Modern Macro, Money, and International Finance

Eco 529

Lecture 11: Cash vs. Cashless Economy – The I Theory of Money with Heterogenous Agents

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Key Takeaways

- Real vs. Nominal Debt/Cashless vs. Cash
  - Inflation risk can improve risk sharing
- Intertemporal unit of account
  - State-contingent Monetary Policy if $\sigma^B \neq 0$
- Equivalence of capital vs. risk allocation setting (κ vs. χ)
- Liquidity and Disinflationary Spiral

- Policy
  - Fiscal Policy
  - Monetary Policy
    - Stealth recapitalization of intermediaries
  - Macroprudential Policy

- Technical Takeaways
  - Two sector money models

The big Roadmap: Towards the I Theory of Money

- One sector model with idio risk - “The I Theory without I” (steady state focus)
  - Store of value
    - Insurance role of money *within sector*
  - Money as bubble or not
  - Fiscal Theory of the Price Level
  - Medium of Exchange Role ⇒ SDF-Liquidity multiplier ⇒ Money bubble
- 2 sector/type model with money and idio risk
  - Generic Solution procedure (compared to earlier lectures)
  - Equivalence btw experts producers and intermediaries
  - Real debt vs. nominal debt/money
    - Implicit insurance role of money *across sectors*
  - I Theory
- Welfare analysis
- Optimal Monetary Policy and Macroprudential Policy
- International Monetary Model

Previous lectures

Today

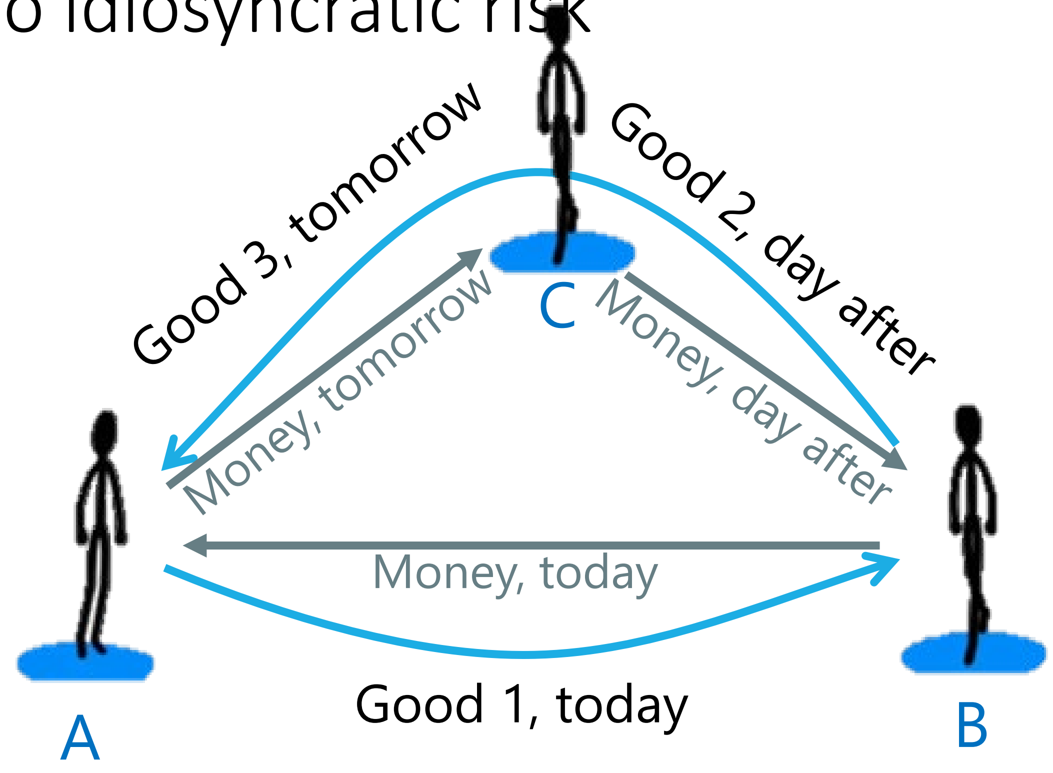
Next lectures

The 4 Roles of Money

- Unit of account
  - Intratemporal: Numeraire — bounded rationality/price stickiness
  - Intertemporal: Debt contract — incomplete markets

- Store of value
  - "I Theory of Money without I"
    Less risky than other "capital" – no idiosyncratic risk
  - Fiscal theory of the price level

- Medium of exchange
  - Overcome double-coincidence of wants problem



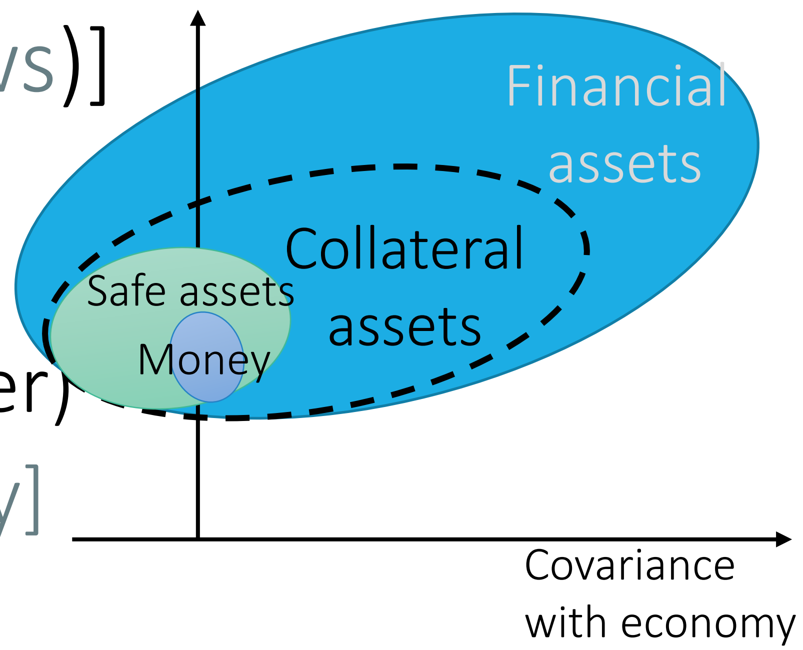
- Record keeping device – money is memory
  - Virtual ledger

