Safe Assets: A Retrading Perspective

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Motivation

- What is a safe asset? What are its features?
  - Precautionary savings
  - Why is (US) government debt a safe asset?

- (Safe) asset pricing formula for incomplete market setting
  - What is service flow? How does it differ from
    - cash flow and convenience yield?
  - Flight-to-safety phenomenon (negative β)
  - Loss of safe-asset status / exorbitant privilege

- Is there a public debt valuation puzzle?
  - Valuation too large? Risk premium too low?

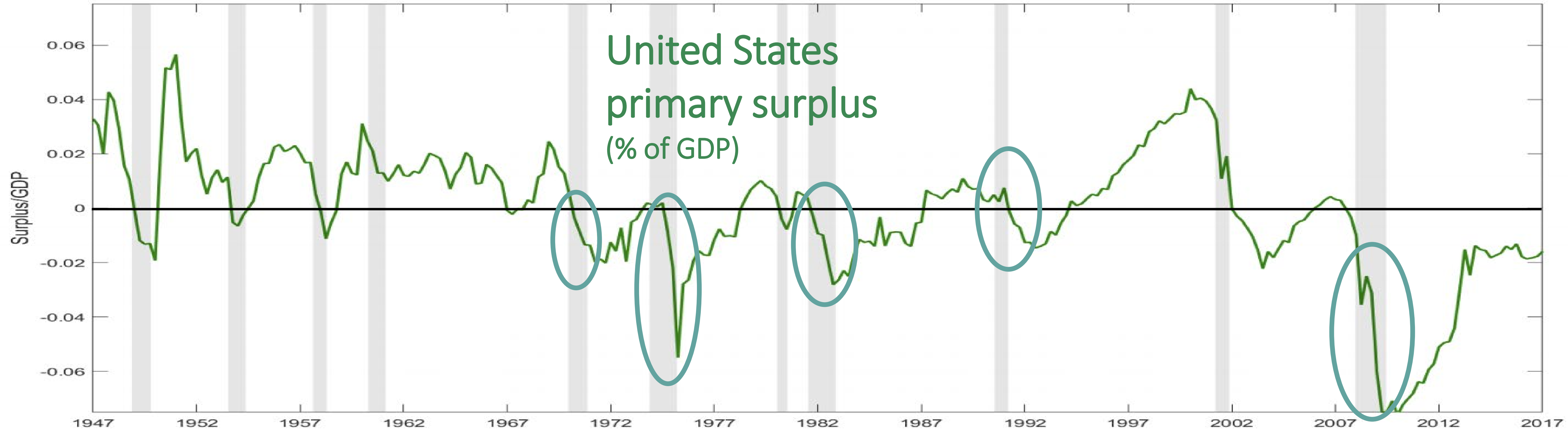
- Can government spend without taxation? How much?
  - Debt sustainability analysis and "Debt Laffer Curve"

Motivation

- Why is there debt valuation puzzle for US, Japan?
- Traditional FTPL equation:

$$\frac{\mathcal{B}_t}{\wp_t} = E_t\left[PV_\xi(\text{primary surpluses})\right] + Bubble$$

- Persistently negative primary surpluses
- Negative surpluses in recessions: Asset Pricing (with SDF ξ_t)



Asset Pricing with Safe Assets

- Standard asset pricing: Buy and Hold Perspective

$$price = \mathbb{E}[PV_{\xi}(cash\ flow)]$$

Asset Pricing with Safe Assets

- Standard asset pricing with bubbles: Buy and Hold Perspective

$$price = \mathbb{E}[PV_{\xi}(cash\ flow)] + bubble$$

  - Bubble if $r < g$ (due to precautionary motive)

Asset Pricing with Safe Assets

- Standard asset pricing with bubbles: Buy and Hold Perspective

$$price = \underbrace{\mathbb{E}[PV_{\xi}(cash\ flow)]}_{=-\infty} + \underbrace{bubble}_{=\infty}$$

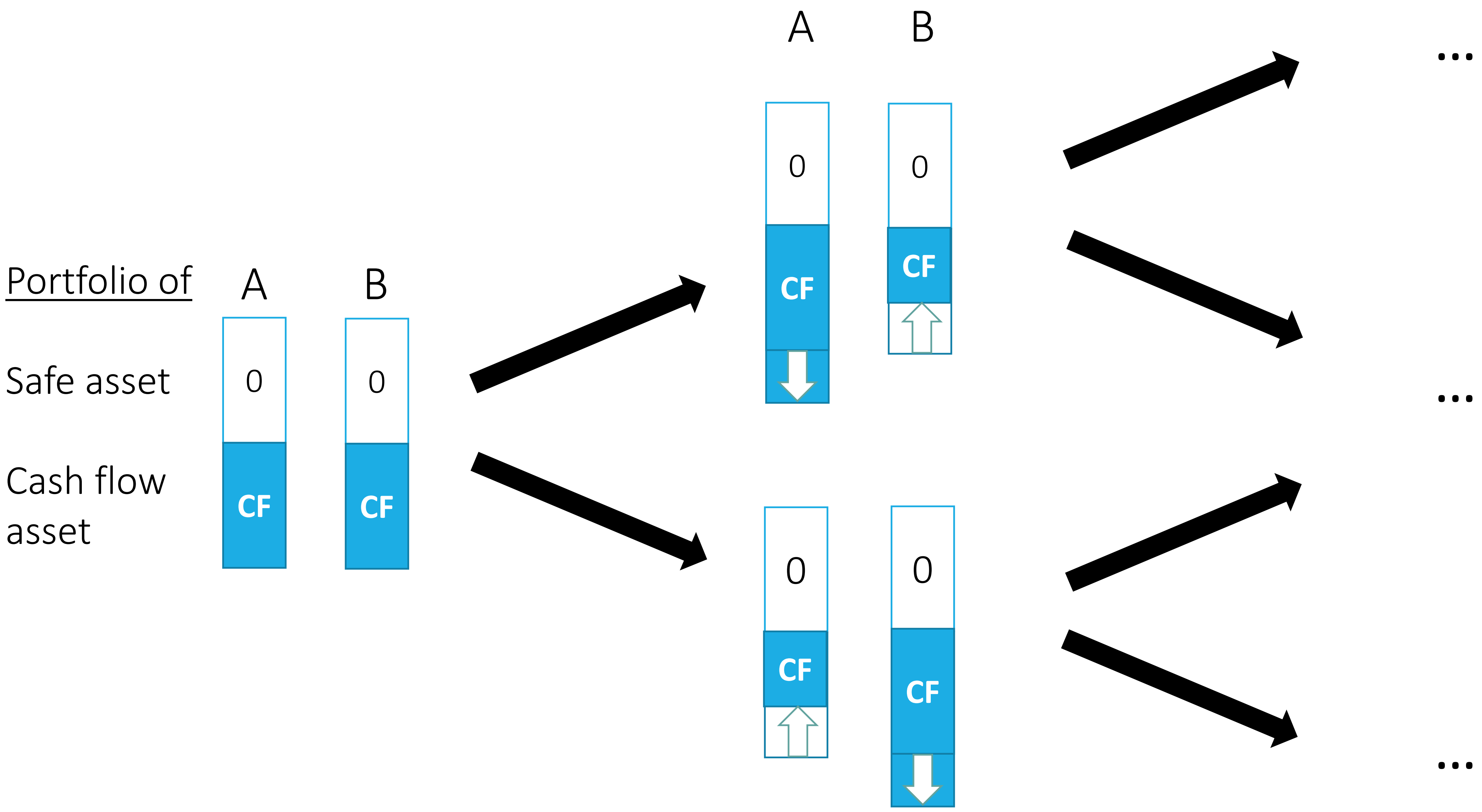
  - Bubble if $r < g$ (due to precautionary motive)
  - Effectively prices a buy-and-hold strategy
  - But in incomplete markets, agents trade
- Alternative approach: Dynamic Trading Perspective
  - Value agents' actual portfolio strategies, then aggregate
  - Discounts at higher effective rate $r^{**} > g$

$$price = \underbrace{\mathbb{E}[PV_{\xi^{**}}(cash\ flow)]}_{>-\infty} + \underbrace{\mathbb{E}[PV_{\xi^{**}}(service\ flow)]}_{<-\infty}$$

Note: SDF ξ^{**} = "representative agent" discount rate $\neq m$ (Reis 2022)

What’s is a Safe Asset Service Flow?

