Strategic Money and Credit Ledgers

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Abstract

This paper studies strategic decision making by a private currency ledger operator, which faces competition from public money and/or other ledgers. A monopoly ledger operator can incentivize contract enforcement across the financial sector by threatening exclusion, but it can also impose markups through its pricing power. Currency competition limits rent extraction, but also makes coordinated contract enforcement more fragile. The emergent market structure bundles the provision of ledger and platform trading technologies. Regulation to ensure platform cooperation on contract enforcement and competition on markup setting is effective so long as agents can easily switch between platforms.

Keywords: Ledgers, currency competition, private currencies, "Industrial Organization of Money", smart contracts, platforms.

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1 Introduction

We have seen increasing competition in the provision of currencies and ledgers for settling transactions and contracts. BigTech consumer platforms have started to offer their own tokens and/or payment services (e.g. Alibaba, Meta, Amazon). Large supply chains have started to move payments and contracting onto shared ledgers (e.g. Corning, Emerson, Hayward). The "DeFi" community has provided decentralized ledgers to replicate traditional financial services (e.g. Ethereum, Solana). These changes make it important to understand what happens to the macroeconomy when a large, strategic, private institution provides the currency ledger. Can the ledger controller improve contract enforcement? Can they extract additional rents? How does a private token compete with public money? What is the likely market structure? How should regulators respond? To address these questions, we build a dynamic general equilibrium production model with private control of settlement assets and ledgers.

In our baseline macroeconomic model, a profit-maximizing ledger operator has monopoly control over a ledger technology that it uses to create its own currency. The currency can be used to make payments and settle the contracts that can be written on the ledger. Our environment is populated by agents who repeatedly interact with the ledger. They arrive as *producers* without collateral, borrow resources from financial intermediaries using contracts written on the ledger, hire labor to produce, sell their inventory, and then finally search as *buyers* for the opportunity to purchase consumption goods. Buyers face a fixed fraction of sellers that exclusively accept tokens, while the remaining fraction only accepts public money. The ledger controller charges a fee for using their payment technology. While searching for buying opportunities, agents deposit their wealth into financial intermediaries, which lend resources to producers and manage depositors' liquidity needs.

We show that the monopoly ledger controller ensures that contracts are enforced but also extracts rents from the economy. In an environment with only government dollars and absent an alternative coordinating device, there is always an equilibrium in which financial intermediaries secretly allow producers to default on loans from other financial intermediaries and store their sales revenue with them. The ledger controller chooses to eliminate this equilibrium by threatening to punish financial intermediaries that do not cooperate on contract enforcement by seizing their token holdings and excluding them from the ledger. The more transactions that are paid for with private tokens on the ledger, the easier it is for the ledger controller to inventivize contract enforcement. So, the type of payment technology matters for the collateralizability of future sales revenue. In other words, there is an enforcement-based rather than an information-based synergy between payments and lending. The downside of this arrangement is that the ledger operator also uses its market power to charge fees that depress output relative to perfect competition.

Section 3 studies the competition between a private ledger operator and public money to explore the fragility of the default-free credit market. Agents can now choose whether to pay with the ledger's token on a private trading technology or pay with public money on a public marketplace. The ledger operator must incentivize agents to use its payment technology. Charging higher markups reduces the fraction of sellers accepting tokens. This tightens the incentive compatibility constraint on financial intermediaries not accepting depositors who have defaulted on other financial intermediaries. If the elasticity of agent substitution is sufficiently high, this forces the ledger to lower markups, potentially making the provision of a no-default ledger unprofitable.

Section 4 studies competition between two ledger providers. We show that the emergent market structure is a platform that bundles ledger and trading technologies because this gives it greater exclusion power. So, tech platforms are the likely providers of payment system ledgers. The dominance of Alibaba and WeChat in the Chinese payment system might reflect their underlying advantages in trading technology. Competing private platforms that bundle ledger and trading technologies will cooperate on contract enforcement so long as the gap between their respective trading technologies is not too large and financial frictions do not prevent the less efficient platform from committing to pay the more efficient platform. Otherwise, a dominant platform emerges that attracts more trading and extracts higher rents. If regulators allow platforms to cooperate on contract enforcement but encourage them to compete on setting markups, then we recover a form of Bertrand competition with low markups so long as sellers can easily switch between platforms. This suggests that regulation of currency ledgers is only effective if policymakers also ensure agents can easily move between tech platforms.

While we focus on currency competition across digital platforms, our macro model can also be applied to the international financial system. In this case, the Federal Reserve Bank (or central bank of another large economy) provides the ledger, and the currencies of small open economies are competing public monies. The Fed can coordinate international banks on enforcement, but it can also use its power to extract rents through fees or inflation taxes.

Literature Review: Our paper is related to the literature on the role of ledgers and settlement assets in organizing trading systems. Aiyagari and Wallace (1991) and Kocherlakota (1998) study how a planner can increase the contracting space by updating a common ledger with trading histories. Freeman (1996b,a) studies how the choice of settlement asset creates or mitigates trading frictions in the currency market. Our model shares many features with these papers. However, we consider an environment where a private, profit maximizing agent controls the ledger. This brings an industrial organization perspective to the literature on ledgers and settlement assets. In Brunnermeier and Payne (2023), we extend our model to study strategic information portability decisions in a contested market setting.

Second, we relate to the literature on currency competition (e.g. Hayek (1976), Kareken and Wallace (1981), Brunnermeier and Sannikov (2019)). Formally, our baseline model in section 2 expands on a continuous time version of the two currency cash-in-advance model from Svensson (1985). Our full model in section 3 endogenizes currency demand using search and trading frictions in the tradition of the new monetarist literature (e.g. Lagos and Wright (2005), Lagos et al. (2017)). To investigate our IO-money perspective, the key features we need in our money model are: a platform that controls both a currency ledger and trading technology, directed search between trading and payment technologies, debt contracts specifying a settlement asset, and government money as an outside payment option. The later feature is shared with Lagos and Zhang (2019).

Third, we relate to the growing field of digital currencies. Instead of focusing on decentralized digital currencies such as cryptocurrencies (e.g. Fernández-Villaverde and Sanches (2018), Benigno et al. (2019), Abadi and Brunnermeier (2018), Schilling and Uhlig (2019), Cong et al. (2021)) or on central bank digital currency (e.g. Fernández-Villaverde et al. (2020), Keister and Sanches (2019), Kahn et al. (2019)), we are part of a less developed literature studying centralized digital currencies supplied by private tech platforms (e.g. Chiu and Wong (2020), Cong et al. (2020), Ahnert et al. (2022)). We argue that what makes "digital" currency special is its connection to a digital ledger and the associated increased contracting space.

Fourth, we relate to the literature on endogenizing debt limits when future income is difficult to pledge. In our model, the ledger operator can incentivize the repayment of debt contracts by threatening to seize assets and exclude agents from using their ledger. In this sense, the type of payment technology used determines the collateralizability of future sales revenue when purchasing inputs. This builds on classic papers on contract enforcement (e.g. Kehoe and Levine (1993), Holmström and Tirole (1998)) and the emerging literature on "digital collateral", (e.g. Garber et al. (2021)). Having a centralized platform can resolve the contracting issues across the supply chain presented in Bigio (2023). It also relates to Kahn and van Oordt (2022), where money is programmable and thereby offers users a commitment technology that stores resources in an escrow account until the payment is automatically executed.

The presence of cash as an alternative unmonitored payment technology in our model potentially allows agents to circumvent contract enforcement though "side-payments", similar to in Jacklin (1987). Rishabh and Schäublin (2021) shows the empirical counterpart. They document that after an Indian fintech company disbursed loans "digitally collateralized" by future digital sales revenue, borrowers' non-cash revenue drops. In our paper, the ledger controller resolves this enforcement difficulty by incentivizing financial intermediaries to report defaulters to other intermediaries so that they can coordinate on enforcement. This is in contrast to the literature, which has focused on incentivizing debtors directly.

Finally, our model relates to the literature on platforms (e.g. Rochet and Tirole (2003, 2006)). In our model, the platform strategically controls not only the trading technology but also a currency ledger.

We structure the paper in the following way. Section 2 solves the model with a monopoly ledger controller and exogenous currency demand. Section 3 introduces platforms that bundle trading and ledger technologies and endogenizes agent choices about where to trade. Section 4 introduces competition between strategic platforms and explores regulation approaches. Section 5 concludes.

2 Monopoly Ledger and Enforcement

In this section, we outline our "baseline" model of a monopoly provider of a common currency ledger. This model is a continuous time version of the two currency cash-in-advance model by Svensson (1985) but with the difference that one of the currencies is provided on a common ledger by a large, profit maximising agent. We use this model to highlight how a large ledger controller can both help and hurt the economy. On the one hand, they can coordinate the financial sector to enforce non-default but, on the other hand, they can use their monopoly power to extract rents and distort production decisions. In this baseline model we limit agent choices to focus on the monopoly ledger problem. In subsequent sections, we extend this model by endogenizing currency demand and allowing competition between ledgers.

2.1 Environment

Time is continuous with infinite horizon. There is a "labor" input and a final "consumption" good. The economy contains a continuum of agents, a continuum of mutual funds, and a monopoly controller of the ledger technology.

Production, Preferences, and Life-Cycle: Each agent follows a "life-cycle". Agents arrive at rate 1 without resources but with a production technology to immediately hire other agents' labor, l, and produce consumption goods to be sold next period according to the production function $y = f(l) = zl^\alpha$, where productivity $z > 0$ and $\alpha \in (0, 1)$. We refer to these agents as "sellers". After selling their production, agents start providing labor inputs to producers at marginal disutility Ξ and searching for consumption opportunities. We refer to these agents as "buyers". Buyers find trades at rate λ and once they find a trade they get log utility, $u(c) = \log(c)$ from consuming $c \geq 0$ consumption goods. After an agent gets a trading opportunity, they consume and exit. All traders have a discount rate ρ throughout their life.

Currencies: At any point in time, there are two currencies circulating in the economy, indexed by $\mathcal{M} \in \{0, 1\}$. Currency $\mathcal{M} = 0$ denotes "dollars" issued by the government. Currency $\mathcal{M} = 1$ denotes digital "tokens" recorded on a centralized digital ledger run by a profit maximizing controller. We assume that the ledger is programmed to supply tokens equal to the supply of dollars, $\bar{M}_t$, and the supply of dollars grows at fixed rate, μ^M. Trades must be transacted using currencies. We start by imposing that a fraction η of goods-sellers only accept tokens and a fraction $1 - \eta$ only accept dollars. We endogenize η in later sections. To focus on issues in the goods market, we assume that labor transactions can use either currency. The ledger operator charges a markup ψ on the goods price to sellers using the ledger.

Funds: After selling their goods, each generation forms a continuum of competitive mutual funds that invest their profits and provide insurance against the idiosyncratic random arrivals of trading opportunities.[1] On the asset side of a fund's balance sheet,

[1] We assume agents within a generation are assigned randomly to funds since agent assignment is not important for the aggregate variables in equilibrium. For simplicity, as in Blanchard (1985), we assume that after a fund forms it does not accept deposits from later generations. This is similar to a "Lucas-family" structure for each generation.

a fund can make short term loans to producers, hold reserves of currency, and hold equity in the ledger. On the liability side, the fund issues deposits. We assume that the fund faces a "reserve-in-advance" constraint that it must hold reserves for at least a fraction κ of searching depositors. Searching agents who do not find a trade bring currency back to the fund. We assume that there is frictionless contracting between the members of a particular fund and so the fund chooses (i) whether to allow their depositors to repay loans to other funds and (ii) the currency allocations given to buyers when they withdraw from the fund.

Ledgers, information, and contracts: The digital token provider organizes their tokens through a centralized ledger, which, in this section, is the only universal record keeping technology in the economy and is updated publicly at the end of each time period. The funds also extend and commit to one period credit contracts via the ledger. We start by assuming the ledger requires contracts to be settled using tokens and then later allow the ledger to choose the settlement asset. We impose that a fund posting a unit stock of wealth on the ledger makes loans at unit rate.[2]

The economy has information, enforcement, and monitoring frictions. Although agents have publicly verifiable identities, there is no legal system that can fully enforce contracts. This means that the ledger can always automatically enforce contracts when trades are paid using tokens but agents can default on contracts when they trade using dollars. If an agent trades in dollars and defaults on a contract, then the fund can recover only a fraction $\chi \in [0, 1)$ of the amount they are owed. The funds do not know whether they are making loans to producers that accept dollars or tokens and so cannot condition contracts on the payment received by the seller. The ledger operator cannot see the identities of all the depositors who enter a particular fund because some bring dollar cash. However, it can see the identities of any depositor who is given tokens when they leave the fund because those currency transactions must take place on the ledger. This means that if the ledger controller requests that

[2] We impose this restriction for technical reasons to ensure that we can have a funds with a positive measure of wealth in loans. Conceptually, it can be interpreted as a requirement that the funds must commit to loans in advance by pledging wealth.

funds provide the identities of their depositors at formation (a "know-your-customer" requirement), then the ledger controller can infer whether the fund is lying as soon they give tokens to any agents withdrawing deposits.

Timing: Each period is divided into a morning and an evening sub-period. In the morning, the timing is the following. (i) Producers start the period with inventory and a (randomly chosen) payment type that they accept. Funds start the period with currency holdings. (ii) The goods market opens. Depositors who get the opportunity to purchase goods withdraw their wealth from the fund in the currency required by the producer. Depositors and producers trading in a particular currency participate in a competitive goods market.[3] Depositors who purchase goods consume and exit.

In the evening, the timing is the following. (iii) Producers repay loans and deposit revenue with the fund (or default and face potential punishment). (iv) The ledger chooses policies for next period. We assume that the ledger operator can commit to a policy one period ahead but cannot commit to a sequence of policies (across multiple periods).[4] (v) New producers arrive and privately learn which currency they accept. (vi) The currency, asset, and labor markets open. Funds choose their asset portfolio for the next period, including their currency reserves. New producers borrow from funds, hire labor, and produce inventory.

Asset and markets: The economy has the following assets: dollars, tokens, producer loans, risk free short term loans amongst funds in zero net supply, and equity in the ledger. All asset markets are competitive. Following the monetary literature, it will be helpful to distinguish between "dollar-goods" traded in dollar transactions and "token-goods" traded in token transactions. Let $P_t^{\mathcal{M}}$ denote the units of currency $\mathcal{M}$

[3]The search literature often studies models where pricing is determined through one-to-one matching and bargaining over prices. Throughout this paper, we instead consider segmented competitive markets. We believe this is a closer approximation to the markets we are studying, especially in later sections when we model trade taking place on platforms such as Amazon or Alibaba.

[4]We make this assumption to simplify the ledger problem. There are many other papers that consider how promising policy paths can influence seigniorage revenue at time 0. We do not address those questions in this paper.

required to purchase an $\mathcal{M}$-good in the morning goods market. Let E_t denote the nominal currency exchange rate in the evening market: the tokens required to purchase 1 dollar. We typically use token-goods as the numeriare and use "real prices" to refer to prices in terms of token-goods (and, where helpful, use "nominal prices" to refer to prices in tokens). The corresponding real exchange rate is denoted by ϵ_t. Let r_t denote the real instantaneous rate on inter-fund loans and let r_t^B denote the real interest rate on producer loans. Let q_t^E denote the real price of ledger equity. Labor is traded in a competitive market at real wage w.

Technical restrictions: We assume that the economy starts with the steady state measure of buyers and funds so the population dynamics are stationary and the total mass of agents is $1/\lambda$. At time 0, the ledger sells M^0 tokens to the funds. We assume that laborers receive wages scaled by their wealth to ensure we can solve fund problem in closed form. Finally, we assume that $\alpha(1 + \mu^M) < 1$ to ensure that markups are positive. Our technical assumptions in this section are guided by the desire to get a closed form solution.

Discussion of the roles of money. The environment has an IO problem for the ledger built on top of a general equilibrium monetary model. The monetary model nests the three key roles of money: (i) store of value, (ii) medium of exchange, and (iii) unit of account or use for contract settlement. Concerning the first role, for $\kappa > 0$, the funds face a "reserve-in-advance" constraint, which in our continuous time setting can be seen as a precautionary liquidity buffer to service depositors. This constraint requires funds to store money across periods and so give up the higher return on bonds or ledger equity. This determines the equilibrium price of both currencies as well as the nominal and real exchange rates. As $\kappa \to 0$, the funds can satisfy depositors with a zero measure of money and so the opportunity cost of holding money becomes irrelevant. In this case, the price of money becomes indeterminate, similar to Kareken and Wallace (1981). Concerning the second role, agents need to use particular currencies for particular trades and so both monies are used as a medium of exchange. The relative currency acceptance ratio $\eta/(1-\eta)$ indexes the relative usefulness of tokens as a medium of exchange. Finally, concerning the third role, the contracts require a denomination or settlement asset, which in our baseline model is tokens.[5]

Modeling the ledger explicitly is required for digital money models because all digital money trades must be recorded. This means these models must give serious consideration to all three roles.

Discussion of the agent life-cycle: In our model, the agents follow a "life-cycle" where they start as good producers, then later provide labor for other producers. This ensures that producers need to borrow before they have any collateral.[6] It also ensures that agents interact with the ledger more than once so that exclusion from the ledger is costly. If sellers did not have to return to use the ledger as buyers, then there would be no way for the ledger operator to punish them for defaulting on loans.

