Macro, Money and (International) Finance – Problem Set 4

October 3, 2022

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Please submit to `alexandrov@princeton.edu` by 1pm on Monday, October 10 (Princeton time).

1 Endogenous Jumps

In this exercise you will add endogenous jumps to the Lecture 5 model with log utility. The main advantage of log utility is that model solution $(q, \sigma^q, \chi, \iota, \kappa)$ remains unchanged, it is only the law of motion for η that is affected by runs. Use the model parameters from Problem Set 2, set $\lambda = 0.05$ and introduce transfers τ, such that whenever there is a run and experts' net wealth goes to zero, they receive a transfer (from households) in the size of τK_t. The transfers are such that after the run $\eta_t = \eta^* = 0.005$. Proceed in the following steps:

1. Obtain solution for the log utility model (no changes needed at this point).
2. Given that after the run η jumps to η^*, write down expressions for $j^q(\eta), j^{\eta^e}(\eta)$ and $j^{\eta^h}(\eta)$.[1]
3. Characterize the vulnerability region in terms of $j^q(\eta), \chi$ and η. We still consider a jump to η^* to be self-fulfilling if it destroys the net worth of experts.[2]
4. Find the value of τ that ensures $\eta_t = \eta^*$ after the run, given that $N_t^e = \tau K_t$ after the run.
5. Express $j^N(\eta)$, $j^{N^e}(\eta)$ and $j^{N^h}(\eta)$ in terms of η, η^* and $j^q(\eta)$.
6. Derive $j^{r^D}(\eta)$. For that, use the balance sheet of experts ($D_t^e = q_t K_t^e - N_t^e - OE_t^e$) to compute the loss that is absorbed by debt when the jump occurs. Remember that the shock destroys experts' net worth and part of the shock loads onto outside equity.

[1] Hint: whenever there is a jump, we can ignore the drift and volatility terms. E.g. for $\frac{dx_t}{x_t} = \mu_t^x dt + \sigma_t^x dZ_t + j_t^x dJ_t$, denote by x_t^+ the value right after the jump and by x_t^- – right before the jump. When the jump occurs and $dJ_t = 1$, $dx_t = x_t^+ - x_t^- = j_t^x x_t^-$.

[2] Destruction of experts' net worth is immediately followed by transfers, resulting in positive η^*, but that does not affect the determination of vulnerability region.

7. Don't forget to set all the $j-s$ to zero outside of the vulnerability region! Compute the (geometric) drift of η:

$$\mu_t^\eta = (1-\eta_t)\Bigg[(\varsigma_t^e-\sigma-\sigma_t^q)(\sigma_t^\eta+\sigma+\sigma_t^q) - (\varsigma_t^h-\sigma-\sigma_t^q)\left(-\frac{\eta_t\sigma_t^\eta}{1-\eta_t}+\sigma+\sigma_t^q\right) - \left(\frac{C_t^e}{N_t^e}-\frac{C_t^h}{N_t^h}\right)$$
$$+\ \lambda\left(\hat{\nu}_t^e\left(j_t^{\eta^e}-\frac{j_t^{r^D}-j_t^N}{1+j_t^N}\right)-\hat{\nu}_t^h\left(j_t^{\eta^h}-\frac{j_t^{r^D}-j_t^N}{1+j_t^N}\right)\right)\Bigg] - \lambda j_t^{\eta^e}$$

where $\hat{\nu}_t^i = \nu_t^i - j_t^N + \nu_t^i j_t^N$, $\nu_t^i = \frac{j^{N^i}}{1+j^{N^i}}$, $\varsigma_t^i = \sigma_t^{N^i}$, and σ_t^η is as before. Plot the (arithmetic) drift $\eta\mu^\eta$, explain the differences with the baseline model ($\lambda = 0$).

8. Given that now the law of motion for η is as follows:

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dZ_t + j_t^\eta dJ_t$$

compute the stationary distribution. In addition to the usual M matrix, you need to construct a Λ matrix that will reflect the jumps, so that the relevant matrix for the KFE will be $M+\Lambda$.