- Safe asset = good friend
  - Idiosyncratic risk: provides partial insurance through re-trading



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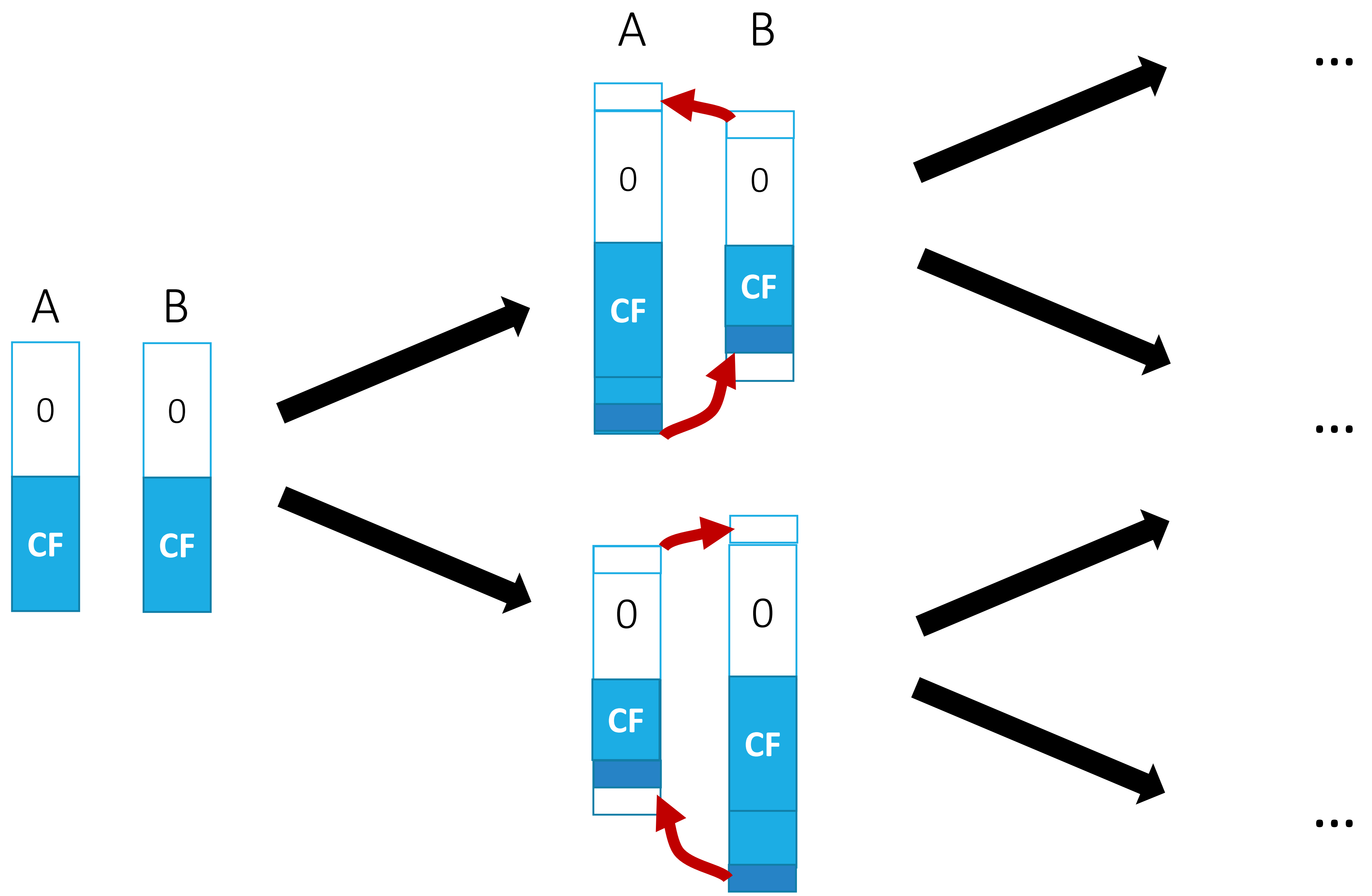
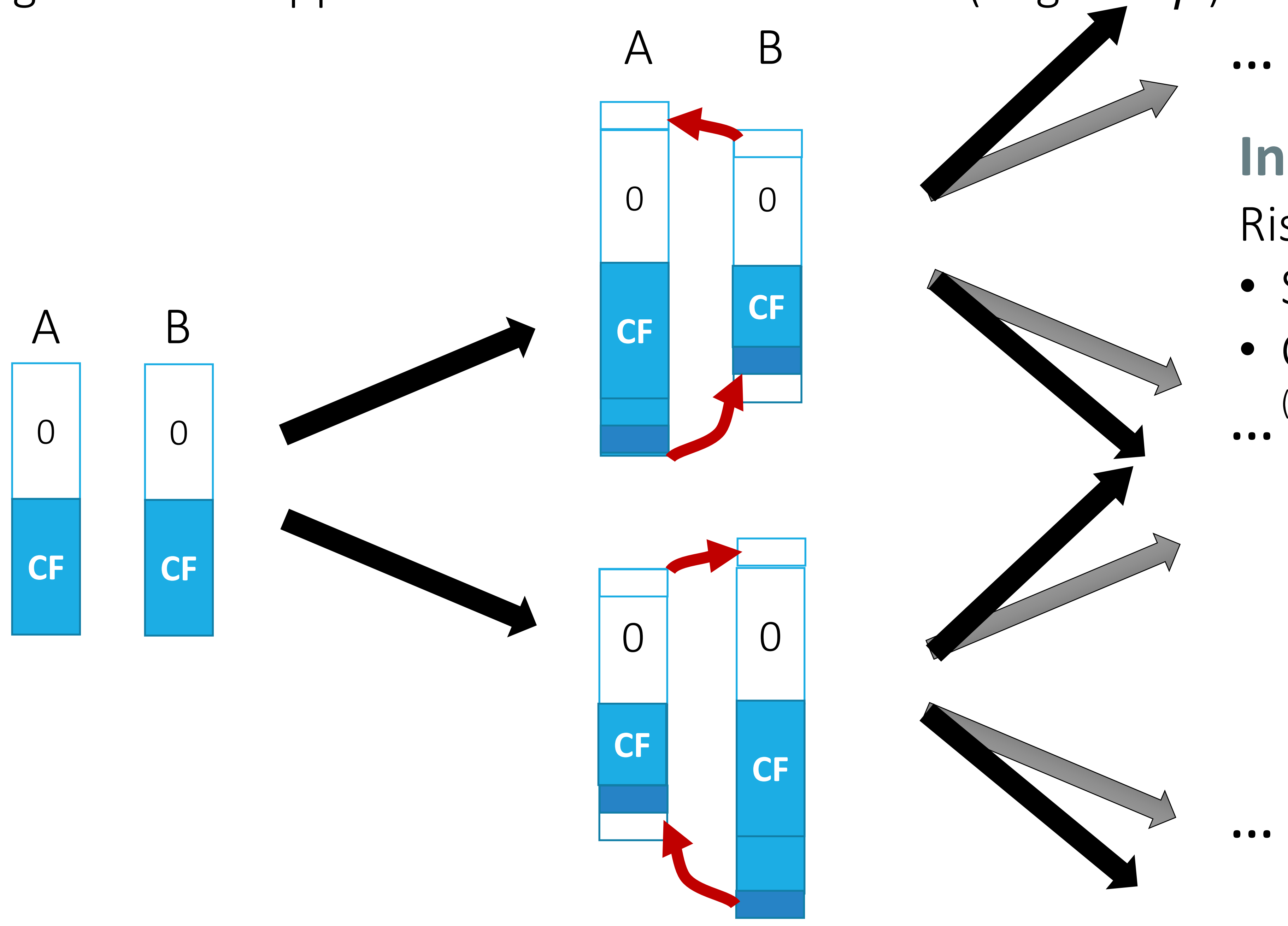