Discussion of contracting difficulties: In our model, the ledger operator sees all the trades that use their ledger but none of the trades that use dollars. By contrast, the funds see money trades when dollars are deposited into their accounts. This means that the ledger operator can enforce contracts when trades take place using their ledger but can only enforce contracts off-ledger if the funds make producers repay loans. This creates a coordination problem for contract enforcement during dollar trades. Collectively, the funds would like the other funds to ensure depositors do not default. However, individually, they would like to secretly accept depositors who have defaulted and hide their sales revenue.

Discussion of ledger controller power: In this model, the ledger operator has a

[5]We do not distinguish between the denomination of the contract and the asset used for settling the contract.

[6]We would get qualitatively similar economic forces in a more "realistic"–but more challenging– environment with long-lived firms so long as some firms have sufficiently little wealth that they need to borrow.

number of ways to "exploit" their control of the ledger. They can take resources from agents or funds holding wealth on the ledger, exclude agents from using the ledger, and charge a fee for using the ledger. Observe that having the ledger technology is only powerful if agents want to use the ledger. If no trade occurred on the ledger, $\eta = 0$, then ledger control would not be useful. In this section, we treat η as exogenous. In the subsequent sections, we focus on endogenizing η by allowing the buyers and sellers to choose how to trade, and the ledger controller has to attract agents to its payment technology.

2.2 Market Equilibrium Without Default

In this subsection, we solve for a recursive characterization of a stationary equilibrium in which the currency-in-advance constraint is binding, the ledger sets a recursive markup policy, and no agent defaults. We use these results to establish the incentive compatibility constraint on not defaulting in the next section.

2.2.1 Fund Problem

Consider a fund that forms in the evening of t_0. The fund starts each period $t > t_0$ with a measure of agents remaining in the fund, I_t, reserves of dollars and tokens, (M_t^0, M_t^1), and holdings of other assets. In the morning goods market, the fund faces the "reserve-in-advance" constraints:

$$\kappa \int_0^{I_t} \lambda(1-\eta) P_t^0 c_t^{0i} \leq M_t^0, \qquad \kappa \int_0^{I_t} \lambda \eta P_t^1 c_t^{1i} \leq M_t^1,$$

where $P_t^{\mathcal{M}} c_t^{\mathcal{M}i}$ denotes the units of currency $\mathcal{M}$ the fund gives to depositor i withdrawing currency $\mathcal{M}$ at time t. It will be convenient to use token-goods as the numeraire for setting up the fund problem. Let $q_t^0 := E_t/P_t^1$ denotes the token-goods price of dollars, $q_t^1 := 1/P_t^1$ denote the token-goods price of tokens, and $\epsilon_t := E_t P_t^0 / P_t^1$ denote the "real" exchange rate from dollars to tokens. Then, the "reserve-in-advance" constraints become:

$$\kappa \int_0^{I_t} \lambda(1-\eta)\epsilon_t c_t^{0i} \leq q_t^0 M_t^0, \qquad \kappa \int_0^{I_t} \lambda\eta P_t^1 c_t^{1i} \leq q_t^1 M_t^1, \tag{2.1}$$

In the evening asset market, let the real wealth of the fund be defined as: $A_t := q_t^0 M_t^0 + q_t^1 M_t^1 + B_t$, where B_t is the real market value of nonmonetary assets. Let $\mu^{q\mathcal{M}}$ denote the growth rate in the price of currency $\mathcal{M}$ and and let μ^{qE} denote the growth rate in the price of ledger equity. Since funds diversify across producer loans, there is no default, and there is no aggregate risk, it must hold that all non-monetary assets, B_t, have the same return r_t:

$$r_t = r_t^B = \frac{\pi_t^E}{q_t^E} + \mu^{qE}$$

where π_t^E are ledger dividends. The wealth of the fund across evening markets evolves according to:

$$\begin{aligned} dA_t = \Bigg(\Big(r_t + (\mu_t^{q0} - r_t)(1-\varphi_t)\theta_t + (\mu_t^{q1} - r_t)\varphi_t\theta_t\Big) A_t \\ - \int_0^{I_t} \lambda \Big((1-\eta)\epsilon_t c_t^{0i} + \eta c_t^{1i}\Big)\, di + w_t N_t A_t \Bigg) dt \end{aligned} \tag{2.2}$$

where $\theta_t := (q_t^0 M_t^0 + q_t^1 M_t^1)/A_t$ is the share of fund wealth in currency, $\varphi_t := q_t^1 M_t^1/(q_t^0 M_t^0 + q_t^1 M_t^1)$ is the share of currency wealth in tokens, and N_t is aggregate labor supply. The measure of agents in the particular fund evolves according to:

$$dI_t = -\lambda I_t dt \tag{2.3}$$

The fund solves problem (2.4) below:

$$\begin{aligned} V(A_{t_0}, I_{t_0}) = \max_{c,\varphi,\theta,l} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left(\int_0^{I_t} \lambda \Big((1-\eta)u(c_t^{0i}) + \eta u(c_t^{1i})\Big)\, di - \Xi N_t \right) dt \right\} \\ s.t. \quad (2.1), (2.2), (2.3) \end{aligned} \tag{2.4}$$

where Ξ is the disutility from provision of labor[7] and the initial conditions are $A_{t_0} = \lambda\left((1-\eta)\pi^0_{t_0} + \eta\pi^1_{t_0}\right)$ and $I_{t_0} = 1$, where $\pi^{\mathcal{M}}_t$ is the profit that a producer makes in currency $\mathcal{M}$ transactions at time t.

Theorem 1. *Fund depositors supply labor elastically $N_t \in [0,\infty)$ when $w_t = 1$. If $\mu^{q0}_t = \mu^{q1}_t = \mu^q_t$ and the reserve-in-advance constraint binds, then all funds choose currency portfolio and consumption:*

$$\theta_t = \frac{\rho+\lambda}{r_t - \mu^q_t + 1/\kappa}, \qquad \varphi_t = \eta, \qquad c^{0i} = \frac{1}{\epsilon_t}\frac{\theta_t A_t}{\kappa\lambda I_t}\frac{a^i_0}{A_0}, \qquad c^{1i} = \frac{\theta_t A_t}{\kappa\lambda I_t}\frac{a^i_0}{A_0}$$

where a^i_0 is the real wealth brought to the fund by agent i. The value function of a fund with wealth A_t and mass of depositors I_t is:

$$V(A_t, I_t) = \left(\beta\log\left(\frac{A_t}{I_t}\right) + v_t\right) I_t, \quad \textit{where} \quad \beta := \frac{\lambda}{\rho+\lambda}$$

and where, in a stationary equilibrium v_t is given by:

$$v = \frac{\lambda}{\rho+\lambda}\left[\log\left(\frac{\theta\eta^\eta(1-\eta)^{1-\eta}}{\lambda\kappa\epsilon^{1-\eta}}\right) + \bar{u}_0 + \beta(r-\rho)\right]$$

where $\bar{u}_0 = \int_0^1 u(a^i_0)di$.

Proof. See Appendix A. □

The solution to the fund problem allows us to solve for the aggregate choices of the fund sector. Let j index the funds and i index the depositors. All funds choose the same asset portfolio so aggregate demand for assets across all funds is given by the following:

$$M^0_t = (1-\varphi_t)\theta_t A_t, \qquad M^1_t = \varphi_t\theta_t A_t, \qquad B_t = \theta_t A_t,$$

where $A_t := \int_j A^j_t dj$ the aggregate wealth of the fund sector. Since the economy starts with the steady state distribution of buyers across funds we also have that aggregate

[7] We choose the disutility of labor to ensure so that we get elastic labor supply and $w_t = 1$.

consumption in $\mathcal{M}$-currency trades is given by:

$$C_t^{\mathcal{M}} = \int_{ij} c^{\mathcal{M}ji} didj = \int_{ij} \mathbb{1}^i_{\mathcal{M}} \frac{\theta A_t^j}{\epsilon^{\mathcal{M}1}\kappa I_t^j} \frac{a_0^{ij}}{A_0^j} didj = \frac{\eta^{\mathcal{M}}\theta}{\kappa\epsilon^{\mathcal{M}1}} A_t.$$

where $\mathbb{1}^i_{\mathcal{M}}$ is an indicator for whether agent i finds a seller that accepts payment $\mathcal{M}$.

2.2.2 Producer Problem

Suppose that producers believe that contracts will be enforced. Taking prices and returns as given, a producer accepting currency $\mathcal{M}$ chooses labor, $l_t^{\mathcal{M}}$, to produce inventory, $n_t^{\mathcal{M}} = z(l_t^{\mathcal{M}})^\alpha$, financed by borrowing, $b_t^{\mathcal{M}} = P_t^1 w_t l_t^{\mathcal{M}}$, to solve (2.5) below:

$$\begin{aligned} &\max_{n_t^{\mathcal{M}}, b_t^{\mathcal{M}}} \left\{\Pi\left(n_t^{\mathcal{M}}, b_t^{\mathcal{M}}\right)\right\}, \quad s.t. \\ &\Pi(n_t^{\mathcal{M}}, b_t^{\mathcal{M}}) := E^{\mathcal{M}1} P_t^{\mathcal{M}} (1-\psi^{\mathcal{M}}) n_t^{\mathcal{M}} - (1+r_t^B) b_t^{\mathcal{M}} \end{aligned} \tag{2.5}$$

where Π is profit in tokens, $\psi^{\mathcal{M}}$ is the markup for trading using currency $\mathcal{M}$, and $E^{\mathcal{M}1}$ is the exchange rate from currency $\mathcal{M}$ to 1 so $E^{01} := E$ and $E^{11} := 1$. Taking the FOC gives producer labor demand, output, and profit to be:

$$\begin{aligned} l_t^{\mathcal{M}} &= \left(\frac{\alpha z \epsilon_t^{\mathcal{M}1}(1-\psi^{\mathcal{M}})}{w(1+r^B)}\right)^{\frac{1}{1-\alpha}}, \qquad y_t^{\mathcal{M}} = z\left(\frac{\alpha z \epsilon_t^{\mathcal{M}1}(1-\psi^{\mathcal{M}})}{w(1+r^B)}\right)^{\frac{\alpha}{1-\alpha}} \\ \pi_t^{\mathcal{M}} &= \left(\frac{\alpha z \epsilon_t^{\mathcal{M}1}(1-\psi^{\mathcal{M}})}{w^\alpha(1+r^B)^\alpha}\right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right) \end{aligned} \tag{2.6}$$

where $\pi^{\mathcal{M}}$ is real profit for producers accepting currency $\mathcal{M}$, w is the real wage, and we have used the following notation $\epsilon_t^{01} := \epsilon_t = E_t P_t^0 / P_t^1$ and $\epsilon_t^{11} := 1$ for the real exchange rate.

2.2.3 Recursive Market Equilibrium

We look for a recursive, stationary, monetary equilibrium[8] with state variables, which we denote by $(\cdot)$, and is simply M_t in our setting.[9] Suppose that the ledger operator chooses a recursive policy $\psi(\cdot)$. Then, equilibrium is defined formally below.

Definition 1. Given a ledger policy rule, $\psi(\cdot)$, a competitive equilibrium is a collection of functions for prices, $(r(\cdot), r^B(\cdot), q^0(\cdot), q^1(\cdot), \epsilon(\cdot), q^E(\cdot))$, fund choices, $(c^{0i}(\cdot)$, $c^{1i}(\cdot)$, $\varphi(\cdot)$, $\theta(\cdot)$, $A(\cdot)$, $V(\cdot))$, producer choices, $(l^0(\cdot), l^1(\cdot))$ such that: (i) given prices, the fund choices solve the HJBE associated with problem (2.4), (ii) given prices, producers solve problem (2.5) and (iii) markets clear:

$$
\begin{aligned}
C^0(\cdot) &= (1-\eta)y^0(\cdot), & C^1(\cdot) &= \eta y^1(\cdot), \\
(1-\varphi(\cdot))\theta(\cdot)A(\cdot) &= q^0(\cdot)\bar{M}, & \varphi(\cdot)\theta(\cdot)A(\cdot) &= q^1(\cdot)\bar{M} \\
(1-\theta(\cdot))A(\cdot) &= w((1-\eta)l^0(\cdot) + \eta l^1(\cdot)) + q^E(\cdot), \\
L(\cdot) &= (1-\eta)l^0(\cdot) + \eta l^1(\cdot)
\end{aligned}
$$

where $C^{\mathcal{M}}(\cdot) = \int_{ij} c^{\mathcal{M}ij}(\cdot)didj$ is aggregate consumption in currency $\mathcal{M}$ trades across all agents i in all funds j, $A(\cdot) = \int_j A^j(\cdot)di$ is aggregate wealth across funds, and $L(\cdot)$ is aggregate labor supply.

We characterize the equilibrium prices in closed form below. As is standard in "currency-in-advance" models, the environment has money neutrality in the sense that the level of money supply does not affect real variables.

[8] We do not consider the possibility of non-stationary equilibria in which the growth rate in the price of money is non-stationary.

[9] In principle, the state variables are: $X_t = (I_t^j, M_t^{0j}, M_t^{1j}, A_t^j)_{j\geq 0}$, where I_t^j is the mass of buyers in the fund j, $M_t^{\mathcal{M}j}$ is the quantity of currency $\mathcal{M}$ in fund j, and A_t^j is the wealth of the fund j. However, we will have aggregation so we will not have to track the distribution of states. We assume that $I_0 = \bar{I}$ so we start in the steady state. Finally, we show that $\int_i A_t^j di$ is a function of M_t^0 and M_t^1.

Corollary 1. *The real prices are:*

$$\begin{aligned} &w(\cdot) = 1, \quad \epsilon(\cdot) = (1-\psi(\cdot))^{\alpha}, \quad q^E(\cdot) = \int_0^{\infty} e^{-\rho s} \frac{\xi(\cdot_{t+s})}{\xi(\cdot)} \pi^E(\cdot_{t+s}) ds, \\ &r(\cdot) = r^B(\cdot) = \rho + \lambda - \left(\frac{1-\eta}{\epsilon(\cdot)} + \eta(1-\psi(\cdot)) \right) \frac{\theta}{\kappa} + \mu^{y1}(\cdot) + \frac{\mathcal{S}(\cdot)\mu^{\mathcal{S}}(\cdot)}{\mathcal{S}(\cdot) + 1/\kappa} \end{aligned} \tag{2.7}$$

where $\xi(\cdot) = \partial_A V(\cdot)$ *is the marginal discount factor,* $\pi^E(\cdot)$ *is the profit from the ledger,* $\mathcal{S}(\cdot) = r(\cdot) - \mu^q$ *is the spread between the return on bonds and money,* $\mu_t^{\mathcal{S}}$ *is the growth rate of this return spread,* y *is the output in token trades, and* μ^{y1} *is the growth of output in token trades. Given price equations* (2.7)*, we can solve for labor and output using equations* (2.6)*. Output is:*

$$y_t^0 = z \left(\frac{\alpha z (1-\psi(\cdot))^{\alpha}}{1+r(\cdot)} \right)^{\frac{\alpha}{1-\alpha}}, \qquad y_t^1 = z \left(\frac{\alpha z (1-\psi(\cdot))}{1+r(\cdot)} \right)^{\frac{\alpha}{1-\alpha}} \tag{2.8}$$

Having solved for the real economy, the currency prices are given by:

$$q^0(\cdot) = \frac{\kappa \epsilon(\cdot)(1-\eta) y^0(\cdot)}{\bar{M}}, \qquad q^1(\cdot) = \frac{\kappa \eta y^1(\cdot)}{\bar{M}}$$

Proof. See Appendix A. □

To understand the equilibrium, observe that the real exchange rate is given implicitly by the ratio of output in token trades to dollar trades:

$$\epsilon_t = \frac{y_t^1}{y_t^0} = \left(\frac{1-\psi}{\epsilon_t} \right)^{\frac{\alpha}{1-\alpha}}$$

since buyers and sellers cannot choose which currency to use. So, if the ledger increases its fee, then it decreases production in token trades, which makes token-goods relatively scarce and so leads to a decrease in the real exchange rate ϵ_t (i.e. a relative real depreciation in dollars and real appreciation in tokens).

2.3 Incentive Compatibility Condition for No-Default

Now, we consider the possibility of default and derive the incentive compatibility condition under which ledger threats can enforce a no-default equilibrium. We first show that without the ledger, the economy has default. We then show that if η is sufficiently high, then the ledger can use threats to eliminate default.

2.3.1 No Ledger Punishment For Default

Suppose the common ledger operator chooses not to punish funds that allow their depositors to default (or there is no common ledger in the economy). Then, there is always an equilibrium in which the funds accept depositors who defaulted on their credit contracts. Why? The agents have public identities, so funds can report agents who default. However, the funds cannot track to which other fund an agent goes after defaulting. Even if funds could track, they are not able to exclude them from future trades. And, even if funds could exclude defaulting depositors, they would not want to do so because they do not internalize their aggregate impact on their actions on the default rate. So, there is always an equilibrium in which each fund accepts depositors who default on loans from other funds.

