>15</sup>

$$\xi_t^H = e^{-\rho t} \frac{1}{\rho n_t^H} \quad (40)$$

$$\frac{\partial r_t^K(v_t, \iota_t, \nu_t)}{\partial \nu_t} = 0 \quad (41)$$

$$\rho b'(v_t) = \nu_t \theta_t^{D,H} \frac{\partial r_t^K(v_t, \iota_t, \nu_t)}{\partial \nu_t} \quad (42)$$

$$r_t^K(v_t, \iota_t, \nu_t) - r_t^L = \zeta_t^H (\sigma_t^{r,K} - \sigma_t^{r,L}) + \tilde{\zeta}_t^H \tilde{\sigma}_t - \nu_t \frac{\partial r_t^K(v_t, \iota_t, \nu_t)}{\partial \nu_t} \quad (43)$$

$$r_t^D - r_t^L = \zeta_t^H (\sigma_t^{r,D} - \sigma_t^{r,L}) + \nu_t^2 \frac{\partial r_t^K(v_t, \iota_t, \nu_t)}{\partial \nu_t} \quad (44)$$

$$r_t^x - r_t^L = \zeta_t^H (\sigma_t^{r,K} - \sigma_t^{r,L}) + \tilde{\zeta}_t^H \tilde{\sigma}_t \quad (45)$$

Combining the FOCs with the co-state equation, we obtain:

$$\frac{d\xi_t^H}{\xi_t^H} = - \left[ r_t^L - \sigma_t^{n,H} \sigma_t^{r,L} + \tau_t^H \right] dt - \sigma_t^{n,H} dZ_t - \tilde{\sigma}_t^{n,H} d\tilde{Z}_t$$

Note that the SDF does not have a jump term because neither net worth nor consumption-to-net worth ratio is affected by the type switch. Using FOCs and the fact that  $\zeta_t^H = \sigma_t^{n,H}$ ,  $\tilde{\zeta}_t^H = \tilde{\sigma}_t^{n,H}$ , net worth of households evolves as follows:

$$\begin{aligned} \frac{dn_t^H}{n_t^H} &= \left[ -\rho + r_t^L + \sigma_t^{n,H} (\sigma_t^{n,H} - \sigma_t^{r,L}) + (\tilde{\sigma}_t^{n,H})^2 + \tau_t^H \right] dt \\ &+ \left[ \sigma_t^{r,L} + \theta_t^{D,H} (\sigma_t^{r,D} - \sigma_t^{r,L} + \nu_t(1 - \chi_t) (\sigma_t^{r,K} - \sigma_t^{r,L})) \right] dZ_t \\ &+ \theta_t^{D,H} \nu_t(1 - \chi_t) \tilde{\sigma}_t d\tilde{Z}_t \end{aligned} \quad (46)$$

## A.2 Intermediaries' problem

Intermediaries invest in outside equity, reserves and long-term bonds, and issue deposits. They face the required reserves constraint and become households with intensity  $\lambda^I$ . Denote by  $\theta_t^{D,I}, \theta_t^{L,I}, \theta_t^{\mathcal{R}}, \theta_t^{x,I}$  intermediaries' portfolio weights on deposits, long-term bonds, reserves and outside equity, respectively. The objective of intermediaries is as follows:

$$\begin{aligned} V_0^I &= \max_{c_t^I, \theta_t^{D,I}, \theta_t^{L,I}, \theta_t^{\mathcal{R}}, \theta_t^{x,I}} \mathbb{E} \left[ \int_0^T e^{-\rho t} \log(c_t^I) dt + e^{-\rho T} V_T^H \right] \quad \text{s.t.} \\ \frac{dn_t^I}{n_t^I} &= -\frac{c_t^I}{n_t^I} dt + dr_t^L + \theta_t^{D,I} (dr_t^D - dr_t^L) + \theta_t^{x,I} (dr_t^{x,I} - dr_t^L) + \theta_t^{\mathcal{R}} (dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) - dr_t^L) + \tau_t^I \\ \theta_t^{\mathcal{R}} &\geq \underline{\theta}_t^{\mathcal{R}} \end{aligned}$$

<sup>15</sup>The  $c_t^H = \rho n_t^H$  result is not immediate despite the log-utility specification because of the type-switching risk and in fact requires equal discount rates for households and intermediaries, as shown in appendix H.

where we have used  $1 = \theta_t^{D,I} + \theta_t^{L,I} + \theta_t^{\mathcal{R}} + \theta_t^{x,I}$ . Write returns in the following way:

$$\begin{aligned} dr_t^{\mathcal{R}} &= r_t^{\mathcal{R}}(\theta_t^{\mathcal{R}})dt + \sigma_t^{r,\mathcal{R}}dZ_t \\ dr_t^D &= r_t^D dt + \sigma_t^{r,D}dZ_t \\ dr_t^L &= r_t^L dt + \sigma_t^{r,L}dZ_t \\ dr_t^{x,I} &= (r_t^x + \tau_t^x) dt + \sigma_t^{r,K}dZ_t + \varphi\tilde{\sigma}_t d\tilde{Z}_t \end{aligned}$$

The state is  $n_t^I$  and let  $\xi_t^I$  be the co-state. As with households, the random type switch does not affect net worth  $n_t^I$  and only introduces idiosyncratic jump risk component in the co-state  $\xi_t^I$ . Then the Hamiltonian and the co-state evolution for intermediaries are given by:

$$\begin{aligned} H_t^I &= e^{-\rho t} \log c_t^I - \xi_t^I c_t^I \\ &+ \xi_t^I n_t^I \left[ r_t^L + \theta_t^{D,I}(r_t^D - r_t^L) + \theta_t^{x,I}(r_t^x + \tau_t^x - r_t^L) + \theta_t^{\mathcal{R}}(r_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) - r_t^L) + \tau_t^I \right] \\ &- \xi_t^I n_t^I \zeta_t^I \left[ \sigma_t^{r,L} + \theta_t^{D,I}(\sigma_t^{r,D} - \sigma_t^{r,L}) + \theta_t^{x,I}(\sigma_t^{r,K} - \sigma_t^{r,L}) + \theta_t^{\mathcal{R}}(\sigma_t^{r,\mathcal{R}} - \sigma_t^{r,L}) \right] \\ &- \xi_t^I n_t^I \tilde{\zeta}_t^I \theta_t^{x,I} \varphi \tilde{\sigma}_t + \lambda_t^{\mathcal{R}} \zeta_t^I n_t^I (\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}}) \end{aligned}$$

with  $j_t^{\xi,I} = \frac{\xi_t^I - \xi_t^H}{\xi_t^I}$ . The FOCs wrt  $c_t^I$ ,  $\theta_t^{D,I}$ ,  $\theta_t^{x,I}$  and  $\theta_t^{\mathcal{R}}$  are:

$$\xi_t^I = e^{-\rho t} \frac{1}{c_t^I} \quad (47)$$

$$r_t^D - r_t^L = \zeta_t^I (\sigma_t^{r,D} - \sigma_t^{r,L}) \quad (48)$$

$$r_t^x + \tau_t^x - r_t^L = \zeta_t^I (\sigma_t^{r,K} - \sigma_t^{r,L}) + \tilde{\zeta}_t^I \varphi \tilde{\sigma}_t \quad (49)$$

$$r_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) - r_t^L + \theta_t^{\mathcal{R}} \frac{\partial r_t^{\mathcal{R}}(\theta_t^{\mathcal{R}})}{\partial \theta_t^{\mathcal{R}}} = \zeta_t^I (\sigma_t^{r,\mathcal{R}} - \sigma_t^{r,L}) - \lambda_t^{\mathcal{R}} \quad (50)$$

$$\lambda_t^{\mathcal{R}} \xi_t^I n_t^I (\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}}) = 0$$

We again use the result that  $c_t^I = \rho n_t^I$  and combine the FOCs with the co-state equation to obtain:

$$\begin{aligned} \xi_t^I &= e^{-\rho t} \frac{1}{\rho n_t^I} \\ \frac{d\xi_t^I}{\xi_t^I} &= - \left[ r_t^L - \sigma_t^{n,I} \sigma_t^{r,L} - \theta_t^{\mathcal{R}} \left( \lambda_t^{\mathcal{R}} + \theta_t^{\mathcal{R}} \frac{\partial r_t^{\mathcal{R}}(\theta_t^{\mathcal{R}})}{\partial \theta_t^{\mathcal{R}}} \right) + \tau_t^I \right] dt - \sigma_t^{n,I} dZ_t - \tilde{\sigma}_t^{n,I} d\tilde{Z}_t \end{aligned}$$

Using FOCs and the fact that  $\zeta_t^I = \sigma_t^{n,I}$ ,  $\tilde{\zeta}_t^I = \tilde{\sigma}_t^{n,I}$ , net worth of households evolves as follows:

$$\begin{aligned} \frac{dn_t^I}{n_t^I} = & \left[ -\rho + r_t^L + \sigma_t^{n,I} (\sigma_t^{n,I} - \sigma_t^{r,L}) + (\tilde{\sigma}_t^{n,I})^2 - \theta_t^{\mathcal{R}} \left( \lambda_t^{\mathcal{R}} + \theta_t^{\mathcal{R}} \frac{\partial r_t^{\mathcal{R}}(\theta_t^{\mathcal{R}})}{\partial \theta_t^{\mathcal{R}}} \right) + \tau_t^I \right] dt \\ & + \left[ \sigma_t^{r,L} + \theta_t^{D,I} (\sigma_t^{r,D} - \sigma_t^{r,L}) + \theta_t^{x,I} (\sigma_t^{r,K} - \sigma_t^{r,L}) + \theta_t^{\mathcal{R}} (\sigma_t^{r,\mathcal{R}} - \sigma_t^{r,L}) \right] dZ_t \\ & + \theta_t^{x,I} \varphi \tilde{\sigma}_t d\tilde{Z}_t \end{aligned} \quad (51)$$

### A.3 Monopolistic firms

The firm setup is standard and follows Li and Merkel (2025). Final goods firms have no market power and aggregate varieties into final consumption good using CES technology:  $Y_t = \left( \int_0^1 (Y_t^j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$  with elasticity of substitution  $\varepsilon > 1$ . Their demand for intermediate good  $j$  is given by  $Y_t^j = \left( \frac{P_t^j}{\mathcal{P}_t} \right)^{-\varepsilon} Y_t$ , with  $P_t^j$  denoting  $j$ 's price and  $\mathcal{P}_t = \left( \int_0^1 (P_t^j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ .

Monopolistic firms purchase the common input good from households and produce a differentiated variety with linear technology  $Y_t^j = y_t^j$ . These firms sell their output to the final good producers at price  $P_t^j$ , which they can only adjust smoothly ( $dP_t^j = \pi_t^j P_t^j dt$ ) and at a flow cost  $\frac{\kappa}{2} (\pi_t^j)^2 Y_t dt$ , á la Rotemberg (1982). Real flow profits (net of adjustment costs) are given by:

$$\frac{P_t^j Y_t^j}{\mathcal{P}_t} - p_t (1 - \tau_t) y_t^j - T_t Y_t$$

where  $\tau_t$  is a rental subsidy that is financed by a lump-sum tax  $T_t$ .<sup>16</sup> They take final good producers' demand as given and use households' sector discount factor to maximize the present value of profits:

$$\int_0^\infty \Xi_t^H \left[ \left( \frac{P_t^j}{\mathcal{P}_t} \right)^{1-\varepsilon} - p_t (1 - \tau_t) \left( \frac{P_t^j}{\mathcal{P}_t} \right)^{-\varepsilon} - \frac{\kappa}{2} (\pi_t^j)^2 - T_t \right] Y_t dt$$

where  $\Xi_t^H = e^{-\rho t} \frac{1}{C_t^H}$  with  $C_t^H$  denoting total consumption of households. Their state is  $P_t^j$  and let  $\lambda_t^F$  be the co-state. Hamiltonian for these firms is given by:

$$H_t^F = \Xi_t^H \left[ \left( \frac{P_t^j}{\mathcal{P}_t} \right)^{1-\varepsilon} - p_t (1 - \tau_t) \left( \frac{P_t^j}{\mathcal{P}_t} \right)^{-\varepsilon} - \frac{\kappa}{2} (\pi_t^j)^2 - T_t \right] Y_t + \lambda_t^F \pi_t^j P_t^j$$

In a symmetric equilibrium, first order condition wrt  $\pi_t^j$  requires  $\pi_t^j = \frac{\lambda_t^F P_t^j}{\kappa \Xi_t^H Y_t}$  and co-state

<sup>16</sup>This type of subsidy is the standard way of correcting monopolistic power of firms.

evolves as follows:

$$d\lambda_t^F = - \left[ \Xi_t^H \frac{\varepsilon Y_t}{\mathcal{P}_t} \left( p_t(1 - \tau_t) - \frac{\varepsilon - 1}{\varepsilon} \right) + \lambda_t^F \pi_t \right] dt + \sigma_t^{\lambda^F} \lambda_t^F dZ_t$$

Letting  $d\Xi_t^H = -r_t^{f,H} \Xi_t^H dt - \varsigma_t^{C,H} \Xi_t^H dZ_t$ , we obtain the New Keynesian Phillips Curve from Ito's Lemma:

$$\begin{aligned} \frac{\mathbb{E}[d\pi_t]}{dt} &= - \frac{\mathbb{E}[d(\Xi_t^H Y_t)]}{(\Xi_t^H Y_t) dt} \pi_t - \frac{\varepsilon}{\kappa} \left( p_t(1 - \tau_t) - \frac{\varepsilon - 1}{\varepsilon} \right) \\ &= \left( r_t^{f,H} - \frac{\mathbb{E}[dY_t]}{Y_t dt} + \varsigma_t^{C,H} \sigma_t^Y \right) \pi_t - \frac{\varepsilon}{\kappa} \left( p_t(1 - \tau_t) - \frac{\varepsilon - 1}{\varepsilon} \right) \end{aligned} \quad (52)$$

where  $Y_t = av_t K_t$ . Rewriting the NKPC in integral form:

$$\pi_t = \frac{\varepsilon}{\kappa Y_t} \mathbb{E}_t \int_t^\infty e^{-\int_t^s r_\tau^f d\tau} Y_s (m_s - m^f) ds$$

where  $r_t^f = r_t^{f,H} + \varsigma_t^{C,H} \sigma_t^Y$ ,  $m_s = p_s(1 - \tau_s)$  is the marginal cost and  $m^f = \frac{\varepsilon - 1}{\varepsilon}$  is the flex-price marginal cost. Note that in the symmetric equilibrium, firms profits are  $(1 - p_t - \frac{\kappa}{2} \pi_t^2) Y_t$ . Firms transfer these profits to households, together with adjustment costs that they had paid, such that:

$$\mathfrak{d}_t = av_t - ap_t v_t \quad (53)$$

## A.4 Government

Denote by  $N_t^I$  and  $N_t^H$  total net worth of intermediaries' and households', respectively, with aggregate net worth  $N_t = N_t^I + N_t^H$ . The fiscal side of the government (treasury) issues infinite-maturity long-term bonds  $L_t^T$  at rate  $\mu_t^{L,T}$  and sets a fixed interest rate on bonds  $i^L$ . The government also imposes a capital tax  $\tau_t^K$ , wealth taxes/subsidies for intermediaries and households  $\tau_t^I$  and  $\tau_t^H$ , intermediation subsidy  $\tau_t^x$ , and a subsidy for monopolistic firms  $\tau_t$ , as well as their lump-sum tax  $T_t = p_t \tau_t$ . For convenience, we let monopolistic firm subsidy to be self-financed, and let wealth and intermediation subsidies to net each other out in the budget:

$$N_t^I (\theta_t^{x,I} \tau_t^x + \tau_t^I) + N_t^H \tau_t^H = 0$$

The remaining treasury's budget constraint is as follows:

$$P_t^L dL_t^T + \mathcal{P}_t \tau_t^K K_t dt + T_t^{CB} dt = i^L L_t^T dt$$

where  $P_t^L$  is the nominal price of bonds ( $dP_t^L = \mu_t^{PL} P_t^L dt + \sigma_t^{PL} P_t^L dZ_t$ ),  $\mathcal{P}_t$  is the price level ( $d\mathcal{P}_t = \pi_t \mathcal{P}_t dt$ ) and  $T_t^{CB}$  is the transfer from the central bank.

