ECO529: Modern Macro, Money and (International) Finance

Precept 5

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I Theory: Setup

Intermediaries

Money	Deposits
Risky Claims	
	Net Worth

Households

Tech. b	Risky Claims
Tech. a	Net Worth
Deposits	

I Theory: Capital

- Two technologies/sectors: a and b

$$\frac{dk_t^a}{k_t^a} = (\Phi(\iota_t) - \delta)dt + \sigma^a dZ_t + \tilde{\sigma} d\tilde{Z}_t$$
$$\frac{dk_t^b}{k_t^b} = (\Phi(\iota_t) - \delta)dt + \sigma^b dZ_t + \tilde{\sigma} d\tilde{Z}_t$$

- Total output: Leontief technology

$$Y_t = AK_t = A_t^a(1-\psi)K_t + A_t^b \psi K_t$$

- Aggregate capital:

$$\frac{dK_t}{K_t} = \underbrace{(\Phi(\iota_t) - \delta)}_{\mu_t^K} dt + \underbrace{\left((1-\psi)\sigma^a + \psi\sigma^b\right)}_{\sigma^K} dZ_t$$

Theory: Government

- Issues Money/Nominal Bonds B_t at rate μ_t^B, pays interest i_t
- Distributes seigniorage (to capital holders) with transfers τ_t

$$\mu_t^B B_t = i_t B_t + \tau_t P_t K_t$$

$$\underbrace{(\mu_t^B - i_t)}_{\check{\mu}_t^B} q_t^B = \tau_t$$

I Theory: Returns

- On technologies a and b:

$$
\begin{aligned}
dr_t^a &= \frac{A_t^a - \iota_t}{q_t^K} dt + \frac{d(q_t^K k_t^a)}{q_t^K k_t^a} + \frac{\tau_t}{q_t^K} dt \\
&= \left(\frac{A_t^a - \iota_t}{q_t^K} + \mu_t^K + \mu_t^{q^K} + \sigma^a \sigma_t^{q^K} + \frac{q_t^B}{q_t^K} \check{\mu}_t^B \right) dt + \left(\sigma^a + \sigma_t^{q^K} \right) dZ_t + \tilde{\sigma} \tilde{Z}_t \\
dr_t^b &= \frac{A_t^b - \iota_t}{q_t^K} dt + \frac{d(q_t^K k_t^b)}{q_t^K k_t^b} + \frac{\tau_t}{q_t^K} dt \\
&= \left(\frac{A_t^b - \iota_t}{q_t^K} + \mu_t^K + \mu_t^{q^K} + \sigma^b \sigma_t^{q^K} + \frac{q_t^B}{q_t^K} \check{\mu}_t^B \right) dt + \left(\sigma^b + \sigma_t^{q^K} \right) dZ_t + \tilde{\sigma} \tilde{Z}_t
\end{aligned}
$$

I Theory: Returns

- On risky claims:

$$dr_t^{OE,H} = r_t^{OE} dt + \left(\sigma^b + \sigma_t^{q^K}\right) dZ_t + \tilde{\sigma}\tilde{Z}_t$$

$$dr_t^{OE,I} = r_t^{OE} dt + \left(\sigma^b + \sigma_t^{q^K}\right) dZ_t + \varphi\tilde{\sigma}\tilde{Z}_t$$

- On bonds and deposits:

$$dr_t^B = i_t dt + \frac{d(1/P_t)}{1/P_t} = \left(\mu_t^K + \mu_t^{q^B} + \sigma^K \sigma_t^{q^B} - \check{\mu}_t^B\right) dt + (\sigma^K + \sigma_t^{q^B}) dZ_t$$

$$dr_t^D = dr_t^B$$

I Theory: Solution

1. Change numeraire to N_t
2. Derive pricing and risk allocation conditions (SMP/Martingale)
3. Derive evolution of η
4. Derive money valuation equation for $\vartheta_t = \frac{q_t^B}{q_t^K + q_t^B}$
5. Solve numerically

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I Theory: Change of Numeraire