Corollary 2. *If the ledger operator does not punish default (or no ledger exists in the economy), then there is always an equilibrium where funds default.*[10]

Proof. Suppose that other funds are defaulting and the ledger is not punishing agents for defaulting. Then, a fund allows its agents to default because it increases the initial wealth per depositor in the fund, A_0/I_0, and the value of the fund is increasing in wealth per depositor. □

2.3.2 Ledger Punishment For Default

Now suppose that the ledger (i) asks funds to report the identities of the agents in their fund, and (ii) excludes any funds that allow their depositors to default. In our

[10]If the funds are able to share information to "reconstruct" the ledger, then it is possible there is an equilibrium where they can sustain cooperation through a "folk-theorem" style agreement. Such an equilibrium would be not be unique and we do not look for such equilibria.

information environment, the ledger can establish if a fund's agents have defaulted as soon as it gives tokens to their depositors. So, if fund allows its depositors to default, then it is excluded from the ledger and can only hold dollars.

Fund Problem Under Default: Suppose that the fund chooses to accept defaulting agents and so is excluded from the ledger. In this case, the fund can only save into dollars and so the cash-in-advance constraint becomes non-binding. Then the fund solves the standard consumption-saving problem (2.9) below:

$$\begin{aligned} V(A_0, I_0) =& \max_{c^0} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left(\lambda \int_0^{I_t} (1-\eta) u(c_t^{0i}) di - \Xi_t N_t \right) dt \right\} \\ s.t. \quad dA_t =& \left(\mu_t^{q0} A_t - \lambda \int_0^{I_t} (1-\eta) \epsilon_t c_t^{0i} di + w_t N_t A_t \right) dt \\ dI_t =& -\lambda I_t dt \end{aligned} \tag{2.9}$$

We can observe that depositors now only trade at the rate $\lambda(1-\eta)$ because they can only trade when they find sellers that accept dollars. In addition, they cannot access the real interest rate, r_t, and so their return on assets is simply the growth rate of the real dollar price.

Corollary 3. *The value function of a defaulting agent satisfies:*

$$\check{V}(A_t, I_t) = \left(\check{\beta} \log \left(\frac{A_t}{I_t} \right) + \check{v}_t \right) I_t, \quad \textit{where} \quad \check{\beta} := \frac{\lambda(1-\eta)}{\rho + \lambda(1-\eta)}$$

and where, in a stationary equilibrium, $\check{v}_t$ *is given by:*

$$\check{v}_t = \check{\beta}(\check{\beta} - \log(\check{\beta})) + \check{u}_0 + \frac{\check{\beta}}{\rho + \lambda(1-\eta)} \left(\mu_t^{q0} - \frac{\lambda(1-\eta)}{\check{\beta}} \right).$$

where $\check{u}_0 = \int_0^1 u(\check{a}_0^i) di$.

Proof of Corollary 3. See Appendix A. □

Note that the discount rate of a fund under default is $\check{\beta} := \frac{\lambda(1-\eta)}{\rho+\lambda(1-\eta)}$ instead of $\beta = \frac{\lambda}{\rho+\lambda}$ because depositors can now only trade at rate $\lambda(1-\eta)$. This implies that, for a given $A_t < \infty$, as $\eta \to 1$, the fund is essentially useless and so fund value goes to zero $\check{V}(A_t, I_t) \to 0$.

Production Under Default: Suppose that the producer believes they will default when they trade in dollars. Then, instead of repaying the full loan $(1 + r_t^B)b_t^0$ tokens, they only repay χb_t^0. Thus, under default, the producers choose labor, output and profit:

$$\check{l}^0 = \left(\frac{\alpha z \epsilon}{\chi}\right)^{\frac{1}{1-\alpha}}, \qquad \check{y}^0 = z\left(\frac{\alpha z \epsilon}{\chi}\right)^{\frac{\alpha}{1-\alpha}} \qquad \check{\pi}^0 = \epsilon\left(\frac{\alpha z \epsilon}{\chi}\right)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)$$

Fund Default Decision: Now, finally, consider the problem of a fund that forms at t_0 and is deciding whether to accept defaulting agents. A fund forces its depositors to repay if the following incentive compatibility condition is satisfied:

$$\begin{aligned} V(A_{t_0}, I_{t_0}) &\geq \check{V}(\check{A}_{t_0}, I_{t_0}) \\ \Rightarrow \beta \log(A_{t_0}/I_{t_0}) + v_t &\geq \check{\beta} \log(\check{A}_{t_0}/I_{t_0}) + \check{v}_t \end{aligned}$$

where A_{t_0} is the initial wealth of a fund that does not allows their depositors to default and $\check{A}_{t_0}$ is the initial wealth of a fund that allows their depositors to default:

$$\begin{aligned} A_{t_0} &= (1-\eta)\pi_{t_0}^0 + \eta \pi_{t_0}^1 \\ \check{A}_{t_0} &= (1-\eta)\check{\pi}_{t_0}^0 + \eta \pi_{t_0}^1 \end{aligned}$$

After rearranging, the incentive compatibility condition becomes:

$$\begin{aligned} \log\left(\frac{\left(\check{A}_0/\check{I}_0\right)^{\frac{\lambda(1-\eta)}{\rho+\lambda(1-\eta)}}}{(A_0/I_0)^{\frac{\lambda}{\rho+\lambda}}}\right) < \lambda(1-\eta)\log\left(\epsilon_t^{\frac{\lambda\eta}{(\rho+\lambda)(\rho+\lambda(1-\eta))}}\right) \\ + \left(\frac{1}{\rho+\lambda} - \frac{1}{\rho+\lambda(1-\eta)}\right)\bar{u}_0 \\ + \beta(r_t - \rho) - \check{\beta}(\mu^{q0} - \rho - \lambda(1-\eta)) \end{aligned}$$

where the left-hand-side is the potential benefit of default and the right-hand-side is opportunity cost of no longer being able to trade using tokens or access the loan market through the ledger.

Corollary 4. *If $\chi > 0$ and z and η are sufficiently high, then funds ensure their depositors repay loans to other funds.*

Proof. Since $\chi > 0$, we have that $\check{A}_{t_0} < \infty$. So, if $\eta \to 0$, then $\check{V}(\hat{A}_{t_0}, I_{t_0}) \to 0$. If z is sufficiently large, then $V(A_{t_0}, I_{t_0}) > 0$ and so the result follows. □

2.4 Ledger Controller Problem

Now, finally, we can consider the problem of the ledger operator. We suppose that $\chi > 0$ and η and z are sufficiently large that the ledger operator can successfully select the no-default equilibrium by threatening to exclude funds that accept depositors who default. We assume that the ledger operator internalizes their impact on the goods market and default rate but takes the fund discount factor, ξ_t, the real interest rate, r_t, and the real exchange rate, ϵ_t, as given.[11]

Ledger problem at $t > 0$: In this case, for each period, $t > 0$, the ledger operator chooses whether funds default on contracts, $\mathcal{D} \in \{0, 1\}$, and sets the fee, ψ, to solve (2.10):

$$\rho V(M_t) = \max_{\mathcal{D},\psi} \left\{ \xi_t(\psi \eta y^1(\psi, \mathcal{D}) + q_t^1 \mu^M M_t) + \partial_M V(M_t) \mu^M M_t \right\} \quad s.t.$$

$$y^1(\psi, \mathcal{D}) = z \left(\frac{\alpha z(1-\psi)}{1 + r_t^B(\mathcal{D})} \right)^{\frac{\alpha}{1-\alpha}}, \quad q^1(M_t) = \frac{\eta y^1(\psi, \mathcal{D})}{M_t}, \tag{2.10}$$

Other market clearing conditions,

[11] We set up the monopoly problem in this way for a number of reasons. Firstly, we believe it is closer to reality to think about ledger operators and trading platforms that have market power in the goods markets they control but do not internalize their impact on aggregate consumption in the economy. Secondly, for log utility, allowing the platform to internalize their impact on the household's discount factor leads to the result that the platform is indifferent about the amount of output produced in the problem because low output is exactly offset by a high household marginal value of output. These issues (and additional issues about the choice of numeraire) are discussed in Kelsey and Milne (2006) and Böhm (1994). A detailed discussion of how to define the objective function for a monopolist is beyond the scope of this paper. Finally, this specification gives us a closed form solution to the ledger problem.

where $\psi\eta y^1(\psi, \mathcal{D})$ is the markup revenue from transactions using the ledger, $q_t^1 \mu^M M_t$ is the seigniorage revenue from the growth in money supply, and $r_t^B(\mathcal{D})$ is the loan interest rate under default decision $\mathcal{D}$ with $\mathcal{D} = 1$ indicating default occurs. Since the ledger operator cannot commit to a markup across periods and its choice of markup and contract enforcement do not influence the evolution of M_t, the ledger operator solves the static problem:

$$\max_{\mathcal{D},\psi} \left\{ \eta(\psi + \mu^M) y^1(\psi, \mathcal{D}) \right\}.$$

Theorem 2. *The ledger operator chooses the no-default equilibrium, sets transaction fee $\psi = 1 - \alpha(1 + \mu^M) > 0$, and earns the flow profit $\pi^E = (1 - \alpha)(1 + \mu^M)\eta y^1$.*

Proof. See Appendix A. □

The ledger operator prefers the no-default equilibrium even though they are not making loans. The intuition is as follows. Substituting the optimal choice of ψ into equilibrium output under no default, (B.3), gives that production for money and token trades is:

$$y^0 = z \left(\frac{\alpha z \epsilon}{1 + r} \right)^{\frac{\alpha}{1-\alpha}} < y^{0,PC}, \quad y^1 = z \left(\frac{\alpha^2 (1 + \mu^M) z}{1 + r} \right)^{\frac{\alpha}{1-\alpha}} < y^{1,PC}$$

where $\epsilon = (1 - \psi)^\alpha = (\alpha(1 + \mu^M))^\alpha$, and $y^{\mathcal{M},PC}$ is production in currency $\mathcal{M}$ trades under perfect competition (with $\psi = 0$). If default occurs, then lenders raise the interest rate to cover the losses on default and so production is:

$$\check{y}^0 = z \left(\frac{\alpha z \check{\epsilon}}{\chi} \right)^{\frac{\alpha}{1-\alpha}}, \quad \check{y}^1 = z \left(\frac{\alpha^2 (1 + \mu^M) z}{1 + r + \delta} \right)^{\frac{\alpha}{1-\alpha}} < y^1 < y^{1,PC},$$

where $\check{\epsilon} = ((1-\psi)\chi/(1+r+\delta))^\alpha = (\alpha(1+\mu^M)\chi/(1+r+\delta))^\alpha$ and $\delta = (1-\eta)(1-\chi) > 0$ is the average default rate. So, the default equilibrium distorts production away from token trades because loans are only repaid in the token trades and so the higher interest rate is only paid on token trades. This ultimately decreases ledger profit.

We can also observe that although the ledger operator implements the first best contract enforcement outcome, they do not deliver the competitive level of output because they charge markups, which distorts the real exchange rate and production in trades on the ledger.

Ledger value at time $t = 0$. At time 0, the ledger sells M^1 tokens and gets value

$$V^0(M_0) = q^1 M^1 + V(M_0) = \kappa\eta y^1 + \frac{\pi^E}{\rho}$$

where the first term is seigniorage revenue, and $\pi^E = (1-\alpha)(1+\mu^M)\eta y^1$ is ledger profit. Since $q^1 M^1 = \kappa\eta y^1$ and y^1 is independent of M^1, the ledger is indifferent about initial token issuance. This means it as without loss of generality to assume that the ledger chooses the same initial money supply as the government. We can also observe that seigniorage revenue goes to 0 as $\kappa \to 0$ because money does not have to be stored in-advance for trading.

Payment technology as a way to collateralize sales revenue: In this economy, the type of payment technology matters for the collateralizability of future sales revenue. Trades made using tokens on the common ledger can always be used for the repayment of contracts and so essentially act as "collateral" for borrowing. Trades made using dollars are not automatically used for the repayment of contracts and so can only be used as collateral if the financial intermediaries coordinate on reporting and excluding defaulting agents. In this sense, the model is setup so the controller of the currency ledger can choose to what extent future sales revenue can be collateralized across the economy.

Trade-off between contract enforcement and rent extraction: This simple equilibrium model illustrates a key trade-off with having a common ledger. On the one hand, the ledger controller takes actions to get agents to cooperate on enforcing default by threatening to seize their resources and exclude them from the ledger. In this sense, we get a "fortuitous" alignment of incentives where the ledger wants to force the funds to behave in the way that funds would choose if they could agree to coordinate. However, on the other hand, the ledger controller has a monopoly over the provision of ledger services and so is able to extract rents by charging a high transaction fee. The inefficiency shows up in the depressed output in token trades compared to the one under perfect competition.

A natural monopoly dilemma: We can also see that there is a type of natural monopoly force in this economy. The more trade that uses the ledger (the higher is η), the more easily the ledger controller can enforce contracts. For example, suppose that the minimum η for which the no-default incentive compatibility constraint holds is greater than 1/2. In this case, there is no way for multiple ledgers to operate in the economy and enforce contracts unless they cooperate on enforcement. In other words, one large ledger provider can better enforce contracts than a collection of non-cooperative smaller ledger providers. So, a regulator in this environment needs to find a way to get a monopoly ledger provider to behave more competitively or have multiple large ledgers compete on markups while coordinating on contract enforcement. We take up these questions in section 4.

Choice of settlement asset: So far, we have assumed that the ledger had a technological constraint that contracts were denominated and settled in tokens.[12] We also assumed that contracts were settled in the evening and so producers could access the competitive currency market during settlement. If we relaxed these assumptions, then the ledger operator would choose tokens as the settlement asset on their ledger if transaction costs were sufficiently high. We formalize this argument in Appendix C.

[12]In many currency ledgers we observe in the world, this is a technological constraint.

2.5 Interpretations of the Ledger Controller

So far, we have not given an interpretation to the ledger provider or an explanation of which institution in the economy would provide the ledger. Some natural interpretations are that the ledger provider is a tech platform, an international payment platform, or government.

Interpretation as tech platform: In the subsequent sections, we will interpret the the ledger controller as a tech platform that also controls a trading technology. In this case, we can think about the agents as participating in a supply chain where the tech platform says that they must use their ledger in order to trade on their platform. This offers a non-bubble explanation of why a particular settlement asset and ledger are used in equilibrium. We are going to focus on the implications of endogenizing ledger demand in this way and argue that they have advantages in providing the ledger technology.

Interpretation as payment system: A related interpretation is that the ledger is provided by an international payment system like Paypal that allows users to hold currency balances. We can think about this interpretation as a special case of the model in the final section in which the ledger is separate from the trading technology. As we will see, there are reasons to think that such a combination would be dominated by a platform bundling ledger and trading technologies.

Interpretation as international currency: An alternative interpretation is that the ledger is provided by a large, foreign government and the money is provided by the domestic economy. For example, we can interpret the ledger controller as the Federal Reserve Bank and the money provider as a small open economy. In this case, we can interpret the results as saying that the Fed can coordinate international banks on enforcement but can also extract rents (e.g. through fees or an inflation tax in an expanded model). This offers a possible resolution to a long-standing puzzle in the international banking literature that it is unclear how international banks coordinate on excluding borrowers. We leave this interpretation for future work.

3 Competition Between Private Platform and Public Currency

The model in the previous section illustrates that a monopoly ledger operator can both incentivize contract enforcement across the financial sector and extract rents. However, it left important questions unanswered. What is the impact of competition when agents can choose their payment method? Does competition mitigate the rent seeking behavior of the ledger operator? Can the ledger still incentivize the financial sector to coordinate on the no-default equilibrium?

To answer these questions, we enrich the model from the previous section. We introduce a platform that controls both a goods trading technology and ledger technology. We allow agents to chose where to trade goods and we allow the platform to force agents to use their ledger when they use their trading technology. This means that currencies are backed by the usefulness of the platform's trading technology.

We use this model to study competition between the ledger and a public marketplace with public money. We show that when agents have a high elasticity of substitution, the need to incentivize no-default disciplines the markup behavior of the platform but does not restore a competitive equilibrium. If elasticity is sufficiently high, then then the platform becomes unwilling to ensure a no-default equilibrium. In the next section, we study competition between private platforms.

3.1 Environment Changes

The environment is the same as in section 2.1 but with the following modifications:

Labor market, goods market, and trading frictions: We keep the labor market competitive and frictionless.[13] By contrast, we introduce frictions into the goods market. Buyers must choose where to trade consumption goods. At any point in time, there are two trading technologies for connecting buyers and sellers, indexed by $\mathcal{L} \in \{0, 1\}$. Trading technology $\mathcal{L} = 0$ is not controlled by anyone and is referred to as the "public" marketplace. Trading technology $\mathcal{L} = 1$ is controlled by a profit maximizing organization, which we refer to as the private platform. In addition to the pecuniary benefits from trading on a platform, each time period, t, for each platform, $\mathcal{L} \in \{0, 1\}$, each agent, i, gets an idiosyncratic, independent amenity draw for trading on that platform.[14] For sellers, the draw is distributed according to $\zeta_t^{s\mathcal{L}i} \sim \log(\zeta^{\mathcal{L}}) + \mathrm{Gu}(1/\gamma^s, -(1/\gamma^s)\mathcal{E})$, where $\mathcal{E}$ is the Euler–Mascheroni constant and $\zeta^{\mathcal{L}}$ is a platform specific component that characterizes the average service quality provided by the platform to sellers. For buyers, the draw is distributed according to $\zeta_t^{b\mathcal{L}i} \sim \log(\zeta^{\mathcal{L}}) + \mathrm{Gu}(1/\gamma^b, -(1/\gamma^b)\mathcal{E})$. For convenience, we normalize $\zeta^0 = 1$ and denote $\zeta^1 = \zeta$. We do not impose a physical interpretation on the amenity values but they could be modeled as idiosyncratic search costs or good quality.[15] In the evening of each period, sellers observe their amenity and then choose on which market to post trading offers. In the morning of each period, buyers observe their amenity and then choose on which trading technology to search for trades. As before, buyers find trades at rate λ. A competitive market then opens on each trading technology amongst the sellers and buyers who found trading opportunities. As before, the platform charges a markup ψ on sellers.