What's a Safe Asset? Exorbitant Privilege rises in Recessions

- Safe asset = good friend
  - Idiosyncratic risk: provides partial insurance through re-trading
  - Aggregate risk: appreciates in value in bad times (negative β)



In recessions:

Risk is higher

- Service flow is more valuable
- Cash flows are lower
  (depends on fiscal policy)

Safe Asset Definition

- Equilibrium concept
- Good friend (relative to own net worth return $dr_t^{n^i}$)
  - Idiosyncratic risk
  - Aggregate risk
  - $\boldsymbol{Cov_t\left[d\xi_t^i/\xi_t^i, dr_t^j - dr_t^{n^i}\right] > 0}$
    - For agent i
      with SDF ξ_t^i is SDF of agent i $\quad (d\xi_t^i/\xi_t^i \quad = -r_t^f dt - \varsigma_t^i dZ_t - \tilde{\varsigma}_t^i d\tilde{Z}_t^{\tilde{\imath}})$
      with net worth return $dr_t^{n^i}$
    - At time t possible loss of safe asset status/exorbitant privilege
- Re-tradeable:
  - No asymmetric info – info insensitive
  - Service flow is derived from "dynamic re-trading"

Service Flow Term vs. Bubble Term

- Service flow
  - Partial insurance via retrading – (partially undo incomplete markets)
    - Bewley … smooth consumption
    - BruSan … retrade capital and safe asset + smooth consumption
  - Remaining (idiosyncratic) risk depresses cash flow return
- Bubble
  - $\lim_{T\to\infty} E[\bar{\xi}_t P_t] > 0$ if $r_t^f + risk\ premium \leq g_t$ (on average)
    - r^f is depressed by precautionary savings (incl. uninsurable idiosyncratic risk)

  - Transversality condition holds for each individual, but not in aggregate
    - $\neq$ complete markets

Related Literature 1

- FTPL literature: Leeper, Sims, Woodford, Cochrane, …
- Cui Bassetto (2018), Sims (2020), Angeletos et al. (2020), Obstfeld Rogoff (1983),

\Friction	OLG	Incomplete Markets + idiosyncratic risk	
Risk	deterministic	endowment risk borrowing constraint	return risk Risk tied up with Individual capital
Only money	Samuelson	Bewley	
			"I Theory without I" Brunnermeier-Sannikov (AER PP 2016)
With capital	Diamond	Aiyagari	Angeletos (2006)

Related Literature 2

- Safe Asset:
  - Gorton-Pennachi (1990), Dang et al. (2017), Caballero et al. (2016,7), …
  - Brunnermeier et al. (2017) , ESBies,
- Equity premium
  - Constantinides-Duffie (1996) – imposes “aggregate” transversality condition
- Public Debt Evaluation Puzzles:
  - Jiang et al. (2020,2021)
- Fiscal debt sustainability r vs. g:
  - OLG: Bohn (1995), Samuelson (1958), Diamond (1965), Tirole (1985), Blanchard (2019), Martin-Ventura (2018)
  - Incomplete markets: Bewley (1980), Aiyagari-McGrattan (1998), Angeletos (2007)

    No capital | Capital is a safe asset (risk-free) | No debt
  - Bassetto-Cui (2018), Reis (2020)

    m instead of ξ^{**}/r^{**}

Roadmap

- Motivation
- Steady state model (closed form solution)
- Stochastic idiosyncratic risk model
- (Safe) asset pricing with SDF ξ_t^{**}
- Flight-to-Safety, $\beta < 0$, and Excess Volatility of capital
- Equity issuance, non-safe mutual fund with $\beta > 0$ } Calibration
- Debt Laffer Curve } Calibration
- Price level determination with a bubble
  - Fiscal space to defend Exorbitant Privilege (crypto, …)

Exorbitant Privilege Model with a Bubble: Safe Asset Model

- Model overview:
  - Continuous time, infinite horizon, one consumption good
  - Continuum of agents
    - Operate capital with time-varying idiosyncratic risk, *AK* production technology
    - Can trade capital, government bond, and diversified equity claims
  - Government
    - Exogenous spending
    - Taxes output
    - Issues (nominal) bonds
  - Financial Frictions: incomplete markets
    - Agents cannot fully insure idiosyncratic risk (must retain skin in the game)
  - Aggregate risk: fluctuations in volatility of idiosyncratic risk (& capital productivity)

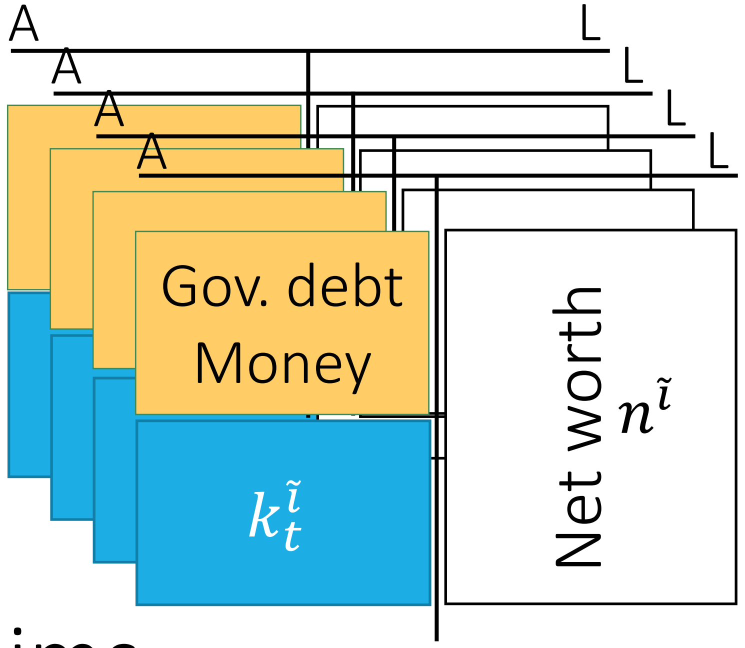
- Calibrated to US data to match macro and asset pricing moments