- Denote $\sigma \equiv \sigma^b - \sigma^a > 0$

$$d\hat{r}_t^a = \frac{A_t^a - \iota_t + \tau_t}{q_t^K} dt + \frac{d\left((1-\vartheta_t)\frac{k_t^a}{K_t}\right)}{(1-\vartheta_t)\frac{k_t^a}{K_t}}$$

$$= \underbrace{\left(\frac{A_t^a - \iota_t + q_t^B \check{\mu}_t^B}{q_t^K} + \mu_t^{1-\vartheta} + \psi\sigma\left(\sigma^K - \sigma_t^{1-\vartheta}\right)\right)}_{\hat{r}_t^a(\iota_t)} dt + \underbrace{\left(\sigma_t^{1-\vartheta} - \psi\sigma\right)}_{\hat{\sigma}_t^a} dZ_t + \tilde{\sigma}\tilde{Z}_t$$

$$d\hat{r}_t^b = \underbrace{\left(\frac{A_t^b - \iota_t + q_t^B \check{\mu}_t^B}{q_t^K} + \mu_t^{1-\vartheta} - (1-\psi)\sigma\left(\sigma^K - \sigma_t^{1-\vartheta}\right)\right)}_{\hat{r}_t^b(\iota_t)} dt + \underbrace{\left(\sigma_t^{1-\vartheta} + (1-\psi)\sigma\right)}_{\hat{\sigma}_t^b} dZ_t + \tilde{\sigma}\tilde{Z}_t$$

I Theory: Change of Numeraire

- Denote $\sigma \equiv \sigma^b - \sigma^a > 0$

$$d\hat{r}_t^a = \frac{A_t^a - \iota_t + \tau_t}{q_t^K} dt + \frac{d\left(\left(1-\vartheta_t\right)\frac{k_t^a}{K_t}\right)}{\left(1-\vartheta_t\right)\frac{k_t^a}{K_t}}$$

$$= \underbrace{\left(\frac{A_t^a - \iota_t + q_t^B \check{\mu}_t^B}{q_t^K} + \mu_t^{1-\vartheta} + \psi\sigma\left(\sigma^K - \sigma_t^{1-\vartheta}\right)\right)}_{\hat{r}_t^a(\iota_t)} dt + \underbrace{\left(\sigma_t^{1-\vartheta} - \psi\sigma\right)}_{\hat{\sigma}_t^a} dZ_t + \tilde{\sigma}\tilde{Z}_t$$

$$d\hat{r}_t^b = \underbrace{\left(\frac{A_t^b - \iota_t + q_t^B \check{\mu}_t^B}{q_t^K} + \mu_t^{1-\vartheta} - (1-\psi)\sigma\left(\sigma^K - \sigma_t^{1-\vartheta}\right)\right)}_{\hat{r}_t^b(\iota_t)} dt + \underbrace{\left(\sigma_t^{1-\vartheta} + (1-\psi)\sigma\right)}_{\hat{\sigma}_t^b} dZ_t + \tilde{\sigma}\tilde{Z}_t$$

$$d\hat{r}_t^B = i_t + \frac{d\left(\vartheta_t / B_t\right)}{\vartheta_t / B_t} = \underbrace{\left(\mu_t^\vartheta - \check{\mu}_t^B\right)}_{\hat{r}_t^B} dt + \underbrace{\sigma_t^\vartheta}_{\hat{\sigma}_t^B} dZ_t$$

I Theory: Solution

1. Change numeraire to N_t
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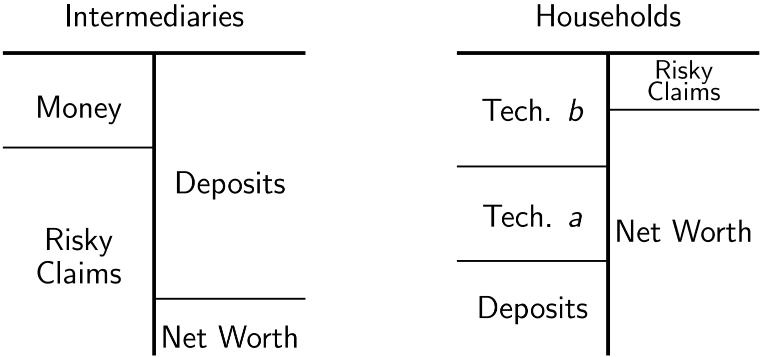
3. Derive evolution of η
4. Derive money valuation equation for $\vartheta_t = \frac{q_t^B}{q_t^K + q_t^B}$
5. Solve numerically