The monetary side of the government (central bank) issues reserves  $\mathcal{R}_t$  ( $d\mathcal{R}_t = \mu_t^{\mathcal{R}} \mathcal{R}_t dt + \sigma_t^{\mathcal{R}} \mathcal{R}_t dZ_t$ ), chooses a floating interest rate  $\underline{i}_t$  on required reserves  $\underline{\mathcal{R}}_t$ , a floating rate  $i_t$  on excess reserves  $\mathcal{R}_t - \underline{\mathcal{R}}_t$ , reserve requirements  $\underline{\theta}_t^{\mathcal{R}}$ , and in addition holds long-term bonds  $L_t^{CB}$ , which evolve according to  $dL_t^{CB} = \mu_t^{L,CB} L_t^{CB} dt + \sigma_t^{L,CB} L_t^{CB} dZ_t$ . Its budget constraint is as follows:

$$d\mathcal{R}_t + i^L L_t^{CB} dt = \underline{i}_t \underline{\mathcal{R}}_t dt + i_t (\mathcal{R}_t - \underline{\mathcal{R}}_t) dt + P_t^L dL_t^{CB} + T_t^{CB} dt + \sigma_t^{PL} \sigma_t^{L,CB} P_t^L L_t^{CB} dt$$

The last term is due to expected losses/gains from stochastic bond purchases.<sup>17</sup> The role of the interest on excess reserves is to control the marginal and the average interest rate on reserves separately. This gives the central bank the ability to set the marginal interest rate in the economy without having to finance the change in interest rate payments and without imposing any fiscal consequences. Effectively, it reproduces the freedom enjoyed by the central bank in a standard New Keynesian model with reserves in zero net supply. The consolidated government budget becomes:

$$d\mathcal{R}_t + P_t^L (dL_t^T - dL_t^{CB}) + \mathcal{P}_t \tau_t^K K_t dt = \underline{i}_t \underline{\mathcal{R}}_t dt + i_t (\mathcal{R}_t - \underline{\mathcal{R}}_t) dt + i^L (L_t^T - L_t^{CB}) dt + \sigma_t^{PL} \sigma_t^{L,CB} P_t^L L_t^{CB} dt$$

or, separating the drift and volatility components:

$$\begin{aligned} \mu_t^{\mathcal{R}} \mathcal{R}_t + P_t^L \left( \mu_t^{L,T} L_t^T - \mu_t^{L,CB} L_t^{CB} \right) + \mathcal{P}_t \tau_t^K K_t &= \underline{i}_t \underline{\mathcal{R}}_t + i_t (\mathcal{R}_t - \underline{\mathcal{R}}_t) \\ &+ i^L (L_t^T - L_t^{CB}) + \sigma_t^{PL} \sigma_t^{L,CB} P_t^L L_t^{CB} \\ \sigma_t^{\mathcal{R}} \mathcal{R}_t - P_t^L \sigma_t^{L,CB} L_t^{CB} &= 0 \end{aligned}$$

Let  $L_t \equiv L_t^T - L_t^{CB}$  be the outstanding stock of long-term bonds held by private agents, then:

$$dL_t = \underbrace{\left( \mu_t^{L,T} L_t^T - \mu_t^{L,CB} L_t^{CB} \right)}_{\mu_t^L L_t} dt + \underbrace{\left( -\sigma_t^{L,CB} L_t^{CB} \right)}_{\sigma_t^L L_t} dZ_t$$

and the budget constraint terms become:

$$\mu_t^{\mathcal{R}} \mathcal{R}_t + P_t^L \mu_t^L L_t + \mathcal{P}_t \tau_t^K K_t = \underline{i}_t \underline{\mathcal{R}}_t + i_t (\mathcal{R}_t - \underline{\mathcal{R}}_t) + i^L L_t - \sigma_t^{PL} \sigma_t^L P_t^L L_t \quad (54)$$

$$\sigma_t^{\mathcal{R}} \mathcal{R}_t + P_t^L \sigma_t^L L_t = 0 \quad (55)$$

<sup>17</sup>If the central bank purchases long-term bonds whenever their price goes up and sells them whenever their price goes down ( $\sigma_t^{PL}$  and  $\sigma_t^{L,CB}$  are of the same sign), then in expectation it is going to make a loss, reflected in the drift component. See Appendix G for more details.

## A.5 Return processes

Capital accumulation by households is subject to idiosyncratic shocks  $d\tilde{Z}_t$  with volatility loading  $\tilde{\sigma}_t$ . Individual capital holding follows:

$$\frac{dk_t}{k_t} = \underbrace{\left( \frac{1}{\phi} \log(1 + \phi \iota_t) - \delta \right)}_{g(\iota_t)} dt + \tilde{\sigma}_t d\tilde{Z}_t \quad (56)$$

where  $\iota_t$  is the investment rate and  $g(\iota_t)$  is the growth rate of capital. Idiosyncratic volatility  $\tilde{\sigma}_t^2$  is a stationary stochastic process:

$$d\tilde{\sigma}_t^2 = \mu(\tilde{\sigma}_t^2)dt + \sigma(\tilde{\sigma}_t^2)dZ_t \quad (57)$$

with drift and diffusion terms potentially depending on  $\tilde{\sigma}_t^2$  and  $dZ_t$  denoting aggregate Brownian shocks to idiosyncratic volatility. Aggregate capital evolves as follows:

$$\frac{dK_t}{K_t} = \underbrace{\left( \frac{1}{\phi} \log(1 + \phi \iota_t) - \delta \right)}_{g(\iota_t)} dt$$

Capital price in consumption numeraire is denoted by  $q_t^K$  and is driven by aggregate shocks:

$$\frac{dq_t^K}{q_t^K} = \mu_t^{q^K} dt + \sigma_t^{q^K} dZ_t$$

Return on capital is given by:

$$\begin{aligned} dr_t^K(\nu_t, \iota_t, \nu_t) &= \left[ \frac{p_t a \nu_t - \iota_t - \tau_t^K + \mathfrak{d}_t}{q_t^K} - \mathfrak{t}_t(\nu_t) \right] dt + \frac{d(q_t^K k_t)}{q_t^K k_t} \\ &= \left[ \frac{p_t a \nu_t - \iota_t - \tau_t^K + \mathfrak{d}_t}{q_t^K} - \mathfrak{t}_t(\nu_t) + \mu_t^{q^K} + g(\iota_t) \right] dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t \end{aligned} \quad (58)$$

where  $a$  is capital productivity,  $\nu_t$  is capital utilization rate,  $p_t$  is the price of a common input good,  $\tau_t^K$  is the capital tax,  $\mathfrak{d}_t$  is the transfer from monopolistic firms,  $\nu_t$  is the velocity of deposits, and  $\mathfrak{t}_t(\nu_t)$  is an increasing and convex transaction cost function. Outside equity inherits the loadings of capital on aggregate and idiosyncratic shocks, with a diversification factor  $\varphi$  for the intermediaries:

$$\begin{aligned} dr_t^{x,H} &= r_t^x dt + \sigma_t^{q^K} dZ_t + \tilde{\sigma}_t d\tilde{Z}_t \\ dr_t^{x,I} &= (r_t^x + \tau_t^x) dt + \sigma_t^{q^K} dZ_t + \varphi \tilde{\sigma}_t d\tilde{Z}_t \end{aligned}$$

Returns on nominal assets are as follows:

$$dr_t^{\mathcal{R}}(\theta_t^{\mathcal{R}}) = i(\theta_t^{\mathcal{R}})dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = \left[ \frac{\theta_t^{\mathcal{R}} i_t + (\theta_t^{\mathcal{R}} - \theta_t^{\mathcal{R}})i_t}{\theta_t^{\mathcal{R}}} - \pi_t \right] dt \quad (59)$$

$$dr_t^L = \frac{i^L}{P_t^L} dt + \frac{d(P_t^L/\mathcal{P}_t)}{P_t^L/\mathcal{P}_t} = \left[ \frac{i^L}{P_t^L} + \mu_t^{PL} - \pi_t \right] dt + \sigma_t^{PL} dZ_t \quad (60)$$

$$dr_t^D = i_t^D dt + \frac{d(1/\mathcal{P}_t)}{1/\mathcal{P}_t} = [i_t^D - \pi_t] dt \quad (61)$$

## A.6 Equilibrium

### A.6.1 Collecting equilibrium conditions

Capital, reserves, long-term bonds, outside equity, deposits, and goods markets clear as follows:

$$\theta_t^K N_t^H = q_t^K K_t \quad (62)$$

$$\theta_t^{\mathcal{R}} N_t^I = \frac{\mathcal{R}_t}{\mathcal{P}_t} \quad (63)$$

$$\theta_t^{L,I} N_t^I + \theta_t^{L,H} N_t^H = \frac{P_t^L L_t}{\mathcal{P}_t} \quad (64)$$

$$\theta_t^{x,I} N_t^I + \theta_t^{x,H} N_t^H = 0 \quad (65)$$

$$\theta_t^{D,I} N_t^I + \theta_t^{D,H} N_t^H = 0 \quad (66)$$

$$C_t = (av_t - \iota_t)K_t \quad (67)$$

And in addition the deposit velocity equation must be satisfied:  $\nu_t \theta_t^{D,H} = \theta_t^K$ . Using the expressions for returns on capital (58) and long-term bonds (60), households' first order conditions (41), and (42) can be written as:

$$\iota_t = \frac{q_t^K - 1}{\phi} \quad (68)$$

$$\rho b'(v_t) = \theta_t^K \frac{P_t^a}{q_t^K} \quad (69)$$

$$(70)$$

Using the expression for returns on reserves (59) and long-term bonds (60), rewrite intermediaries FOC (50) as follows:

$$\mu_t^{PL} = \underbrace{i_t + \lambda_t^{\mathcal{R}}}_{i_t^m} - \frac{i^L}{P_t^L} + \sigma_t^{n,I} \sigma_t^{PL} \quad (71)$$

where we denote the marginal interest rate by  $i_t^m = i_t + \lambda_t^{\mathcal{R}}$ . This is the short-term rate is intermediaries use to price long-term bonds. Combining intermediaries' and households'

FOC (44), (45), (48) and (49) we obtain:

$$\left(\sigma_t^{n,I} - \sigma_t^{n,H}\right) \sigma_t^{PL} = \nu_t^2 \mathbf{t}'(\nu_t) \quad (72)$$

$$\left(\sigma_t^{n,I} - \sigma_t^{n,H}\right) \left(\sigma_t^{PL} - \sigma_t^{q^K}\right) + \tau_t^x = \left(\varphi \tilde{\sigma}_t^{n,I} - \tilde{\sigma}_t^{n,H}\right) \tilde{\sigma}_t \quad (73)$$

Using the expressions for return on capital (58), monopolistic firms' transfer to households (53), goods market clearing condition (67), and the facts that  $C_t = \rho N_t$  and  $\mathbf{t}_t(\nu_t) = 0$  in equilibrium, the FOC wrt capital holdings (43) becomes:

$$\frac{\rho N_t}{q_t^K K_t} - \frac{\tau_t^K}{q_t^K} + \mu_t^{q^K} + g(\iota_t) - \frac{i^L}{P_t^L} - \mu_t^{PL} + \pi_t = \sigma_t^{n,H} (\sigma_t^{q^K} - \sigma_t^{PL}) + \tilde{\sigma}_t^{n,H} \tilde{\sigma}_t + \nu_t \mathbf{t}'(\nu_t)$$

Using (71) and (72), we obtain:

$$\frac{\rho N_t}{q_t^K K_t} - \frac{\tau_t^K}{q_t^K} + \mu_t^{q^K} + g(\iota_t) - i_t^m + \pi_t = \sigma_t^{n,H} \sigma_t^{q^K} + \frac{\nu_t + 1}{\nu_t} \left(\sigma_t^{n,I} - \sigma_t^{n,H}\right) \sigma_t^{PL} + \tilde{\sigma}_t^{n,H} \tilde{\sigma}_t \quad (74)$$

Total household sector net worth evolves as:

$$dN_t^H = \left[ \mu_t^{n,H} N_t^H + \lambda^I N_t^I - \lambda^H N_t^H \right] dt + \sigma_t^{n,H} N_t^H dZ_t$$

Using the expression for individual net worth evolution (46), (71), (72) and  $C_t^H = \rho N_t^H$ , we can write the drift of monopolistic firm's discount factor  $\Xi_t^H = e^{-\rho t} \frac{1}{C_t^H}$  as follows:

$$\begin{aligned} r_t^{f,H} &= \rho + \mu_t^{N,H} - \left(\sigma_t^{N,H}\right)^2 \\ &= i_t^m - \pi_t + \left(\tilde{\sigma}_t^{n,H}\right)^2 + \nu_t^2 \mathbf{t}'(\nu_t) + \tau_t^H + \lambda^I \frac{N_t^I}{N_t^H} - \lambda^H \end{aligned} \quad (75)$$

We can then rewrite the NKPC (52) as follows:

$$\mu_{\pi,t} = \left(r_t^{f,H} - \mu_t^Y + \sigma_t^{n,H} \sigma_t^Y\right) \pi_t - \frac{\varepsilon}{\kappa} \left(p_t(1 - \tau_t) - \frac{\varepsilon - 1}{\varepsilon}\right) \quad (76)$$

where  $r_t^{f,H}$  is given by (75) and:

$$\mu_t^Y = \mu_t^v + g(\iota_t) \quad \sigma_t^Y = \sigma_t^v$$

The key forward-looking equations are (71), (74) and (76) which determine the nominal price of a long-term bond  $P_t^L$ , capital price  $q_t^K$  and inflation  $\pi_t$ . In addition, a Markov equilibrium formulation requires specifying a set of state variables. Given that technology is linear in capital, we can work only with scaled versions of aggregate variables that grow with capital (consumption, net worth, output, etc), and recover their original values

using the equilibrium dynamics for capital ex-post. In particular, we will use exogenous idiosyncratic volatility  $\tilde{\sigma}_t$ , utilization  $v_t$  and net worth share of intermediaries  $\eta_t$ :

$$\eta_t = \frac{N_t^I}{N_t}$$

as our state variables. The set of state variables is not unique, for example, we can use the scaled real value of nominal assets  $\frac{\mathcal{R}_t + P_t^L L_t}{\mathcal{P}_t K_t}$  (in the following defined as  $q_t^{\mathcal{B}}$ ) instead of utilization  $v_t$ . However, the  $\{\tilde{\sigma}, \eta, v\}$  state space allows for a direct connection to the constrained efficient allocation and is more economically meaningful. For the same reason, we will reformulate the forward-looking equation for  $q_t^K$  (74) as a forward-looking equation for the share of nominal assets in total net worth  $\frac{\mathcal{R}_t + P_t^L L_t}{\mathcal{P}_t N_t}$ , in the following defined as  $\vartheta_t$ . In the next section we rewrite the equilibrium conditions using this (and additional) new variables.