Model with Capital + Safe Asset

- Each heterogenous citizen $\tilde{\imath} \in [0,1]$

$$E\left[\int_0^\infty e^{-\rho t} \log c_t^{\tilde{\imath}}\, dt\right] \text{ s.t. } \frac{dn_t^{\tilde{\imath}}}{n_t^{\tilde{\imath}}} = -\frac{c_t^{\tilde{\imath}}}{n_t^{\tilde{\imath}}} dt + dr_t^{\mathcal{B}} + (1-\theta_t^{\tilde{\imath}})\left(dr_t^{K,\tilde{\imath}}(\iota_t^{\tilde{\imath}}) - dr_t^{\mathcal{B}}\right)$$ & No Ponzi

- Each citizen operates physical capital $k_t^{\tilde{\imath}}$
  - Output (net investment) $a_t k_t^{\tilde{\imath}} dt - \iota_t^{\tilde{\imath}} k_t^{\tilde{\imath}} dt$
  - Output tax $\tau_t a_t k_t^{\tilde{\imath}} dt$
  - $\frac{dk_t^{\tilde{\imath}}}{k_t^{\tilde{\imath}}} = \left(\Phi(\iota_t^{\tilde{\imath}}) - \delta\right)dt + \tilde{\sigma}_t d\tilde{Z}_t^{\tilde{\imath}} + d\Delta_t^k$
    - $d\tilde{Z}_t^{\tilde{\imath}}$ idiosyncratic Brownian



- Financial Friction: Incomplete markets: no $d\tilde{Z}_t^{\tilde{\imath}}$ claims
- Aggregate risk:
  $\tilde{\sigma}_t, a_t, g_t$ exogenous process by aggregate Brownian dZ_t
  - E.g. Heston model: $d\tilde{\sigma}_t^2 = -\psi\left(\tilde{\sigma}_t^2 - (\tilde{\sigma}^0)^2\right)dt - \sigma\tilde{\sigma}_t dZ_t$
  - $a_t = a(\tilde{\sigma}_t)$, $g_t = g(\tilde{\sigma}_t)$

Government: Taxes, Bond/Money Supply, Gov. Budget

- Policy Instruments ($K_t := \int k_t^{\tilde{\imath}} d\tilde{\imath}$)
  - Government spending $\mathscr{g}_t K_t$ (with exogenous $\mathscr{g}_t$)
  - Proportional output tax $\tau_t a_t K_t$
  - Nominal government debt supply $\frac{d\mathcal{B}_t}{\mathcal{B}_t} = \mu_t^{\mathcal{B}} dt$
  - Floating nominal interest rate i_t on outstanding bonds
- Government budget constraint (BC)

$$\underbrace{\left(\mu_t^{\mathcal{B}} - i_t\right)}_{\breve{\mu}_t^B :=} \mathcal{B}_t + \wp_t K_t \underbrace{\left(\tau_t a_t - \mathscr{g}_t\right)}_{s_t :=} = 0$$

Not market clearing,
Payment/redistribution to bond holders
q^B clears bond market

Primary surplus (per K_t)

  - Equilibrium selection: No No-Ponzi constraint

Optimal Choices & Market Clearing

- Optimal **investment rate**
  - $\Phi(\iota) = \frac{1}{\phi}\log(1+\phi\iota)$ $\qquad\qquad$ $\boldsymbol{\iota_t} = \frac{1}{\phi}(q_t^K - 1)$
- **Consumption** $\qquad\qquad$ Goods market
  - $\frac{\boldsymbol{c_t}}{n_t} =: \rho \qquad \Rightarrow C_t = \rho\big(q_t^K K_t + \underbrace{q_t^{\mathcal{B}} K_t}_{\mathcal{B}_t/\wp_t}\big) = (a - \iota_t - g)K_t$
- **Portfolio** $\qquad\qquad$ Capital market
  - Solve for θ_t later $\qquad 1 - \boldsymbol{\theta_t} = \frac{q_t^K}{q_t^K + q_t^B} =: 1 - \vartheta_t$

Bond market

clears by Walras law

ϑ= fraction of wealth in nominal claims

Equilibrium (before solving for portfolio choice)

Equilibrium
$q_t^{\mathcal{B}} = \vartheta_t \dfrac{1+\phi\check{a}}{(1-\vartheta_t)+\phi\check{\rho}_t}$
$q_t^{K} = (1-\vartheta_t)\dfrac{1+\phi\check{a}}{(1-\vartheta_t)+\phi\check{\rho}_t}$
$\iota_t = \dfrac{(1-\vartheta_t)\check{a}-\check{\rho}_t}{(1-\vartheta_t)+\phi\check{\rho}_t}$

$\check{a} = a - \mathcal{g}$

For log utility

$\check{\rho}_t = \rho$

- Moneyless equilibrium with $q_t^{\mathcal{B}} = 0 \Rightarrow \vartheta_t = 0$
- Next, determine portfolio choice.

Portfolio choice θ ⇒ Evaluation Equation

Agent i's SDF, ξ_t^i: $d\xi_t^i/\xi_t^i \quad = -r_t^f dt - \varsigma_t dZ_t - \tilde{\varsigma}_t^i d\tilde{Z}_t^i$

Price of risk

- Asset pricing equation (martingale method): $\frac{\mathbb{E}_t\left[dr_t^{K,\tilde{\imath}}\right]}{dt} - \frac{\mathbb{E}_t\left[dr_t^{\mathcal{B}}\right]}{dt} = \varsigma\sigma^K$

$$\frac{\mathbb{E}_t\left[dr_t^{K,\tilde{\imath}}\right]}{dt} = \frac{\breve{a} - \iota_t}{q_t^K} + \frac{q_t^{\mathcal{B}}}{q_t^K}\breve{\mu}^{\mathcal{B}} + \Phi(\iota_t) - \delta + \mu_t^{q^K} \qquad = r_t^f \quad + \tilde{\varsigma}_t\tilde{\sigma}$$

$$\frac{\mathbb{E}_t\left[dr_t^{\mathcal{B}}\right]}{dt} = \qquad -\breve{\mu}^{\mathcal{B}} + \Phi(\iota_t) - \delta + \mu_t^{q^{\mathcal{B}}} \qquad = r_t^f$$

$$\frac{\mathbb{E}_t\left[dr_t^{K,\tilde{\imath}}\right]}{dt} - \frac{\mathbb{E}_t\left[dr_t^{\mathcal{B}}\right]}{dt} = \frac{\breve{a} - \iota_t}{q_t^K} + \frac{1}{1-\vartheta_t}\breve{\mu}^{\mathcal{B}} \qquad + \underbrace{\mu_t^{q^K} - \mu_t^{q^{\mathcal{B}}}}_{= -\mu_t^{\vartheta}/(1-\vartheta_t)} = \qquad \tilde{\varsigma}_t\tilde{\sigma}$$

Seigniorage redistribution

- Goods market clearing: $\breve{\rho}(q_t^{\mathcal{B}} + q_t^K)K_t \quad = (\breve{a} - \iota_t)K_t \Rightarrow \frac{\breve{a}-\iota_t}{q_t^K} = \frac{\breve{\rho}}{1-\vartheta_t}$
- Price of idiosyncratic risk: $\tilde{\varsigma}_t = \gamma\tilde{\sigma}_t^n = \gamma(1-\theta_t)\gamma\tilde{\sigma}$
- Capital market clearing: $1 - \theta_t = 1 - \vartheta_t$
- FTPL/Money Valuation Equation: $\mu_t^{\vartheta} = \rho + \breve{\mu}_t^{\mathcal{B}} - (1-\vartheta_t)^2\tilde{\sigma}^2$
  - In steady state $\mu_t^{\vartheta} = 0$: $\qquad (1-\vartheta) = \sqrt{\rho + \breve{\mu}^{\mathcal{B}}}/\tilde{\sigma}$