Safe Assets ⊇ (Narrow) Money

- Asset Price = E[PV(cash flows)] + E[PV(service flows)]

  dividends/interest

  - Service flows/convenience yield
    1. Collateral: relax constraints (Lagrange multiplier)
    2. Safe asset: [good friend analogy]
       - When one needs funds, one can sell at stable price
         … since others buy
       - Partial insurance through **retrading** - market liquidity!
    3. Money (narrow): relax double-coincidence of wants
  - Higher Asset Price = lower expected return



- Problem: safe asset + money status might burst like a bubble
  - Multiple equilibria: [safe asset tautology]

Models on Money as Store of Value

\Friction	OLG	Incomplete Markets + idiosyncratic risk	
Risk	deterministic	endowment risk borrowing constraint	return risk Risk tied up with Individual capital
Only money	Samuelson	Bewley	
			"I Theory without I" Brunnermeier-Sannikov (AER PP 2016)
With capital	Diamond	Aiyagari	

(New) Keynesian Demand Management		**I Theory of Money Risk (Premium) Management**
Stimulate aggregate consumption		Alleviate balance sheet constraints
Woodford (2003)	Tobin (1982), HANK	BruSan
Price stickiness & ZLB Perfect capital markets	Both	Financial frictions Incomplete markets
Representative Agent	Heterogeneous Agents	
Cut i Reduces r due to price stickiness Consumption c rises	Cut i Changes bond prices Redistributes from low MPC to high MPC consumers	Cut i or QE Changes asset prices Ex-post: Redistributes to balance sheet impaired sector
		Price of Risk Dynamics

“Money and Banking” (in macro-finance)

- Money — store of value/safe asset/Gov. bond
- Banking — “diversifier”
  holds risky assets, issues inside money

Watch “Money and Banking”
YouTube Video Channel: “markus.economicus”
https://www.youtube.com/channel/UCV8DKoTKvJtuvkI4UsRYIqA/videos?pbjreload=10

Money and Banking, part 3:
Redistributive Monetary...

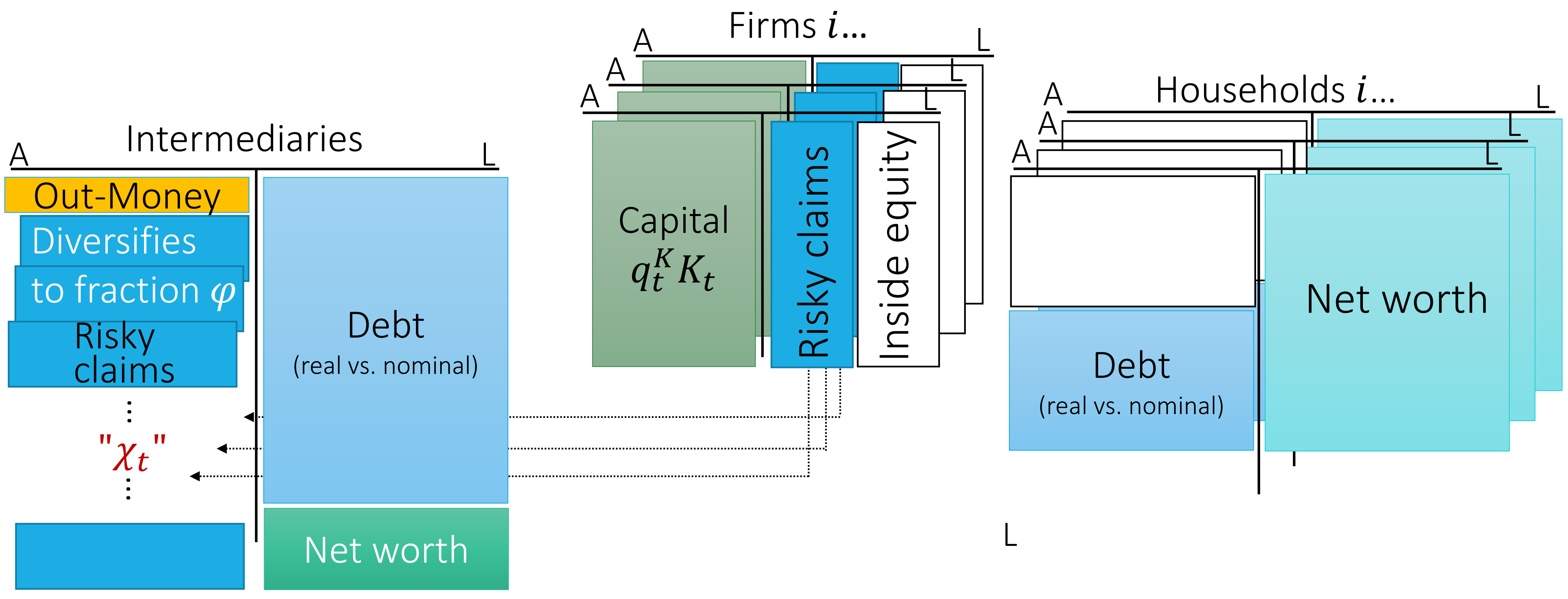
“Money and Banking” (in macro-finance)

- Money store of value/safe asset/Gov. bond
- Banking “diversifier”
  holds risky assets, issues inside money
- Amplification/endogenous risk dynamics
  - Value of capital declines due to fire-sales **Liquidity spiral**
    - Flight to safety
  - Value of money rises **Disinflation spiral** a la Fisher
    - Demand for money rises – less idiosyncratic risk is diversified
    - Supply for inside money declines – less creation by intermediaries
      - Endogenous money multiplier = f(capitalization of critical sector)
  - Paradox of Prudence ~~Paradox of Thrift~~ (in risk terms)
- Monetary Policy (redistributive)