2 Money Model with Stochastic Volatility

Consider the model of Lecture 10 with log utility and without government policy ($\mu^B = i = \sigma^B = \breve{g} = \tau = 0$).[3] In this problem, we add stochastic volatility to the model. Suppose idiosyncratic risk $\tilde{\sigma}_t$ evolves according to the exogenous stochastic process

$$d\tilde{\sigma}_t = b(\tilde{\sigma}^{ss} - \tilde{\sigma}_t)dt + \nu\sqrt{\tilde{\sigma}_t}dZ_t,$$

where $\tilde{\sigma}^{ss}$, b and ν are positive constants. There are no aggregate capital shocks, i.e. $\sigma = 0$.

1. Use goods market clearing and optimal investment to express q^K, q^B and ι in terms of $\vartheta := \frac{q^B}{q^K+q^B}$
2. Derive the "money valuation equation" using martingale method and change of numeraire:
   (a) First, write down:
      - Returns on capital ($dr_t^{k,\tilde{i}}$) and bonds (dr_t^B), laws of motion for individual wealth $n_t^{\tilde{i}}$ and aggregate wealth $N_t = \int n_t^{\tilde{i}}$ (simply integrate over all agents), and return on individual wealth $dr_t^{n,\tilde{i}} = \frac{dn_t^{\tilde{i}}}{n_t^{\tilde{i}}} + \frac{c_t^{\tilde{i}}}{n_t^{\tilde{i}}}dt$, all in consumption numeraire.
      - Laws of motion for SDF in consumption numeraire ($\xi_t^{\tilde{i}}$) and N_t-numeraire ($\hat{\xi}_t^{\tilde{i}}$), you don't need to plug in for μ_t^N and σ_t^N at this point.

   (b) Express the return on bonds in N_t-numeraire in terms of μ_t^ϑ and σ_t^ϑ. Note that $dr_t^B = \frac{d(q_t^BK_t/B_t)}{q_t^BK_t/B_t}$, which in N_t-numeraire becomes $d\hat{r}_t^B = \frac{d(q_t^BK_t/(B_tN_t))}{q_t^BK_t/(B_tN_t)}$.
   (c) Write down the return on indiviual wealth in N_t-numeraire $d\hat{r}_t^{n,\tilde{i}}$

[3] There can still be a (constant) supply of bonds $B_t \neq 0$.

(d) Use the martinagale pricing condition:[4]

$$\frac{\mathbb{E}[d\hat{r}_t^{n,\tilde{i}}]}{dt} - \frac{\mathbb{E}[d\hat{r}_t^{B}]}{dt} = \hat{\varsigma}_t(\sigma_t^{\hat{r}^{n,\tilde{i}}} - \sigma_t^{\hat{r}^{B}}) + \hat{\tilde{\varsigma}}_t(\tilde{\sigma}_t^{\hat{r}^{n,\tilde{i}}} - \tilde{\sigma}_t^{\hat{r}^{B}})$$

and capital market clearing condition to derive an expression of the form $\mu_t^{\vartheta} = f(\vartheta_t, \tilde{\sigma}_t)$, where function f only depends on model parameters (the "money valuation equation")

3. Solve the model numerically:

- Apply Ito's lemma to $\vartheta_t = \vartheta(\tilde{\sigma}_t)$, and equate the drift term with $\vartheta_t \mu_t^{\vartheta}$. This gives you an HJB-looking equation for $\vartheta(\tilde{\sigma})$.
- Set $a = 0.2$, $\phi = 1$, $\delta = 0.05$, $\rho = 0.01$, $\tilde{\sigma}^{ss} = 0.2$, $b = 0.05$, $\nu = 0.02$.
- Before solving for $\vartheta(\tilde{\sigma})$, suggest a grid for $\tilde{\sigma}$ (you can work with its law of motion). Motivate your choice of the upper and the lower boundaries.
- Solve the model using value function iteration.
- Plot $\vartheta, q^B, q^K, r^f, \varsigma, \tilde{\varsigma}$ as functions of $\tilde{\sigma}$.[5]

[4] Where $\hat{\varsigma}_t$ and $\hat{\tilde{\varsigma}}_t$ are the prices of aggregate and individual risk in N_t-numeraire, respectively.

[5] To compute r^f you would be using Ito's formula and the martingale pricing formula for $dr^{k,\tilde{i}}$ or dr^B.