I Theory: Setup



I Theory: Household's Problem

- Households chose c_t^H, ι_t, $\theta_t^{D,H}$, θ_t^a, θ_t^b, $\theta_t^{OE,H}$
- Can not offload risk in technology a
- Can offload at most fraction $\frac{\bar{\chi}}{\psi} < 1$ of risk in technology b

$$\max \mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho t} \log(c_t^H) dt\right] \qquad \text{s.t.}$$

$$\frac{dn_t^H}{n_t^H} = -\frac{c_t^H}{n_t^H} dt + \theta_t^{D,H} dr_t^D + \theta_t^a dr_t^a(\iota_t) + \theta_t^b dr_t^b(\iota_t) + \theta_t^{OE,H} dr_t^{OE,H}$$

$$1 = \theta_t^{D,H} + \theta_t^a + \theta_t^b + \theta_t^{OE,H}$$

$$-\theta_t^{OE,H} = \frac{\chi_t}{\psi}\theta_t^b \leq \frac{\bar{\chi}}{\psi}\theta_t^b$$

I Theory: Household's Problem

- Write in N_t-numeraire and substitute $\theta_t^{OE,H}$ with $-\frac{\chi_t}{\psi}\theta_t^b$

$$\max \mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho t}\log(c_t^H)dt\right] \qquad \text{s.t.}$$

$$\frac{d\hat{n}_t^H}{\hat{n}_t^H} = -\frac{\hat{c}_t^H}{\hat{n}_t^H}dt + d\hat{r}_t^D + \theta_t^a\left(d\hat{r}_t^a(\iota_t) - d\hat{r}_t^D\right) + \theta_t^b\left(d\hat{r}_t^b(\iota_t) - d\hat{r}_t^D - \frac{\chi_t}{\psi}\left(d\hat{r}_t^{OE,H} - d\hat{r}_t^D\right)\right)$$

$$\chi_t \leq \bar{\chi}$$

I Theory: Household's Problem

- Write in N_t-numeraire and substitute $\theta_t^{OE,H}$ with $-\frac{\chi_t}{\psi}\theta_t^b$

$$\max \mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho t}\log(c_t^H)dt\right] \qquad \text{s.t.}$$

$$\frac{d\hat{n}_t^H}{\hat{n}_t^H} = -\frac{\hat{c}_t^H}{\hat{n}_t^H}dt + d\hat{r}_t^D + \theta_t^a\left(d\hat{r}_t^a(\iota_t) - d\hat{r}_t^D\right) + \theta_t^b\left(d\hat{r}_t^b(\iota_t) - d\hat{r}_t^D - \frac{\chi_t}{\psi}\left(d\hat{r}_t^{OE,H} - d\hat{r}_t^D\right)\right)$$

$$\chi_t \leq \bar{\chi}$$

$$H_t^H = e^{-\rho t}\log c_t^H - \hat{\xi}_t^H\frac{c_t^H}{N_t} + \hat{\xi}_t^H\hat{n}_t^H\left[\hat{r}_t^D + \theta_t^a\left(\hat{r}_t^a(\iota_t) - \hat{r}_t^D\right) + \theta_t^b\left(\hat{r}_t^b(\iota_t) - \hat{r}_t^D - \frac{\chi_t}{\psi}\left(\hat{r}_t^{OE} - \hat{r}_t^D\right)\right)\right]$$
$$- \hat{\xi}_t^H\hat{n}_t^H\hat{\varsigma}_t^H\left[\hat{\sigma}_t^D + \theta_t^a\left(\hat{\sigma}_t^a - \hat{\sigma}_t^D\right) + \theta_t^b\left(\hat{\sigma}_t^b - \hat{\sigma}_t^D\right)\left(1 - \frac{\chi_t}{\psi}\right)\right]$$
$$- \hat{\xi}_t^H\hat{n}_t^H\tilde{\varsigma}_t^H\left[\theta_t^a + \theta_t^b\left(1 - \frac{\chi_t}{\psi}\right)\right]\tilde{\sigma} + \lambda_t^H\hat{\xi}_t^H\hat{n}_t^H\theta_t^b\frac{\bar{\chi} - \chi_t}{\psi}$$