Roadmap

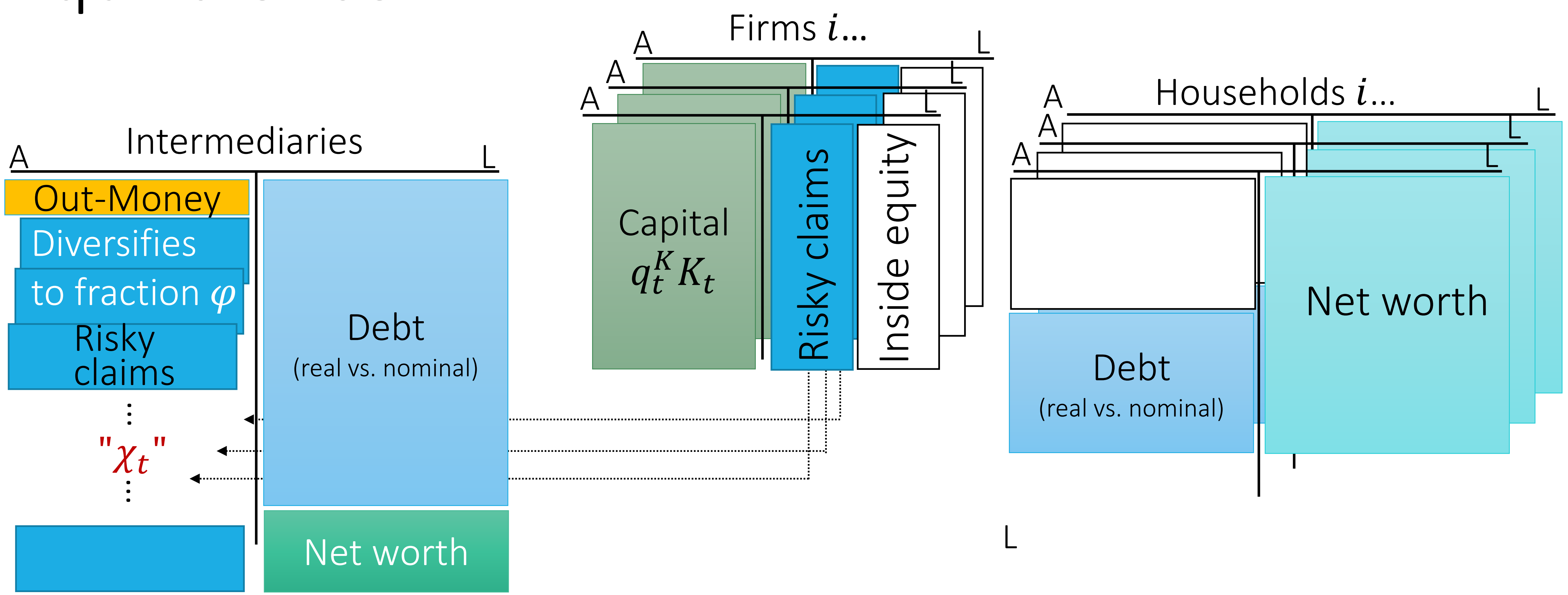
- Intro
- Equivalence btw experts producers and intermediaries
- Real vs. Nominal Debt
- I Theory of Money
- Policy