### A.6.2 Change of variables

We first introduce some new variables. Let  $\mathcal{B}_t$  be the total nominal value of nominal assets, and  $q_t^{\mathcal{B}}$  be the real value scaled by aggregate capital:

$$\begin{aligned}\mathcal{B}_t &\equiv \mathcal{R}_t + P_t^L L_t \\ q_t^{\mathcal{B}} &\equiv \frac{\mathcal{B}_t}{\mathcal{P}_t K_t}\end{aligned}$$

Then we can write total net worth as:

$$N_t = \frac{\mathcal{B}_t}{\mathcal{P}_t} + q_t^K K_t = (q_t^{\mathcal{B}} + q_t^K) K_t$$

Let  $\vartheta_t$  be the share of nominal assets in total net worth:

$$\vartheta_t \equiv \frac{\mathcal{B}_t / \mathcal{P}_t}{N_t} = \frac{q_t^{\mathcal{B}}}{q_t^{\mathcal{B}} + q_t^K}$$

Analogously, define  $\vartheta_t^L$  to be the share of outstanding long-term bonds in total nominal assets:

$$\vartheta_t^L \equiv \frac{P_t^L L_t}{\mathcal{B}_t}$$

Let  $\vartheta_t^{\mathcal{ER}}$  be the fraction of excess reserves in total reserves:

$$\vartheta_t^{\mathcal{ER}} \equiv \frac{\mathcal{R}_t - \underline{\mathcal{R}}_t}{\mathcal{R}_t} = \frac{\theta_t^{\mathcal{R}} - \underline{\theta}_t^{\mathcal{R}}}{\theta_t^{\mathcal{R}}} \quad (77)$$

and  $s_t$  be the surplus-to-debt ratio:

$$s_t \equiv \frac{\tau_t^K K_t}{\mathcal{B}_t/\mathcal{P}_t} = \frac{\tau_t^K}{q_t^B}$$

With this, consider the budget constraint drift and volatility components (54) and (55) and divide through by  $\mathcal{B}_t$ :

$$(1 - \vartheta_t^L)\mu_t^R + \vartheta_t^L\mu_t^L + s_t = (1 - \vartheta_t^L)\overbrace{\left((1 - \vartheta_t^{\mathcal{E}\mathcal{R}})\dot{i}_t + \vartheta_t^{\mathcal{E}\mathcal{R}}i_t\right)}^{i_t^a} + \vartheta_t^L\frac{i^L}{P_t^L} - \vartheta_t^L\sigma_t^{P^L}\sigma_t^L \quad (78)$$

$$(1 - \vartheta_t^L)\sigma_t^R + \vartheta_t^L\sigma_t^L = 0 \quad (79)$$

where we define the average interest rate on reserves as  $i_t^a = (1 - \vartheta_t^{\mathcal{E}\mathcal{R}})\dot{i}_t + \vartheta_t^{\mathcal{E}\mathcal{R}}i_t$ . In the following, we use the average and the marginal ( $i_t^m = i_t + \lambda_t^{\mathcal{R}}$ ) interest rates instead of the rates on required reserves ( $\dot{i}_t$ ) and excess reserves ( $i_t$ ), because it is the former two that matter in equilibrium. Total nominal wealth  $\mathcal{B}_t$  follows:

$$\begin{aligned} \frac{d\mathcal{B}_t}{\mathcal{B}_t} &= \left[ (1 - \vartheta_t^L)\mu_t^R + \vartheta_t^L \left( \mu_t^L + \mu_t^{P^L} + \sigma_t^{P^L}\sigma_t^L \right) \right] dt + \left[ (1 - \vartheta_t^L)\sigma_t^R + \vartheta_t^L \left( \sigma_t^L + \sigma_t^{P^L} \right) \right] dZ_t \\ &= \underbrace{\left[ (1 - \vartheta_t^L)i_t^a - s_t + \vartheta_t^L \left( \mu_t^{P^L} + \frac{i^L}{P_t^L} \right) \right]}_{\mu_t^B} dt + \underbrace{\vartheta_t^L\sigma_t^{P^L}}_{\sigma_t^B} dZ_t \end{aligned} \quad (80)$$

Let  $\eta_t^L = \frac{\theta_t^{L,I}N_t^I}{P_t^L L_t/\mathcal{P}_t}$  denote the share of outstanding long-term bonds held by intermediaries. With  $\eta_t = N_t^I/N_t$  denoting the intermediaries' share in total net worth, market clearing conditions become:

$$\theta_t^K N_t^H = q_t^K K_t \implies \theta_t^K = \frac{1 - \vartheta_t}{1 - \eta_t} \quad (81)$$

$$\theta_t^R N_t^I = \frac{\mathcal{R}_t}{\mathcal{P}_t} \implies \theta_t^R = (1 - \vartheta_t^L)\frac{\vartheta_t}{\eta_t} \quad (82)$$

$$\theta_t^{L,I} N_t^I + \theta_t^{L,H} N_t^H = \frac{P_t^L L_t}{\mathcal{P}_t} \implies \theta_t^{L,I} = \eta_t^L \frac{\vartheta_t^L \vartheta_t}{\eta_t}, \quad \theta_t^{L,H} = (1 - \eta_t^L) \frac{\vartheta_t^L \vartheta_t}{1 - \eta_t} \quad (83)$$

$$\theta_t^{x,I} N_t^I + \theta_t^{x,H} N_t^H = 0 \implies \theta_t^{x,H} = -\chi_t \frac{1 - \vartheta_t}{1 - \eta_t}, \quad \theta_t^{x,I} = \chi_t \frac{1 - \vartheta_t}{\eta_t} \quad (84)$$

$$\theta_t^{D,I} = 1 - (1 - \vartheta_t^L)\frac{\vartheta_t}{\eta_t} - \eta_t^L \frac{\vartheta_t^L \vartheta_t}{\eta_t} - \chi_t \frac{1 - \vartheta_t}{\eta_t} \quad (85)$$

$$\theta_t^{D,H} = 1 - (1 - \chi_t)\frac{1 - \vartheta_t}{1 - \eta_t} - (1 - \eta_t^L)\frac{\vartheta_t^L \vartheta_t}{1 - \eta_t} \quad (86)$$

$$C_t = (av_t - \iota_t)K_t \implies \rho(q_t^K + q_t^B) = av_t - \iota_t \quad (87)$$

And the deposit velocity constraint can be written as:

$$\nu_t (\chi_t - \eta_t + (1 - \chi_t)\vartheta_t - (1 - \eta_t^L)\vartheta_t^L\vartheta_t) = 1 - \vartheta_t \quad (88)$$

First order condition (69) can now be stated as:

$$\rho b'(v_t) = a \frac{1 - \vartheta_t}{1 - \eta_t} \frac{p_t}{q_t^K} \quad (89)$$

Note that since  $N_t = \frac{\mathcal{B}_t}{P_t \vartheta_t}$  and  $\vartheta_t = \frac{q_t^{\mathcal{B}}}{q_t^{\mathcal{B}+q^K}}$ , due to price stickiness ( $\sigma_t^{\mathcal{P}} = 0$ ) and by Ito's lemma:

$$\begin{aligned} \sigma_t^N &= \sigma_t^{\mathcal{B}} - \sigma_t^{\vartheta} \\ \sigma_t^\eta &= \sigma_t^{n,I} - \sigma_t^N = \sigma_t^{n,I} + \sigma_t^{\vartheta} - \sigma_t^{\mathcal{B}} \\ \sigma_t^{1-\eta} &= \sigma_t^{n,H} - \sigma_t^N = \sigma_t^{n,H} + \sigma_t^{\vartheta} - \sigma_t^{\mathcal{B}} \\ \eta_t \sigma_t^\eta + (1 - \eta_t) \sigma_t^{1-\eta} &= 0 \\ \sigma_t^{\vartheta} &= \sigma_t^{q^{\mathcal{B}}} - \sigma_t^{q^{\mathcal{B}+q^K}} = (1 - \vartheta_t) (\sigma_t^{\mathcal{B}} - \sigma_t^{q^K}) \end{aligned}$$

Note also that:

$$\begin{aligned} \mu_t^{\vartheta} &= \mu_t^{q^{\mathcal{B}}} - \mu_t^{q^{\mathcal{B}+q^K}} + \sigma_t^{q^{\mathcal{B}+q^K}} (\sigma_t^{q^{\mathcal{B}+q^K}} - \sigma_t^{q^{\mathcal{B}}}) \\ &= (1 - \vartheta) (\mu_t^{q^{\mathcal{B}}} - \mu_t^{q^K}) - (\sigma_t^{\mathcal{B}} - \sigma_t^{\vartheta}) \sigma_t^{\vartheta} \\ &= (1 - \vartheta) (\mu_t^{\mathcal{B}} - \pi_t - g(\iota_t) - \mu_t^{q^K}) - (\sigma_t^{\mathcal{B}} - \sigma_t^{\vartheta}) \sigma_t^{\vartheta} \end{aligned}$$

Analogously to aggregate volatility loadings and for convenience, we denote the idiosyncratic volatility loadings of intermediaries' and households' net worth as  $\tilde{\sigma}_t^\eta$  and  $\tilde{\sigma}_t^{1-\eta}$ , receptively:

$$\begin{aligned} \tilde{\sigma}_t^\eta &= \tilde{\sigma}_t^{n,I} = (1 - \vartheta_t) \frac{\chi_t}{\eta_t} \varphi \tilde{\sigma}_t \\ \tilde{\sigma}_t^{1-\eta} &= \tilde{\sigma}_t^{n,H} = (1 - \vartheta_t) \frac{1 - \chi_t}{1 - \eta_t} \tilde{\sigma}_t \end{aligned}$$

The long-term bond pricing equation (71), the equilibrium deposit velocity equation (72), and the outside equity pricing equation (73) then become:

$$\mu_t^{PL} = i_t^m - \frac{i_t^L}{P_t^L} + \sigma_t^{PL} (\sigma_t^\eta - \sigma_t^\vartheta + \sigma_t^B) \quad (90)$$

$$\frac{\sigma_t^\eta}{1 - \eta_t} \sigma_t^{PL} = \nu_t^2 \mathbf{t}'(\nu_t) \quad (91)$$

$$\frac{\sigma_t^\eta}{1 - \eta_t} \left( \sigma_t^{PL} - \sigma_t^B + \frac{\sigma_t^\vartheta}{1 - \vartheta_t} \right) + \tau_t^x = (1 - \vartheta_t) \left( \frac{\chi_t}{\eta_t} \varphi^2 - \frac{1 - \chi_t}{1 - \eta_t} \right) \tilde{\sigma}_t^2 \quad (92)$$

where (90) is a forward-looking equation that pins down  $P_t^L$ . Rewriting (74), we obtain:

$$\frac{\rho}{1 - \vartheta_t} - \frac{\vartheta_t}{(1 - \vartheta_t)} s_t + \mu_t^B - \frac{\mu_t^\vartheta + \sigma_t^\vartheta (\sigma_t^B - \sigma_t^\vartheta)}{1 - \vartheta_t} - i_t^m \quad (93)$$

$$= (\sigma_t^{1-\eta} - \sigma_t^\vartheta + \sigma_t^B) \left( \sigma_t^B - \frac{\sigma_t^\vartheta}{1 - \vartheta_t} \right) + \frac{\sigma_t^\eta}{1 - \eta_t} \sigma_t^{PL} + \nu_t \mathbf{t}'(\nu_t) + \tilde{\sigma}_t^{1-\eta} \tilde{\sigma}_t \quad (94)$$

and, plugging in (80), (90) and (91) yields:

$$\mu_t^\vartheta = \rho - s_t + \sigma_t^{1-\eta} \sigma_t^\vartheta - (1 - \vartheta_t) \left( (1 - \vartheta_t^L) (i_t^m - i_t^a) + \left( 1 - \vartheta_t^L + \frac{1}{\nu_t} \right) \nu_t^2 \mathbf{t}'(\nu_t) + \tilde{\sigma}_t^{1-\eta} \tilde{\sigma}_t \right) \quad (95)$$

which is a forward-looking equation that pins down  $\vartheta_t$ . The discount factor used by monopolistic firms is then:

$$r_t^{f,H} = i_t^m - \pi_t + (\tilde{\sigma}_t^{1-\eta})^2 + \nu_t^2 \mathbf{t}'(\nu_t) + \tau_t^H + \lambda^I \frac{\eta_t}{1 - \eta_t} - \lambda^H \quad (96)$$

and can rewrite the NKPC (76) as follows:

$$\mu_{\pi,t} = \left( r_t^{f,H} - \mu_t^v - g(\nu_t) + (\sigma_t^{1-\eta} - \sigma_t^\vartheta + \sigma_t^B) \sigma_t^v \right) \pi_t - \frac{\varepsilon}{\kappa} \left( p_t (1 - \tau_t) - \frac{\varepsilon - 1}{\varepsilon} \right) \quad (97)$$

which is a forward-looking equation that pins down  $\pi_t$ . It remains to specify the laws of motion for the two endogenous state variables:  $\eta_t$  and  $\nu_t$ . Recall that  $\sigma_t^\eta = \sigma_t^{n,I} + \sigma_t^\vartheta - \sigma_t^B$  and from (51) we get:

$$\begin{aligned} \sigma_t^{n,I} &= \sigma_t^{r,L} + \theta_t^{D,I} (\sigma_t^{r,D} - \sigma_t^{r,L}) + \theta_t^{x,I} (\sigma_t^{r,K} - \sigma_t^{r,L}) + \theta_t^{\mathcal{R}} (\sigma_t^{r,\mathcal{R}} - \sigma_t^{r,L}) \\ &= \frac{1}{\eta_t} \left( (\eta_t^L \vartheta_t + \chi_t (1 - \vartheta_t)) \sigma_t^B - \chi_t \sigma_t^\vartheta \right) \end{aligned}$$

and therefore:

$$\eta_t \sigma_t^\eta = (\eta_t - \chi_t) \sigma_t^\vartheta + (\chi_t - \eta_t + \vartheta_t (\eta_t^L - \chi_t)) \sigma_t^B \quad (98)$$

Similarly,  $\mu_t^\eta = \mu_t^{N,I} - \mu_t^N + \sigma_t^N (\sigma_t^N - \sigma_t^{n,I})$ , where:

$$\mu_t^N = \mu_t^{\mathcal{B}} - \pi_t - \mu_t^\vartheta + \sigma_t^\vartheta (\sigma_t^\vartheta - \sigma_t^{\mathcal{B}})$$

and:

$$\begin{aligned} \mu_t^{N,I} &= \mu_t^{n,I} + \lambda^H \frac{1 - \eta_t}{\eta_t} - \lambda^I \\ &= -\rho + r_t^L + \sigma_t^{n,I} (\sigma_t^{n,I} - \sigma_t^{r,L}) + (\tilde{\sigma}_t^{n,I})^2 - \theta_t^{\mathcal{R}} \left( \lambda_t^{\mathcal{R}} + \theta_t^{\mathcal{R}} \frac{\partial r_t^{\mathcal{R}}(\theta_t^{\mathcal{R}})}{\partial \theta_t^{\mathcal{R}}} \right) + \tau_t^I + \lambda^H \frac{1 - \eta_t}{\eta_t} - \lambda^I \\ &= -\rho + i_t^m - \pi_t + (\sigma_t^\eta - \sigma_t^\vartheta + \sigma_t^{\mathcal{B}})^2 + (\tilde{\sigma}_t^{1-\eta})^2 - (1 - \vartheta_t^L) \frac{\vartheta_t}{\eta_t} (i_t^m - i_t^a) + \tau_t^I + \lambda^H \frac{1 - \eta_t}{\eta_t} - \lambda^I \end{aligned}$$