Theory: Household's Problem

- FOCs:

$$\hat{r}_t^a - \hat{r}_t^D = \hat{\varsigma}_t^H(\hat{\sigma}_t^a - \hat{\sigma}_t^D) + \tilde{\varsigma}_t^H \tilde{\sigma}$$

$$\hat{r}_t^{OE} - \hat{r}_t^D = \hat{\varsigma}_t^H(\hat{\sigma}_t^b - \hat{\sigma}_t^D) + \tilde{\varsigma}_t^H \tilde{\sigma} - \lambda_t^H$$

$$\hat{r}_t^b - \hat{r}_t^D - \frac{\chi_t}{\psi}\left(\hat{r}_t^{OE} - \hat{r}_t^D\right) = \hat{\varsigma}_t^H(\hat{\sigma}_t^b - \hat{\sigma}_t^D)\left(1 - \frac{\chi_t}{\psi}\right) + \tilde{\varsigma}_t^H \tilde{\sigma}\left(1 - \frac{\chi_t}{\psi}\right)$$

$$\lambda_t^H(\bar{\chi} - \chi_t) = 0$$

- Recall $\hat{\varsigma}_t^H = \sigma_t^{1-\eta}$, $\tilde{\varsigma}_t^H = \tilde{\sigma}_t^{1-\eta}$

I Theory: Intermediaries Problem

- Intermediaries chose c_t^I, θ_t^B, $\theta_t^{D,I}$, $\theta_t^{OE,I}$
- Since $dr_t^B = dr_t^D$, effectively choose c_t^I, $\theta_t^{OE,I}$, $\theta_t^{M,I} \equiv \theta_t^B + \theta_t^{D,I}$

$$\max \mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho t} \log(c_t^I) dt\right] \qquad \text{s.t.}$$

$$\frac{d\hat{n}_t^I}{\hat{n}_t^I} = -\frac{\hat{c}_t^I}{\hat{n}_t^I} dt + d\hat{r}_t^B + \theta_t^{OE,I}\left(d\hat{r}_t^{OE,I} - d\hat{r}_t^B\right)$$

I Theory: Intermediaries Problem

- Intermediaries chose c_t^I, θ_t^B, $\theta_t^{D,I}$, $\theta_t^{OE,I}$
- Since $dr_t^B = dr_t^D$, effectively choose c_t^I, $\theta_t^{OE,I}$, $\theta_t^{M,I} \equiv \theta_t^B + \theta_t^{D,I}$

$$\max \mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho t} \log(c_t^I) dt\right] \qquad \text{s.t.}$$

$$\frac{d\hat{n}_t^I}{\hat{n}_t^I} = -\frac{\hat{c}_t^I}{\hat{n}_t^I} dt + d\hat{r}_t^B + \theta_t^{OE,I}\left(d\hat{r}_t^{OE,I} - d\hat{r}_t^B\right)$$

$$H_t^I = e^{-\rho t} \log c_t^I - \hat{\xi}_t^I \frac{c_t^I}{N_t} + \hat{\xi}_t^I \hat{n}_t^I \left[\hat{r}_t^B + \theta_t^{OE,I}\left(\hat{r}_t^{OE} - \hat{r}_t^B\right)\right]$$

$$- \hat{\xi}_t^I \hat{n}_t^I \hat{\varsigma}_t^I \left[\hat{\sigma}_t^B + \theta_t^{OE,I}\left(\hat{\sigma}_t^b - \hat{\sigma}_t^B\right)\right] - \hat{\xi}_t^I \hat{n}_t^I \tilde{\varsigma}_t^I \theta_t^{OE,I} \varphi \tilde{\sigma}$$