Recall: ϑ= fraction of wealth in nominal claims

Two Stationary Equilibria – in closed form

Non-Monetary	Monetary
$q_0^{\mathcal{B}} = 0$	$\frac{\mathcal{B}_0}{\wp_0}/K_t = q^{\mathcal{B}} = \frac{\left(\tilde{\sigma} - \sqrt{\rho + \breve{\mu}^{\mathcal{B}}}\right)(1 + \phi\breve{a})}{\sqrt{\rho + \breve{\mu}^{\mathcal{B}}} + \phi\tilde{\sigma}\rho}$
$q_0^K = \frac{1 + \phi\breve{a}}{1 + \phi\rho}$	$q^K = \frac{\sqrt{\rho + \breve{\mu}^{\mathcal{B}}}\ (1 + \phi\breve{a})}{\sqrt{\rho + \breve{\mu}^{\mathcal{B}}} + \phi\tilde{\sigma}\rho}$
$\iota = \frac{\breve{a} - \rho}{1 + \phi\rho_0}$	$\iota = \frac{\breve{a}\sqrt{\rho + \breve{\mu}^{\mathcal{B}}} - \tilde{\sigma}\breve{\rho}}{\sqrt{\rho + \breve{\mu}^{\mathcal{B}}} + \phi\tilde{\sigma}\breve{\rho}}$

- ρ time preference rate
- ϕ adjustment cost for investment rate
- $\breve{\mu}_t^{\mathcal{B}} = \mu_t^{\mathcal{B}} - i_t$ bond issuance rate beyond interest rate
- $\breve{a} = a - g$ part of TFP not spend on gov.

Remarks for steady state case:

- Real risk-free rate
  - $r^f = \underbrace{\left(\Phi\left(\iota(\breve{\mu}^{\mathcal{B}})\right) - \delta\right)}_{=g} - \breve{\mu}^{\mathcal{B}}$

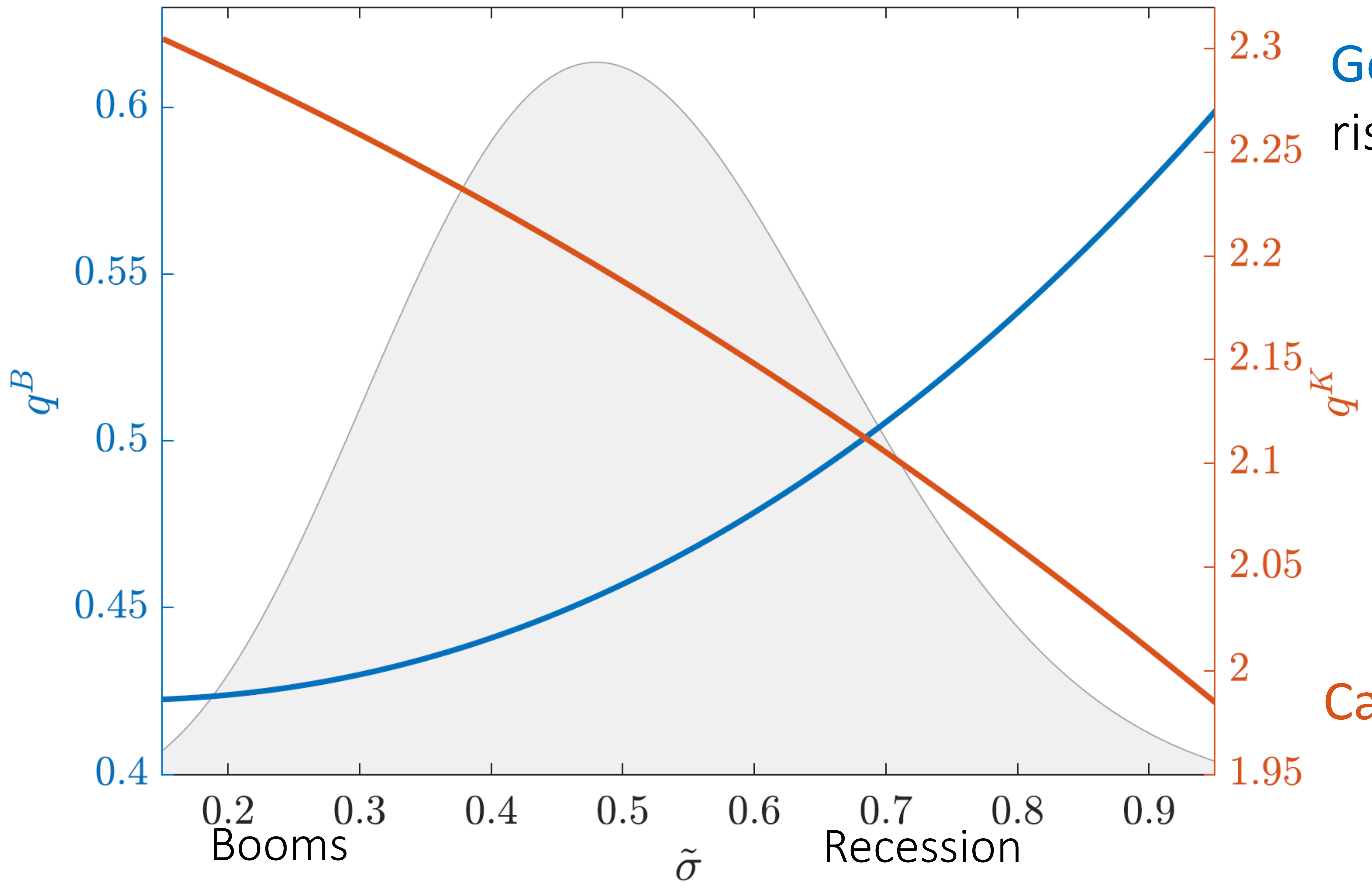
Recall $\breve{\mu}_t^B \mathcal{B}_t + \wp_t K_t \underbrace{(\tau_t a_t - g_t)}_{s_t :=} = 0$

- $\breve{\mu}^{\mathcal{B}} = 0 \Rightarrow s = 0$ no primary surplus (no cash payoff for bond)
  - $q^{\mathcal{B}} K = \frac{\mathcal{B}}{P} > 0$ bond trades at a **bubble** due to service flow

- $\breve{\mu}^{\mathcal{B}} > 0 \Rightarrow s < 0$ primary deficit (constant fraction of GDP)
  - As long as $q^{\mathcal{B}} > 0$ "mine the bubble"

- $\breve{\mu}^{\mathcal{B}} < 0 \Rightarrow s > 0$ and $r > g$ primary surplus (constant fraction of GDP)
  - $q^{\mathcal{B}} K_t = E_t[PV_{r^f}(sK_t)]$ no bubble, but service flow
  - $\frac{\mathcal{B}_0}{\wp_0} = \mathbb{E}\left[\int_0^\infty e^{-r^f t} s K_t dt\right]$

Flight-to-Safety when idiosyncratic risk is $\tilde{\sigma}_t$ high $\Rightarrow$ negative β for Gov. Bond