Note that  $\mu_t^{\mathcal{B}}$  can be written as:

$$\mu_t^{\mathcal{B}} = (1 - \vartheta_t^L) i_t^a + \vartheta_t^L i_t^m - s_t + \sigma_t^{\mathcal{B}} (\sigma_t^\eta - \sigma_t^\vartheta + \sigma_t^{\mathcal{B}}) \quad (99)$$

and therefore:

$$\begin{aligned} \mu_t^\eta &= -\rho + i_t^m - \pi_t + (\sigma_t^\eta - \sigma_t^\vartheta + \sigma_t^{\mathcal{B}})^2 + (\tilde{\sigma}_t^{1-\eta})^2 - (1 - \vartheta_t^L) \frac{\vartheta_t}{\eta_t} (i_t^m - i_t^a) + \tau_t^I + \lambda^H \frac{1 - \eta_t}{\eta_t} - \lambda^I \\ &\quad - \mu_t^{\mathcal{B}} + \pi_t + \mu_t^\vartheta - \sigma_t^\vartheta (\sigma_t^\vartheta - \sigma_t^{\mathcal{B}}) - (\sigma_t^{\mathcal{B}} - \sigma_t^\vartheta) \sigma_t^\eta \\ &= \mu_t^\vartheta + s_t - \rho + (1 - \vartheta_t^L) (i_t^m - i_t^a) \left( 1 - \frac{\vartheta_t}{\eta_t} \right) + (\sigma_t^\eta - \sigma_t^\vartheta) \sigma_t^\eta + (\tilde{\sigma}_t^{1-\eta})^2 + \tau_t^I + \lambda^H \frac{1 - \eta_t}{\eta_t} - \lambda^I \end{aligned} \quad (100)$$

The law of motion for utilization can be obtained from the goods market clearing (87) and optimal investment condition (68) using Ito's lemma and satisfies:

$$v_t \mu_t^v = \frac{q_t^{\mathcal{B}}}{a\phi\vartheta_t} \left[ (1 - \vartheta_t + \rho\phi) (\mu_t^{\mathcal{B}} - \pi_t - g(\iota_t)) + (1 + \rho\phi) (\sigma_t^\vartheta (\sigma_t^\vartheta - \sigma_t^{\mathcal{B}}) - \mu_t^\vartheta) \right] \quad (101)$$

$$v_t \sigma_t^v = \frac{q_t^{\mathcal{B}}}{a\phi\vartheta_t} \left[ (1 - \vartheta_t + \rho\phi) \sigma_t^{\mathcal{B}} - (1 + \rho\phi) \sigma_t^\vartheta \right] \quad (102)$$

where  $q_t^{\mathcal{B}}$  can be determined from:

$$av_t - \iota_t = \rho \frac{q_t^{\mathcal{B}}}{\vartheta_t} \quad (103)$$

$$\iota_t = \frac{(1 - \vartheta_t) q_t^{\mathcal{B}}}{\vartheta_t \phi} - \frac{1}{\phi} \quad (104)$$

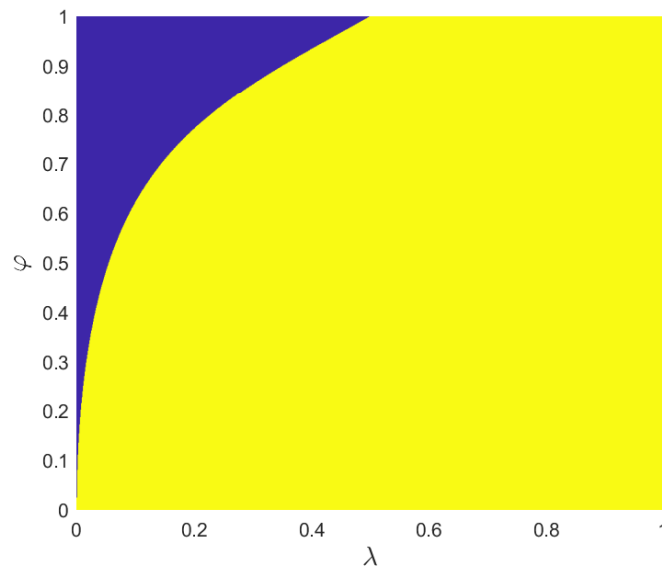
Altogether, the model's solution consists of laws of motion for state variables – exogenous state  $\tilde{\sigma}_t$ , and two endogenous states:  $\eta_t$  ((100) and (98)) and  $v_t$  ((101) and (102)) and forward-looking equations for  $\vartheta_t$  (95),  $\pi_t$  (97) and  $P_t^L$  (90). The other variables that

appear in the laws of motion and forward-looking equations are  $\mu_t^B$  (99),  $q_t^B$  (103),  $\chi_t$  (92),  $\nu_t$  (91),  $\eta_t^L$  (88),  $\iota$  (104),  $r^{f,H}$  (96),  $p_t$  (89),  $i_t^m$  (71),  $i_t^a$  (78). For simplicity, we assume that reserve requirements are sufficiently low (but never zero) at all times such that  $\vartheta_t^{\mathcal{E}\mathcal{R}} \in (0, 1)$  and  $\lambda_t^{\mathcal{R}} = 0$ .

## B Constrained Efficiency

### B.1 Parameter restriction

Figure 6: Assumption A1



Yellow region on Figure 6 corresponds to  $\lambda$  and  $\varphi$  combinations satisfying (A1), blue region – to those combinations for which (A1) does not hold. The assumption is satisfied if  $\lambda > 0.5$ , or if  $\lambda < 0.5$  and intermediaries are sufficiently efficient at risk diversification ( $\varphi$  is sufficiently low). Note that the assumption is a sufficient but not a necessary condition, meaning that Proposition 1 may hold even if (A1) is not satisfied. Indeed, assumption A1 can be further relaxed by a numerical application of Sturm’s theorem.

## B.2 Static representation with physical Pareto weights

Recall the planner's objective:

$$\begin{aligned}
W_0 &= \max_{\{v_t, \vartheta_t, \eta_t, \chi_t\}_{t=0}^{\infty}} \int_0^1 \left[ \mathbb{E} \int_0^{\infty} e^{-\rho t} \left( \log(\eta_t^{i_t(\tilde{i})} \tilde{\eta}_t^{\tilde{i}} c_t K_t) - b(v_t) \mathbb{1}_{i_t(\tilde{i})=H} \right) dt \right] d\tilde{i} \quad \text{s.t.} \\
c_t &= av_t - \iota_t = \rho \frac{q_t^K}{1 - \vartheta_t}, \quad q_t^K = (1 + \phi \iota_t) \\
\frac{d\tilde{\eta}_t^{\tilde{i}}}{\tilde{\eta}_t^{\tilde{i}}} &= \begin{cases} \left( \lambda^I - \lambda^H \frac{1-\eta_t}{\eta_t} \right) dt + (1 - \vartheta_t) \frac{\chi_t}{\eta_t} \varphi \tilde{\sigma}_t d\tilde{Z}_t + \left( \frac{\eta_t}{1-\eta_t} - 1 \right) d\tilde{J}_t^I, & \text{if } i_t(\tilde{i}) = I \\ \left( \lambda^H - \lambda^I \frac{\eta_t}{1-\eta_t} \right) dt + (1 - \vartheta_t) \frac{1-\chi_t}{1-\eta_t} \tilde{\sigma}_t d\tilde{Z}_t + \left( \frac{1-\eta_t}{\eta_t} - 1 \right) d\tilde{J}_t^H, & \text{if } i_t(\tilde{i}) = H \end{cases}
\end{aligned}$$

Denote by  $W_0^{\tilde{i}} = \mathbb{E} \int_0^{\infty} e^{-\rho t} \left( \log(\eta_t^{i_t(\tilde{i})} \tilde{\eta}_t^{\tilde{i}} c_t K_t) - b(v_t) \mathbb{1}_{i_t(\tilde{i})=H} \right) dt$  the welfare of individual agent. Using the fact that for  $dX_t = \mu_t^X X_t dt + \sigma_t^X X_t dZ_t + j_t^X X_t dJ_t$ :

$$\begin{aligned}
\mathbb{E} \int_0^{\infty} e^{-\rho t} \log(X_t) dt &= \frac{1}{\rho} \log(X_0) + \frac{1}{\rho} \mathbb{E} \int_0^{\infty} e^{-\rho t} d \log(X_t) \\
&= \frac{1}{\rho} \log(X_0) + \frac{1}{\rho} \mathbb{E} \int_0^{\infty} e^{-\rho t} \left( \mu_t^X - \frac{(\sigma_t^X)^2}{2} + \lambda^J \log(1 + j_t^X) \right) dt
\end{aligned}$$

where  $\lambda^J$  is the intensity of Poisson process  $J_t$ , we can rewrite  $W_0^{\tilde{i}}$  as:

$$\begin{aligned}
W_0^{\tilde{i}} &= \mathbb{E} \int_0^{\infty} e^{-\rho t} \log(av_t - \iota_t) dt - \mathbb{E} \int_0^{\infty} e^{-\rho t} b(v_t) \mathbb{1}_{i_t(\tilde{i})=H} dt \\
&\quad + \frac{1}{\rho} \log(K_0) + \frac{1}{\rho} \mathbb{E} \int_0^{\infty} e^{-\rho t} \left( \frac{1}{\phi} \log(1 + \phi \iota_t) - \delta \right) dt \\
&\quad + \mathbb{E} \int_0^{\infty} e^{-\rho t} \log(\eta_t^{i_t(\tilde{i})}) dt + \frac{1}{\rho} \log(\tilde{\eta}_0^{\tilde{i}}) - \frac{1}{2\rho} \mathbb{E} \int_0^{\infty} e^{-\rho t} \left( \tilde{\sigma}_t^{i_t(\tilde{i})} \right)^2 dt \\
&\quad + \frac{1}{\rho} \mathbb{E} \int_0^{\infty} e^{-\rho t} \left( \tilde{\mu}_t^{i_t(\tilde{i})} + \lambda^{i_t(\tilde{i})} \log(1 + j_t^{i_t(\tilde{i})}) \right) dt
\end{aligned}$$

where  $\tilde{\mu}_t^{i_t(\tilde{i})}$ ,  $\tilde{\sigma}_t^{i_t(\tilde{i})}$  and  $j_t^{i_t(\tilde{i})}$  correspond to the drift, idiosyncratic risk and jump risk loadings of  $\tilde{\eta}_t^{i_t(\tilde{i})}$  as outlined above. Note that expectations in the previous expression are with respect to three stochastic processes – the aggregate shocks driving aggregate variables ( $dZ_t$  driving  $\tilde{\sigma}_t$ ), idiosyncratic shocks driving agent  $\tilde{i}$ 's type ( $dJ_t^{i_t(\tilde{i})}$ ), and idiosyncratic shocks affecting agent  $\tilde{i}$ 's consumption share within a sector ( $d\tilde{Z}_t$ ). All of these processes are independent from each other. We can now integrate across individual agents, and will do it line-by-line, starting from the first one:

$$\begin{aligned}
&\int_0^1 \left[ \mathbb{E} \int_0^{\infty} e^{-\rho t} \log(av_t - \iota_t) dt \right] d\tilde{i} - \int_0^1 \left[ \mathbb{E} \int_0^{\infty} e^{-\rho t} b(v_t) \mathbb{1}_{i_t(\tilde{i})=H} dt \right] d\tilde{i} \\
&= \mathbb{E} \int_0^{\infty} e^{-\rho t} \log(av_t - \iota_t) dt - (1 - \lambda) \mathbb{E} \int_0^{\infty} e^{-\rho t} b(v_t) dt
\end{aligned}$$

where the first term does not depend on  $\tilde{i}$  and we have used that aggregate and idiosyncratic shocks are independent, with  $\int_0^1 \mathbb{1}_{i_t(\tilde{i})=H} d\tilde{i} = (1 - \lambda)$  being the physical mass of households at any given time. The second line is independent of  $\tilde{i}$  as well, and averaging across agents does not affect it either. The third line becomes:

$$\begin{aligned}
& \frac{1}{\rho} \int_0^1 \log(\tilde{\eta}_0^{\tilde{i}}) d\tilde{i} + \int_0^1 \left[ \mathbb{E} \int_0^\infty e^{-\rho t} \log(\eta_t^{i_t(\tilde{i})}) dt \right] d\tilde{i} - \frac{1}{2\rho} \int_0^1 \left[ \mathbb{E} \int_0^\infty e^{-\rho t} (\tilde{\sigma}_t^{i_t(\tilde{i})})^2 dt \right] d\tilde{i} \\
&= \frac{1}{\rho} \int_0^1 \log(\tilde{\eta}_0^{\tilde{i}}) d\tilde{i} + \int_0^1 \left[ \mathbb{E} \int_0^\infty e^{-\rho t} \left( \log(\eta_t) \mathbb{1}_{i_t(\tilde{i})=I} + \log(1 - \eta_t) \mathbb{1}_{i_t(\tilde{i})=H} \right) dt \right] d\tilde{i} \\
&\quad - \frac{1}{2\rho} \int_0^1 \left[ \mathbb{E} \int_0^\infty e^{-\rho t} \left( (\tilde{\sigma}_t^I)^2 \mathbb{1}_{i_t(\tilde{i})=I} + (\tilde{\sigma}_t^H)^2 \mathbb{1}_{i_t(\tilde{i})=H} \right) dt \right] d\tilde{i} \\
&= \frac{1}{\rho} \int_0^1 \log(\tilde{\eta}_0^{\tilde{i}}) d\tilde{i} + \lambda \mathbb{E} \int_0^\infty e^{-\rho t} \log(\eta_t) dt + (1 - \lambda) \mathbb{E} \int_0^\infty e^{-\rho t} \log(1 - \eta_t) dt \\
&\quad - \frac{1}{2\rho} \left[ \lambda \mathbb{E} \int_0^\infty e^{-\rho t} (\tilde{\sigma}_t^I)^2 dt + (1 - \lambda) \mathbb{E} \int_0^\infty e^{-\rho t} (\tilde{\sigma}_t^H)^2 dt \right]
\end{aligned}$$

where we have again used independence of shocks. Similarly, the last line becomes:

$$\begin{aligned}
& \frac{1}{\rho} \int_0^1 \left[ \mathbb{E} \int_0^\infty e^{-\rho t} \left( \tilde{\mu}_t^{i_t(\tilde{i})} + \lambda^{i_t(\tilde{i})} \log(1 + j_t^{i_t(\tilde{i})}) \right) dt \right] d\tilde{i} \\
&= \frac{1}{\rho} \int_0^1 \left[ \mathbb{E} \int_0^\infty e^{-\rho t} \left( (\tilde{\mu}_t^I + \lambda^I \log(1 + j_t^I)) \mathbb{1}_{i_t(\tilde{i})=I} + (\tilde{\mu}_t^H + \lambda^H \log(1 + j_t^H)) \mathbb{1}_{i_t(\tilde{i})=H} \right) dt \right] d\tilde{i} \\
&= \frac{1}{\rho} \left[ \mathbb{E} \int_0^\infty e^{-\rho t} \left( \lambda \left( \tilde{\mu}_t^I + \lambda^I \log\left(\frac{\eta_t}{1 - \eta_t}\right) \right) + (1 - \lambda) \left( \tilde{\mu}_t^H + \lambda^H \log\left(\frac{1 - \eta_t}{\eta_t}\right) \right) \right) dt \right] \\
&= \frac{1}{\rho} \left[ \mathbb{E} \int_0^\infty e^{-\rho t} (\lambda \tilde{\mu}_t^I + (1 - \lambda) \tilde{\mu}_t^H) dt \right]
\end{aligned}$$

since  $\lambda \lambda^I = (1 - \lambda) \lambda^H$ . Putting the terms together:

$$\begin{aligned}
W_0 &= \max_{\{\iota_t, v_t, \vartheta_t, \eta_t, \chi_t\}_{t=0}^\infty} \left[ \mathbb{E} \int_0^\infty e^{-\rho t} \log(av_t - \iota_t) dt - (1 - \lambda) \mathbb{E} \int_0^\infty e^{-\rho t} b(v_t) dt \right. \\
&\quad + \frac{1}{\rho} \mathbb{E} \int_0^\infty e^{-\rho t} \left( \frac{1}{\phi} \log(1 + \phi \iota_t) - \delta \right) dt + \lambda \mathbb{E} \int_0^\infty e^{-\rho t} \log(\eta_t) dt + (1 - \lambda) \mathbb{E} \int_0^\infty e^{-\rho t} \log(1 - \eta_t) dt \\
&\quad - \frac{1}{2\rho} \left[ \lambda \mathbb{E} \int_0^\infty e^{-\rho t} (\tilde{\sigma}_t^I)^2 dt + (1 - \lambda) \mathbb{E} \int_0^\infty e^{-\rho t} (\tilde{\sigma}_t^H)^2 dt \right] \\
&\quad \left. + \frac{1}{\rho} \left[ \mathbb{E} \int_0^\infty e^{-\rho t} (\lambda \tilde{\mu}_t^I + (1 - \lambda) \tilde{\mu}_t^H) dt \right] \right] + \frac{1}{\rho} \log(K_0) + \frac{1}{\rho} \int_0^1 \log(\tilde{\eta}_0^{\tilde{i}}) d\tilde{i}
\end{aligned}$$

where we have taken the initial conditions out of the maximization problem. Finally, note that maximizing the above objective is equivalent to static maximization for every  $t$  since planner is not bound by any additional intertemporal constraints. Therefore, solution to

the above problem satisfies:

$$\begin{aligned}
& \max_{\{\iota_t, v_t, \vartheta_t, \eta_t, \chi_t\}} \log(av_t - \iota_t) - (1 - \lambda)b(v_t) + \frac{1}{\rho} \left( \frac{1}{\phi} \log(1 + \phi \iota_t) - \delta \right) \\
& + \lambda \log(\eta_t) + (1 - \lambda) \log(1 - \eta_t) - \frac{(1 - \vartheta_t)^2 \tilde{\sigma}_t^2}{2\rho} \left[ \lambda \frac{\chi_t^2}{\eta_t^2} \varphi^2 + (1 - \lambda) \frac{(1 - \chi_t)^2}{(1 - \eta_t)^2} \right] \\
& + \frac{1}{\rho} \left[ \lambda \left( \lambda^I - \lambda^H \frac{1 - \eta_t}{\eta_t} \right) + (1 - \lambda) \left( \lambda^H - \lambda^I \frac{\eta_t}{1 - \eta_t} \right) \right]
\end{aligned}$$

This recovers the static objective from the main text.

### B.3 Existence and uniqueness

We now proceed with establishing existence and uniqueness conditions for solutions to the above problem. First, rewrite the planner's objective in the following way:

$$\begin{aligned}
& \max_{\vartheta_t, v_t, \eta_t, \chi_t} \log(\rho) - \frac{\delta}{\rho} + \frac{1 + \rho\phi}{\rho\phi} \log(1 + a\phi v_t) - (1 - \lambda)b(v_t) \\
& + \frac{1}{\rho\phi} (\log(1 - \vartheta_t) - (1 + \rho\phi) \log(1 - \vartheta_t + \rho\phi)) \\
& + \lambda \log(\eta_t) + (1 - \lambda) \log(1 - \eta_t) - \frac{(1 - \vartheta_t)^2 \tilde{\sigma}_t^2}{2\rho} \left[ \lambda \frac{\chi_t^2}{\eta_t^2} \varphi^2 + (1 - \lambda) \frac{(1 - \chi_t)^2}{(1 - \eta_t)^2} \right] \\
& - \frac{1}{\rho} \left[ \lambda \lambda^H \frac{1 - \eta_t}{\eta_t} + (1 - \lambda) \lambda^I \frac{\eta_t}{1 - \eta_t} \right]
\end{aligned}$$

where we used  $av_t - \iota_t = \rho \frac{1 + \phi \iota_t}{1 - \vartheta_t}$  to substitute  $\iota_t$ . Note that the first line is only a function of  $v_t$ , which neither interacts with  $\tilde{\sigma}_t$  nor appears in the next three lines, implying that optimal utilization is independent of  $\tilde{\sigma}_t$  and can be solved for independently from other variables. Furthermore, convexity of  $b(v_t)$  guarantees that the FOC with respect to  $v_t$  provides the unique global maximum of planner's objective with respect to  $v_t$ . Next, note that  $\chi_t$  only appears in the bracket in the third line, and the last term of that line is concave in  $\chi_t$ . Therefore, FOC with respect to  $\chi_t$  provides the unique global maximum of planner's objective with respect to  $\chi_t$ , given  $\eta_t$ :

$$\chi_t = \frac{(1 - \lambda)\eta_t^2}{(1 - \lambda)\eta_t^2 + \lambda\varphi^2(1 - \eta_t)^2} \in [0, 1]$$

We can now plug the above expression into Planner's objective and analyze the terms containing the two remaining variables ( $\vartheta_t$  and  $\eta_t$ ). To ease notation, we drop the  $t$

subscript:

$$\begin{aligned} \max_{\vartheta, \eta} W(\vartheta, \eta) &= \max_{\vartheta, \eta} \frac{1}{\rho\phi} (\log(1 - \vartheta) - (1 + \rho\phi) \log(1 - \vartheta + \rho\phi)) \\ &\quad + \lambda \log(\eta) + (1 - \lambda) \log(1 - \eta) - \frac{(1 - \vartheta)^2 \tilde{\sigma}^2}{2\rho} \frac{\lambda(1 - \lambda)\varphi^2}{(1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2} \\ &\quad - \frac{1}{\rho} \left[ \lambda\lambda^H \frac{1 - \eta}{\eta} + (1 - \lambda)\lambda^I \frac{\eta}{1 - \eta} \right] \end{aligned}$$

Note that  $\vartheta \in [0, 1]$ ,  $\eta \in [0, 1]$  and  $W(\vartheta, \eta)$  is defined on  $[0, 1] \times (0, 1)$ . Furthermore,  $\lim_{\eta \rightarrow 1} W(\vartheta, \eta) = \lim_{\eta \rightarrow 0} W(\vartheta, \eta) = -\infty$  for all  $\vartheta \in [0, 1]$ . Note also that  $\lim_{\vartheta \rightarrow 1} W(\vartheta, \eta) = -\infty$  for any  $\eta \in (0, 1)$ . In addition:

$$\frac{\partial W(\vartheta, \eta)}{\partial \vartheta} = \frac{1}{\rho\phi} \left( -\frac{1}{1 - \vartheta} + \frac{1 + \rho\phi}{1 - \vartheta + \rho\phi} \right) + \frac{(1 - \vartheta) \tilde{\sigma}^2}{\rho} \frac{\lambda(1 - \lambda)\varphi^2}{(1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2} \quad (105)$$

such that  $\lim_{\vartheta \rightarrow 0} \frac{\partial W(\vartheta, \eta)}{\partial \vartheta} > 0$  for any  $\eta \in (0, 1)$ . Altogether, this implies that the maximum of  $W(\vartheta, \eta)$  always exists and is achieved in the interior for some  $\vartheta \in (0, 1)$ ,  $\eta \in (0, 1)$ .

Rearrange (105) and set to zero:

$$\frac{\lambda(1 - \lambda)\varphi^2 \tilde{\sigma}^2}{(1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2} (1 - \vartheta)^3 + \frac{\rho\phi\lambda(1 - \lambda)\varphi^2 \tilde{\sigma}^2}{(1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2} (1 - \vartheta)^2 + \rho(1 - \vartheta) - \rho = 0 \quad (106)$$

Note that for any given  $\eta \in (0, 1)$ , the above expression is strictly positive for  $\vartheta = 0$  and strictly negative for  $\vartheta = 1$ . Furthermore, the derivative of the above expression with respect to  $\vartheta$  is strictly negative for all  $\vartheta \in [0, 1]$ , meaning that there is exactly one root on the  $(0, 1)$  interval, which defines a function  $\vartheta(\eta)$ . Since the second derivative of  $W(\vartheta, \eta)$  with respect to  $\vartheta$  is always negative:

$$\frac{\partial^2 W(\vartheta, \eta)}{\partial \vartheta^2} = \frac{1}{\rho\phi} \underbrace{\left( -\frac{1}{(1 - \vartheta)^2} + \frac{1 + \rho\phi}{(1 - \vartheta + \rho\phi)^2} \right)}_{<0} - \underbrace{\frac{\tilde{\sigma}^2}{\rho} \frac{\lambda(1 - \lambda)\varphi^2}{(1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2}}_{>0} < 0$$

the above  $\vartheta(\eta)$  indeed delivers the maximum of  $W(\vartheta, \eta)$  given  $\eta$ . Define  $\tilde{W}(\eta) = W(\vartheta(\eta), \eta)$ , which is the maximum welfare attainable for a given  $\eta$  (ignoring constants and terms coming from utilization). By definition of  $\vartheta(\eta)$ ,  $\frac{\partial \tilde{W}(\eta)}{\partial \eta} = \frac{\partial W(\vartheta, \eta)}{\partial \eta}$  and it is straightforward to show that  $\frac{\partial \tilde{W}(\eta)}{\partial \eta} \Big|_{\eta=\lambda} > 0$  and  $\lim_{\eta \rightarrow 1} \frac{\partial \tilde{W}(\eta)}{\partial \eta} = -\infty$ . From continuity of  $\frac{\partial \tilde{W}(\eta)}{\partial \eta}$  it then follows that there always exists a (local) maximum point at some  $\eta \in (\lambda, 1)$ .

From now on, we focus on local maxima s.t.  $\eta > \lambda$ . There might exist local maxima for  $\eta < \lambda$ , but we do not consider them because these maxima (i) do not always exist and (ii) never happen to be global in our numerical simulations, even though we can not show this analytically. To see why there might not be a local maximum to the left of  $\lambda$ , consider the limit as idiosyncratic risk vanishes. It is straightforward to see that  $\lim_{\tilde{\sigma} \rightarrow 0} \gamma = 0$  and

therefore  $\lim_{\tilde{\sigma} \rightarrow 0} \eta = \lambda$  for the optimal  $\eta$ . More importantly, optimal  $\eta$  approaches  $\lambda$  from the right – in the vicinity of  $\tilde{\sigma} = 0$ ,  $\gamma$  is slightly positive, meaning that  $\eta - \lambda$  and  $(1 - \lambda)\eta - \lambda\varphi^2(1 - \eta)$  must be of the same sign (follows from (108)). This is only possible if  $\eta$  approaches  $\lambda$  from the right, since otherwise  $\eta < \lambda$  implies  $\eta < \lambda\varphi^2/(1 - \lambda + \lambda\varphi^2)$  which bounds  $\eta$  away from  $\lambda$  and prevents convergence. It follows that for sufficiently small values of  $\tilde{\sigma}$  all critical points are such that  $\eta > \lambda$  and the global maximum is achieved in the same region.

In the following, we show that there is a unique maximum with  $\eta > \lambda$  under a certain assumption on  $\{\lambda, \varphi, \lambda^I, \lambda^H\}$ . The maximum has to satisfy the following FOCs:

$$\vartheta - \gamma(1 - \vartheta + \rho\phi) = 0 \quad (107)$$

$$\frac{\lambda}{\eta} - \frac{1 - \lambda}{1 - \eta} + \gamma \frac{(1 - \lambda)\eta - \lambda\varphi^2(1 - \eta)}{(1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2} + \frac{\lambda\lambda^H}{\rho\eta^2} - \frac{(1 - \lambda)\lambda^I}{\rho(1 - \eta)^2} = 0 \quad (108)$$

$$\gamma = \frac{1}{\rho} \frac{\lambda(1 - \lambda)\varphi^2(1 - \vartheta)^2}{(1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2} \tilde{\sigma}^2 \quad (109)$$

where  $\gamma$  is an auxiliary variable. Express  $\gamma$  as a function of  $\eta$  from (108):<sup>18</sup>

$$\gamma(\eta) = \left( \frac{1 - \lambda}{1 - \eta} - \frac{\lambda}{\eta} + \frac{(1 - \lambda)\lambda^I}{\rho(1 - \eta)^2} - \frac{\lambda\lambda^H}{\rho\eta^2} \right) \frac{(1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2}{(1 - \lambda)\eta - \lambda\varphi^2(1 - \eta)}$$

Plugging  $\gamma(\eta)$  and (107) into (109) gives that the maximum must satisfy the following condition:

$$d(\eta) \equiv \gamma(\eta) - \frac{1}{\rho} \frac{\lambda(1 - \lambda)\varphi^2\tilde{\sigma}^2}{(1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2} \frac{(1 - \rho\phi\gamma(\eta))^2}{(1 + \gamma(\eta))^2} = 0$$

Note that  $\gamma(\lambda) = 0$ , and therefore  $d(\lambda) < 0$ . Furthermore,  $\gamma(\eta)$  is continuous on  $[\lambda, 1)$  with  $\lim_{\eta \rightarrow 1} \gamma(\eta) = \infty$  and also  $\lim_{\eta \rightarrow 1} d(\eta) > 0$ . To establish uniqueness, it suffices to show that  $d(\eta)$  is strictly increasing in  $\eta$  on the interval  $\eta \in (\lambda, 1)$ , as that would imply  $d(\eta)$  crosses zero exactly once. This is clearly a sufficient but not a necessary condition for the uniqueness of the maximum. First, differentiate  $d(\eta)$ :

$$\begin{aligned} \frac{\partial d(\eta)}{\partial \eta} &= \frac{\partial \gamma(\eta)}{\partial \eta} + \underbrace{\frac{2}{\rho} \frac{\lambda(1 - \lambda)\varphi^2\tilde{\sigma}^2}{(1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2}}_{>0} \underbrace{\frac{(1 - \rho\phi\gamma(\eta))(2 + (1 - \rho\phi)\gamma(\eta))}{(1 + \gamma(\eta))^3}}_{>0} \frac{\partial \gamma(\eta)}{\partial \eta} \\ &+ \underbrace{\frac{2}{\rho} \frac{\lambda(1 - \lambda)\varphi^2\tilde{\sigma}^2}{((1 - \lambda)\eta^2 + \lambda\varphi^2(1 - \eta)^2)^2}}_{>0} \underbrace{\frac{(1 - \rho\phi\gamma(\eta))^2}{(1 + \gamma(\eta))^2} \underbrace{((1 - \lambda)\eta - \lambda\varphi^2(1 - \eta))}_{>0}}_{>0} \end{aligned}$$

where the second term in front of  $\frac{\partial \gamma(\eta)}{\partial \eta}$  is positive because otherwise  $\vartheta$  in (107) is larger than one,<sup>19</sup> and the last term in the second line is positive since we consider  $\eta > \lambda$ .