I Theory: Intermediaries Problem

- FOCs:

$$\hat{r}_t^{OE} - \hat{r}_t^{B} = \hat{\varsigma}_t^{I}(\hat{\sigma}_t^{b} - \hat{\sigma}_t^{B}) + \tilde{\varsigma}_t^{I}\varphi\tilde{\sigma}$$

- Recall $\hat{\varsigma}_t^{I} = \sigma_t^{\eta}$, $\tilde{\varsigma}_t^{I} = \tilde{\sigma}_t^{\eta}$

I Theory: Risk Allocation

- Combine Households' and Intermediaries' FOCs:

$$\hat{r}_t^{OE} - \hat{r}_t^{D} = \sigma_t^{1-\eta}(\hat{\sigma}_t^b - \hat{\sigma}_t^D) + \tilde{\sigma}_t^{1-\eta}\tilde{\sigma} - \lambda_t^H$$

$$\hat{r}_t^{OE} - \hat{r}_t^{B} = \sigma_t^{\eta}(\hat{\sigma}_t^b - \hat{\sigma}_t^B) + \tilde{\sigma}_t^{\eta}\varphi\tilde{\sigma}$$

- Recall $\hat{r}_t^D = \hat{r}_t^B$, $\hat{\sigma}_t^D = \hat{\sigma}_t^B = \sigma_t^{\vartheta}$:

$$\left(\sigma_t^{\eta} - \sigma_t^{1-\eta}\right)(\hat{\sigma}_t^b - \sigma_t^{\vartheta}) + \left(\varphi\tilde{\sigma}_t^{\eta} - \tilde{\sigma}_t^{1-\eta}\right)\tilde{\sigma} + \lambda_t^H = 0$$

$$\tilde{\sigma}_t^{\eta} = (1-\vartheta_t)\frac{\chi_t}{\eta_t}\varphi\tilde{\sigma} \qquad \tilde{\sigma}_t^{1-\eta} = (1-\vartheta_t)\frac{1-\chi_t}{1-\eta_t}\tilde{\sigma}$$

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5. Solve numerically

I Theory: Evolution of η

- Combine Intermediaries' FOCs or use Martingale Pricing on net worth:

$$\frac{d\eta_t}{\eta_t} = \frac{d\hat{N}_t^I}{\hat{N}_t^I} = \underbrace{\left[-\rho + \hat{r}_t^B + \theta_t^{OE,I} \left(\hat{r}_t^{OE,I} - \hat{r}_t^B \right) \right]}_{\mu_t^\eta} dt + \underbrace{\left[\hat{\sigma}_t^B + \theta_t^{OE,I} \left(\hat{\sigma}_t^b - \hat{\sigma}_t^B \right) \right]}_{\sigma_t^\eta} dZ_t$$

$$= \underbrace{\left[-\rho + \hat{r}_t^B + \theta_t^{OE,I} \left[\hat{\varsigma}_t^I \left(\hat{\sigma}_t^b - \hat{\sigma}_t^B \right) + \tilde{\varsigma}_t^I \varphi \tilde{\sigma} \right] \right]}_{\mu_t^\eta} dt + \sigma_t^\eta dZ_t \quad \Longrightarrow$$

$$\mu_t^\eta + \rho = \hat{r}_t^B + \hat{\varsigma}_t^I \left(\sigma_t^\eta - \hat{\sigma}_t^B \right) + \theta_t^{OE,I} \tilde{\varsigma}_t^I \varphi \tilde{\sigma} = \hat{r}_t^B + \sigma_t^\eta \left(\sigma_t^\eta - \hat{\sigma}_t^B \right) + \left(\tilde{\sigma}_t^\eta \right)^2$$

I Theory: Evolution of η

- Same for Households:

$$\mu_t^{1-\eta} + \rho = \hat{r}_t^B + \sigma_t^{1-\eta}\left(\sigma_t^{1-\eta} - \hat{\sigma}_t^B\right) + \left(\tilde{\sigma}_t^{1-\eta}\right)^2$$