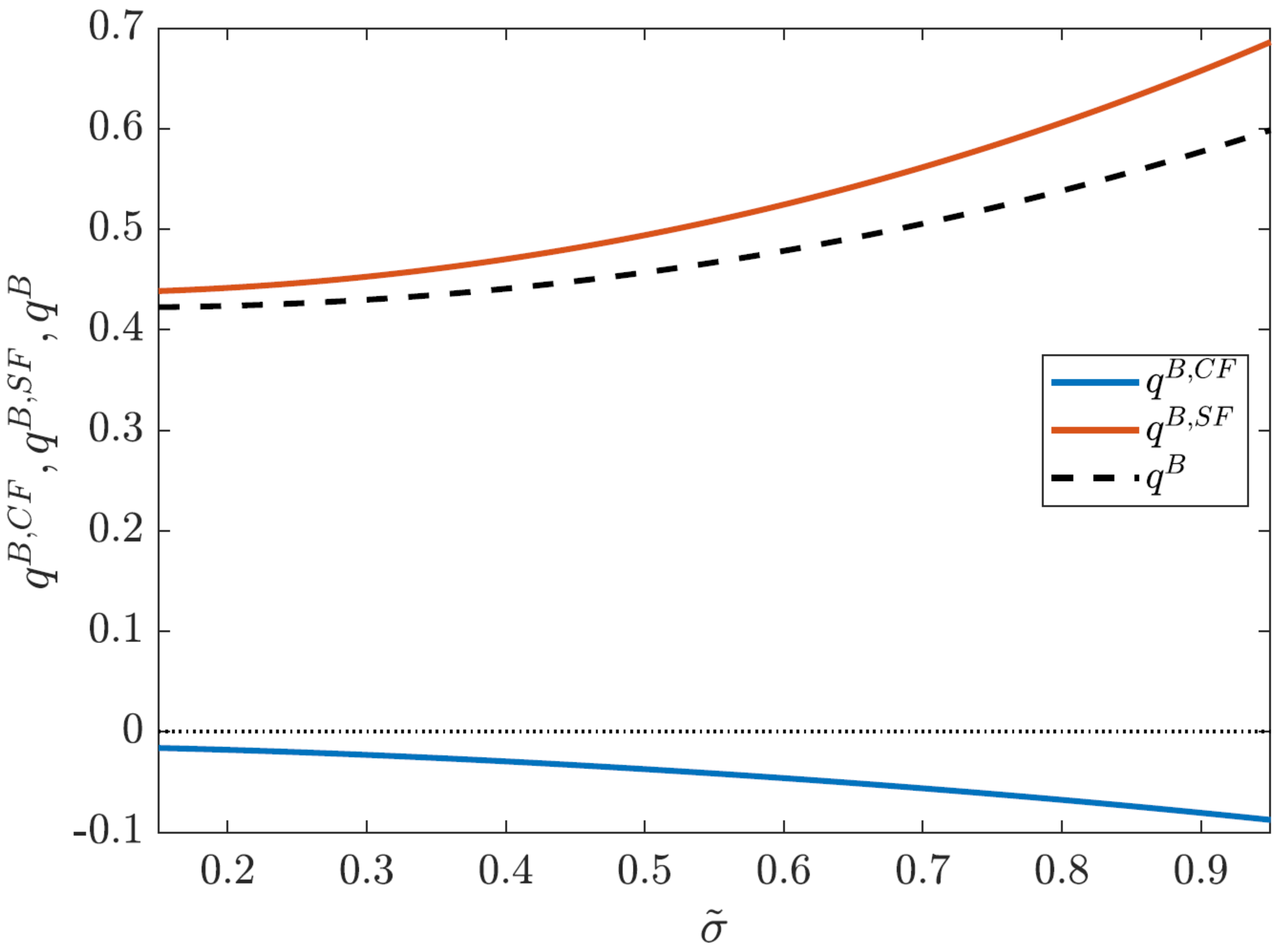


Gov. debt value rises in recessions

Capital price

Safe Asset – Service flow >> Cash flow

- Asset Price = E[PV(primary surplus/cash flows)] + E[PV(service flows)]



Debt Valuation (FTPL) – Two Perspectives

- **Buy and Hold Perspective:**
  - $\frac{\mathcal{B}_0}{\wp_0} = \lim_{T\to\infty}\left(\mathbb{E}\left[\int_0^T \xi_t^i s_t K_t dt\right] + \mathbb{E}\left[\xi_t^i \frac{\mathcal{B}_T}{\wp_T}\right]\right)$
    - Valuation of strategy that buys and holds a fixed fraction of outstanding debt

- Agent i's SDF, ξ_t^i: $d\xi_t^i/\xi_t^i \quad = -r_t^f dt - \varsigma_t dZ_t - \tilde{\varsigma}_t^i d\tilde{Z}_t^i$, idiosyncratic consumption vol. $\tilde{\sigma}_t^c$

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- **Dynamic Trading Perspective:**
  - $\eta_0^i \frac{\mathcal{B}_0}{\wp_0} = \mathbb{E}[\int_0^\infty \xi_t^i \eta_t^i s_t K_t dt] + \mathbb{E}[\int_0^\infty \xi_t^i \eta_t^i (\tilde{\sigma}_t^c)^2 \frac{\mathcal{B}_t}{\wp_t} dt]$
  - Valuation of equilibrium cash flows from individual bond portfolios, incl. trading cash flows (aggregated over all agents i to obtain total value of debt)
  - Agent i's SDF, ξ_t^i: $d\xi_t^i/\xi_t^i = -r_t^f dt - \varsigma_t dZ_t - \tilde{\varsigma}_t^i d\tilde{Z}_t^i$, idiosyncratic consumption vol. $\tilde{\sigma}_t^c$

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    - Valuation of strategy that buys and holds a fixed fraction of outstanding debt

- **Dynamic Trading Perspective:**
  - $\frac{\mathcal{B}_0}{\wp_0} = \mathbb{E}[\int_0^\infty \underbrace{\left(\int \xi_t^i \eta_t^i \, di\right)}_{=\xi_t^{**}} s_t K_t dt] + \mathbb{E}[\int_0^\infty \underbrace{\left(\int \xi_t^i \eta_t^i \, di\right)}_{=\xi_t^{**}} (\tilde{\sigma}_t^c)^2 \frac{\mathcal{B}_t}{\wp_t} dt]$

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Capital price "Excess" Volatility due to Flight to Safety

- "Aggregate Intertemporal Budget Constraint

Consumption share

$$\underbrace{q_t^K K_t + q_t^B K_t}_{\text{total (net) wealth}} = \mathbb{E}_t[\int_t^\infty \underbrace{\frac{\int \xi_s^i \eta_s^i di}{\int \xi_t^i \eta_t^i di}}_{=\xi_s^{**}/\xi_t^{**}} C_s ds] \quad (*)$$

- Lucas-type models: $q^B = 0$ (also $C_t = Y_t$, no idiosyncratic risk)
  - Value of equity (Lucas tree) = PV of consumption claim
  - Volatility equity values require (low) volatile RHS of (*)
- This model: even for constant RHS of (*), $q_t^K K_t$ can be volatile due to flight to safety:
  - increase in $\tilde{\sigma}_t$ $\Rightarrow$ Portfolio reallocation from capital to bonds, $q_t^K K_t \downarrow, \frac{\mathcal{B}_t}{\wp_t} = q_t^B K_t \uparrow$,
- Quantitatively relevant? Yes
  - Excess return volatility
    - 2.9% in equivalent bondless model ($s = 0$ and no bubble)
    - 12.9% in our model

Service Flow Term, Convenience Yield, Ponzi Scheme

- Service flow
  - Convenience yield: relaxes collateral constraint or CIA constraint (money)
    - Traditional measure: BAA-US Treasury spread
  - Here: Partially completing markets through retrading
    - Low interest rate (cash flow) asset can be issued by everyone
      Hence, corporate-Treasury spread = 0
- Ponzi scheme is not feasibly for everyone
  No Ponzi constraint may be binding
  - Who can run a Ponzi scheme? **exorbitant privilege**
    … assigned by equilibrium selection
    - Likely to government, private entities are subject to solvency constraint
      - … still there is a Debt Laffer Curve