Currencies and Currency Market Frictions: Once again, at any point in time, there are two currencies circulating in the economy, indexed by $\mathcal{M} \in \{0, 1\}$. However, now

[13]This assumption is without loss of generality. We could also have included a frictional labor market. Leaving the labor market competitive simplifies the algebra.

[14]We introduce idiosyncratic risk in order to avoid "bang-bang" solutions to the platform choice problem. Our model uses tools from the discrete choice literature. This is analogous to assuming a CES preference function across the trading technologies.

[15]For the cost interpretation, note that the Gumbel distribution takes values across the real line and so $\zeta^{b\mathcal{L}i}$ would represent a normalized cost. For the good quality interpretation, observe that we can write the total utility of a buyer receives as: $\log(e^{\zeta^{\mathcal{L}i}}\zeta^{b\mathcal{L}}c)$ and so $e^{\zeta^{\mathcal{L}i}}\zeta^{b\mathcal{L}}$ is essentially scaling the utility that the buyer gets from the good they consume.

currency $\mathcal{M} = 1$ denotes digital "tokens" issued by the private platform and organized on a digital ledger controlled by the platform. We assume that when buyers choose where to trade consumption goods, they request to withdraw their deposits from the fund in the currency that is used on the trading technology. We assume that sellers accept dollars in the public marketplace and the platform mandates that sellers can only accept tokens when trading on the private platform.

Discussion of platform role: The platform controls the two key technologies in this economy: a trading technology for connecting buyers and sellers in the goods market and the common record keeping ledger technology. They use these technologies to ensure three services are provided to this economy: uncollateralized credit extensions to producers, a trading technology in the consumption goods market, and a token currency that can used for storage and exchange:

(i) Similar to before, the platform is able to use their control of the trading technology and the ledger technology to disincentivize default by threatening to take resouces from funds who accept defaulting depositors.

(ii) The platform provides a differentiated trading service. As in other discrete choice models, the scale parameters, γ^b and γ^s, determine the elasticity of substitution between the trading service offered by the private platform and the public marketplace. The ratio ζ characterizes the relative average advantage (or disadvantage) of the private platform. Conceptually, we can think about γ and ζ as characterizing the "movability" or "interoperability" between platforms.

(iii) In the previous section, we exogenously imposed that agents valued the tokens on the ledger. Here, the platform tokens are valued because the platform requires them to be used on their trading technology. In this sense, the tokens are "use-tokens" backed by the benefit of being able to access the trading platform (instead of being a pure "bubble").

Discussion of feedback and network effects: While our environment has feedback through the real exchange rate between currencies, it does not have feedback through a matching function, as is common in the platform literature. The later could be introduced into the model by having the relative trading advantage of the platform, ζ, depend on the fraction of buyers and sellers trading on the platform.

3.2 Market Equilibrium

In this subsection, we solve for the recursive characterization of a stationary equilibrium with agent choice about where to trade. We first characterize the buyer and seller problems under no-default. We then characterize the market prices. Finally, we characterize the incentive compatibility constraint for no-default when agents can choose where to trade.

3.2.1 Buyer and Seller Problems

The agent problems are very similar to those in subsection 2.2. The main difference is that now both the buyers and sellers choose a technology on which to trade goods and funds internalize this choice when choosing their currency reserves.

Buyer search problem: Suppose that a buyer has deposits with token-goods value a_t in a fund. Taking price processes as given, a buyer chooses on which platform to search to solve problem (3.1) below:

$$\max_{c,\mathcal{L}} \left\{ \zeta_\tau^{b\mathcal{L}i} + u(c_\tau^{\mathcal{L}}) \right\} \quad s.t. \quad c_\tau^{\mathcal{L}} \le \epsilon_\tau^{\mathcal{L}1} a_\tau, \quad \forall \mathcal{L} \in \{0,1\}, \tag{3.1}$$

where $\mathcal{L} \in \{0,1\}$ is the platform on which the buyer is searching at time t, $\zeta_t^{b\mathcal{L}i}$ is the idiosyncratic amenity of searching on platform $\mathcal{L}$ at time t, and $\epsilon_\tau^{\mathcal{L}1} a_\tau$ is depositor wealth in terms of goods on platform $\mathcal{L}$.

Seller posting problem: The seller problem is similar to in section 2.1 except that now the seller also chooses a platform on which to trade (and so a currency to accept).

That is, the seller chooses platform, $\mathcal{L} \in \{0, 1\}$, to solve problem (3.2) below:

$$V_t^s = \max_{\mathcal{L}} \left\{ \mathbb{E}_t \left[\zeta_t^{s\mathcal{L}i} + V^b(\pi_t^{\mathcal{L}}, A_t/I_t) \right] \right\} \quad s.t. \tag{3.2}$$
$$\pi_t^{\mathcal{L}} = \left(\frac{\alpha z \epsilon_t^{\mathcal{L}1}(1-\psi^{\mathcal{L}})}{w_t^{\alpha}(1+r_t^B)^{\alpha}} \right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right),$$

where $V^b(\cdot, A_t/I_t)$ is the value of being in a fund with A_t/I_t and $\pi_t^{\mathcal{L}}$ is the profit a seller earns under their optimal choice of labor when selling on platform $\mathcal{L}$.

Lemma 1 summarizes the buyer and seller decisions about where to search.

Lemma 1. *Let η^b and η^s denote the fraction of buyers and sellers directing trade towards platform* 1.[16] *Then, under no-default we have:*

$$\eta^b = \left(1 + \left(\frac{1}{\zeta \epsilon_t} \right)^{\gamma^b} \right)^{-1}, \qquad \eta^s = \left(1 + \left(\frac{\epsilon_t}{\zeta(1-\psi)} \right)^{\beta^b \gamma^s} \right)^{-1} \tag{3.3}$$

Proof. This is a standard discrete choice problem. See Appendix B.2 for the details. □

We can see that, holding everything else constant, an increase in the private platform trading advantage, $\uparrow \zeta$, and an improvement in the real currency exchange rate for buyers on the private platform, $\uparrow \epsilon_t$, leads to an increase in the fraction of buyers that choose to trade on the private platform. Likewise, an improvement in the currency exchange rate for the seller receiving tokens, $\downarrow \epsilon_t$, leads to an increase in the fraction of producers choosing to sell on the private platform.

3.2.2 Fund Problem

The fund problem is the same as problem (2.4) in subsection 2.2.1 except that the fund now anticipates the buyer choice about where to trade. We set up the problem in detail in appendix B.2. Here, we just state the key results about portfolio choice in

[16]Where convenient, we will use $\eta^{b\mathcal{L}}$ and $\eta^{s\mathcal{L}}$ to denote the fraction of buyers and sellers directing trade towards platform $\mathcal{L}$.

Lemma 2 below. Essentially the fund makes the same portfolio choice as before but now sets the fraction of currency reserves in tokens to match the buyer choice about where to trade.

Lemma 2. *The fund excludes defaulting depositors if:*

$$\beta \log\left(\frac{\check{A}_t}{A_t}\right) \leq \beta \log\left(\beta\theta\left(\left(\frac{\zeta^0}{\epsilon}\right)^{\gamma^b} + (\zeta^1)^{\gamma^b}\right)^{1/\gamma^b} \frac{\epsilon}{\zeta^0}\right) + \frac{\beta}{\rho+\lambda}(r-\mu^q) \tag{3.4}$$

where $\check{A}_t$ and A_t denote the wealth brought to the fund if depositors do and do not default on loans. In this case, the fund chooses the portfolio:

$$\theta_t = \frac{\lambda}{\beta(r_t - \mu_t^q + 1/\kappa)}, \qquad \varphi_t = \eta_t^b$$

Proof. See Appendix B.2. □

The key difference is that now the IC constraint on deterring funds from defaulting depends upon ψ in a way that the platform will internalize because the buyers choose where to trade. This means that as the platform changes ψ, it must consider how this impacts the no-default IC-constraint.

3.2.3 Market Equilibrium

The market equilibrium is defined the same way as in Section 2.2.3 so we leave the full characterization to Appendix B.3. The only difference is that now the fraction of buyers and sellers trading on each marketplace is endogenous. This means that goods market clearing conditions now become:

$$(1-\eta_t^s)y_t^0 = \int_{ij} c_t^{0ji} didj, \qquad \eta_t^s y_t^1 = \int_{ij} c_t^{1ji} didj$$

Combining the goods market clearing equations with the asset market clearing and the fund reserve-in-advance requirement gives that:

$$\left(\frac{\eta_t^s}{1-\eta_t^s}\right)\left(\frac{y_t^1}{y_t^0}\right) = \epsilon_t \left(\frac{\eta_t^b}{1-\eta_t^b}\right)$$

This equation tells us how buyer and seller demand for platform trading services end up influence the real exchange rate. It is helpful to recall that ϵ_t is the real exchange rate from dollars to tokens. So, an increase (decrease) in ϵ_t is an an appreciation (depreciation) of dollars and a depreciation (appreciation) of tokens. An increase in production on the private platform, $\uparrow \eta_t^s y_t^1$, leads to an increase in ϵ_t because it makes token-goods relatively plentiful. By contrast, an increase in the fraction of buyers trading on the private platform, $\uparrow \eta_t^b$, drives up demand for tokens and so leads to a decrease in $\downarrow \epsilon_t$. Ultimately, this generates endogenous "feedback" through the real exchange rate: if buyer want to hold tokens, then tokens appreciate and more sellers want to trade in tokens.

Substituting in the expressions for η_t^b, η^s, y_t^0, and y_t^1 and then rearranging gives that the equilibrium real exchange rate is:

$$\epsilon_t = \left[\zeta^{\beta^b\gamma^s-\gamma^b}(1-\psi)^{\beta\gamma^s+\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{\frac{1}{1-\alpha}+\beta^b\gamma^s+\gamma^b}}$$

This nests the previous section as a special case when $\gamma^b = \gamma^s = 0$ and so $\eta^b = \eta^s$.

3.3 Platform Problem

The ledger controller now faces a more complicated problem because its choices impact the fraction of buyers and sellers trading on their platform, (η^b, η^s), and so the incentive compatibility constraint on agents not defaulting. We also allow the platform to internalize their impact on ϵ. We restrict attention to the case where the ledger operator implements the no-default equilibrium and only discuss the problem for $t > 0$ since the value at $t = 0$ is the same as before. In this case, each period

$t > 0$, the ledger operator chooses the fee, ψ, to solve problem (3.5) below:

$$\begin{aligned} \rho V(M_t) = \ & \max_{\psi} \left\{ \xi_t \left(\psi \eta^s(\psi) y^1(\psi) + q^1(\psi) \mu^M M_t \right) + D_M V(M_t) \mu^M M_t \right\} \\ s.t. \quad & y^1(\psi) = z \left(\frac{\alpha z (1-\psi)}{1+r_t} \right)^{\frac{\alpha}{1-\alpha}}, \quad q^1(\psi) = \frac{\kappa \eta^s(\psi) y^1(\psi)}{M_t}, \\ & \text{Fractions } \eta^b \text{ and } \eta^s \text{ satisfy (3.3)}, \\ & \text{The IC constraint (3.4) is satisfied}, \\ & \text{Other market clearing conditions}, \end{aligned} \tag{3.5}$$

where we can observe that we now have additional equations for the fraction of trade on the ledger and the incentive compatibility constraint on the funds forcing their depositors to repay loans. As before, since the ledger operator cannot commit to a markup process across periods and its choice of markup and contract enforcement do not influence the evolution of M_t, the ledger operator solves the static problem:

$$\begin{aligned} & \max_{\psi} \left\{ \xi_t \eta^s(\psi, \mathcal{S}) \left(\psi + \mu^M \right) y^1(\psi, \mathcal{S}) \right\} \quad s.t. \\ & \text{The IC constraint (3.4) is satisfied} \end{aligned}$$

Corollary 5. *For γ^s, γ^b sufficiently small and ζ^1 sufficiently large, the IC on non-default constraint does not bind and the platform chooses markup:*

$$\psi^* = \arg\max_{\psi} \left\{ \eta(\psi, \mathcal{S})(\psi + \mu^M) y^1(\psi, \mathcal{S}) \right\}$$

Proof. As $\gamma^b, \gamma^s \to 0$, $\eta^b, \eta^s \to 1/2$ and so the IC constraint becomes:

$$\beta \log \left(\frac{\check{A}_t}{A_t} \right) \leq \beta \log \left(\beta \theta \zeta^1 \right) + \frac{\beta}{\rho + \lambda} (r - \mu^q)$$

For ζ^1 sufficiently large, this constraint must be satisfied.

□

The intuition for the solution to the platform problem is as follows. As $\gamma^b, \gamma^s \to 0$, the extreme value shocks become infinitely disperse and so the agent choices of trading platform becomes perfectly inelastic with respect to markups. In this case, we return to the model in section 2.4 where the ledger operator sets the markup to be:

$$\psi = \arg\max_{\psi} \{\xi_t(\psi + \mu^M) y^1(\psi)\}$$

At the other limit, as $\gamma^b, \gamma^s \to \infty$, the extreme value shocks become infinitely concentrated and so the seller choice of currency becomes perfectly elastic with respect to markups and trading costs. In this case, the ledger operator cannot impose markups without getting $\eta = 0$ and violating the IC constraint.

To make these observations more concrete, Figure 1 plots the numerical solution to the equilibrium for different values of buyer elasticity. Evidently, for low γ^b, the constraint on incentivising default is non-binding because it is easy for the platform to attract buyers. However, as γ^b increases, it becomes more difficult for the platform to attract buyers and so the constraint eventually binds. Once this occurs, the platform must significantly decrease ψ in order to attract enough buyers to achieve the no-default equilibrium. In this sense, buyer elasticity is the key variable for understanding to what extent platform rent extraction is disciplined by the platform needing to attract traders.

Credit Fragility: The key result in section 2 is that the ledger operator is able and willing to coordinate the financial system to ensure "uncollateralized" contracts are enforced. Once we allows buyers and sellers to direct their search, this is not necessarily the case. Figure 1 shows that for very large γ^b, the platform actually needs to sets a negative markup (i.e. a subsidy) in order to attract sufficiently many traders which allows it to enforce contracts. This means that the platform derives negative value from running a non-default ledger and so would not choose to ensure contract enforcement. In this sense, the uncollateralized credit equilibrium is "fragile"; strong competition from the dollar and public marketplace make it too costly for the platform to set up a no-default ledger. This implies that regulatory changes that

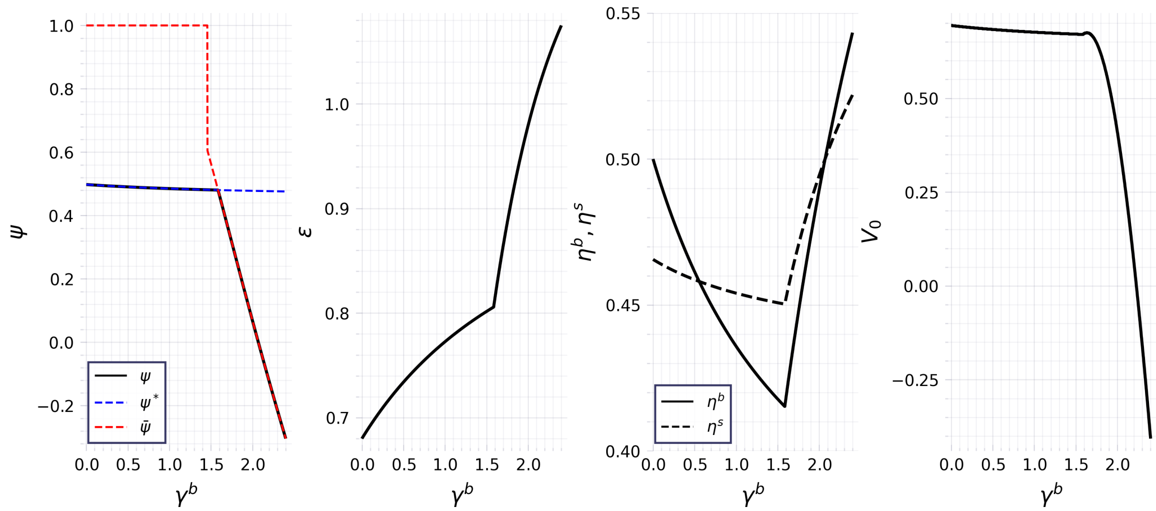

Figure 1: Ledger choice for $\gamma^b \in [0, 2]$.