- Combine the two with weights η_t, $1 - \eta_t$:

$$\rho = \hat{r}_t^B + \eta_t\left(\sigma_t^\eta\right)^2 + (1-\eta_t)\left(\sigma_t^{1-\eta}\right)^2 + \eta_t\left(\tilde{\sigma}_t^\eta\right)^2 + (1-\eta_t)\left(\tilde{\sigma}_t^{1-\eta}\right)^2$$

- Plug into expression for μ_t^η, recall $\hat{\sigma}_t^B = \sigma_t^\vartheta$:

$$\mu_t^\eta = (1-\eta_t)\left[\left(\sigma_t^\eta\right)^2 + \left(\tilde{\sigma}_t^\eta\right)^2 - \left(\sigma_t^{1-\eta}\right)^2 - \left(\tilde{\sigma}_t^{1-\eta}\right)^2\right] - \sigma_t^\eta \sigma_t^\vartheta$$

I Theory: Evolution of η

- Volatility loading σ_t^η:

$$\sigma_t^\eta = \hat{\sigma}_t^B + \theta_t^{OE,I}\left(\hat{\sigma}_t^b - \hat{\sigma}_t^B\right)$$

- Market Clearing:

$$\theta_t^{OE,I} = -\theta_t^{OE,H}\frac{\eta_t}{1-\eta_t} = \frac{\chi_t}{\psi}\theta_t^b\frac{\eta_t}{1-\eta_t} = \chi_t\frac{1-\vartheta_t}{\eta_t}$$

- Recall $\hat{\sigma}_t^B = \sigma_t^\vartheta$, $\hat{\sigma}_t^b = \sigma_t^{1-\vartheta} + (1-\psi)\sigma$, $\sigma_t^\vartheta = \frac{\partial_\eta \vartheta_t}{\vartheta_t}\eta_t\sigma_t^\eta$:

$$\eta_t\sigma_t^\eta = \frac{(1-\vartheta_t)\chi_t(1-\psi)\sigma}{1+(\chi_t-\eta_t)\frac{\partial_\eta\vartheta_t}{\vartheta_t}}$$

Theory: Solution

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I Theory: Money Valuation Equation

- Use expression for $\hat{r}^B_t = \mu^\vartheta_t - \check{\mu}^B_t$ from previous derivations:

$$\mu^\vartheta_t = \rho + \check{\mu}^B_t - \left[\eta_t \left(\sigma^\eta_t\right)^2 + (1-\eta_t)\left(\sigma^{1-\eta}_t\right)^2 + \eta_t \left(\tilde{\sigma}^\eta_t\right)^2 + (1-\eta_t)\left(\tilde{\sigma}^{1-\eta}_t\right)^2\right]$$

I Theory: Solution

1. Change numeraire to N_t
2. Derive pricing and risk allocation conditions (SMP/Martingale)
3. Derive evolution of η
4. Derive money valuation equation for $\vartheta_t = \frac{q_t^B}{q_t^K + q_t^B}$
5. Solve numerically

I Theory: Summary

- Main equation for $\vartheta(\eta)$:

$$\mu^{\vartheta} = \rho + \check{\mu}^{B} - \left[\eta\left(\sigma^{\eta}\right)^{2} + (1-\eta)\left(\sigma^{1-\eta}\right)^{2} + \eta\left(\tilde{\sigma}^{\eta}\right)^{2} + (1-\eta)\left(\tilde{\sigma}^{1-\eta}\right)^{2}\right]$$

$$\mu^{\vartheta}\vartheta = \eta\mu^{\eta}\partial_{\eta}\vartheta(\eta) + \frac{\left(\eta\sigma^{\eta}\right)^{2}}{2}\partial_{\eta\eta}\vartheta(\eta)$$

$$\rho\vartheta(\eta) = \left[\sum_{i\in\{I,H\}}\eta^{i}\left(\sigma^{\eta^{i}}\right)^{2} + \sum_{i\in\{I,H\}}\eta^{i}\left(\tilde{\sigma}^{\eta^{i}}\right)^{2} - \check{\mu}^{B}\right]\vartheta(\eta) + \eta\mu^{\eta}\partial_{\eta}\vartheta(\eta) + \frac{\left(\eta\sigma^{\eta}\right)^{2}}{2}\partial_{\eta\eta}\vartheta(\eta)$$