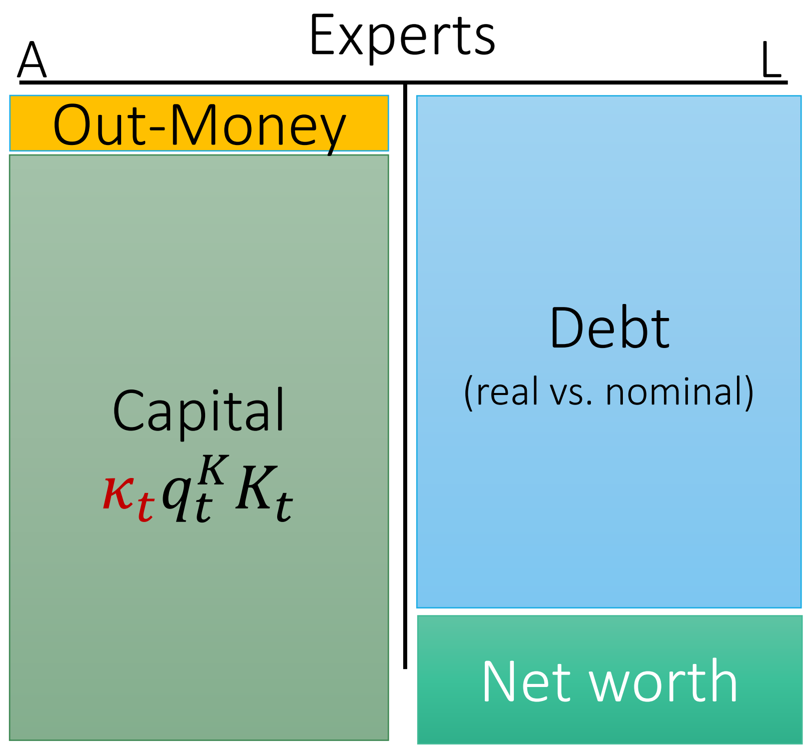
Intermediaries



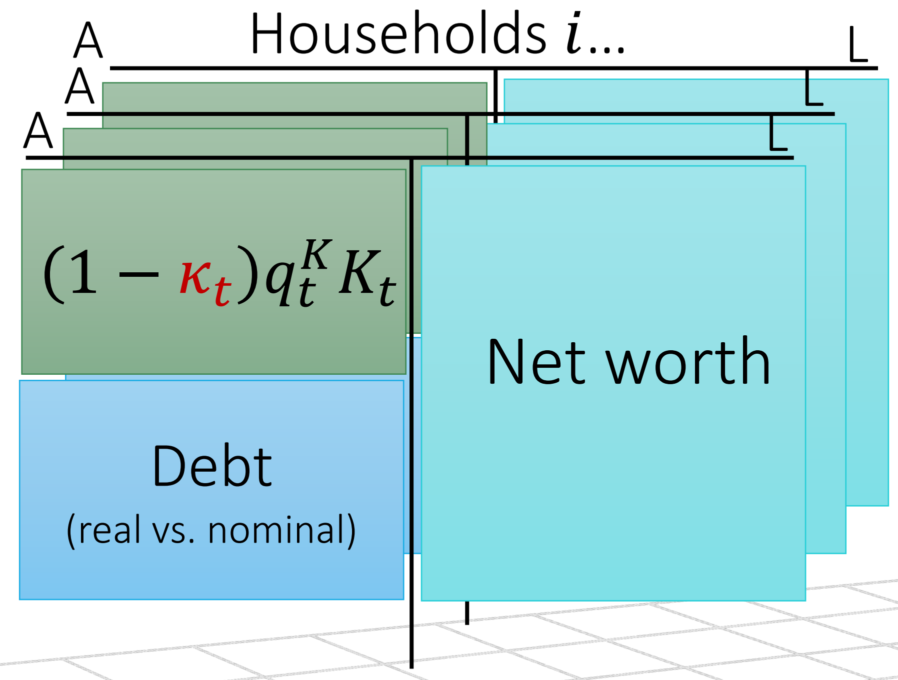
- Frictions:
  - Household cannot diversify idio risk
  - Limited risky claims issuance

Equivalence





- $a^e = a^h$
- $\tilde{\sigma}^e < \tilde{\sigma}^h$



Equivalence

- Why equivalence btw.
  Intermediaries χ-risk allocation model and
  experts κ-capital allocation model?

Poll 13: Why are both models equivalent?

a) *Since $a^e = a^h$.*
b) *Intermediary sector doesn't produce any output*
c) *Risk χ and capital allocation κ are fundamentally different.*

- Next: Contrast Real Debt with Nominal Debt/Money Model
  - solve generic model and highlight the differences

Roadmap

- Intro
- Equivalence btw experts producers and intermediaries
- Real vs. Nominal Debt
- I Theory of Money
- Policy

Model with Intermediary Sector

Intermediary sector

- Hold equity up to $\bar{\chi} \leq 1$
- Diversify idio risk to $\varphi\tilde{\sigma}$
- Consumption rate: c_t^I
- $E_0[\int_0^\infty e^{-\rho t} \log c_t^I \, dt]$

Household sector

- Output: $y_t^h = a^h k_t^h$
- Investment rate: ι_t^h

$$\frac{dk_t^{h,\tilde{\imath}}}{k_t^{h,\tilde{\imath}}} = (\Phi(\iota_t^h) - \delta^h)dt + \sigma dZ_t + \tilde{\sigma}^h d\tilde{Z}_t^{\tilde{\imath}} + d\Delta_t^{k,h,\tilde{\imath}}$$

- Consumption rate: c_t^h
- $E_0[\int_0^\infty e^{-\rho t} \log c_t^h dt]$

- Friction: Can only issue debt
  - 2 Models:
    1. Real debt issuance only (and money has no value)
    2. Nominal debt issuance
- Bond/money supply $\frac{dB_t}{B_t} = (\check{\mu}_t^B + i_t)dt + \sigma_t^B dZ_t$
- seigniorage distribution as in previous lecture (no fiscal impact – per period balanced budget)

Solving MacroModels Step-by-Step

0. Postulate aggregates, price processes & obtain return processes
1. For given C/N-ratio and SDF processes for each i *finance block*
   a. Real investment ι + Goods market clearing *(static)*
   - *Toolbox 1:* Martingale Approach, HJB vs. Stochastic Maximum Principle Approach

   b. Portfolio choice θ + Asset market clearing
      Asset allocation κ & risk allocation χ
   - *Toolbox 2:* "price-taking social planner approach" – Fisher separation theorem

   c. "Money evaluation equation" ϑ
   - *Toolbox 3:* Change in numeraire to total wealth (including SDF)
2. Evolution of state variable η (and K) *forward equation*
3. Value functions *backward equation*
   a. Value fcn. as fcn. of individual investment opportunities ω
   - *Special cases: log-utility, con… ment opportunities*

   Log-utility

   b. Separating value fcn. $V^i(n^{\tilde{i}};\eta,K)$ into $v^i(\eta)u(K)\left(n^{\tilde{i}}/n^i\right)^{1-\gamma}$
   c. Derive $\check{\rho} = C/N$-ratio and $\varsigma, \tilde{\varsigma}$ prices of risks
4. Numerical model solution
   a. Transform BSDE for separated value fcn. $v^i(\eta)$ into PDE
   b. Solve PDE via value function iteration
5. KFE: Stationary distribution, Fan charts

0. Postulate Aggregates and Processes

- $q_t^K K_t$ value of physical capital
- $q_t^B K_t$ value of nominal capital/outside money/gov. debt
  - $\wp_t := B_t / q_t^B K_t$ price level (inverse of "value of money")
- $N_t \coloneqq (q_t^K + q_t^B) K_t$ is total wealth in the economy
- $\vartheta_t := \frac{q_t^B}{q_t^K + q_t^B}$ fraction of nominal wealth

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0. Postulate in the N_t-numeraire!

  - ϑ-price process $\quad d\vartheta_t / \vartheta_t = \mu_t^\vartheta dt + \sigma_t^\vartheta dZ_t,$
  - SDF for each $\tilde{\iota}$ agent $\quad \frac{d\xi_t^{\tilde{\iota}}}{\xi_t^{\tilde{\iota}}} = -r_t^{\tilde{\iota}} dt - \varsigma_t^{\tilde{\iota}} dZ_t - \tilde{\varsigma}_t^{\tilde{\iota}} d\tilde{Z}_t^{\tilde{\iota}}$
    - Change of notation (dropped “hat”) compared to previous lectures!