Panel 1: The blue dashed line depicts the platform's markup choice, ψ^* if they are not constrained by having to ensure no-default. The red dashed line depicts the minimum value $\bar{\psi}$ required to deter funds from defaulting. The black line depicts the markup for the equilibrium chosen by the platform, ψ. Panel 2: shows the equilibrium real exchange rate under the platform's choice of ψ. Panel 3: shows the fractions of buyers and sellers that choose platform 1 given ψ. Panel 4: shows the time 0 value of a platforms that sets up a ledger and chooses the no-default equilibrium. Other variables are $\gamma^s = 0.1$, $z = 1.0$, $\alpha = 0.45$, $\rho = 0.2$, $\lambda = 1.0$, $\zeta = 1$, $\kappa = 0.1$, and $\chi = 0.5$.

increase agent ability to switch away from trading platforms would not necessarily be welfare improving.

4 Competition between Private Platforms

So far, we have only considered competition between a private platform and a public marketplace with cash. What if the private platform instead faces competition with another private platform? Can a sole ledger operator survive competing with a platform that also offers a trading technology? Will they cooperate on contract enforcement? Which platform's token will be used as the settlement asset for credit contracts?

To answer these questions, in this section, we study the strategic competition among two competing private ledgers/platforms. Our analysis yields several interesting findings. First, a ledger controller without an associated platform and trading technology is destined to lose out against a competing ledger controller who also offers a platform. In other words, bundling ledger and trading services is an equilibrium outcome. Second, regulation to ensure platform cooperation on contract enforcement and competition on markup setting is effective so long as agents can easily switch between platforms.

4.1 Environment and Equilibrium Changes

The environment is the same as in subsection 3.1 but with the following changes.

Goods market: There are now two private platforms, labeled $\mathcal{L} \in \{1, 2\}$. There is no public marketplace or public currency 0. Both platforms manage their own ledger, charge a markup $\psi^{\mathcal{L}}$, and have average trading advantage $\zeta^{\mathcal{L}}$. Platforms choose their markups simultaneously during the evening market each period. We let (η^b, η^s) denote the fraction of buyers and sellers choosing platform 1.

Ledgers and loans: Contracts on ledger $\mathcal{L}$ use currency $\mathcal{L}$ for the denomination of the contracts. Since there is no public dollar, agents cannot undertake side payments; all transactions are observed by one of the two platforms. In other words, in this new environment the only way producers can default is by writing a contract on ledger $\mathcal{L}$, then defaulting and trading on the other platform $\mathcal{L}'$. We also now assume that χ captures the dead-weight loss producers incur when they default and lenders get nothing when default occurs.

Currency market: We introduce settlement frictions to help understand the strategic advantage of becoming the dominant ledger. Agents can trade currencies during the morning but only get a fraction $\varsigma \leq 1$ of the overnight market exchange rate (with the remaining $1 - \varsigma$ fraction a deadweight transaction cost). Producers need to settle debt contracts during the morning market and so potentially face the ς cost. We assume that these transaction cost is sufficiently large so that producers would also choose loan arrangements in the ledger's token to avoid the cost. We use the currency provided by ledger 1 as the numeraire for asset pricing. So, ϵ_t now refers to the real exchange rate from tokens provided by platform 1 to tokens provided by platform 2.

Regulation: The regulator allows the platforms to bargain at time $t = 0$ over committing to exclude funds who allow their depositors to default on contracts on the other ledger. We assume that funds face no borrowing constraints or commitment problems during this bargaining and the the Nash bargaining protocol is followed. The regulator does not allow the platforms to collude on setting markups at times $t > 0$.

4.2 Symmetric Platforms

We start by looking at symmetric equilibria for an environment with symmetric platforms that have $\zeta^1 = \zeta^2$. We show platforms negotiate to cooperate on contract enforcement and seller elasticity governs the extent to which we recover the competitive allocation.

4.2.1 Market Equilibrium Given Platform Policies

There are two possible symmetric equilibria in this economy. First, there is an equilibrium with repayment on both ledgers. In this case, producers trading on platform $\mathcal{L}$ also borrow using ledger $\mathcal{L}$ so that they can avoid the transaction cost, ς, during contract settlement. This means the currency market frictions do not end up appearing in the equilibrium conditions. Given (ψ^1, ψ^2), the buyer, seller, and fund problems are the same as before. Likewise, the equilibrium definition is the same as before. The equilibrium characterization is the same as in subsection 3.2 but with $1 - \psi$ replaced by $(1 - \psi^1)/(1 - \psi^2)$ and without the sellers ever facing the exchange rate in order to settle contracts. The key equilibrium variables are:

$$y^1 = z\left(\frac{\alpha z(1-\psi^1)}{1+r}\right)^{\frac{\alpha}{1-\alpha}} \qquad y^2 = z\left(\frac{\alpha z(1-\psi^2)}{1+r}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\eta^b = \left(1+\left(\frac{\zeta^2}{\zeta^1\epsilon_t}\right)^{\gamma^b}\right)^{-1}, \qquad \eta^s = \left(1+\left(\frac{\zeta^2(1-\psi^2)}{\zeta^1(1-\psi^1)}\right)^{\beta^b\gamma^s}\right)^{-1}$$

$$\epsilon = \left[\left(\frac{\zeta^1}{\zeta^2}\right)^{\gamma^s\beta-\gamma^b}\left(\frac{1-\psi^1}{1-\psi^2}\right)^{\gamma^s\beta}\right]^{\frac{1}{1+\gamma^b}}$$

where η^b and η^s are the fraction buyers and sellers that trade on platform 1 and ϵ_t is the real exchange rate from tokens on ledger 1 to tokens on ledger 2. The other possible symmetric equilibrium is that there is default on both ledgers. In this case, no fund is willing to lend and so production shuts down in the economy.

4.2.2 Platform Competition

Platform competition $t > 0$: Given the contract enforcement equilibrium that has been bargained at $t = 0$ and the markup policy of the other platform, $\mathcal{L}'$, platform $\mathcal{L}$ chooses fee $\psi^{\mathcal{L}}$ to solve the problem (4.1) below:

$$\psi^{\mathcal{L}}(\psi^{\mathcal{L}'}) = \arg\max_{\psi^{\mathcal{L}}}\left\{\eta^{s\mathcal{L}}(\psi^{\mathcal{L}},\psi^{\mathcal{L}'})\left(\psi^{\mathcal{L}}+\mu^M\right)y^{\mathcal{L}}(\psi^{\mathcal{L}},\psi^{\mathcal{L}'})\right\} \tag{4.1}$$

where $\eta^{s\mathcal{L}}(\psi^{\mathcal{L}},\psi^{\mathcal{L}'})$ and $y^{\mathcal{L}}(\psi^{\mathcal{L}},\psi^{\mathcal{L}'})$ are the equilibrium fraction of sellers who trade on the platform $\mathcal{L}$ and the production on platform on $\mathcal{L}$. We look for the simultaneous Nash equilibrium in the evening of each period in which platforms each play a best response, $\psi^{\mathcal{L}}(\psi^{\mathcal{L}'})$ to each other.

Platform competition $t = 0$: At $t = 0$, platforms bargain over the contract enforcement regime. We assume that platform 0 makes a (positive or negative) transfer T to platform 1 time 0 and the payment is determined by a Nash Bargaining protocol.

In particular, we have:

$$
\begin{aligned}
T = & \arg\max_{T} \left\{ \left(q^{E2} - T - \tilde{q}^{E2} \right) \left(q^{E1} + T - \tilde{q}^{E1} \right) \right\} \\
& = q^{E2} - \tilde{q}^{E2} - \left(q^{E1} - \tilde{q}^{E1} \right)
\end{aligned}
$$

where $q^{E\mathcal{L}}$ is the price of equity in platform $\mathcal{L}$ under cooperation on enforcement and $q^{E\mathcal{L}}$ is price of equity in platform $\mathcal{L}$ if there is no cooperation on enforcement. Proposition 1 shows that, when platforms are symmetric, the outcome of the bargaining is contract enforcement on both ledgers.

Proposition 1. *The outcome of the bargaining at time* 0 *is that contracts are enforced on both ledgers and no transfer is made.*

Proof. If χ is sufficiently large that the threat of exclusion from either platform is sufficient to incentive funds to repay loans on that ledger, then $q^{E\mathcal{L}} = \tilde{q}^{E\mathcal{L}}$ and there is no need to bargain over enforcement because it doesn't require cooperation. If χ is sufficiently low that only exclusion from both platforms is sufficient to incentivize repayment, then for both $\mathcal{L}$, we have $q^{E\mathcal{L}} = q^{E}$ and $\tilde{q}^{E\mathcal{L}} = 0$ so outcome of the Nash Bargaining is cooperation on enforcement without a transfer $T = 0$. □

Given the outcome of the bargaining over contract enforcement, we show the numerical solution to platform competition for $t > 0$ in Figure 2. Evidently, we recover the perfectly competitive market outcome as $\gamma^{s} \to \infty$ and the agents become highly elastic. In this sense, seller elasticity governs the extent to which we recover Bertrand style competition.

4.3 Asymmetric Platforms and Technology Bundling

We now study the case where the platforms have different trading technologies and consider asymmetric equilibria. Without loss of generality, we assume platform 1 has the superior trading technology so that $\zeta := \zeta^{1}/\zeta^{2} > 1$. There are now potentially equilibria where one ledger becomes the dominant ledger for the economy. This means

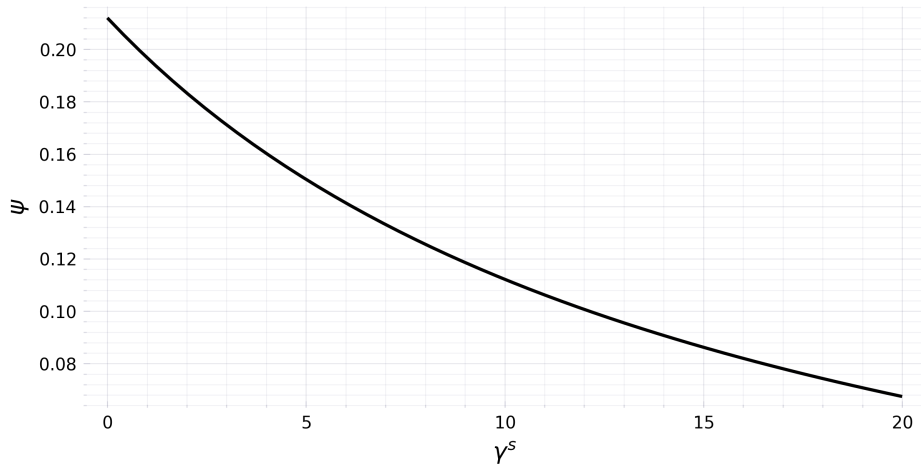

Figure 2: Ledger choice for $\gamma^s \in [0, 20]$.

Other variables are $\gamma^b = 0.5$, $z = 1.0$, $\alpha = 0.75$, $\rho = 1.0$, $\lambda = 1.0$, $\zeta = 1$, $\kappa = 0.1$, and $\chi = 0.5$.

that the frictions in the currency market impact the strategic decision making of the platforms.

4.3.1 Market Equilibrium Given Platform Polices

For ζ close to 1, when the trading advantage of platform 1 is not too large, the possible outcomes look like those in subsection 4.2.1. That is, if χ is large, then both platforms are able to enforce contract without cooperation and if χ is small, then cooperation is required for any contract enforcement. However, when χ and ζ are large, it is possible that, under non-cooperation, platform 1 can enforce contracts while platform 2 cannot. In this case, ledger 1 becomes the dominant ledger and so the currency market frictions become relevant.

Suppose we are in the asymmetric equilibrium where ledger 1 is the dominant ledger. Let the contract be specified so that $P_t^1(1 + r_t^B)w_t l_t^{\mathcal{M}}$ is the total repayment in units of currency 1. Then, the producer profit from trading using currency $\mathcal{M}$ and repaying loans is:

$$\Pi_t^{\mathcal{M}} := P_t^{\mathcal{M}}(1 - \psi^{\mathcal{M}})z(l_t^{\mathcal{M}})^{\alpha} - \frac{P_t^{\mathcal{N}}}{\varsigma^{\mathcal{M}1}E^{1\mathcal{M}}}(1 + r_t^B)w_t l_t^{\mathcal{M}}$$

where $\Pi_t^{\mathcal{M}}$ is the profit in units of currency $\mathcal{M}$ and we use the notation $\varsigma^{\mathcal{M}1} = \varsigma$ if $1 \neq \mathcal{M}$ and 1 otherwise. The potential difference between the trading currency price, $P_t^{\mathcal{M}}$, and the settlement currency price, P_t^1 reflects a potential exchange rate cost or benefit when settling transactions.[17] The key equilibrium variables are:

$$y^1 = z\left(\frac{\alpha z(1-\psi^1)}{1+r}\right)^{\frac{\alpha}{1-\alpha}} \qquad y^2 = z\left(\frac{\alpha z(1-\psi^2)\varsigma\epsilon_t}{1+r}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\eta^b = \left[1+\left(\frac{\zeta^2}{\zeta^1\epsilon_t}\right)^{\gamma^b}\right]^{-1}, \qquad \eta^s = \left[1+\left(\frac{\zeta^2(1-\psi^2)\varsigma\epsilon_t}{\zeta^1(1-\psi^1)}\right)^{\beta^b\gamma^s}\right]^{-1}$$

$$\epsilon_t = \left[\left(\frac{\zeta^1}{\zeta^2}\right)^{\beta^b\gamma^s-\gamma^b}\left(\frac{1-\psi^1}{\varsigma(1-\psi^2)}\right)^{\beta\gamma^s+\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{\frac{1}{1-\alpha}+\beta^b\gamma^s+\gamma^b}},$$

where we can observe that traders on platform 2 now face exchange rate and transaction costs when settling contracts. This introduces additional complexity into the real exchange rate. If $\varsigma\epsilon_t < 1$, then this is an advantage for platform 1 because sellers on platform 2 must make costly currency transactions to settle contracts, which increases trade on the platform.

4.3.2 Platform Competition

Platform bargaining at $t = 0$ is now more complicated because the outside option for platform 1 is more complicated. If χ and ζ are sufficiently large that ledger 1 can incentivize contract enforcement on their ledger without cooperation and $\varsigma\epsilon < 1$, then $\tilde{q}^{E1} > q^{E1}$ and so ledger 1 prefers the non-cooperative outcome. This means that the transfer platform 2 would have to pay to get enforcement leads to negatives surplus:

$$q^{E0} - \tilde{q}^{E2} - T = q^{E1} - \tilde{q}^{E1} < 0$$

and so the bargaining breaks down. This leads to the following corollary.

[17]The reason that settlement choice impacts profits is because the lenders cannot discern which currency a borrower is going to trade in and so all borrowers face the same repayment. Depending on the exchange rate between dollars and tokens, this potentially leads to redistribution between the token and dollar sellers.

Corollary 6. *For χ and ζ sufficiently high and ς is sufficiently low, the outcome of the bargaining at $t = 0$ is that platform 1 provides the monopoly ledger for the economy.*

Implications for market structure: This result has important implications for the market structure. Ultimately, it shows that the only ledger operators that are viable are those that also possess a platform trading technology. In other words, there is a natural bundling between offering ledger and trading services. This implies that a financial intermediary with no trading technology (which would be modeled as $\zeta^2 = 0$ in our environment) would never provide the ledger in equilibrium. In this sense, "BigTech" platforms are more natural providers of currency ledgers and "FinTech" services. So, the dominance of Alibaba and WeChat in the Chinese payment system might reflect their underlying advantage in providing payments.

Regulatory lessons: First, a plausible regulatory option is to ensure there are multiple platforms that can cooperate on contract enforcement but compete on markups. However, this potentially relies on small platforms being able to commit to making payments to large platforms. If this bargaining breaks down, then the economy ends up in an equilibrium with one dominant ledger that attracts more trade and extracts higher rents. In this case, regulating markups on a monopoly platform may be preferable.

A second lesson is that financial competition regulators should focus on understanding the elasticity with which buyers and sellers are able substitute between trading technologies. This is ultimately what determines to what extent competition from public dollars or other platforms discipline platform markups. If the elasticity of sellers is sufficiently high, then we recover Bertrand competition and the economy has low markups.

5 Conclusion

In this paper, we model the strategic decision making of a private controller of the currency ledger used for settling transactions and writing contracts. We find that in an unregulated economy "BigTech" platforms are likely to provide "FinTech" services. This brings both benefits and costs. Tech platforms can expand uncollateralized credit across a supply chain by exploiting their control of the payment system to better coordinate the financial system to enforce contracts. However, Tech platforms will also use their control of the ledger to increase their market power and charge high markups. We see these issues playing out in China where tech platforms Alibaba and WeChat have created a well-functioning payment system with very limited competition.

Ultimately, our model suggests that currency ledgers may need to be regulated like other natural monopolies. This could include restrictions on when ledgers can cooperate on contract enforcement and compete on markups. It could also include a competing public option in the form a programmable Central Bank Digital Currency (CBDC) ledger. We consider further modeling of the government's regulatory options as important future work.