<sup>18</sup>We can divide by  $(1 - \lambda)\eta - \lambda\varphi^2(1 - \eta)$  since this term is always positive for  $\eta \in [\lambda, 1)$ .

<sup>19</sup>In fact, since  $\gamma(\lambda) = 0$  and  $\lim_{\eta \rightarrow 1} \gamma(\eta) = \infty$ , there exists  $\eta' \in (\lambda, 1)$  such that for any  $\eta > \eta'$ :  $\gamma(\eta) > 1/\rho\phi$  and from (107)  $\vartheta > 1$ , meaning that the optimal  $\eta$  is bounded away from 1 by  $\eta'$ .

Therefore, if  $\frac{\partial \gamma(\eta)}{\partial \eta} > 0$ , then  $\frac{\partial d(\eta)}{\partial \eta} > 0$ . Consider (108) and take the total derivative:

$$\begin{aligned} & \left[ \frac{\lambda}{\eta^2} + \frac{1-\lambda}{(1-\eta)^2} + \frac{2\lambda\lambda^H}{\rho\eta^3} + \frac{2(1-\lambda)\lambda^I}{\rho(1-\eta)^3} + \gamma \frac{((1-\lambda)\eta - \lambda\varphi^2(1-\eta))^2 - \lambda(1-\lambda)\varphi^2}{((1-\lambda)\eta^2 + \lambda\varphi^2(1-\eta)^2)^2} \right] d\eta \\ & = \underbrace{\frac{(1-\lambda)\eta - \lambda\varphi^2(1-\eta)}{(1-\lambda)\eta^2 + \lambda\varphi^2(1-\eta)^2}}_{>0} d\gamma \end{aligned}$$

Therefore, as long as the bracket in front of  $d\eta$  is positive, so is the derivative  $\frac{\partial \gamma(\eta)}{\partial \eta}$ . Plug in for  $\gamma$  and split the bracket into two parts:

$$\begin{aligned} & \frac{\lambda}{\eta^2} + \frac{1-\lambda}{(1-\eta)^2} + \left( \frac{1-\lambda}{1-\eta} - \frac{\lambda}{\eta} \right) \frac{((1-\lambda)\eta - \lambda\varphi^2(1-\eta))^2 - \lambda(1-\lambda)\varphi^2}{((1-\lambda)\eta - \lambda\varphi^2(1-\eta))((1-\lambda)\eta^2 + \lambda\varphi^2(1-\eta)^2)} \\ & + \frac{\lambda^H + \lambda^I}{\rho} \left[ \frac{2\lambda^2}{\eta^3} + \frac{2(1-\lambda)^2}{(1-\eta)^3} + \left( \frac{(1-\lambda)^2}{(1-\eta)^2} - \frac{\lambda^2}{\eta^2} \right) \frac{((1-\lambda)\eta - \lambda\varphi^2(1-\eta))^2 - \lambda(1-\lambda)\varphi^2}{((1-\lambda)\eta - \lambda\varphi^2(1-\eta))((1-\lambda)\eta^2 + \lambda\varphi^2(1-\eta)^2)} \right] \end{aligned} \quad (110)$$

Rewrite the first line as:

$$\frac{C(\eta)}{\eta^2(1-\eta)^2((1-\lambda)\eta - \lambda\varphi^2(1-\eta))((1-\lambda)\eta^2 + \lambda\varphi^2(1-\eta)^2)}$$

with

$$\begin{aligned} C(\eta) &= ((1-\lambda)\eta^2 + \lambda(1-\eta)^2)((1-\lambda)\eta - \lambda\varphi^2(1-\eta))((1-\lambda)\eta^2 + \lambda\varphi^2(1-\eta)^2) \\ &+ \eta(1-\eta)(\eta - \lambda) \left( ((1-\lambda)\eta - \lambda\varphi^2(1-\eta))^2 - \lambda(1-\lambda)\varphi^2 \right) \end{aligned}$$

The denominator is positive, and in the following we show that the numerator  $C(\eta)$  is positive as well, under some assumptions. Since  $C(\eta)$  is a polynomial in  $\eta$ , we can rewrite it as an exact Taylor expansion:

$$\begin{aligned} C(\eta) &= \lambda^3(1-\lambda)^3(1-\varphi^2)(\lambda + (1-\lambda)\varphi^2) + 4\lambda^3(1-\lambda)^3(1-\varphi^2)^2(\eta - \lambda) \\ &+ \lambda(1-\lambda)(6\lambda(1-\lambda)(1-\varphi^2)(1-\lambda + \lambda\varphi^2) - (1-2\lambda)\varphi^2)(\eta - \lambda)^2 \\ &+ 4\lambda(1-\lambda)(1-\lambda + \lambda\varphi^2)^2(\eta - \lambda)^3 \\ &+ (1-\lambda + \lambda\varphi^2)(1-2\lambda + \lambda^2(1-\varphi^2))(\eta - \lambda)^4 \end{aligned}$$

It is easy to verify that  $C(\lambda) > 0$  and  $C(1) > 0$ . Furthermore, the first-order term and the third-order terms have positive coefficients. If

$$6\lambda(1-\lambda)(1-\varphi^2)(1-\lambda + \lambda\varphi^2) - (1-2\lambda)\varphi^2 \geq 0$$

then the coefficient in front of the second-order term is non-negative. The sign of the

fourth-order term is irrelevant, since the non-negative signs of the first three terms, together with  $C(1) > 0$  implies  $C(\eta) > 0$  for all  $\eta \in (\lambda, 1)$ .<sup>20</sup> Since the first line in (110) is bounded away from zero, the entire term is strictly positive for sufficiently small switching intensities  $\lambda^H$  and  $\lambda^I$ .<sup>21</sup> Therefore, both  $\frac{\partial \gamma(\eta)}{\partial \eta}$  and  $\frac{\partial d(\eta)}{\partial \eta}$  are strictly positive in  $\eta \in (\lambda, 1)$  and there exists a unique maximum in that region.

## B.4 Properties

Finally, we show properties of optimal allocations  $\eta^*, \chi^*, \vartheta^*, \iota^*, v^*$  as functions of  $\tilde{\sigma}$ . As already noted before, utilization  $v^*$  is independent of  $\tilde{\sigma}$ . Differentiating (107), (108), and (109):

$$(1 + \gamma^*)d\vartheta^* = (1 - \vartheta^* + \rho\phi)d\gamma^* \quad (111)$$

$$\begin{aligned} & \left[ \frac{\lambda}{(\eta^*)^2} + \frac{1-\lambda}{(1-\eta^*)^2} + \frac{2\lambda\lambda^H}{\rho(\eta^*)^3} + \frac{2(1-\lambda)\lambda^I}{\rho(1-\eta^*)^3} + \gamma^* \frac{((1-\lambda)\eta^* - \lambda\varphi^2(1-\eta^*))^2 - \lambda(1-\lambda)\varphi^2}{((1-\lambda)(\eta^*)^2 + \lambda\varphi^2(1-\eta^*)^2)} \right] d\eta^* \\ &= \frac{(1-\lambda)\eta^* - \lambda\varphi^2(1-\eta^*)}{(1-\lambda)(\eta^*)^2 + \lambda\varphi^2(1-\eta^*)^2} d\gamma^* \end{aligned} \quad (112)$$

$$d\gamma^* = \gamma^* \left[ \frac{d\tilde{\sigma}^2}{\tilde{\sigma}^2} - 2\frac{d\vartheta^*}{1-\vartheta^*} - 2\frac{(1-\lambda)\eta^* - \lambda\varphi^2(1-\eta^*)}{(1-\lambda)(\eta^*)^2 + \lambda\varphi^2(1-\eta^*)^2} d\eta^* \right] \quad (113)$$

To simplify notation, rewrite these conditions as:

$$\begin{aligned} d\vartheta^* &= \overbrace{f(\tilde{\sigma}^2)}^{>0} d\gamma^*, & d\eta^* &= g(\tilde{\sigma}^2) d\gamma^* \\ d\gamma^* &= \gamma^* \left[ \frac{d\tilde{\sigma}^2}{\tilde{\sigma}^2} - 2\frac{d\vartheta^*}{1-\vartheta^*} - 2\underbrace{h(\tilde{\sigma}^2)}_{>0} d\eta^* \right] \end{aligned}$$

where  $f(\tilde{\sigma}^2)$  is positive since  $\gamma^* > 0$  and  $\vartheta^* \in (0, 1)$ , and  $h(\tilde{\sigma}^2)$  is positive since we consider  $\eta^* > \lambda$ . Under assumption (A1),  $g(\tilde{\sigma}^2) > 0$  and we can solve for:

$$\frac{\partial \gamma^*}{\partial \tilde{\sigma}^2} = \frac{\gamma^*}{\tilde{\sigma}^2} \left[ 1 + 2\gamma^* \frac{f(\tilde{\sigma}^2)}{1-\vartheta^*} + 2\gamma^* h(\tilde{\sigma}^2) g(\tilde{\sigma}^2) \right]^{-1} > 0 \quad (114)$$

It follows that  $\frac{d\vartheta^*}{d\tilde{\sigma}^2} > 0$  and  $\frac{d\eta^*}{d\tilde{\sigma}^2} > 0$ . Since  $\iota^*$  is a strictly decreasing function of  $\vartheta^*$  and  $\chi^*$  is a strictly increasing function of  $\eta^*$ , it follows that  $\frac{d\iota^*}{d\tilde{\sigma}^2} < 0$  and  $\frac{d\chi^*}{d\tilde{\sigma}^2} > 0$ .

<sup>20</sup>Even if the coefficient in front of the fourth-order term is negative,  $C(\eta_t) = 0$  for some  $\eta \in (\lambda, 1)$  would require a positive fifth-order term to ensure  $C(1) > 0$ . Since there is no fifth-order term, this is ruled out.

<sup>21</sup>One can show non-negativity of the bracket by applying the Sturm's theorem and imposing much weaker restrictions on  $\lambda$  and  $\varphi$  that would rule out any roots on the  $(\lambda, 1)$  interval. This however can only be done numerically.

## C Proof of Proposition 3

Consider the limit as  $\vartheta_t^L \rightarrow 1$ . Optimality requires:

$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta,*} + (\chi_t^* - \eta_t^* + \vartheta_t^* (\alpha_t - \chi_t^*)) \sigma_t^{PL} \quad (115)$$

$$\frac{\sigma_t^{\eta,*}}{1 - \eta_t^*} \sigma_t^{PL} = \nu_t^2 \mathfrak{t}'(\nu_t) \quad (116)$$

$$\nu_t [\chi_t^* - \eta_t^* + \vartheta_t^* (1 - \chi_t^*) - (1 - \alpha_t) \vartheta_t^*] = 1 - \vartheta_t^* \quad (117)$$

where the first line is (33), the second line is bond-deposit pricing condition (??) and the third line is the transaction cost constraint of the households. Note that  $\eta_t^*$ ,  $\sigma_t^{\eta,*}$ ,  $\vartheta_t^*$  and  $\chi_t^*$  are given by the planner's allocation and are considered exogenous from the government's perspective. We can express the required  $\alpha_t$  as a function of velocity  $\nu_t$  from the third line:

$$\alpha_t = \frac{1 - \vartheta_t^* - \nu_t (\chi_t^* - \eta_t^* - \vartheta_t^* \chi_t^*)}{\nu_t \vartheta_t^*}$$

and the required  $\sigma_t^{PL}$  as a function of  $\alpha_t$  (and therefore  $\nu_t$ ) from the first line:

$$\sigma_t^{PL} = \frac{\eta_t^* \sigma_t^{\eta,*} - (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta,*}}{\chi_t^* - \eta_t^* - \vartheta_t^* \chi_t^* + \alpha_t \vartheta_t^*} = \frac{(\eta_t^* \sigma_t^{\eta,*} - (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta,*}) \nu_t}{1 - \vartheta_t^*}$$

Plugging this in the second line gives:

$$d(\nu_t) \equiv \nu_t \mathfrak{t}'(\nu_t) - \underbrace{\frac{\sigma_t^{\eta,*}}{1 - \eta_t^*} \frac{(\eta_t^* \sigma_t^{\eta,*} - (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta,*})}{1 - \vartheta_t^*}}_{>0} = 0$$

where the second term is positive because  $\sigma_t^{\eta,*} > 0$ ,  $\sigma_t^{\vartheta,*} > 0$  and  $\chi_t^* > \eta_t^*$ , as established in 1. It follows that  $d(\nu_t) < 0$  for a sufficiently small  $\nu_t$ . Since  $\mathfrak{t}(\nu_t)$  is a continuous increasing convex function, there exists  $\nu_t$  such that  $d(\nu_t) = 0$ . The required bond price volatility  $\sigma_t^{PL}$  and intermediaries' bond share  $\alpha_t$  can be computed from the equations above.

## D Proof of Proposition 4

Recall that optimal balance sheet and interest rate policy must ensure:

$$\vartheta_t^L \sigma_t^{PL} = \sigma_t^B = \frac{(1 + \rho\phi)}{(1 - \vartheta_t^* + \rho\phi)} \sigma_t^{\vartheta,*} \quad (118)$$

$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta,*} + (\chi_t^* - \eta_t^* + \vartheta_t^* (\alpha_t - \chi_t^*)) \sigma_t^B \quad (119)$$

alongside with efficient drifts of  $\vartheta_t$ ,  $\nu_t$  and  $\eta_t$ , and risk allocation  $\chi_t$ . The latter can always be achieved by appropriately set fiscal policy  $s_t$ ,  $\tau_t^I$ ,  $\tau_t^x$  and average interest rate  $i_t^m$ . The existence of an optimal policy mix therefore depends on the existence of a solution to (118) - (119) subject to equilibrium conditions:

$$\frac{\sigma_t^{\eta,*}}{1 - \eta_t^*} \sigma_t^{PL} = \nu_t^2 \mathbf{t}'(\nu_t) \implies \frac{\sigma_t^{\eta,*}}{1 - \eta_t^*} \sigma_t^{\mathcal{B}} = \nu_t^2 \mathbf{t}'(\nu_t) \vartheta_t^L \quad (120)$$

$$\nu_t [\chi_t^* - \eta_t^* + \vartheta_t^*(1 - \chi_t^*) - (1 - \alpha_t) \vartheta_t^L \vartheta_t^*] = 1 - \vartheta_t^* \quad (121)$$

Since we consider a Markov equilibrium in which policy variables are chosen as functions of the exogenous state  $\tilde{\sigma}$ , we are looking for state-by-state solutions to (118) - (121) in  $\{\vartheta^L, \sigma^{PL}, \nu, \alpha\}$  with additional constraints  $\vartheta^L \in [0, 1]$ ,  $\nu \geq 0$ . Note first that (118) determines uniquely optimal  $\sigma^{\mathcal{B}}$  (denote it by  $\sigma^{\mathcal{B},*}$ ) as a function of  $\tilde{\sigma}$  given the planner's solution, and (119) in turn uniquely pins down the required  $\alpha$ , denoted by  $\alpha^*$ . An optimal policy mix therefore exists if and only if there is a pair  $\{\vartheta^{L,*}, \nu^*\} \in [0, 1] \times \mathbb{R}_+$  satisfying (120) and (121) given the planner's solution and  $\{\sigma^{\mathcal{B},*}, \alpha^*\}$ . Note that by proposition 3  $\sigma^{\mathcal{B},*} > 0$ . We consider several cases separately.