0. Postulate Aggregates and Processes

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0. Postulate in the N_t-numeraire!

- ϑ-price process $\quad d\vartheta_t / \vartheta_t = \mu_t^\vartheta dt + \sigma_t^\vartheta dZ_t,$
- SDF for each $\tilde{\imath}$ agent $\quad \frac{d\xi_t^{\tilde{\imath}}}{\xi_t^{\tilde{\imath}}} = -r_t^i dt - \varsigma_t^{\tilde{\imath}} dZ_t - \tilde{\varsigma}_t^{\tilde{\imath}} d\tilde{Z}_t^{\tilde{\imath}}$
  - Change of notation (dropped "hat") compared to previous lectures!

Poll 19: Why is the drift $-r_t^i$ and not simply $-r_t^f$?

a) With only nominal debt a real risk-free rate might not be in asset span.

b) Negative drift of the SDF in N_t-numeraire is not risk-free rate.

1a. Optimal ι + Goods Market

- Use optional real investment ι and goods market clearing
- Same as in Lecture 10
- Price of physical capital

$$q_t^K = (1-\vartheta_t)\frac{1+\phi a}{(1-\vartheta_t)+\phi\rho}$$

- Price of nominal capital

$$q_t^B = \vartheta_t\frac{1+\phi a}{(1-\vartheta_t)+\phi\rho}$$

- Optimal investment rate

$$\iota_t = \frac{(1-\vartheta_t)a-\rho}{(1-\vartheta_t)+\phi\rho}$$

- Moneyless equilibrium with $q_t^B = 0 \Rightarrow \vartheta_t = 0 \Rightarrow q_t^K = \frac{1+\phi a}{1+\phi\rho}$

1b. Price-taking Planner's Allocation

- $\max_{\{\boldsymbol{\kappa}_t, \boldsymbol{\chi}_t, \tilde{\boldsymbol{\chi}}_t\}} E_t[dr_t^N(\boldsymbol{\kappa}_t)] - \boldsymbol{\varsigma}_t \boldsymbol{\sigma}(\boldsymbol{\kappa}_t, \boldsymbol{\chi}_t) - \tilde{\boldsymbol{\varsigma}}_t \tilde{\boldsymbol{\sigma}}(\boldsymbol{\kappa}_t, \tilde{\boldsymbol{\chi}}_t)$

vectors

- In our model(s):
  - $\kappa = 0$ (households manage all physical capital)
  - $\tilde{\chi}_t = \chi_t$
  - $E_t[dr_t^N(\kappa_t)] = 0$

Poll 21: Why is $E_t[dr_t^N(\kappa_t)] = 0$?

a) Because capital is not reallocated, i.e. $\kappa = 0$ all the time.

b) In the N_t-numeraire return of total wealth $dr_t^N = 0$.

1b. Price-taking Planner's Allocation

- $$\max_{\{\boldsymbol{\psi}_t,\boldsymbol{\chi}_t,\tilde{\boldsymbol{\chi}}_t\}} E_t[dr_t^N(\boldsymbol{\kappa}_t)] - \boldsymbol{\varsigma}_t\boldsymbol{\sigma}(\boldsymbol{\kappa}_t,\boldsymbol{\chi}_t) - \tilde{\boldsymbol{\varsigma}}_t\tilde{\boldsymbol{\sigma}}(\boldsymbol{\kappa}_t,\tilde{\boldsymbol{\chi}}_t)$$

  vectors

- In our model(s):
  - $\kappa = 0$ (households manage all physical capital)
  - $\tilde{\chi}_t = \chi_t$
  - $E_t[dr_t^N(\kappa_t)] = 0$
  - $\boldsymbol{\sigma} = (\chi_t\sigma_t^{xK}, (1-\chi_t)\sigma_t^{xK})$,
    - where σ_t^{xK} =Risk of the excess return of capital beyond benchmark asset
  - $\tilde{\boldsymbol{\sigma}} = (\chi_t\varphi\tilde{\sigma}, (1-\chi_t)\tilde{\sigma})$

    $\varphi < 1$

1b. Price-taking Planner's Allocation

- Minimize weighted average cost of financing

$$\min_{\chi_t \le \bar{\chi}} \left(\varsigma_t^I \chi_t + \varsigma_t^h (1-\chi_t)\right) \sigma_t^{xK} + \left(\tilde{\varsigma}_t^I \varphi \chi_t + \tilde{\varsigma}_t^h (1-\chi_t)\right) \tilde{\sigma}$$

- FOC: (equality if $\chi_t < \bar{\chi}$)

$$\varsigma_t^I \sigma_t^{xK} + \tilde{\varsigma}_t^I \varphi \tilde{\sigma} \le \varsigma_t^h \sigma_t^{xK} + \tilde{\varsigma}_t^h \tilde{\sigma}$$

- Real debt model:
  - $\sigma_t^{xK} = \sigma + \cancel{\sigma^{q^K}}$ (recall q_t^K is constant)
- Nominal debt model
  - $\sigma_t^{xK} = (-\sigma_t^{\vartheta} + \sigma_t^B)/(1-\vartheta_t)$
    - Risk of capital $\sigma + \sigma_t^{q^K} + \vartheta_t \sigma_t^B/(1-\vartheta_t) - \sigma_t^N$ (in N_t-numeraire)
    - Risk of bond/money $\sigma + \sigma_t^{q^B} + \sigma_t^B - \sigma_t^N$ (in N_t-numeraire)

"Benchmark Asset Evaluation Equation"