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A Supplementary Proofs to Section 2 (Online Appendix)

Proof of Theorem 1. The fund problem is:

$$V(A_{t_0}, I_{t_0}) = \max_{c,\varphi,\theta,l} \left\{ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \left(\int_0^{I_t} \lambda \left((1-\eta) u(c_t^{0i}) + \eta u(c_t^{1i}) \right) di - \Xi N_t \right) dt \right\}$$

$$\begin{aligned} s.t. \quad dA_t = & \Big(\left(r_t + (\mu_t^{q0} - r_t)(1-\varphi_t)\theta_t + (\mu_t^{q1} - r_t)\varphi_t\theta_t \right) A_t \\ & - \lambda \int_0^{I_t} \left((1-\eta)\epsilon_t c_t^{0i} + \eta c_t^{1i} \right) di + w_t N_t A_t \Big) dt \\ \kappa \int_0^{I_t} & \lambda \eta^{\mathcal{M}} c_t^{0i} di \leq \varphi_t^{\mathcal{M}} \theta_t A_t, \quad \forall \mathcal{M} \in \{0,1\} \end{aligned}$$

Let $\hat{c}_t^{\mathcal{M}} := c_t^{\mathcal{M}i}/a_0^i$ denote the consumption per unit of wealth brought to the fund. If the reserve-in-advance constraints bind, then for currency $\mathcal{M}$:

$$\begin{aligned} \kappa \int_0^{I_t} \lambda \eta^{\mathcal{M}} \hat{c}_t^{\mathcal{M}} a_{t_0}^i di &= \varphi_t^{\mathcal{M}} \theta_t A_t \\ \Rightarrow \hat{c}_t^{\mathcal{M}} &= \left(\frac{\varphi_t^{\mathcal{M}} \theta_t}{\lambda \kappa \eta^{\mathcal{M}}} \right) \frac{1}{\epsilon_t^{\mathcal{M}1}} \frac{A_t}{I_t} \frac{1}{A_{t_0}} \\ \Rightarrow c_t^{\mathcal{M}i} = \hat{c}_t^{\mathcal{M}} a_0^i &= \left(\frac{\varphi_t^{\mathcal{M}} \theta_t}{\lambda \kappa \eta^{\mathcal{M}}} \right) \frac{1}{\epsilon_t^{\mathcal{M}1}} \frac{A_t}{I_t} \frac{a_{t_0}^i}{A_{t_0}} \end{aligned}$$

where $\epsilon_t^{\mathcal{M}1} = \epsilon$ if $\mathcal{M} = 0$ and $\epsilon_t^{\mathcal{M}1} = 1$ otherwise. The budget constraint becomes:

$$dA_t = \left(r_t + (\mu_t^q - r_t - 1/\kappa)\theta_t + w_t N_t \right) A_t$$

The objective of the of the fund becomes:

$$\begin{aligned} \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \Bigg(\int_0^{I_t} \lambda \Bigg(& (1-\eta) u \left(\frac{(1-\varphi_t)\theta_t}{\lambda \kappa (1-\eta)} \frac{1}{\epsilon_t} \frac{A_t}{I_t} \frac{a_0^i}{A_0} \right) \\ & + \eta u \left(\frac{\varphi_t \theta_t}{\lambda \kappa \eta} \frac{A_t}{I_t} \frac{a_0^i}{A_0} \right) \Bigg) di - \Xi N_t \Bigg) dt \end{aligned}$$

The HJBE under a binding cash-in-advance constraint becomes:

$$
\begin{aligned}
\rho V_t(A_t, I_t) = \max_{\varphi,\theta,L} \Big\{ & \lambda I_t \Big((1-\eta)\log(1-\varphi) + \eta \log(\varphi) \\
& - (1-\eta)\log((1-\eta)\epsilon_t) - \eta\log(\eta) + \log(\theta_t/(\lambda\kappa)) + \log(A_t/I_t) \\
& + \bar{u}_0 - \Xi N_t \Big) + \partial_A V(A_t, I_t)(r_t + (\mu_t^q - r_t - 1/\kappa)\theta_t + w_t N_t) A_t \\
& - \partial_I V(A_t, I_t) I_t + \partial_t V_t(A_t, I_t) \Big\}
\end{aligned}
$$

where $\bar{u}_0 := \lambda \int_0^{I_t} \log(a_0^i/A_0)di$. The first order conditions are:

$$
\begin{aligned}
[\varphi_t]: \qquad & 0 = -\left(\frac{1-\eta}{1-\varphi}\right) + \frac{\eta}{\varphi} \\
[\theta_t]: \qquad & 0 = \frac{\lambda I_t}{\theta_t} + \partial_A V(A_t, I_t)(\mu_t^q - r_t - 1/\kappa) A_t \\
[N_t]: \qquad & \max_{N_t}\{(-I_t \Xi + w_t \partial_A V(A_t, I_t) A_t) N_t\}
\end{aligned}
$$

which imply the portfolio choices:

$$
\varphi_t = \eta, \qquad \theta_t = \frac{\lambda I_t}{A_t \partial_A V(A_t, I_t)(r_t - \mu_t^q + 1/\kappa)}
$$

We guess and verify the functional form:

$$
\begin{aligned}
V_t(A_t, I_t) &= (\beta \log(A_t/I_t) + v_t) I_t \\
\partial_A V_t(A_t, I_t) &= \beta I_t / A_t \\
\partial_I V_t(A_t, I_t) &= \beta \log(I_t/A_t) - \beta + v_t \\
\partial_t V_t(A_t, I_t) &= I_t \partial_t v_t
\end{aligned}
$$

So, the first order conditions become:

$$
\varphi_t = \eta, \qquad \theta_t = \frac{\lambda}{\beta(r_t - \mu_t^q + 1/\kappa)}
$$

and $N_t \in [0, \infty)$ if $\beta w_t = \Xi$. We set $\Xi = \beta$ so that the fund is indifferent about providing labor if and only if $w_t = 1$, which must be true in equilibrium. The HBJE becomes:

$$\rho(\beta\log(A_t/I_t)+v_t)I_t = \lambda I_t\Big(\log\left(\eta^\eta(1-\eta)^{1-\eta}\right)-(1-\eta)\log(\epsilon_t)+\log\left(\frac{\theta_t}{\lambda\kappa}\right)+\log\left(\frac{A_t}{I_t}\right)+\bar{\mu}_0\Big)$$

$$\beta I_t(r_t-\lambda/\beta)-\lambda(\beta\log(A_t/I_t)-\beta+v_t)I_t+I_t\partial v_t$$

Collecting coefficients gives:

$$\begin{aligned}
\beta &= \frac{\lambda}{\rho+\lambda}\\
v_t &= \frac{1}{\rho+\lambda}\left[\log\left(\frac{\theta\eta^\eta(1-\eta)^{1-\eta}}{\lambda\kappa\epsilon_t^{1-\eta}}\right)+\bar{u}_0+\beta(r_t-\rho)+\partial_t v_t\right]
\end{aligned}$$

So, in a stationary equilibrium we have that:

$$v=\frac{1}{\rho+\lambda}\left[\log\left(\frac{\theta\eta^\eta(1-\eta)^{1-\eta}}{\lambda\kappa\epsilon^{1-\eta}}\right)+\bar{u}_0+\beta(r-\rho)\right]$$

It will also be convenient to derive the "asset pricing equation" for the fund. Taking the envelope condition of the HJBE gives:

$$\begin{aligned}
\rho\partial_A V_t(A_t,I_t) = \frac{\lambda I_t}{A_t} &+ \partial_A V(A_t,I_t)\mu_t^A+\partial_{AA}V(A_t,I_t)\mu_t^A\\
&-\partial_{IA}V(A_t,I_t)I_t+\partial_{tA}V_t(A_t,I_t) \qquad \text{(A.1)}
\end{aligned}$$

Define the current value discount factor by $\xi_t := \partial_A V_t(A_t,I_t)$. Using Ito's Lemma we have:

$$\mu_t^\xi\xi_t = \partial_{AA}V(A_t,I_t)\mu_t^A-\partial_{IA}V(A_t,I_t)\lambda I_t+\partial_{tA}V_t(A_t,I_t) \qquad \text{(A.2)}$$

Using the first order condition for θ, we have that:

$$\begin{aligned}
&\frac{\lambda I_t}{A_t}+\partial_A V_t(A_t,I_t)\mu_t^A\\
&\quad=\theta_t\partial_A V_t(A_t,I_t)(r_t-\mu_t^q+1/\kappa)+\partial_A V(A_t,I_t)(r_t+(\mu_t^q-r_t-1/\kappa)\theta_t+wN_t)\\
&\quad=\xi_t(r_t+w_tN_t) \qquad \text{(A.3)}
\end{aligned}$$

Substituting (A.2) and (A.3) into the into the envelope condition (A.1) and dividing by ξ_t gives the familiar asset pricing equation:

$$\rho = r_t + w_t N_t + \mu_t^{\xi}. \tag{A.4}$$

Equivalently, we can write equation (A.4) in terms of average consumption per agent in the fund, which gives the so-called continuous time "Euler equation". The average consumption by depositors at the fund is:

$$C_t^{\mathcal{M}} := \frac{1}{I_t}\int_0^{I_t} c_t^{\mathcal{M}i} di = \frac{1}{I_t}\int_0^{I_t}\left(\frac{\theta_t}{\lambda\kappa}\right)\frac{1}{\epsilon_t^{\mathcal{M}1}}\frac{A_t}{I_t}\frac{a_{t_0}^i}{A_{t_0}} di = \frac{1}{\kappa\epsilon^{\mathcal{M}1}}\frac{\theta_t A_t}{\lambda I_t}$$

From the first order condition for θ_t, we have that:

$$1/\kappa = \xi_t \epsilon_t^{\mathcal{M}1} C_t^{\mathcal{M}}(\mathcal{S}_t + 1/\kappa)$$

where $\mathcal{S}_t := r_t - \mu_t^q$ is the excess return on bonds. By Ito's Lemma we have that:

$$\begin{aligned} 0 = {} & \epsilon_t^{\mathcal{M}1} C_t^{\mathcal{M}}(\mathcal{S}_t + 1/\kappa) d\xi_t + \xi_t C_t^{\mathcal{M}}(\mathcal{S}_t + 1/\kappa) d\epsilon_t^{\mathcal{M}1} \\ & + \xi_t \epsilon_t^{\mathcal{M}1}(\mathcal{S}_t + 1/\kappa) dC_t^{\mathcal{M}} + \xi_t \epsilon_t^{\mathcal{M}1} C_t^{\mathcal{M}} d(\mathcal{S}_t + 1/\kappa) \\ \Rightarrow \mu_t^{\xi} = {} & -\mu_t^{C\mathcal{M}} - \mu_t^{\epsilon\mathcal{M}} - \mu_t^{\mathcal{S}_t}\left(\frac{\mathcal{S}_t}{\mathcal{S}_t + 1/\kappa}\right) \end{aligned}$$

where μ_t^{ξ}, $\mu_t^{C\mathcal{M}}$, $\mu_t^{\epsilon\mathcal{M}1}$, and $\mu_t^{\mathcal{S}_t}$ are geometric drift for the SDF, of average consumption using currency $\mathcal{M}C$, real exchange rate from currency $\mathcal{M}$ to currency 1 and of excess return on bonds $\mathcal{S}$. Substituting into (A.4) and rearranging gives the Euler equation:

$$r_t = \rho - w_t N_t + \mu_t^{C\mathcal{M}} + \mu_t^{\epsilon\mathcal{M}1} + \mu_t^{\mathcal{S}_t}\left(\frac{\mathcal{S}_t}{\mathcal{S}_t + 1/\kappa}\right)$$

□

Proof of Corollary 1. Summarising the equilibrium and optimization equations gives the following characterization of equilibrium. Given $(\bar{M}_t, \mu^M)$, stationary distribution of depositors across funds, $\{I^j\}_j$, and growth rates $(\mu^{C\mathcal{M}1}, \mu^{\epsilon\mathcal{M}}, \mu^q, \mu^{\mathcal{S}})$, we can solve for the equilibrium variables:

$$(c^{0ij}, c^{1ij}, y^0, y^1, l^0, l^1, \xi, \varphi, \theta, A, w, r, q^0, q^1, \epsilon, q^E)$$

using the equations:

$$
\begin{aligned}
(1-\eta)y^0 &= \int\int_0^{I^j} \lambda(1-\eta)c^{0ji}didj, & \eta y^1 &= \int\int_0^{I^j} \lambda\eta c^{1ji}didj,\\
(1-\varphi)\theta A_t &= q^0\bar{M} & \varphi\theta A_t &= q^1\bar{M}\\
(1-\theta)A &= w((1-\eta)l^0+\eta l^1)+q^E, & &\\
y^0 &= z\left(\frac{\alpha z\epsilon}{1+r}\right)^{\frac{\alpha}{1-\alpha}} & y^1 &= z\left(\frac{\alpha z(1-\psi)}{1+r}\right)^{\frac{\alpha}{1-\alpha}}\\
l^0 &= \left(\frac{\alpha z\epsilon}{1+r}\right)^{\frac{1}{1-\alpha}} & l^1 &= \left(\frac{\alpha z(1-\psi)}{1+r}\right)^{\frac{1}{1-\alpha}}\\
\varphi &= \eta & \theta &= \frac{1}{\beta(r-\mu^q-1/\kappa)}\\
r &= \rho - wL^j + \mu^{C\mathcal{M}j} + \mu^{\epsilon\mathcal{M}1} + \frac{\mathcal{S}\mu_t^{\mathcal{S}}}{\mathcal{S}+1/\kappa} & &\\
w &= 1 & &\\
q^E &= \frac{\psi\eta y^1 + q^1\mu^M\bar{M}^1}{\rho} & &\\
c^{0ij} &= \frac{1}{\epsilon}\frac{\theta A^j}{\kappa\lambda I^j}\frac{a_0^i}{A_0}, & c^{1ij} &= \frac{\theta A^j}{\kappa\lambda I^j}\frac{a_0^i}{A_0}\\
\frac{q^0\bar{M}^0}{\kappa} &= \int\int_0^{I^j} \lambda(1-\eta)\epsilon c^{0ji}didj, & \frac{q^1\bar{M}^1}{\kappa} &= \int\int_0^{I^j} \lambda\eta c^{0ji}didj
\end{aligned}
$$

We can solve explicitly for the key prices. First consider the real exchange rate. Combining the first, second, and last lines gives:

$$
y^0 = \frac{\theta A}{\kappa\epsilon_t}, \qquad y^1 = \frac{\theta A}{\kappa}
$$

Dividing through gives that the real exchange rate, ϵ, is solved by:

$$
\epsilon = \frac{y^1}{y^0} = \left(\frac{1-\psi}{\epsilon}\right)^{\frac{\alpha}{1-\alpha}} \quad \Rightarrow \quad \epsilon = (1-\psi)^{\alpha}
$$

Now, we solve for the risk free rate. Define the average consumption of fund j by $C_t^{\mathcal{M}j} = \frac{1}{I^j}\int_0^{I^j} c^{\mathcal{M}ij}di$. Using the expression for consumption, we have that for any funds j and k:

$$I^j C^{\mathcal{M}j} = \left(\frac{A^j}{A^k}\right) I^k C^{\mathcal{M}k}$$

Combining with the goods market clearing condition gives:

$$\begin{aligned} y^{\mathcal{M}} = \int I^k C^{\mathcal{M}k} dk = \int \left(\frac{A^k}{A^j}\right) I^j C^{\mathcal{M}j} dk \\ \Rightarrow I^j C_t^{\mathcal{M}j} = \left(\frac{A^j}{A}\right) y^{\mathcal{M}} \\ \Rightarrow \mu_t^{C\mathcal{M}j} = \mu_t^{Ai} - \mu_t^A - \mu_t^I + \mu_t^{y\mathcal{M}} \end{aligned}$$

Substituting this into the Euler equation for $\mathcal{M} = 1$ gives:

$$r = \rho - L^j + \mu^{Aj} - \mu^A - \mu_t^I + \mu^{y1} + \frac{\mathcal{S}\mu^{\mathcal{S}}}{\mathcal{S} + 1/\kappa}$$

Multiplying the Euler equation by A_t^j / A_t and integrating over the distribution of funds gives:

$$r = \rho + \lambda - \int \left(\frac{A_t^j}{A_t}\right) N_t^j dj + \int \left(\frac{A_t^j}{A_t}\right) \mu^{Aj} dj - \mu^A + \mu^{y1} + \frac{\mathcal{S}\mu^{\mathcal{S}}}{\mathcal{S} + 1/\kappa}$$

Since dA_t is the evolution of total wealth and $\int dA_t^j dj$ is the evolution of total wealth in the existing funds, the difference is wealth brought by new funds: We have that:

$$\begin{aligned} dA_t - \int dA_t^j dj &= ((1-\eta)\pi^0 + \eta\pi^1) dt \\ \Rightarrow \int \left(\frac{A_t^j}{A_t}\right) \mu^{Aj} dj - \mu^A &= -\frac{1}{A_t}((1-\eta) y_t^0 + \eta(1-\psi) y_t^1) + N_t \\ &= -\left(\frac{1-\eta}{\epsilon_t} + \eta(1-\psi)\right) \frac{\theta}{\kappa} + N_t \end{aligned}$$

Combining the equations gives that the risk free rate is:

$$r = \rho + \lambda - \left(\frac{1-\eta}{\epsilon} + \eta(1-\psi)\right) \frac{\theta}{\kappa} + \mu^{y1} + \frac{\mathcal{S}\mu^{\mathcal{S}}}{\mathcal{S} + 1/\kappa}$$