1. Case  $\alpha^* = 1$ . Then  $\nu^* > 0$  is uniquely pinned down by (121) and a unique optimal policy mix exists if and only if:

$$\frac{\sigma_t^{\eta,*}}{1 - \eta_t^*} \sigma^{\mathcal{B},*} \leq (\nu^*)^2 \mathbf{t}'(\nu^*)$$

2. Case  $\alpha^* > 1$ . From (121) express:

$$\nu^* \vartheta^{L,*} = \underbrace{\frac{\chi^* - \eta^* + \vartheta^*(1 - \chi^*)}{(1 - \alpha^*) \vartheta^*}}_{\equiv b_1 < 0} \cdot \nu^* + \underbrace{\frac{1 - \vartheta^*}{(\alpha^* - 1) \vartheta^*}}_{\equiv b_2 > 0}$$

and plug in (120):

$$\frac{\sigma_t^{\eta,*}}{1 - \eta_t^*} \sigma^{\mathcal{B},*} = \mathbf{t}'(\nu^*) \cdot (b_1 (\nu^*)^2 + b_2 \nu^*) \quad (122)$$

Therefore an optimal policy mix exists if there is  $\nu^* > 0$  satisfying the above condition and such that  $0 \leq b_1 + \frac{b_2}{\nu^*} \leq 1$ , ensuring  $\vartheta^{L,*} \in [0, 1]$ . The latter condition bounds  $\nu^* \in \left[-\frac{b_2}{b_1 - 1}, -\frac{b_2}{b_1}\right]$ . For convenience, define  $g(\nu) = \mathbf{t}'(\nu) \cdot (b_1 \nu^2 + b_2 \nu)$ . Note that since  $\mathbf{t}(\nu)$  is a strictly increasing strictly convex function,  $\mathbf{t}'(\nu)$  is strictly increasing and strictly positive for all  $\nu > 0$ . Since  $b_1 < 0$  and  $b_2 > 0$ , polynomial  $b_1 \nu^2 + b_2 \nu$  is strictly positive in  $\left(0, -\frac{b_2}{b_1}\right)$  and strictly negative for all  $\nu > -\frac{b_2}{b_1}$ . Therefore,  $g(\nu)$  is strictly positive in  $\left(0, -\frac{b_2}{b_1}\right)$ , attains its maximum in this region and  $g\left(-\frac{b_2}{b_1}\right) = 0$ . Since the LHS of (122) is strictly positive, an optimal policy mix

exists if and only if:

$$\max_{\nu \in \left[-\frac{b_2}{b_1-1}, -\frac{b_2}{b_1}\right]} g(\nu) \geq \frac{\sigma_t^{\eta,*}}{1 - \eta_t^*} \sigma_t^{\mathcal{B},*}$$

If the above condition is satisfied, there is not necessarily a unique solution, as  $g(\nu)$  is increasing on the interval  $\nu \in [0, -\frac{b_2}{2b_1}]$  and can therefore attain the LHS value of (122) multiply times. For a specific functional form of transaction cost  $\mathfrak{t}'(\nu) = c\nu^{p-1}$  with  $c > 0$  and  $p \geq 2$ , we can bound the maximal number of solutions by two. Under this functional form,  $g'(\nu)$  is a polynomial:

$$g'(\nu) = c\nu^{p-1} (b_1(p+1)\nu + b_2p)$$

such that  $g'(0) = 0$ ,  $g'(\nu)$  is positive for small  $\nu$ ,  $g'(-\frac{b_2}{b_1}) < 0$  and there is only one root in the  $(0, -\frac{b_2}{b_1})$  interval at  $\nu = -\frac{b_2}{b_1} \frac{p}{p+1}$ , meaning that the derivative of  $g(\nu)$  crosses zero only once on that interval. Therefore,  $g(\nu)$  has only one extremum (a maximum) on that interval and there can be at most two solutions for (122). More generally, as long as  $\mathfrak{t}'(\nu)$  is sufficiently well-behaved (i.e. grows sufficiently smoothly),  $g(\nu)$  has only one peak on  $(0, -\frac{b_2}{b_1})$  and there are most two solutions to the optimal policy mix.

3. Case  $\alpha^* < 1$ . Similar to the previous case, express from (121)

$$\nu^* \vartheta^{L,*} = \underbrace{\frac{\chi^* - \eta^* + \vartheta^*(1 - \chi^*)}{(1 - \alpha^*)\vartheta^*}}_{\equiv b_1 > 0} \cdot \nu^* + \underbrace{\frac{1 - \vartheta^*}{(\alpha^* - 1)\vartheta^*}}_{\equiv b_2 < 0}$$

and plug in (120):

$$\frac{\sigma^{\eta,*}}{1 - \eta^*} \sigma^{\mathcal{B},*} = \mathfrak{t}'(\nu^*) \cdot (b_1(\nu^*)^2 + b_2\nu^*) \quad (123)$$

An optimal policy mix exists if there is  $\nu^* > 0$  satisfying the above condition and such that  $0 \leq b_1 + \frac{b_2}{\nu^*} \leq 1$ , ensuring  $\vartheta^{L,*} \in [0, 1]$ . The latter condition bounds  $\nu^* \in \left[-\frac{b_2}{b_1}, -\frac{b_2}{b_1-1}\right]$  with  $b_1 > 1$  following from the optimal value of  $\alpha^*$ .<sup>22</sup> As before, define  $g(\nu) = \mathfrak{t}'(\nu) \cdot (b_1\nu^2 + b_2\nu)$ . Note again that  $\mathfrak{t}'(\nu)$  is strictly increasing and strictly positive for all  $\nu > 0$ . Since  $b_1 > 0$  and  $b_2 < 0$ , polynomial  $b_1\nu^2 + b_2\nu$  is strictly negative on  $(0, -\frac{b_2}{b_1})$  and strictly positive and increasing for all  $\nu > -\frac{b_2}{b_1}$ . Therefore,  $g(\nu)$  is strictly positive and strictly increasing on  $(-\frac{b_2}{b_1}, \infty)$ , with  $g(-\frac{b_2}{b_1}) = 0$ . Since the LHS of (123) is strictly positive, a unique optimal policy mix exists if and only

<sup>22</sup>The bracket containing  $\alpha^*$  in (119) must be strictly positive since by proposition 3,  $\chi^* > \eta^*$ ,  $\sigma^{\eta,*} > 0$ ,  $\sigma^{\vartheta,*} > 0$  and  $\sigma^{\mathcal{B},*} > 0$ . This is then equivalent to  $b_1 > 1$ .

if:

$$g\left(-\frac{b_2}{b_1-1}\right) \geq \frac{\sigma^{\eta,*}}{1-\eta^*} \sigma^{\mathcal{B},*} \quad (124)$$

Notice that condition (124) is a sufficient (and almost surely necessary) condition for existence of a unique optimal policy mix in all three cases, provided  $g(\nu)$  has only one extremum on the  $\left(0, -\frac{b_2}{b_1}\right)$  interval. Consider limit as  $\alpha^* \rightarrow 1$ , then  $\nu^*$  from case 1 converges to  $-\frac{b_2}{b_1-1}$ , and (124) becomes equivalent to (1) – the condition ensuring existence and uniqueness in case 1. For case 2, note that by assumption,  $g(\nu)$  has only one extremum on the  $\left(0, -\frac{b_2}{b_1}\right)$  interval, and if (124) is satisfied, there is only one solution to (120) on the relevant interval  $\nu \in \left[-\frac{b_2}{b_1-1}, -\frac{b_2}{b_1}\right]$ . The converse is almost always true: if there is a unique solution under case 2, it must be that (124) is satisfied, since  $g\left(-\frac{b_2}{b_1}\right) < \frac{\sigma^{\eta,*}}{1-\eta^*} \sigma^{\mathcal{B},*}$  and by continuity it must be  $g\left(-\frac{b_2}{b_1-1}\right) \geq \frac{\sigma^{\eta,*}}{1-\eta^*} \sigma^{\mathcal{B},*}$ , except for a knife-edge case when  $g(\nu) = \frac{\sigma^{\eta,*}}{1-\eta^*} \sigma^{\mathcal{B},*}$  at the extremum point  $\nu$  (in which case  $g\left(-\frac{b_2}{b_1-1}\right) < \frac{\sigma^{\eta,*}}{1-\eta^*} \sigma^{\mathcal{B},*}$ ). For reference:

$$-\frac{b_2}{b_1-1} = \frac{1-\vartheta^*}{\chi^* - \eta^* + \vartheta^*(1-\chi^*)}$$

In addition, note that neither  $b_1$  nor  $b_2$  depend on the transaction cost function  $\mathfrak{t}_t(\nu)$ . Therefore, the sufficient condition for existence of a unique optimal policy mix can be rewritten as:

$$\mathfrak{t}'\left(\frac{1-\vartheta^*}{\chi^* - \eta^* + \vartheta^*(1-\chi^*)}\right) \geq \left(\frac{\chi^* - \eta^* + \vartheta^*(1-\chi^*)}{1-\vartheta^*}\right)^2 \frac{\sigma^{\eta,*}}{1-\eta^*} \sigma^{\mathcal{B},*} \quad (125)$$

## E Proof of Proposition 5

First, we characterize the behavior of planner's allocation for small values of  $\tilde{\sigma}_t$ . Recall that random variables  $\vartheta_t^*$ ,  $\gamma_t^*$ ,  $\eta_t^*$  can be written as function of the underlying state  $\tilde{\sigma}_t^2$ . Using Ito's lemma, we can write:

$$\sigma^{\vartheta,*}(\tilde{\sigma}_t^2) = \frac{\partial \log \vartheta^*(\tilde{\sigma}_t^2)}{\partial \tilde{\sigma}_t^2} \sigma(\tilde{\sigma}_t^2)$$

Differentiating and manipulating the optimality condition (107) yields:

$$\frac{\partial \log \vartheta^*(\tilde{\sigma}_t^2)}{\partial \tilde{\sigma}_t^2} = \frac{1}{1 + \gamma^*(\tilde{\sigma}_t^2)} \frac{\partial \log \gamma^*(\tilde{\sigma}_t^2)}{\partial \tilde{\sigma}_t^2}$$

In turn, from (114) it follows:

$$\frac{\partial \log \gamma^*(\tilde{\sigma}_t^2)}{\partial \tilde{\sigma}_t^2} = \frac{1}{\tilde{\sigma}_t^2} \left[ 1 + 2\gamma^*(\tilde{\sigma}_t^2) \frac{f(\tilde{\sigma}_t^2)}{1 - \vartheta^*(\tilde{\sigma}_t^2)} + 2\gamma^*(\tilde{\sigma}_t^2) h(\tilde{\sigma}_t^2) g(\tilde{\sigma}_t^2) \right]^{-1} \quad (126)$$

where  $f(\tilde{\sigma}_t^2)$ ,  $g(\tilde{\sigma}_t^2)$  and  $h(\tilde{\sigma}_t^2)$  are provided in Appendix B.4. Then we can rewrite:

$$\sigma^{\vartheta,*}(\tilde{\sigma}_t^2) = \frac{1}{1 + \gamma^*(\tilde{\sigma}_t^2)} \left[ 1 + 2\gamma^*(\tilde{\sigma}_t^2) \frac{f(\tilde{\sigma}_t^2)}{1 - \vartheta^*(\tilde{\sigma}_t^2)} + 2\gamma^*(\tilde{\sigma}_t^2) h(\tilde{\sigma}_t^2) g(\tilde{\sigma}_t^2) \right]^{-1} \frac{\sigma(\tilde{\sigma}_t^2)}{\tilde{\sigma}_t^2}$$

It can be shown that as  $\tilde{\sigma}_t^2$  converges to zero:

$$\begin{aligned} \gamma^*(\tilde{\sigma}_t^2) &\rightarrow 0, & \vartheta^*(\tilde{\sigma}_t^2) &\rightarrow 0 \\ f(\tilde{\sigma}_t^2) &\rightarrow 1 + \rho\phi \\ h(\tilde{\sigma}_t^2) &\rightarrow \frac{1 - \varphi^2}{\lambda + \varphi^2(1 - \lambda)} \\ g(\tilde{\sigma}_t^2) &\rightarrow \frac{1 - \varphi^2}{\lambda + \varphi^2(1 - \lambda)} \frac{\rho\lambda(1 - \lambda)}{\rho + 2(\lambda^I + \lambda^H)} \end{aligned}$$

And therefore  $\sigma^{\vartheta,*}(\tilde{\sigma}_t^2)$  converges to the geometric volatility loading of  $\tilde{\sigma}_t^2$  as it approaches zero. Consider the special case of constant geometric volatility loading  $\sigma > 0$ :  $\sigma(\tilde{\sigma}_t^2) = \sigma\tilde{\sigma}_t^2$ . Then, since  $\partial\gamma^*(\tilde{\sigma}_t^2)/\partial\tilde{\sigma}_t^2 > 0$ :

$$\sigma^{\vartheta,*}(\tilde{\sigma}_t^2) \rightarrow \sigma, \quad \lim_{\tilde{\sigma}_t^2 \rightarrow 0} \frac{\partial\sigma^{\vartheta,*}(\tilde{\sigma}_t^2)}{\partial\tilde{\sigma}_t^2} < 0$$

Meaning that in the vicinity of zero, as the idiosyncratic risk increases, the geometric volatility of optimal  $\vartheta_t^*$  goes down. In the same way, we can write:

$$\sigma^{\eta,*}(\tilde{\sigma}_t^2) = \gamma^*(\tilde{\sigma}_t^2) \frac{g(\tilde{\sigma}_t^2)}{\eta^*(\tilde{\sigma}_t^2)} \left[ 1 + 2\gamma^*(\tilde{\sigma}_t^2) \frac{f(\tilde{\sigma}_t^2)}{1 - \vartheta^*(\tilde{\sigma}_t^2)} + 2\gamma^*(\tilde{\sigma}_t^2) h(\tilde{\sigma}_t^2) g(\tilde{\sigma}_t^2) \right]^{-1} \sigma$$

And show that:

$$\sigma^{\eta,*}(\tilde{\sigma}_t^2) \rightarrow 0, \quad \lim_{\tilde{\sigma}_t^2 \rightarrow 0} \frac{\partial\sigma^{\eta,*}(\tilde{\sigma}_t^2)}{\partial\tilde{\sigma}_t^2} > 0$$

Meaning that in the vicinity of zero, as the idiosyncratic risk increases, the geometric volatility of optimal  $\eta_t^*$  goes up.