- In N_t-numeraire η_t^i takes on role of sector net worth N_t^i
- Return on individual agent's net worth return (in N_t-numeraire)

$$\underbrace{\frac{d\eta_t^i}{\eta_t^i}}_{sector\ share} + \underbrace{\frac{d\tilde{\eta}_t^{\tilde{\imath}}}{\tilde{\eta}_t^{\tilde{\imath}}}}_{\substack{within \\ sector\ share}} + \underbrace{\rho dt}_{consumption} \quad dt$$

- Martingale condition relative to benchmark asset is

$$\mu_t^{\eta^i} + \rho - r_t^{bm} = \varsigma_t^i \left(\sigma_t^{\eta^i} - \sigma_t^{bm}\right) + \tilde{\varsigma}_t^i \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{\imath}}}$$

- Take η_t^i-weighted sum (across 2 types $i = I, h$ here)

$$\rho - r_t^{bm} = \eta_t \varsigma_t^I \left(\sigma_t^\eta - \sigma_t^{bm}\right) + (1-\eta_t)\varsigma_t^h \left(-\tfrac{\eta_t}{1-\eta_t}\sigma_t^\eta - \sigma_t^{bm}\right) + \eta_t \tilde{\varsigma}_t^I \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}} + (1-\eta_t)\tilde{\varsigma}_t^h \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}$$

- For log utility: $\varsigma_t^I = \sigma_t^\eta, \varsigma_t^h = -\frac{\eta_t}{1-\eta_t}\sigma_t^\eta, \tilde{\varsigma}_t^I = \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}}, \tilde{\varsigma}_t^h = \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}$

$$\rho - r_t^{bm} = \eta_t \left(\sigma_t^\eta\right)^2 + (1-\eta_t)\left(-\tfrac{\eta_t}{1-\eta_t}\sigma_t^\eta\right)^2 + \eta_t\left(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}}\right)^2 + (1-\eta_t)\left(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}\right)^2$$

"Benchmark Asset Evaluation Equation"

- Real debt = benchmark asset bm
  - Redundant equation for allocation just useful for deriving risk-free rate in c-numeraire r_t^f (expressed in N_t-numeraire)

- Nominal debt/money = benchmark asset bm
  - Money evaluation equation (bubble)
  - Replace $r_t^{bm} = \mu_t^{\vartheta/B} := \mu_t^{\vartheta} - \mu_t^{B} - \sigma_t^{B}(\sigma_t^{\vartheta} - \sigma_t^{B})$ (and $\sigma_t^{bm} = \sigma_t^{\vartheta}$)

$$\underbrace{\rho - \mu_t^{\vartheta/B}}_{\text{excess return of } N_t} = \underbrace{\eta_t(\sigma_t^{\eta})^2 + (1-\eta_t)\left(-\frac{\eta_t}{1-\eta_t}\sigma_t^{\eta}\right)^2 + \eta_t\left(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}}\right)^2 + (1-\eta_t)\left(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}\right)^2}_{\text{(required) "net worth weighted risk premium" (for holding risk in excess of money risk)}}$$

excess return = of N_t

(required) "net worth weighted risk premium"
(for holding risk in excess of money risk)

"Benchmark Asset Evaluation Equation" (FTPL Equation)

- Nominal debt/money = benchmark asset bm
  - Money evaluation equation
  - Replace $r_t^{bm} = \mu_t^{\vartheta/B} := \mu_t^{\vartheta} - \mu_t^B - \sigma_t^B(\sigma_t^{\vartheta} - \sigma_t^B)$ (and $\sigma_t^{bm} = \sigma_t^{\vartheta}$)

$$\rho - \mu_t^{\vartheta/B} = \eta_t(\sigma_t^{\eta})^2 + (1-\eta_t)\left(-\frac{\eta_t}{1-\eta_t}\sigma_t^{\eta}\right)^2 + \eta_t\left(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}}\right)^2 + (1-\eta_t)\left(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}\right)^2$$

- Integrate

$$\vartheta_t = E_t\left[\int_t^{\infty} e^{-\rho(s-t)}\left(\eta_s\left(\sigma_s^{\eta^I}\right)^2 + (1-\eta_s)\left(\sigma_s^{\eta^h}\right)^2 + \eta_s\left(\tilde{\sigma}_s^{\tilde{\eta}^{\tilde{I}}}\right)^2 + (1-\eta_s)\left(\tilde{\sigma}_s^{\tilde{\eta}^{\tilde{h}}}\right)^2 - \check{\mu}_s^B - \sigma_s^B(\sigma_s^{\vartheta} - \sigma_s^B)\right)\vartheta_s\, ds\right]$$

Because μ_t^{ϑ} is the "geometric drift"

2. η-Evolution: Drift μ_t^η (in N_t-numeraire)

- Take difference from two earlier equations

$$\mu_t^\eta + \rho - r_t^{bm} = \varsigma_t^I(\sigma_t^\eta - \sigma_t^{bm}) + \tilde{\varsigma}_t^I \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}}$$

$$\rho - r_t^{bm} = \eta_t \varsigma_t^I(\sigma_t^\eta - \sigma_t^{bm}) + (1-\eta_t)\varsigma_t^h\left(-\frac{\eta_t}{1-\eta_t}\sigma_t^\eta - \sigma_t^{bm}\right) + \eta_t \tilde{\varsigma}_t^I \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}} + (1-\eta_t)\tilde{\varsigma}_t^h \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}$$

- $\mu_t^\eta = (1-\eta_t)\left[\varsigma_t^I(\sigma_t^\eta - \sigma_t^{bm}) - \varsigma_t^h\left(-\frac{\eta_t}{1-\eta_t}\sigma_t^\eta - \sigma_t^{bm}\right) + \tilde{\varsigma}_t^I \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}} - \tilde{\varsigma}_t^h \tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}\right]$

- Real Debt
  - $\sigma_t^{bm} = -\sigma_t^N = -\sigma$ (Recall $\sigma_t^q = 0$)