I Theory: Summary

$$\rho\vartheta(\eta) = \left[\sum_{i\in\{I,H\}} \eta^i \left(\sigma^{\eta^i}\right)^2 + \sum_{i\in\{I,H\}} \eta^i \left(\tilde{\sigma}^{\eta^i}\right)^2 - \check{\mu}^B\right] \vartheta(\eta) + \eta\mu^\eta\partial_\eta\vartheta(\eta) + \frac{(\eta\sigma^\eta)^2}{2}\partial_{\eta\eta}\vartheta(\eta)$$

$$\mu^\eta = (1-\eta)\left[(\sigma^\eta)^2 + (\tilde{\sigma}^\eta)^2 - \left(\sigma^{1-\eta}\right)^2 - \left(\tilde{\sigma}^{1-\eta}\right)^2\right] - \sigma^\eta\sigma^\vartheta$$

$$\eta\sigma^\eta = \frac{(1-\vartheta)\chi(1-\psi)\sigma}{1+(\chi-\eta)\frac{\partial_\eta\vartheta}{\vartheta}}, \quad \tilde{\sigma}^\eta = (1-\vartheta)\frac{\chi}{\eta}\varphi\tilde{\sigma}, \quad \tilde{\sigma}^{1-\eta} = (1-\vartheta)\frac{1-\chi}{1-\eta}\tilde{\sigma}, \quad \sigma^\vartheta = \eta\sigma^\eta\frac{\partial_\eta\vartheta}{\vartheta}$$

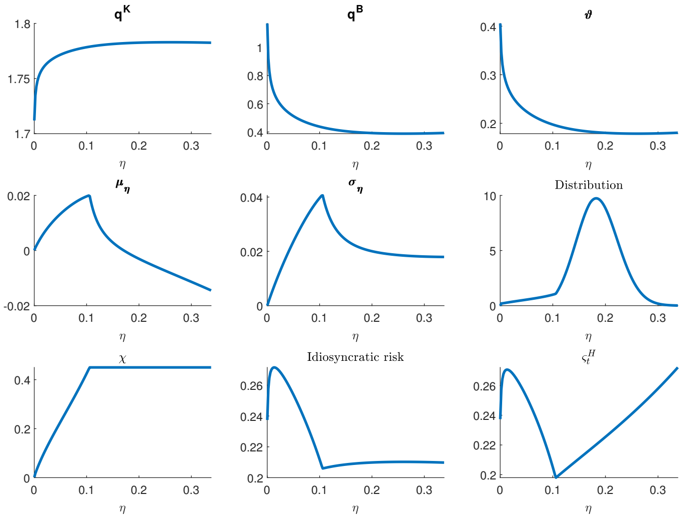
$$\left(\sigma^\eta - \sigma^{1-\eta}\right)\left((1-\psi)\sigma - \frac{\sigma^\vartheta}{1-\vartheta}\right) + \left(\varphi\tilde{\sigma}^\eta - \tilde{\sigma}^{1-\eta}\right)\tilde{\sigma} + \lambda^H = 0$$

$$\lambda^H(\bar{\chi}-\chi) = 0, \qquad \chi \leq \bar{\chi}$$

I Theory: Numerical Algorithm

- Outer Loop:
    1. Guess $\vartheta(\eta)$
    2. Inner loop:
        - Compute $\sigma^{\eta}(\eta)$ and $\chi(\eta)$ (parallelize!)
    3. Compute $\mu^{\eta}(\eta)$, $\tilde{\sigma}^{\eta}$, $\tilde{\sigma}^{1-\eta}$
    4. Update $\vartheta(\eta)$

I Theory



q^K
q^B
ϑ
μ_η
σ_η
Distribution
χ
Idiosyncratic risk
ς_t^H
η