We now move to the optimal policy in the competitive equilibrium. Recall the four conditions that need to be satisfied by the optimal policy mix:

$$\vartheta_t^L \sigma_t^{PL} = \sigma_t^B = \frac{(1 + \rho\phi)}{(1 - \vartheta_t^* + \rho\phi)} \sigma_t^{\vartheta,*} \quad (127)$$

$$\eta_t^* \sigma_t^{\eta,*} = (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta,*} + (\chi_t^* - \eta_t^* + \vartheta_t^*(\alpha_t - \chi_t^*)) \sigma_t^B \quad (128)$$

$$\frac{\sigma_t^{\eta,*}}{1 - \eta_t^*} \sigma_t^B = \vartheta_t^L \nu_t^2 \mathfrak{t}'(\nu_t) \quad (129)$$

$$\nu_t [\chi_t^* - \eta_t^* + \vartheta_t^*(1 - \chi_t^*) - (1 - \alpha_t) \vartheta_t^L \vartheta_t^*] = 1 - \vartheta_t^* \quad (130)$$

Using the fact that  $(1 - \vartheta_t^* + \rho\phi)(1 + \gamma_t^*) = 1 + \rho\phi$  and (127), we can conclude that:

$$\sigma^{\mathcal{B}}(\tilde{\sigma}_t^2) \rightarrow \sigma, \quad \lim_{\tilde{\sigma}_t^2 \rightarrow 0} \frac{\partial \sigma^{\mathcal{B}}(\tilde{\sigma}_t^2)}{\partial \tilde{\sigma}_t^2} < 0$$

We can express the required distribution of bonds  $\alpha_t^*$  from (128) as:

$$\alpha_t^* = \frac{\frac{\eta_t^* \sigma_t^{\eta, *} + (\chi_t^* - \eta_t^*) \sigma_t^{\vartheta, *}}{\sigma_t^{\mathcal{B}}} + \eta_t^* - \chi_t^*}{\vartheta_t^*} + \chi_t^* \quad (131)$$

$$= \chi_t^* + \frac{\gamma_t^* g(\tilde{\sigma}_t^2)}{\vartheta_t^*} + \frac{(\eta_t^* - \chi_t^*)}{(1 + \rho\phi)} \quad (132)$$

Such that as idiosyncratic risk vanishes,  $\alpha_t^*$  converges to a finite number. From (130) it then follows that the required deposit velocity  $\nu_t^*$  also converges to a finite number. This implies that  $\vartheta_t^L$  must go to zero to ensure that the LHS of (129) vanishes together with the RHS. Since  $\vartheta_t^L \sigma_t^{PL} = \sigma_t^{\mathcal{B}}$  must converges to a finite value  $\sigma$ , optimal policy requires  $\sigma_t^{PL}$  to explode as  $\tilde{\sigma}_t^2 \rightarrow 0$ . Altogether, in the vicinity of zero, an increase in idiosyncratic risk calls for more moderate interest rate policy (lower  $\sigma_t^{PL}$ ) and a decrease in the central bank balance sheet (higher  $\vartheta_t^L$ ).

## F Proof of Proposition 6

Note that the planner's solution is unaffected by changes in aggregate risk since the planner chooses allocations state-by-state – all  $\vartheta^*(\tilde{\sigma})$ ,  $\eta^*(\tilde{\sigma})$ , etc, remain the same (as functions of  $\tilde{\sigma}$ ). However, stochastic processes  $\vartheta_t^*$ ,  $\eta_t^*$ , etc, are now different, in particular their volatility loadings are homogeneous of degree one in  $\sigma$ . For convenience, let us introduce  $\sigma$  explicitly as an argument of any function of interest:

$$\begin{aligned} \vartheta^*(\tilde{\sigma}_t; \sigma) &= \vartheta^*(\tilde{\sigma}_t) \\ \sigma_t^{\vartheta, *}(\sigma) &= \sigma^{\vartheta, *}(\tilde{\sigma}_t; \sigma) = \underbrace{\frac{\partial \vartheta^*(\tilde{\sigma}_t^2)}{\partial \tilde{\sigma}_t^2} \frac{\tilde{\sigma}_t^2}{\vartheta^*(\tilde{\sigma}_t^2)}}_{\text{independent of } \sigma} \sigma = \sigma \sigma_t^{\vartheta, *}(1) \end{aligned}$$

The same property then applies to the efficient volatility of nominal wealth:

$$\sigma_t^{\mathcal{B}}(\sigma) = \frac{(1 + \rho\phi)}{(1 - \vartheta_t^* + \rho\phi)} \sigma_t^{\vartheta, *}(\sigma) = \sigma \sigma_t^{\mathcal{B}}(1)$$

implying that the optimal distribution of long-term bonds  $\alpha_t$  is unaffected by  $\sigma$ :

$$\eta_t^* \sigma_t^{\eta, *}(\sigma) = (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta, *}(\sigma) + (\chi_t^* - \eta_t^* + \vartheta_t^*(\alpha_t(\sigma) - \chi_t^*)) \sigma_t^{\mathcal{B}}(\sigma) \quad (133)$$

$$\eta_t^* \sigma_t^{\eta, *}(1) = (\eta_t^* - \chi_t^*) \sigma_t^{\vartheta, *}(1) + (\chi_t^* - \eta_t^* + \vartheta_t^*(\alpha_t(\sigma) - \chi_t^*)) \sigma_t^{\mathcal{B}}(1) \implies \quad (134)$$

$$\alpha_t(\sigma) = \alpha_t \quad (135)$$

The optimal balance sheet composition  $\vartheta_t^L(\sigma)$ , volatility of bond price  $\sigma_t^{PL}(\sigma)$  and velocity  $\nu(\sigma)$  can then be solved from:

$$\begin{aligned} \vartheta_t^L(\sigma) \sigma_t^{PL}(\sigma) &= \sigma \sigma_t^{\mathcal{B}}(1) \\ \frac{\sigma \sigma_t^{\eta, *}(1)}{1 - \eta_t^*} \sigma_t^{PL}(\sigma) &= \nu_t(\sigma)^2 \mathfrak{t}'(\nu_t(\sigma)) \\ \nu_t(\sigma) [\chi_t^* - \eta_t^* + \vartheta_t^*(1 - \chi_t^*) - (1 - \alpha_t) \vartheta_t^L(\sigma) \vartheta_t^*] &= 1 - \vartheta_t^* \end{aligned}$$

Plugging the first condition into the second and applying the Implicit Function Theorem:

$$\begin{aligned} \frac{\partial \log \vartheta_t^L}{\partial \log \sigma} + \frac{\partial \log \sigma_t^{PL}}{\partial \log \sigma} &= 1 \\ \left[ 2 + \nu(\sigma) \frac{\mathfrak{t}''(\nu(\sigma))}{\mathfrak{t}'(\nu(\sigma))} \right] \frac{\partial \log \nu_t}{\partial \log \sigma} + \frac{\partial \log \vartheta_t^L}{\partial \log \sigma} &= 2 \\ \frac{\partial \log \nu_t}{\partial \log \sigma} (1 - \vartheta_t^*) &= (1 - \alpha_t) \nu_t(\sigma) \vartheta_t^L(\sigma) \vartheta_t^* \frac{\partial \log \vartheta_t^L}{\partial \log \sigma} \end{aligned}$$

Since  $\mathfrak{t}(\nu)$  is convex and increasing, the bracket in the second line is positive. The result then follows immediately. Note that even if  $\alpha_t = 0$ , then still  $\frac{\partial \log \vartheta_t^L}{\partial \log \sigma} > 0$ , and as long as  $\alpha_t > \underline{\alpha}_t$  for some  $\underline{\alpha}_t < 1$ ,  $\frac{\partial \log \vartheta_t^L}{\partial \log \sigma} > 0$  and  $\frac{\partial \log \sigma_t^{PL}}{\partial \log \sigma} < 0$ . We stress that the value of  $\underline{\alpha}_t$  can be inferred from the planner's allocation, given a fixed set of parameter values.

## G Expected Gains/Losses from Balance Sheet Management

To derive the extra drift term in the central bank's budget constraint, stemming from the covariance between long-term bond price movements and bond purchases, write the bond-purchase term in discrete time first:

$$P_{t+\Delta t}^L (L_{t+\Delta t} - L_t)$$

which highlights that bonds issued within period  $\Delta t$  are sold at the end of this period. Now take the expectation of the above term, divide by  $\Delta t$  and take the limit as  $\Delta t \rightarrow 0$ :

$$\begin{aligned}
& \lim_{\Delta t \rightarrow 0} \frac{\mathbb{E} [P_{t+\Delta t}^L (L_{t+\Delta t} - L_t)]}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{\mathbb{E} [(P_t^L + (P_{t+\Delta t}^L - P_t^L))(L_{t+\Delta t} - L_t)]}{\Delta t} \\
& = \lim_{\Delta t \rightarrow 0} \frac{P_t^L \mathbb{E} [L_{t+\Delta t} - L_t] + \mathbb{E} [(P_{t+\Delta t}^L - P_t^L)(L_{t+\Delta t} - L_t)]}{\Delta t} \\
& = P_t^L \frac{\mathbb{E} [dL_t]}{dt} + \frac{\mathbb{E} [dP_t^L L_t]}{dt} \\
& = P_t^L \mu_t^L L_t + P_t^L L_t \frac{\mathbb{E} [(\mu_t^{P^L} dt + \sigma_t^{P^L} dZ_t)(\mu_t^L dt + \sigma_t^L dZ_t)]}{dt} \\
& = P_t^L \mu_t^L L_t + P_t^L L_t \frac{\mathbb{E} [\mu_t^{P^L} \mu_t^L (dt)^2 + (\mu_t^L \sigma_t^{P^L} + \mu_t^{P^L} \sigma_t^L) dt dZ_t + \sigma_t^{P^L} \sigma_t^L (dZ_t)^2]}{dt} \\
& = P_t^L \mu_t^L L_t + P_t^L L_t \sigma_t^{P^L} \sigma_t^L
\end{aligned}$$

since  $\mathbb{E}[dZ_t] = 0$  and  $\mathbb{E}[(dZ_t)^2] = dt$ .

## H C/N with Type-switching

To ease exposition, we abstract from the choice of utilization, investment, deposit velocity and portfolios, focusing on the optimal consumption decision. Suppose that returns on net worth are given by  $dr_t^I = r_t^I dt + \sigma_t^I dZ_t + \tilde{\sigma}_t^I d\tilde{Z}_t$  for intermediaries and by  $dr_t^H = r_t^H dt + \sigma_t^H dZ_t + \tilde{\sigma}_t^H d\tilde{Z}_t$  for households. Intermediaries have a switching intensity  $\lambda^I$  and discount rate  $\rho^I$ , and households –  $\lambda^H$  and  $\rho^H$ . Their objectives are given by:

$$\begin{aligned}
V_t^I &= \max_{\{c_s^I\}_t^\infty} \mathbb{E} \left[ \int_t^T e^{-\rho^I(s-t)} \log(c_s^I) + e^{-\rho^I T} V_T^H \right] \\
& \quad dn_s^I = -c_s^I + n_s^I dr_s^I \\
V_t^H &= \max_{\{c_s^H\}_t^\infty} \mathbb{E} \left[ \int_0^T e^{-\rho^H(s-t)} \log(c_s^H) + e^{-\rho^H T} V_T^I \right] \\
& \quad dn_s^H = -c_s^H + n_s^H dr_s^H
\end{aligned}$$

The Hamiltonians take the form:

$$\begin{aligned}
H_t^I &= e^{-\rho^I t} \log(c_t^I) - \xi_t^I c_t^I + \xi_t^I n_t^I r_t^I - \xi_t^I n_t^I \zeta_t^I \sigma_t^I - \xi_t^I n_t^I \tilde{\zeta}_t^I \tilde{\sigma}_t^I \\
H_t^H &= e^{-\rho^H t} \log(c_t^H) - \xi_t^H c_t^H + \xi_t^H n_t^H r_t^H - \xi_t^H n_t^H \zeta_t^H \sigma_t^H - \xi_t^H n_t^H \tilde{\zeta}_t^H \tilde{\sigma}_t^H
\end{aligned}$$

with co-states following:

$$\begin{aligned}\frac{d\xi_t^I}{\xi_t^I} &= \mu_t^{\xi,I} dt - \varsigma_t^{\xi,I} dZ_t - \tilde{\varsigma}_t^{\xi,I} d\tilde{Z}_t + j_t^{\xi,I} (dJ_t^I - \lambda^I dt) \\ \frac{d\xi_t^H}{\xi_t^H} &= \mu_t^{\xi,H} dt - \varsigma_t^{\xi,H} dZ_t - \tilde{\varsigma}_t^{\xi,H} d\tilde{Z}_t + j_t^{\xi,H} (dJ_t^H - \lambda^H dt)\end{aligned}$$

with  $j_t^{\xi,I} = \frac{\xi_t^H - \xi_t^I}{\xi_t^I}$  and  $j_t^{\xi,H} = \frac{\xi_t^I - \xi_t^H}{\xi_t^H}$ . FOCs wrt  $c_t^I$  and  $c_t^H$  are given by:

$$e^{-\rho^I t} \frac{1}{c_t^I} = \xi_t^I \quad e^{-\rho^H t} \frac{1}{c_t^H} = \xi_t^H \quad (136)$$

and the co-state equations are:

$$\mu_t^{\xi,I} \xi_t^I = -\frac{\partial H_t^I}{\partial n_t^I} = -\xi_t^I (r_t^I - \varsigma_t^I \sigma_t^I - \tilde{\varsigma}_t^I \tilde{\sigma}_t^I) \quad (137)$$

$$\mu_t^{\xi,H} \xi_t^H = -\frac{\partial H_t^H}{\partial n_t^H} = -\xi_t^H (r_t^H - \varsigma_t^H \sigma_t^H - \tilde{\varsigma}_t^H \tilde{\sigma}_t^H) \quad (138)$$

Now guess that  $c_t^I = \alpha_t^I n_t^I$  and  $c_t^H = \alpha_t^H n_t^H$ , where:<sup>23</sup>

$$da_t^I = (a_t^H - a_t^I) dJ_t^I \quad da_t^H = (a_t^I - a_t^H) dJ_t^H$$

Applying the Ito's lemma on the FOCs yields:

$$\begin{aligned}\mu_t^{\xi,I} - \lambda^I j_t^{\xi,I} &= -\rho^I + \alpha_t^I - r_t^I + (\sigma_t^I)^2 + (\tilde{\sigma}_t^I)^2 \\ \varsigma_t^{\xi,I} &= \sigma_t^I \quad \tilde{\varsigma}_t^{\xi,I} = \tilde{\sigma}_t^I \\ \mu_t^{\xi,H} - \lambda^H j_t^{\xi,H} &= -\rho^H + \alpha_t^H - r_t^H + (\sigma_t^H)^2 + (\tilde{\sigma}_t^H)^2 \\ \varsigma_t^{\xi,H} &= \sigma_t^H \quad \tilde{\varsigma}_t^{\xi,H} = \tilde{\sigma}_t^H\end{aligned}$$

and combining with co-state equations:

$$\begin{aligned}\alpha_t^I &= \rho^I + \lambda^I \left(1 - \frac{\xi_t^H}{\xi_t^I}\right) = \rho^I + \lambda^I \left(1 - \frac{\alpha_t^I}{\alpha_t^H}\right) \\ \alpha_t^H &= \rho^H + \lambda^H \left(1 - \frac{\xi_t^I}{\xi_t^H}\right) = \rho^H + \lambda^H \left(1 - \frac{\alpha_t^H}{\alpha_t^I}\right)\end{aligned}$$

It is straightforward to verify that  $\alpha_t^I = \alpha_t^H = \rho$  if and only if  $\rho^I = \rho^H = \rho$ . One can arrive at the same conclusion using the HJB approach.

---

<sup>23</sup>One might also guess that  $\alpha_t^I$  and  $\alpha_t^H$  have drift and volatility components and arrive at the conclusion that they are indeed zero.