- Nominal Debt/Money
  - $\sigma_t^{bm} = \sigma_t^\vartheta - \sigma^B$

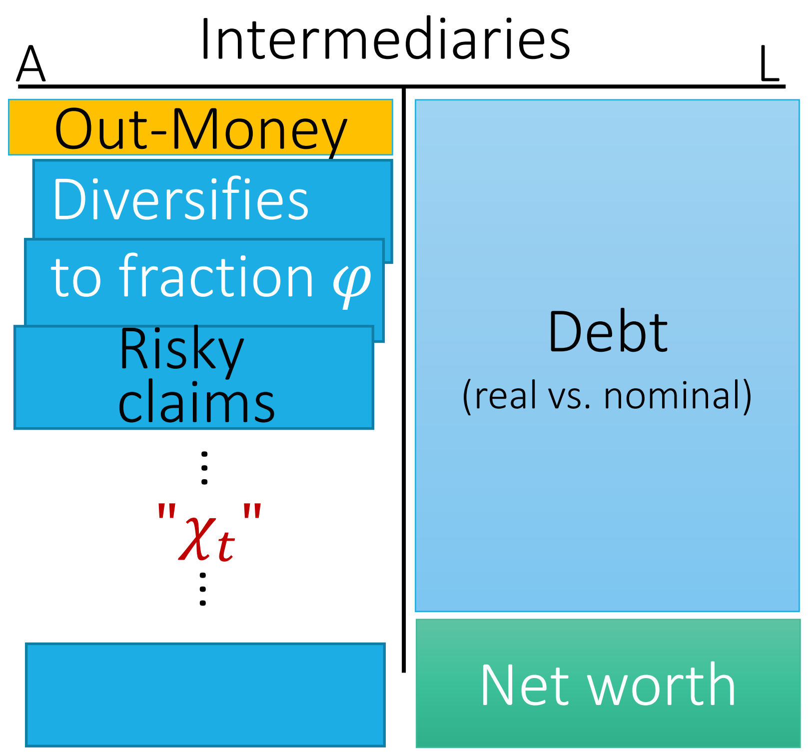
2. η-Evolution: η-Aggregate Risk

- $\sigma_t^\eta = \quad \sigma_t^{r^{bm}} + (1-\theta_t^I)\left(\sigma_t^{r^K} - \sigma_t^{r^{bm}}\right)$
  - Where portfolio share $1-\theta_t^I = \frac{\chi_t}{\eta_t}(1-\vartheta_t)$
- Real Debt
  - Note $\sigma_t^{r^K} = 0$ given $N_t = q_t^K K_t$-numeraire
  - $\sigma_t^\eta = \frac{\chi_t - \eta_t}{\eta_t}\sigma$ (recall $\vartheta_t = 0$)
  - No amplification since q^K is constant
  - Imperfect risk-sharing for $\chi_t \neq \eta_t$

Inflation Risk allows Perfect Risk Sharing

- Nominal Debt
  - Note $\sigma_t^{r^K} = \sigma_t^{1-\vartheta} = -\frac{\vartheta_t}{1-\vartheta_t}\sigma_t^{\vartheta}$
  - $\sigma_t^{\eta} = \sigma_t^{\vartheta} - \sigma^B + \frac{\chi_t}{\eta_t}(1-\vartheta_t)\left(-\frac{\vartheta_t}{1-\vartheta_t}\sigma_t^{\vartheta} - \sigma_t^{\vartheta} + \sigma^B\right)$
  - Use $\sigma_t^{\vartheta} = \frac{\vartheta'(\eta_t)}{\vartheta(\eta_t)}\eta_t\sigma_t^{\eta}$ and solve for $\eta_t\sigma_t^{\eta}$ yields

$$\eta_t\sigma_t^{\eta} = \frac{(\chi_t-\eta_t)\sigma_t^B}{1-\frac{\chi_t-\eta_t}{\eta_t}\left(\frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)}\right)}$$



- Intermediaries' balance sheet perfectly hedges agg. risk for $\sigma^B = 0$!

- Proposition: Aggregate risk is perfectly shared for $\sigma^B = 0$!
  - Via inflation risk
  - Stable inflation (targeting) would ruin risk-sharing
    - Example: Brexit uncertainty. Use inflation reaction to share risks within UK

2. Within Type $\tilde{\eta}$-Risk

- Within intermediary sector

$$\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}} = (1-\theta_t^I)\varphi\tilde{\sigma} = \frac{\chi_t}{\eta_t}(1-\vartheta_t)\varphi\tilde{\sigma}$$

- Within household sector

$$\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}} = \left(1-\theta_t^h\right)\tilde{\sigma} = \frac{1-\chi_t}{1-\eta_t}(1-\vartheta_t)\tilde{\sigma}$$

Solving for χ_t

- Recall planner condition: (equality if $\chi_t < \bar{\chi}$)

$$\varsigma_t^I \sigma_t^{xK} + \tilde{\varsigma}_t^I \phi \tilde{\sigma} \le \varsigma_t^h \sigma_t^{xK} + \tilde{\varsigma}_t^h \tilde{\sigma}$$

Price of Risks	Real Debt	Nominal Debt with $\sigma^B = 0$
$\varsigma_t^I = \sigma_t^\eta$	$= \frac{\chi_t - \eta_t}{\eta_t} \sigma$	$= 0$
$\varsigma_t^h = -\frac{\eta_t}{1-\eta_t} \sigma_t^\eta$	$= \frac{\chi_t - \eta_t}{1-\eta_t} \sigma$	$= 0$
$\tilde{\varsigma}_t^I = \frac{\chi_t}{\eta_t}(1-\vartheta_t)\varphi\tilde{\sigma}$	$= \frac{\chi_t}{\eta_t} \varphi \tilde{\sigma}$	$= \frac{\chi_t}{\eta_t}(1-\vartheta_t)\varphi\tilde{\sigma}$
$\tilde{\varsigma}_t^h = \frac{1-\chi_t}{1-\eta_t}(1-\vartheta_t)\tilde{\sigma}$	$= \frac{1-\chi_t}{1-\eta_t}\tilde{\sigma}$	$= \frac{1-\chi_t}{1-\eta_t}(1-\vartheta_t)\tilde{\sigma}$