Roadmap

- Motivation
- Steady state model (closed form solution)
- Stochastic idiosyncratic risk model
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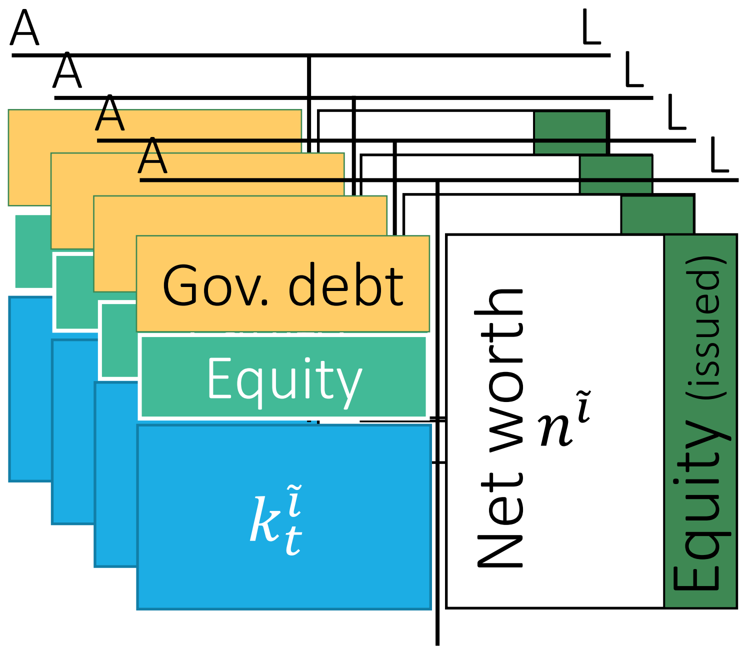
- Flight-to-Safety, $\beta < 0$, and Excess Volatility of capital
- Equity issuance, non-safe mutual fund with $\boldsymbol{\beta > 0}$
- Debt Laffer Curve

} Calibration

- Price level determination with a bubble
  - Fiscal space to defend Exorbitant Privilege (crypto, …)

Equity Markets (ETF)

- Equity Market
  - Each citizen $\tilde{\imath}$ can sell off a fraction $(1-\bar{\chi})$ of capital risk to outside equity holders
  - Return $dr_t^{E,\tilde{\imath}}$
    - Same risk as $dr_t^{K,\tilde{\imath}}$
    - But $\mathbb{E}_t\left[dr_t^{E,\tilde{\imath}}\right] < \mathbb{E}_t\left[dr_t^{K,\tilde{\imath}}\right]$… due to insider premium
  - Prop.: Model equations as before but replace $\tilde{\sigma}$ *with* $\bar{\chi}\tilde{\sigma}$



Calibration with Epstein-Zin Preferences

- Epstein-Zin preferences for calibration (EIS=1)
  - Citizen $\tilde{\imath}$ maximizes $V_0^{\tilde{\imath}}$ where $V_t^{\tilde{\imath}}$ is recursively defined by

$$V_t^i = E_t \left[\int_t^{\infty} (1-\gamma)\rho V_s^i \left(\log(c_s^i) - \frac{1}{1-\gamma} \log\left((1-\gamma) V_s^i \right) \right) ds \right]$$

  - Needed to generate realistic prices of risk (Sharpe ratio)

Numerical Illustration (Calibration)

- Exogenous processes:
  Recessions feature high idiosyncratic risk and low consumption
  - $\tilde{\sigma}_t$: Heston (1993) model of stochastic volatility
    $$d\tilde{\sigma}_t^2 = -\psi\left(\tilde{\sigma}_t^2 - \left(\tilde{\sigma}^0\right)^2\right)dt - \sigma^{\tilde{\sigma}}\tilde{\sigma}_t dZ_t$$
    CIR – ensures that $\tilde{\sigma}$ stays positive
  - a_t: $a_t = a(\tilde{\sigma}_t)$
    $$a_t(\tilde{\sigma}_t) = a^0 - \alpha^a(\tilde{\sigma}_t - \tilde{\sigma}^0)$$
  - $g_t = 0$

- Government (bubble-mining policy)
  $$\check{\mu}_t^{\mathcal{B}} = \check{\mu}_t^{\mathcal{B},0} + \alpha^{\mathcal{B}}(\tilde{\sigma}_t - \tilde{\sigma}^0)$$

- Calibration to US data (1970-2019, period length is one year)

Parameters

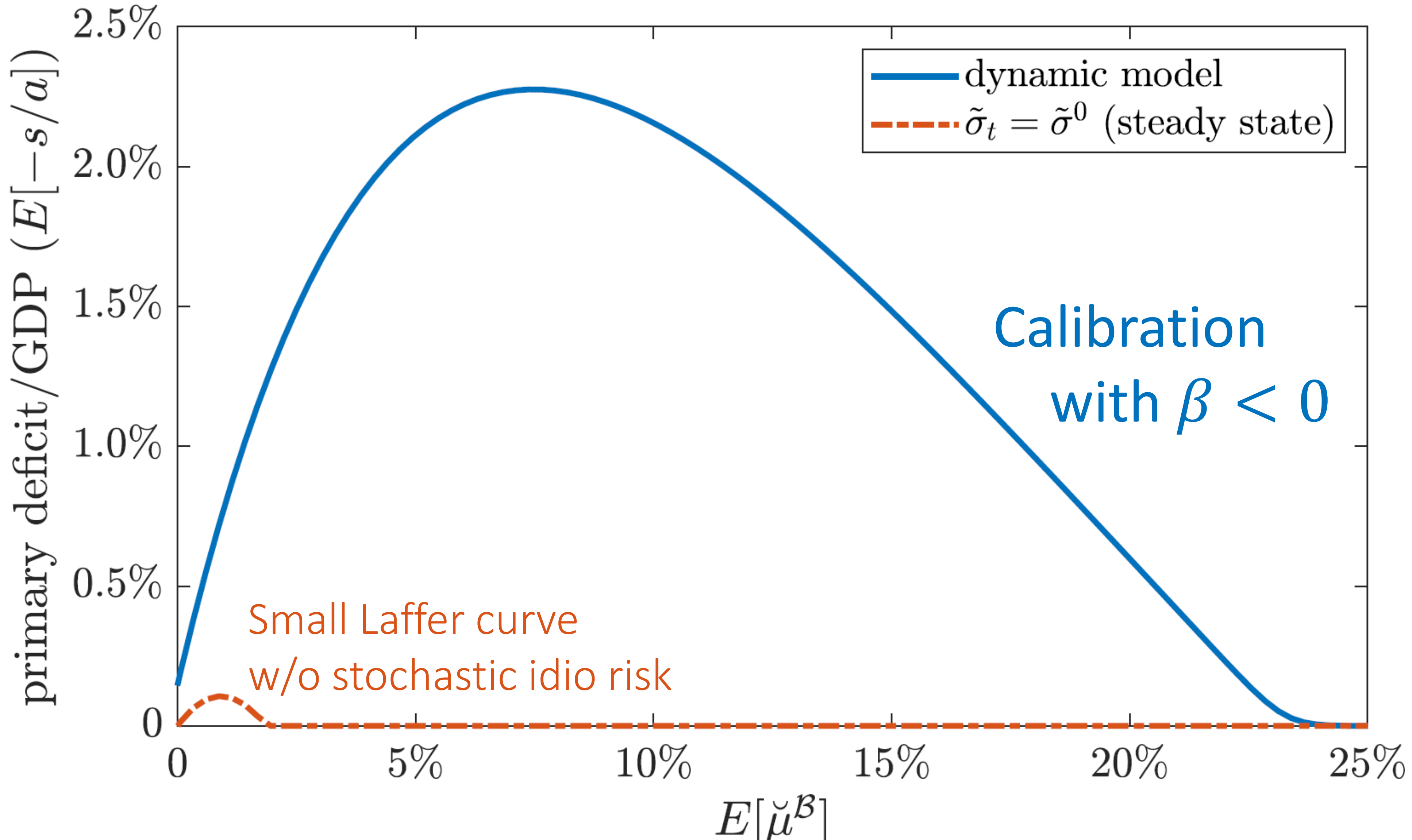
parameter	description	value	target
			external calibration
$\tilde{\sigma}^0$	$\tilde{\sigma}_t$ stoch. steady state	0.54	
ψ	$\tilde{\sigma}_t^2$ mean reversion	0.67	MLE targeting common idiosyncratic volatility (Herskovic et al. 2018)
σ	$\tilde{\sigma}_t^2$ volatility	0.4	
$\bar{\chi}$	undiversifiable idio. risk	0.3	Heaton, Lucas (1996, 2000, 2001), Angeletos (2007) (range $[0.2, 0.6]$)
			calibration to match model moments
γ	risk aversion	6	
ρ	time preference	0.138	
a^0	a_t stoch. steady state	0.63	chosen jointly to match (approximately)
$\mathfrak{g}$	gov. expenditures	0.138	- volatility of Y, C, I, S/Y
$\breve{\mu}^{\mathcal{B},0}$	$\breve{\mu}_t^{\mathcal{B}}$ stoch. steady state	0.0023	- average C/Y, G/Y, S/Y, I/K, $q^K K/Y$, $q^B K/Y$
α^a	a_t slope	0.071	- mean equity premium
$\alpha^{\mathcal{B}}$	$\breve{\mu}_t^{\mathcal{B}}$ slope	0.12	- equity Sharpe ratio
ϕ	capital adjustment cost	8.5	
			other parameters
δ	depreciation rate	0.055	economic growth rate (*ultimately irrelevant for all results*)