□

Proof of Corollary 3. As in the proof of the fund problem under repayment, let $c^0_t := c^{0i}_t/a^i_0$ denote the consumption per unit of wealth brought to the fund. Then, the objective is given by:

$$\begin{aligned}
&\int_{t_0}^{\infty} e^{-\rho(t-t_0)}\left(\int_0^{I_t}\lambda(1-\eta)u(c^0_t a^i_0)di - \check{\Xi}N_t\right)dt \\
&= \int_{t_0}^{\infty} e^{-\rho(t-t_0)}\left(\int_0^{I_t}\lambda(1-\eta)u(c^0_t)di + \bar{u}_0 - \check{\Xi}N_t\right)dt
\end{aligned}$$

where $\bar{u}_0 = \lambda(1-\eta)\int_0^1 u(a^i_0)di$. The Hamiltonian-Jacobi-Bellman equation (HJBE) is given by:

$$\begin{aligned}
\rho V_t(A_t, I_t) = \max_{c^0,l}\Big\{&\lambda I_t(1-\eta)u(c^0_t) + \bar{u}_0 - \check{\Xi}N_t \\
&+ \partial_A V_t(A_t, I_t)\Big(\mu^{q0}_t - \lambda(1-\eta)I_t\epsilon_t c^0_t/A_t + w_t N_t\Big)A_t \\
&- \partial_I V_t(A_t, I_t)\lambda(1-\eta)I_t + \partial_t V_t(A_t)\Big\}
\end{aligned}$$

The first order conditions are:

$$\begin{aligned}
(c^0_t):&\qquad 0 = \lambda\zeta I_t(1-\eta)u'\left(c^0_t\right) - \partial_A V_t(A_t, I_t)\lambda I_t(1-\eta)\epsilon_t \\
(N_t):&\qquad \max_{N_t}\left\{(-\check{\Xi}_t + w_t\partial_A V_t(A_t, I_t))N_t\right\}
\end{aligned}$$

Guess functional form:

$$\begin{aligned}
\check{V}_t(A_t, I_t) &= (\check{\beta}\log(A_t/I_t) + \check{v}_t)I_t \\
\partial_A \check{V}_t(A_t, I_t) &= \check{\beta}I_t/A_t \\
\partial_I \check{V}_t(A_t, I_t) &= \check{\beta}\log(I_t/A_t) - \check{\beta} + \check{v}_t \\
\partial_t \check{V}_t(A_t, I_t) &= I_t\partial_t\check{v}_t
\end{aligned}$$

Then, the first order condition becomes:

$$\frac{1}{c^0_t} = \frac{\check{\beta}I_t\epsilon_t}{A_t} \quad\Rightarrow\quad c^0_t = \frac{1}{\check{\beta}\epsilon}\frac{A_t}{I_t}$$

and agents are indifferent about supplying labor if and only if $\check{\beta}w_t = \check{\Xi}$. As before we set $\check{\Xi} = \check{\beta}$ so that the fund is indifferent if and only if $w_t = 1$. Substituting into the HJBE gives:

$$\begin{aligned}\rho(\check{\beta}v\log(A_t/I_t)+v_t)I_t = {} & \lambda I_t(1-\eta)\log\left(\frac{1}{\check{\beta}\epsilon}\right)+\bar{u}_0 \\ & +\lambda I_t(1-\eta)\log(A_t/I_t)-(\check{\beta}\log(I_t/A_t)-\check{\beta}+v_t)\lambda(1-\eta)I_t \\ & +\check{\beta}I_t\left(\mu_t^{q0}-\lambda(1-\eta)/\check{\beta}\right)+I_t\partial_t v_t\end{aligned}$$

Equating coefficients:

$$\begin{aligned}\rho\check{\beta} &= \lambda(1-\eta)-\check{\beta}\lambda(1-\eta)\Rightarrow\check{\beta}=\frac{\lambda(1-\eta)}{\rho+\lambda(1-\eta)} \\ \rho\check{v}_t &= -\lambda(1-\eta)\log(\check{\beta}\epsilon)+\bar{u}_0-\lambda(1-\eta)(-\check{\beta}+\check{v}_t) \\ & \quad +\check{\beta}\left(\mu_t^{q0}-\lambda(1-\eta)/\check{\beta}\right)+\partial_t\check{v}_t\end{aligned}$$

In a stationary equilibrium, we have that $\partial_t\check{v}_t^0=0$ and so:

$$\check{v}_t=\check{\beta}(\check{\beta}-\log(\check{\beta}\epsilon))+\bar{u}_0+\frac{\check{\beta}}{\rho+\lambda(1-\eta)}\left(\mu_t^{q0}-\lambda(1-\eta)/\check{\beta}\right)$$

□

Proof of Theorem 2. Taking r and ϵ as given, the first order condition for ledger problem is:

$$\begin{aligned}y^1(\psi)+(\psi+\mu^M)(y^1)'(\psi) &= 0 \\ \Rightarrow y^1(\psi)-y^1(\psi)\left(\frac{\psi+\mu^M}{1-\psi}\right)\left(\frac{\alpha}{1-\alpha}\right) &= 0 \\ \Rightarrow\frac{\psi+\mu^M}{1-\psi} &= \frac{1-\alpha}{\alpha} \\ \Rightarrow\psi &= 1-\alpha(1+\mu^M)\end{aligned}$$

We can observe that this first order condition holds regardless of whether agents default or not because it does not depend upon r or ϵ, which are two prices that change in the default equilibrium.

Substituting the optimal choice of ψ into equilibrium output under no default, (B.3), gives that production for money and token trades is:

$$y^1 = z\left(\frac{\alpha^2(1+\mu^M)z}{1+r}\right)^{\frac{\alpha}{1-\alpha}}$$

where $\epsilon = (1-\psi)^\alpha = (\alpha(1+\mu^M))^\alpha$, $y^{\mathcal{M}PC}$ is production in currency $\mathcal{M}$ trades under perfect competition (with $\psi = 0$). If default occurs, then lenders raise the interest rate to cover the losses on default and so production is:

$$\check{y}^1 = z\left(\frac{\alpha^2(1+\mu^M)z}{1+r+\delta}\right)^{\frac{\alpha}{1-\alpha}} < y^1,$$

where $\delta = (1-\eta)(1-\chi) > 0$ is the average default rate. Thus, the ledger prefers the no-default equilibrium. □

B Supplementary Proofs for Section 3 (Online Appendix)

B.1 Discrete Choice Problems

This section of the appendix contains working for the discrete choice problems. Since these are standard results, we provided limited detail.

Lemma 3. *Let $\{\zeta^n\}_{n\leq N}$ be a collection of independent draws from $Gu(\gamma,\mu)$, where $\mu = -\gamma\mathcal{E}$ and $\mathcal{E}$ represents the Euler–Mascheroni constant. Let $u(c) = \log(c)$. Then:*

$$\max_{n\leq N}\{\zeta^n + \varphi^n u(\pi^n)\} \sim Gu\left(\gamma, \mu + \gamma\log\left(\sum_n (\pi^n)^{\varphi^n/\gamma}\right)\right) \tag{B.1}$$

and so we have:

$$\mathbb{E}[\max_n\{\zeta^n + \varphi^n\log(\pi^n)\}] = \gamma\log\left(\sum_n(\pi^n)^{\varphi^n/\gamma}\right),$$

$$\mathbb{P}\left(n = argmax_{n'}\left\{\zeta^{n'} + \varphi^{n'}\log(\pi^{n'})\right\}\right) = \frac{(\pi^n)^{\varphi^n/\gamma}}{\sum_{n'}(\pi^{n'})^{\varphi^{n'}/\gamma}}$$

Proof. Using the definition of the Gumbel distribution and the independence of the

N draws, we have that:

$$
\begin{aligned}
\mathbb{P}(\max_n \{\zeta^n + \varphi^n u(\pi^n)\} \le k) &= \prod_n \mathbb{P}(\zeta^n + \varphi^n u(\pi^n) \le k) \\
&= \exp\left(\sum_n -e^{-(k-\mu)/\gamma} e^{\varphi^n u(\pi^n)/\gamma}\right) \\
&= \exp\left(-e^{-\left(k-\mu-\gamma\log\left(\sum_n e^{\varphi^n u(\pi^n)/\gamma}\right)\right)/\gamma}\right)
\end{aligned}
$$

which implies result (B.1). From the properties of the Gumbel distribution, the expectation is:

$$
\begin{aligned}
\mathbb{P}\left(n = \text{argmax}_{n'}\left\{\zeta^{n'} + \varphi^{n'}\log(\pi^{n'})\right\}\right) &= \left[\mu + \gamma\log\left(\sum_n (\pi^n)^{\varphi^n/\gamma}\right)\right] + \gamma\mathcal{E} \\
&= \gamma\log\left(\sum_n (\pi^n)^{\varphi^n/\gamma}\right)
\end{aligned}
$$

and the probability of choosing n is:

$$
\begin{aligned}
\mathbb{P}(n = \text{argmax}\{\zeta^{ni} + \varphi^n\log(\pi^n)\}) &= \frac{e^{\varphi^n u(\pi^n)/\gamma}}{\sum_{n'} e^{\varphi^{n'} u(\pi^{n'})/\gamma}} \\
&= \frac{(\pi^n)^{\varphi^n/\gamma}}{\sum_{n'}(\pi^{n'})^{\varphi^{n'}/\gamma}}
\end{aligned}
$$

□

B.2 Additional Proofs For Agent and Fund Problems

Proof of Lemma 1. Buyer problem: Instead of just solving for the buyer's choice of trading platform, we instead solve for the value functions of a buyer in a fund since we need this for the seller problem. The HJBE for buyer who deposited a_0^i into a fund that currently has $\hat{A} := A/I$ is given by:

$$
\begin{aligned}
\rho V_t^{bi}(\hat{A}) = \lambda\left(\mathbb{E}\left[\max_{\mathcal{L}}\left\{\zeta_t^{b\mathcal{L}i} + \log\left(\epsilon_t^{\mathcal{L}1} a_t^i\right)\right\}\right] - V_t^{bi}(\hat{A})\right) \\
+ \partial_{\hat{A}} V^{bi}(\hat{A})\mu^{\hat{A}}(\hat{A}) + \partial_t V_t(\hat{A})
\end{aligned}
$$

where a_t^i is the agent's claim to real wealth when they withdraw. From the fund's reserve-in-advance constraint, we must have that $a_t^i = (a_0^i/A_0)(\theta A_t/I_t)$ is the agent's share of currency reserves per depositor at t.

Lemma 3 gives that the fraction of buyers who choose $\mathcal{L}$ is given by:

$$\eta^{b\mathcal{L}} = \frac{\left(\zeta^{\mathcal{L}} \epsilon_t^{\mathcal{L}1}\right)^{\gamma^b}}{\sum_{\mathcal{L}'} \left(\zeta^{\mathcal{L}'} \epsilon_t^{\mathcal{L}'1}\right)^{\gamma^b}}$$

and so the HJBE becomes:

$$\rho V^{bi}(\hat{A}) = \lambda \left(\log \left(\bar{\nu}^b(\epsilon) \theta \frac{a_0^i}{A_0} \hat{A} \right) - V^{bi}(\hat{A}) \right) + \partial_{\hat{A}} V^{bi}(\hat{A}) \mu^{\hat{A}}(\hat{A}) + \partial_t V_t^{bi}(\hat{A})$$

where the average currency purchasing power is:

$$\bar{\nu}^b(\epsilon) := \left(\sum_{\mathcal{L}'} \left(\zeta_\tau^{\mathcal{L}'} \epsilon_t^{\mathcal{L}1} \right)^{\gamma^b} \right)^{1/\gamma^b}$$

In equilibrium, the evolution of fund net-worth is:

$$\begin{aligned} \mu^A A_t &= (r_t + (\mu_t^q - r_t - 1/\kappa)\theta_t + w_t N_t) A_t \\ &= \left(r_t - \frac{\lambda}{\beta} + w_t N_t \right) A_t \end{aligned}$$

where $\beta = \lambda/(\rho + \lambda)$ like earlier in the document and we have imposed the first order condition of the fund in advance. So, we have that:

$$\begin{aligned} d(\hat{A}_t) &= \hat{A}_t (dA_t/A_t - dI_t/I_t) \\ &= \left(r_t - \lambda \left(\frac{1-\beta}{\beta} \right) + w_t N_t \right) \hat{A}_t dt \end{aligned}$$

Guess that the value for a buyer is:

$$V^{bi}(\hat{A}) = V^b(a_0^i, \hat{A}) = \beta^{ba} \log(a_0) + \beta^{bA} \log(\hat{A}) + \nu_t^b$$

Substituting into the HJBE gives:

$$
\begin{aligned}
&\rho(\beta^{ba}\log(a_0) + \beta^{bA}\log(\hat{A}) + \nu^b) \\
&\quad = \lambda^b\Big(\log(a_0) + \log(\hat{A}) + \log\left(\frac{\bar{\nu}^b(\epsilon)\theta}{A_0}\right) - (\beta^{ba}\log(a_0) + \beta^{bA}\log(\hat{A}) + \nu^b)\Big) \\
&\qquad + \beta^{bA}\left(r_t - \lambda\left(\frac{1-\beta}{\beta}\right) + w_t N_t\right)\hat{A}_t + \partial_t \nu_t^b
\end{aligned}
$$

Equating coefficients we have:

$$
\begin{aligned}
\rho\beta^{ba} &= \lambda - \lambda\beta^{ba} \quad \Rightarrow \quad \beta^{ba} = \frac{\lambda}{\rho+\lambda} =: \beta \\
\rho\beta^{bA} &= \lambda - \lambda\beta^{bA} \quad \Rightarrow \quad \beta^{bA} = \frac{\lambda}{\rho+\lambda} =: \beta \\
\nu_t^b &= \frac{1}{\rho+\lambda}\left(\lambda\log\left(\frac{\bar{\nu}^b(\epsilon)\theta}{A_0}\right) + \beta\left(r_t - \lambda\left(\frac{1-\beta}{\beta}\right) + w_t N_t\right) + \partial_t \nu_t^b\right)
\end{aligned}
$$

Seller problem: Using the functional form for V^b, the seller problem becomes the standard discrete choice problem:

$$
\begin{aligned}
&\max_{\mathcal{L}}\left\{\mathbb{E}_t\left[\zeta^{s\mathcal{L}i} + V^b(\pi^{\mathcal{L}}, \hat{A}_t)\right]\right\} \\
&\quad = \max_{\mathcal{L}}\left\{\mathbb{E}_t\left[\zeta^{s\mathcal{L}i} + \beta^b\log(\pi^{\mathcal{L}}) + \beta^b\log(\hat{A}) + \nu_t^b\right]\right\} \\
&\quad = \max_{\mathcal{L}}\left\{\mathbb{E}_t\left[\zeta^{s\mathcal{L}i} + \beta^b\log(\pi^{\mathcal{L}})\right] + \beta^b\log(\hat{A}) + \nu_t^b\right\}
\end{aligned}
$$

Lemma 3 implies that

$$
\eta^{s\mathcal{L}} = \frac{\left(\zeta^{\mathcal{L}}\pi^{\mathcal{L}}\right)^{\beta^b\gamma^s}}{\sum_{\mathcal{L}'}\left(\zeta^{\mathcal{L}'}\pi^{\mathcal{L}'}\right)^{\beta^b\gamma^s}}.
$$

Substituting in the required condition for profit gives the required result.

□

Proof of Lemma 2. Fund Not Accepting Defaulting Depositors: Consider a fund that forms in the evening of t_0. The fund faces the reserve-in-advance constraints:

$$
\kappa\int_0^{I_t}\lambda(1-\eta_t^b)P_t^0 c_t^{0i}di \le M_t^0, \qquad \kappa\int_0^{I_t}\lambda\eta_t^b P_t^1 c_t^{1i}di \le M_t^1
$$

which can be rearranged as:

$$\kappa \int_0^{I_t} \lambda(1-\eta_t^b)\epsilon_t c_t^{0i} di \le q_t^0 M_t^0, \qquad \kappa \int_0^{I_t} \lambda \eta_t^b c_t^{1i} di \le q_t^1 M_t^1$$

Let a_t denote the claim to real wealth of depositors withdrawing to purchase goods. Let $c_t^{\mathcal{M}}$ denote the average consumption of depositors withdrawing and trading on platform $\mathcal{M}$. Then, we have that $c_t^1 = \epsilon_t c_t^0 = a_t$ and $c_t^{1i} = \epsilon_t c_t^{0i} = a_t(a_0^i/A_0)$. So the reserve-in-advance constraints become:

$$\kappa\lambda(1-\eta_t^b)c_t^1 I_t \le (1-\varphi_t)\theta_t A_t, \qquad \kappa\lambda\eta_t^b c_t^1 I_t \le \varphi_t \theta_t A_t$$

If the reserve-in-advance constraints bind, then the HJBE becomes:

$$\begin{aligned} \rho V_t(A_t, I_t) = \max_{\theta} \Big\{ & \lambda I_t \left(\log(\bar{\nu}^b(\epsilon_t))\right) + \log\left(\frac{\theta_t A_t}{\kappa\lambda I_t}\right) + \bar{u}_0 - \Xi N_t \\ & + \partial_A V(A_t, I_t)(r_t + (\mu_t^q - r_t - 1/\kappa)\theta_t + w_t N_t)A_t \\ & - \lambda \partial_I V(A_t, I_t) I_t + \partial_t V_t(A_t, I_t) \Big\} \end{aligned} \tag{B.2}$$

The first order condition is:

$$\theta_t = \frac{\lambda}{\frac{A_t}{I_t}\partial_A V_t(A_t, I_t)(r_t - \mu_t^q + 1/\kappa)}$$

Guessing the functional form:

$$V_t(A_t, I_t) = (\beta \log(A_t/I_t) + \nu_t) I_t,$$

substituting into the HJBE and collecting coefficients gives:

$$\begin{aligned} \beta &= \frac{\lambda}{\rho+\lambda} \\ \nu_t &= \frac{1}{\rho+\nu_t}\left(\lambda \log\left(\bar{\nu}^b(\epsilon)\frac{\theta_t}{\kappa\lambda}\right) + \bar{u}_0 + \beta\left(r_t - \lambda\left(\frac{1-\beta}{\beta}\right)\right) + \partial_t \nu_t\right) \end{aligned}$$

Fund Accepting Defaulting Depositors: An individual fund facing exclusion now no longer has a binding cash-in-advance constraint. So, the HJBE is:

$$\begin{aligned}\rho\check{V}_t(A_t, I_t) = \max_{c^0, l} \Big\{ & \lambda I_t u\left(c_t^0\right) + \bar{u}_0 - \nu \Xi_t N_t \\ & + \partial_A \check{V}_t(A_t, I_t)\Big(\mu_t^{q0} - \lambda I_t \epsilon_t c_t^0 / A_t + w_t N_t\Big) A_t \\ & - \partial_I \check{V}_t(A_t, I_t)\lambda I_t + \partial_t V_t(A_t)\Big\}\end{aligned}$$

Following the same steps as Theorem 1, we get that:

$$\begin{aligned}\check{V}(A_t, I_t) &= (\check{\beta}\log(A_t/I_t) + \check{v}^0)I_t, \quad \text{where} \\ \check{\beta} &= \frac{\lambda}{\rho+\lambda} = \beta \\ \check{v}_t^0 &= \beta(\beta - \log(\beta\epsilon)) + \frac{\beta}{\rho+\lambda}\left(\mu_t^{q0} - \lambda/\check{v}\right)\end{aligned}$$

Incentive compatibility constraint: Finally, the IC constraint for no-default for a fund forming at t becomes:

$$\begin{aligned} V(A_t, I_t) &\geq \check{V}(\check{A}_t, I_t) \\ \Rightarrow (\beta\log(A_t/I_t) + v^0)I_t &\geq (\beta\log(\check{A}_t/I_t) + \check{v}^0)I_t \\ \Rightarrow \beta\log(A_t) + v^0 &\geq \beta\log(\hat{A}_t) + \check{v}^0 \\ \Rightarrow \beta\log\left(\frac{\check{A}_t}{A_t}\right) &\leq v^0 - \check{v}^0 \end{aligned}$$

where we have that:

$$\begin{aligned} v^0 - \check{v}^0 = {} & \beta\left(\log(\nu(\epsilon)\theta) + \beta + \bar{u}_0\right) + \frac{\beta}{\rho+\lambda}\left(r - \lambda/\beta\right) \\ & - \beta(\beta - \log(\beta\epsilon) + \bar{u}_0) - \frac{\beta}{\rho+\lambda}\left(\mu_t^{q0} - \lambda/\beta\right) \\ = {} & \beta\log(\beta\bar{\nu}(\epsilon)\epsilon\theta) + \frac{\beta}{\rho+\lambda}\left(r - \mu^q\right) \end{aligned}$$

where

$$\bar{\nu}(\epsilon) = ((\zeta^0/\epsilon)^{\gamma^b} + (\zeta^1)^{\gamma^b})^{1/\gamma^b}$$

□

B.3 Definition and Characterization of Equilibrium

In this section of the appendix, we define and characterize the equilibrium for Section 3. This definition is very similar to the definition from Section 2.2.3 except that buyers and sellers choose where to trade. As before, look for a recursive, stationary, monetary equilibrium with state variables, which we denote by $(\cdot)$, and is simply M_t in our setting. Suppose that the ledger operator chooses a recursive policy $\psi(\cdot)$. Then, equilibrium is defined formally below.

Definition 2. Given a ledger policy rule, $\psi(\cdot)$, a competitive equilibrium is a collection of functions for prices, $(r(\cdot), r^B(\cdot), q^0(\cdot), q^1(\cdot), \epsilon(\cdot), q^E(\cdot))$, fund choices, $(c^{0i}(\cdot)$, $c^{1i}(\cdot)$, $\varphi(\cdot)$, $\theta(\cdot)$, $A(\cdot)$, $V(\cdot))$, producer choices, $(\eta^s(\cdot), l^0(\cdot), l^1(\cdot))$, and buyer choices, $(\eta^b(\cdot))$ such that: (i) given prices, the fund choices solve the HJBE (B.2), (ii) given prices, producers solve problem (3.2), (iii) given prices, buyers solve problem (3.1), and (iv) markets clear:

$$
\begin{aligned}
C^0(\cdot) &= (1-\eta^s(\cdot))y^0(\cdot), & C^1(\cdot) &= \eta^s(\cdot)y^1(\cdot),\\
(1-\varphi(\cdot))\theta(\cdot)A(\cdot) &= q^0(\cdot)\bar{M}, & \varphi(\cdot)\theta(\cdot)A(\cdot) &= q^1(\cdot)\bar{M}\\
(1-\theta(\cdot))A(\cdot) &= w((1-\eta)l^0(\cdot)+\eta l^1(\cdot)) + q^E(\cdot), & &\\
L(\cdot) &= (1-\eta)l^0(\cdot)+\eta l^1(\cdot) & &
\end{aligned}
$$

where $C^{\mathcal{M}}(\cdot) = \int_{ij} \eta^{b\mathcal{M}} c^{\mathcal{M}ij}(\cdot)didj$ is aggregate consumption in currency $\mathcal{M}$ trades across all agents i in all funds j, $A(\cdot) = \int_j A^j(\cdot)di$ is aggregate wealth across funds, and $L(\cdot)$ is aggregate labor supply.

We characterize the equilibrium prices below. As is standard in "currency-in-advance" models, the environment has money neutrality in the sense that the level of money supply does not affect real variables.

Corollary 7. *The real prices are:*

$$
\begin{aligned}
&w(\cdot) = 1, \quad \epsilon(\cdot) = \left[\zeta^{\beta^b\gamma^s-\gamma^b}(1-\psi(\cdot))^{\beta\gamma^s+\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{\frac{1}{1-\alpha}+\beta^b\gamma^s+\gamma^b}},\\
&q^E(\cdot) = \int_0^\infty e^{-\rho s}\frac{\xi(\cdot_{t+s})}{\xi(\cdot)}\pi^E(\cdot_{t+s})ds,\\
&r(\cdot) = r^B(\cdot) = \rho+\lambda-\left(\frac{1-\eta^b(\cdot)}{\epsilon(\cdot)}+\eta^b(\cdot)(1-\psi(\cdot))\right)\frac{\theta}{\kappa}+\mu^{y1}(\cdot)+\frac{\mathcal{S}(\cdot)\mu^{\mathcal{S}}(\cdot)}{\mathcal{S}(\cdot)+1/\kappa}
\end{aligned}
$$

where $\xi(\cdot) = \partial_A V(\cdot)$ *is the marginal discount factor,* $\pi^E(\cdot)$ *is the profit from the ledger,* $\mathcal{S}(\cdot) = r(\cdot) - \mu^q$ *is the spread between the return on bonds and money,* $\mu_t^{\mathcal{S}}$ *is the growth rate of this return spread,* y *is the output in token trades, and* μ^{y1} *is the growth of output in token trades. Given price equations* (2.7)*, we can solve for labor and output using equations* (2.6)*. Output is:*

$$y_t^0 = z\left(\frac{\alpha z \epsilon(\cdot)}{1 + r(\cdot)}\right)^{\frac{\alpha}{1-\alpha}}, \qquad y_t^1 = z\left(\frac{\alpha z(1 - \psi(\cdot))}{1 + r(\cdot)}\right)^{\frac{\alpha}{1-\alpha}} \tag{B.3}$$

Having solved for the real economy, the currency prices are given by:

$$q^0(\cdot) = \frac{\epsilon(\cdot)(1 - \eta^s(\cdot))y^0(\cdot)}{\bar{M}/\kappa}, \qquad q^1(\cdot) = \frac{\eta^s(\cdot)y^1(\cdot)}{\bar{M}/\kappa}$$

Proof. Summarising the equilibrium and optimization equations gives the following characterization of equilibrium. Given $(\bar{M}_t, \mu^M)$, stationary distribution of depositors across funds, $\{I^j\}_j$, and $(\mu^{C\mathcal{M}1}, \mu^{\epsilon\mathcal{M}}, \mu^q, \mu^{\mathcal{S}})$, we can solve for the equilibrium variables:

$$(c^{0ij}, c^{1ij}, y^0, y^1, l^0, l^1, \xi, \varphi, \theta, \eta^b, \eta^s, A, w, r, q^0, q^1, \epsilon, q^E)$$

using the equations:

$$(1-\eta^s)y^0 = \int\int_0^{I^j} \lambda(1-\eta^b)c^{0ji}didj, \qquad \eta^s y^1 = \int\int_0^{I^j} \lambda\eta^b c^{1ji}didj,$$

$$(1-\varphi)\theta A_t = q^0\bar{M} \qquad \varphi\theta A_t = q^1\bar{M}$$

$$(1-\theta)A = w((1-\eta^s)l^0 + \eta^s l^1) + q^E,$$

$$y^0 = z\left(\frac{\alpha z\epsilon}{1+r}\right)^{\frac{\alpha}{1-\alpha}} \qquad y^1 = z\left(\frac{\alpha z(1-\psi)}{1+r}\right)^{\frac{\alpha}{1-\alpha}}$$

$$l^0 = \left(\frac{\alpha z\epsilon}{1+r}\right)^{\frac{1}{1-\alpha}} \qquad l^1 = \left(\frac{\alpha z(1-\psi)}{1+r}\right)^{\frac{1}{1-\alpha}}$$

$$\varphi = \eta^b \qquad \theta = \frac{\lambda}{\beta(r-\mu^q+1/\kappa)}$$

$$\eta^s = \left(1+\left(\frac{\epsilon}{\zeta(1-\psi)}\right)^{\beta^b\gamma^s}\right)^{-1}, \qquad \eta^b = \left(1+(\zeta\epsilon)^{-\gamma^b}\right)^{-1},$$

$$r = \rho - wL^j + \mu^{C\mathcal{M}j} + \mu^{\epsilon\mathcal{M}1} + \frac{\mathcal{S}\mu_t^{\mathcal{S}}}{\mathcal{S}+1/\kappa}$$

$$w = 1$$

$$q^E = \frac{\psi\eta^b y^1 + q^1\mu^M\bar{M}^1}{\rho}$$

$$c^{0ij} = \frac{1}{\epsilon}\frac{\theta A^j}{\kappa\lambda I^j}\frac{a_0^i}{A_0}, \qquad c^{1ij} = \frac{\theta A^j}{\kappa\lambda I^j}\frac{a_0^i}{A_0}$$

$$\frac{q^0\bar{M}^0}{\kappa} = \int\int_0^{I^j} \lambda(1-\eta^b)\epsilon c^{0ji}didj, \qquad \frac{q^1\bar{M}^1}{\kappa} = \int\int_0^{I^j} \lambda\eta^b c^{0ji}didj$$

We can solve for the key prices. First consider the real exchange rate. Combining the first, second, and last lines gives:

$$(1-\eta^s)y^0 = \frac{(1-\eta^b)\theta A}{\kappa\epsilon_t}, \qquad \eta^s y^1 = \frac{\eta^b\theta A}{\kappa}$$

Dividing through gives that the real exchange rate, ϵ, is solved by:

$$\epsilon_t\left(\frac{\eta^b}{1-\eta^b}\right) = \left(\frac{\eta^s}{1-\eta^s}\right)\left(\frac{y^1}{y^0}\right)$$

$$\Rightarrow \epsilon = \left[\zeta^{\beta^b\gamma^s-\gamma^b}(1-\psi)^{\beta\gamma^s+\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{\frac{1}{1-\alpha}+\beta^b\gamma^s+\gamma^b}}$$

We can derive the risk free rate in the same way as before to get:

$$r = \rho + \lambda - \left(\frac{1-\eta^b}{\epsilon} + \eta^b(1-\psi)\right)\frac{\theta}{\kappa} + \mu^{y1} + \frac{\mathcal{S}\mu^{\mathcal{S}}}{\mathcal{S}+1/\kappa}$$

□

C Choice of Settlement Asset (Online Appendix)

We now assume that both buyers and sellers can trade currencies during the morning but only get a fraction $\varsigma \leq 1$ of the overnight market exchange rate (with the remaining $1-\varsigma$ fraction a deadweight transaction cost). We also assume that producers need to settle contracts during the morning market and so potentially face the ς cost. We assume that the ledger operator now internalizes their impact on the real exchange rate, as we will in the full model in Section 3. Let $\mathcal{N}$ denote the currency in which contracts are settled and let the contract be specified so that $P_t^{\mathcal{N}}(1+r_t^B)w_t l_t^{\mathcal{M}}$ is the total repayment in units of currency $\mathcal{N}$. Then, the producer profit from trading using currency $\mathcal{M}$ and repaying loans is:

$$\Pi_t^{\mathcal{M}} := P_t^{\mathcal{M}}(1-\psi^{\mathcal{M}})z(l_t^{\mathcal{M}})^{\alpha} - \frac{P_t^{\mathcal{N}}}{\varsigma^{\mathcal{M}\mathcal{N}}E^{\mathcal{N}\mathcal{M}}}(1+r_t^B)w_t l_t^{\mathcal{M}}$$

where $\Pi_t^{\mathcal{M}}$ is the profit in units of currency $\mathcal{M}$ and we use the notation $\varsigma^{\mathcal{M}\mathcal{N}} = \varsigma$ if $\mathcal{N} \neq \mathcal{M}$ and 1 otherwise. The potential difference between the trading currency price, $P_t^{\mathcal{M}}$, and the settlement currency price, $P_t^{\mathcal{N}}$ reflects a potential exchange rate cost or benefit when settling transactions. The reason that settlement choice impacts profits is because the lenders cannot discern which currency a borrower is going to trade in and so all borrowers face the same repayment. Depending on the exchange rate between dollars and tokens, this potentially leads to redistribution between the token and dollar sellers.

The firm problem is solved the same way as before. When contracts are denominated and settled in tokens, firm output in dollar and token trades (under default and non-default) is given by:

$$y_t^0 = z\left(\frac{\alpha z \epsilon_t \varsigma}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}}, \qquad y_t^1 = z\left(\frac{\alpha z(1-\psi)}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\check{y}_t^0 = z\left(\frac{\alpha z \check{\epsilon}_t \varsigma}{\chi}\right)^{\frac{\alpha}{1-\alpha}}, \qquad \check{y}_t^1 = z\left(\frac{\alpha z(1-\psi)}{1+r_t+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

where now $\epsilon_t = ((1-\psi)/\varsigma)^\alpha$ and $\check{\epsilon}_t = ((1-\psi)\chi/((1+r+\delta)/\varsigma))^\alpha$ incorporate the ς. By contrast, if contracts are denominated and settled in dollars, then output in dollar and token trades is (under default and non-default) given by:

$$\tilde{y}_t^0 = z\left(\frac{\alpha z}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}}, \qquad \tilde{y}_t^1 = z\left(\frac{\alpha z(1-\psi)\varsigma}{(1+r_t)\tilde{\epsilon}_t}\right)^{\frac{\alpha}{1-\alpha}}$$

$$\check{\tilde{y}}_t^0 = z\left(\frac{\alpha z}{\chi}\right)^{\frac{\alpha}{1-\alpha}}, \qquad \check{\tilde{y}}_t^1 = z\left(\frac{\alpha z(1-\psi)\varsigma}{(1+r_t+\delta)\check{\tilde{\epsilon}}_t}\right)^{\frac{\alpha}{1-\alpha}}$$

We get the same real exchange rate in each case: $\epsilon_t = \tilde{\epsilon}_t$ and $\check{\epsilon}_t = \check{\tilde{\epsilon}}_t$. The key difference between the cases is in which traders must exchange currencies in order to settle contracts. When contracts are settled in tokens, the dollar sellers need to pay exchange rate adjustment costs whereas when contracts are settled in dollars, the token sellers need to pay exchange rate costs.

Corollary 8. *Under no default, the ledger operator chooses tokens as the settlement asset if $\varsigma < \epsilon_t$ and dollars as the settlement asset otherwise.*

Proof. The proof follows immediately from the expressions for y_t. If $\varsigma/\epsilon_t < 1$, then setting tokens as the settlement asset allows the ledger operator to charge a higher ψ and still get the same output on in token trades. □

The intuition is as follows. By setting tokens as the settlement asset, the ledger allows token sellers to avoid exchanging currencies during settlement. If they set dollars as the settlement asset, then token sellers face the distortion ς/ϵ_t, when they exchange currencies and settle contracts. For $\mu^M < (1-\alpha)/\alpha$, we have $\psi > 0$ and $\epsilon_t < 1$, which indicates that tokens are relatively valuable. So the token traders gain from making sales in tokens, converting them into dollars, and then settling the contracts in dollars. This distortion is partially offset by having to pay the implicit transaction costs associated with $\varsigma < 1$. If the transaction costs dominate, then the ledger chooses settlement in tokens.