Quantitative Model Fit

moment symbol	description	model	data
$\sigma(Y)$	output volatility	1.3%	1.3%
$\sigma(C)/\sigma(Y)$	relative consumption volatility	0.63	0.64
$\sigma(S/Y)$	surplus volatility	1.1%	1.1%
$\mathbb{E}[S/Y]$	average surplus-output ratio	-0.0004	-0.0005
$\mathbb{E}[q^K K/Y]$	average capital-output ratio	3.48	3.73
$\mathbb{E}[q^B K/Y]$	average debt-output ratio	0.74	0.71
$\mathbb{E}[d\bar{r}^E - dr^{\mathcal{B}}]$	average (unlevered) equity premium	3.62%	3.40%
$\frac{\mathbb{E}[dr^E - dr^{\mathcal{B}}]}{\sigma\left(dr^E - dr^{\mathcal{B}}\right)}$	equity sharpe ratio	0.31	0.31

Fiscal Sustainability *given* Exorbitant Privilege: Debt Laffer Curve

- Issue bonds at a faster rate $\check{\mu}^B$ (esp. in recessions)
  - $\Rightarrow$ tax precautionary self insurance $\Rightarrow$ tax rate $\uparrow$
  - $\Rightarrow$ real value of bonds, $\frac{\mathcal{B}}{\wp}$, $\downarrow$ $\Rightarrow$ "tax base" $\downarrow$
    - Less so in recession due to flight-to-safety



Sizeable revenue only if Gov. debt has negative β

Two Debt Valuation Puzzles

- Properties of US primary surpluses
  - Average surplus ≈ 0
  - Procyclical surplus (> 0 in booms, < 0 in recessions)

- Two valuation puzzles from standard perspective:
  (Jiang, Lustig, van Nieuwerburgh, Xiaolan, 2019, 2020)
  1. "Public Debt Valuation Puzzle"
     - Empirical: $E[PV(surpluses)] < 0$, yet $\frac{\mathcal{B}}{\wp} > 0$
     - Our model: bubble/service flow component overturns results
  2. "Gov. Debt Risk Premium Puzzle"
     - Debt should be positive β asset, but market don't price it this way
     - Our model: can be rationalized with countercyclical bubble/service flow

Roadmap

- Motivation
- Steady state model (closed form solution)
- Stochastic idiosyncratic risk model
- (Safe) asset pricing with SDF ξ_t^{**}
- Flight-to-Safety, $\beta < 0$, and Excess Volatility of capital
- Equity issuance, non-safe mutual fund with $\beta > 0$ } Calibration
- Debt Laffer Curve } Calibration
- Price level determination with a bubble
  - Fiscal space to defend Exorbitant Privilege (crypto, …)

Why Does Gov. Safe Asset Survive in Presence of ETFs?

- Diversified stock portfolio is free of idiosyncratic risk
  - Trading in stocks (ETF) can also self-insure idiosyncratic risk
    - Good friend in idiosyncratically bad times

- But: poor hedge against aggregate risk, losses value in recessions
  - Positive β
    - Bad friend in aggregate bad times
  - Why positive β? (after all r^f goes down in recessions, lowers discount rate)
    - Equity are claims to capital, but marginal capital holder is insider
    - Insider bears idiosyncratic risk, must be compensated
    - $\tilde{\sigma}_t \uparrow \Rightarrow$ insider premium $E_t[dr_t^K] - E_t[dr_t^E] \uparrow \Rightarrow$ payouts to stockholders fall

  - **Share** of inside equity relative to outside equity compensation increases with $\tilde{\sigma}_t$
    - E.g. time of promise to issuance of new shares diluting outside equity holders

Exorbitant Privilege

1. Pay **low** real interest rate r (cash flow) on safe asset

2. Run **Ponzi scheme**
   Issue more bonds to fund primary deficit
   - Dilute existing bond holdings - "mine the bubble"
   - Tax on "precautionary savings"/self-insurance

   … but it is limited ⇒ Debt Laffer Curve

Relaxes Fiscal space

- Safe-asset status = exorbitant privilege is like a bubble (it can pop)
  - Jump to bad equilibrium
  - Safe-asset status can jump to foreign safe asset or crypto asset

Needs Fiscal space to preserve safe asset status

Conclusion

- Safe Asset = good friend
  - **Individually:** allows self-insurance through retrading
  - **Aggregate:** appreciates in bad times (negative β)
- Asset pricing with safe assets
  - Service Flow term >> convenience yield (BAA-Treasury spread)
  - Flight to Safety creates
    - Countercyclical safe asset valuation
    - Large stock market volatility
- Exorbitant privilege:
  - "Safe-asset status":
    low cash flow due to service flow (partially completing market via re-trading)
  - Extra space, but Debt Laffer Curve ($\neq$ MMT)
  - Power to run Ponzi scheme
  - Debt sustainability analysis (off-equilibrium)
- Fiscal space to ensure that bubble is attached to gov. bond (not on crypto)
- **Remark:** Competing Safe Assets
  - Within country private bonds are partial safe assets
  - Across countries $\Rightarrow$ Spillover of US Monetary Policy

Extra Slides