Solving for χ_t

- Real debt

$$\chi_t = \min\left\{\frac{\eta_t(\sigma^2 + \tilde{\sigma}^2)}{\sigma^2 + [(1-\eta_t)\phi^2 + \eta_t]\tilde{\sigma}^2}, \bar{\chi}\right\}$$

- Nominal debt

$$\chi_t = \min\left\{\frac{\eta_t}{(1-\eta_t)\phi^2 + \eta_t}, \bar{\chi}\right\}$$

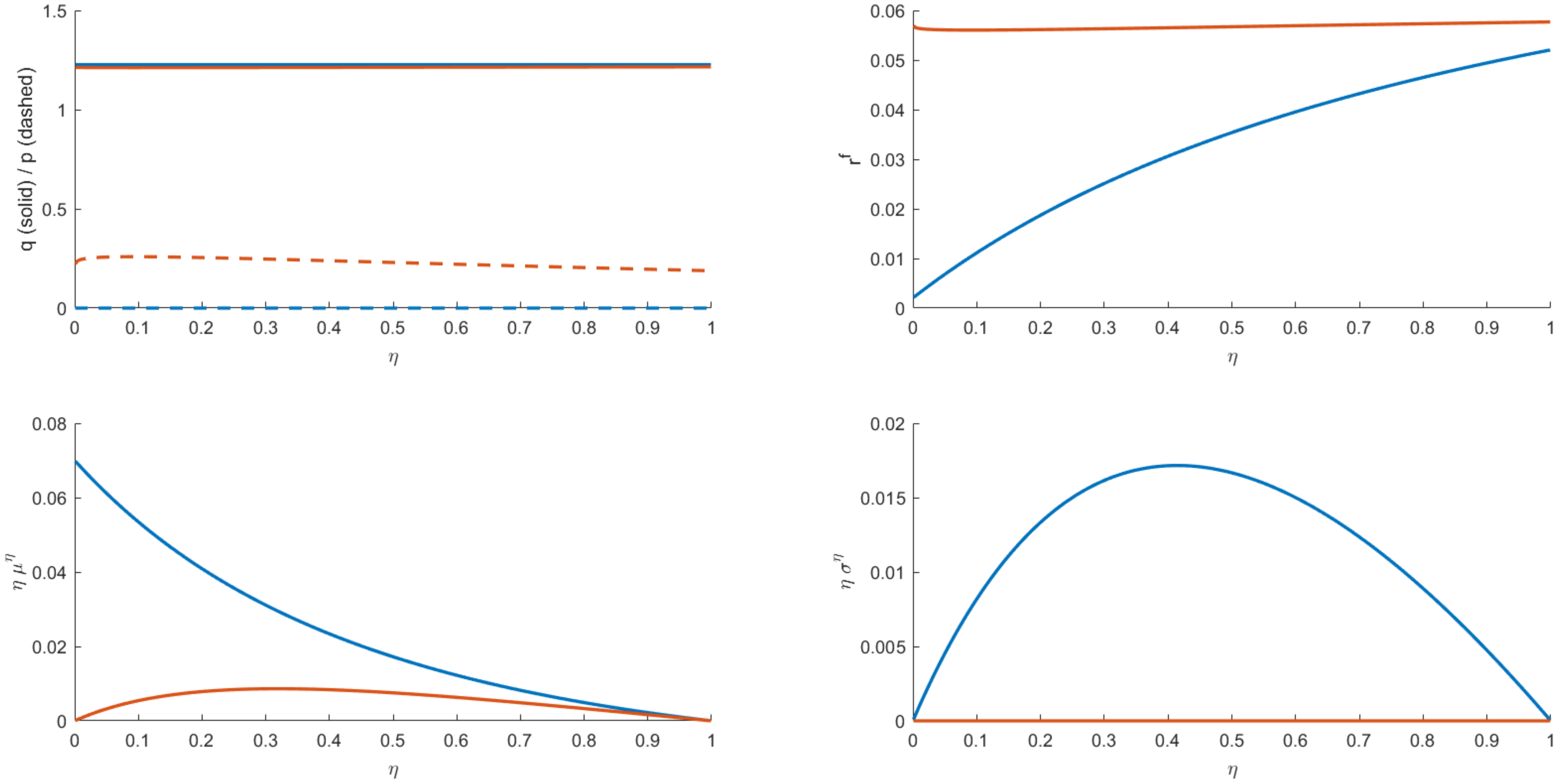
	Real Debt	Nominal Debt with $\sigma^B = 0$
χ_t	$\min\left\{\frac{\eta_t(\sigma^2+\tilde{\sigma}^2)}{\sigma^2+[(1-\eta_t)\varphi^2+\eta_t]\tilde{\sigma}^2}, \bar{\chi}\right\}$	$\min\left\{\frac{\eta_t}{(1-\eta_t)\varphi^2+\eta_t}, \bar{\chi}\right\}$
μ_t^η	$\frac{\chi_t-\eta_t}{\eta_t}\frac{\chi_t-2\chi_t\eta_t+\eta_t^2}{\eta_t(1-\eta_t)}\sigma^2 + $ $+(1-\eta_t)\left(\left(\frac{\chi_t}{\eta_t}\right)^2\varphi^2-\left(\frac{1-\chi_t}{1-\eta_t}\right)^2\right)\tilde{\sigma}^2$	$(1-\eta_t)(1-\vartheta)^2\left(\left(\frac{\chi_t}{\eta_t}\right)^2\varphi^2-\left(\frac{1-\chi_t}{1-\eta_t}\right)^2\right)\tilde{\sigma}^2$
σ_t^η	$\frac{\chi_t-\eta_t}{\eta_t}\sigma$	0
q_t^K	$\frac{1+\phi a}{1+\phi\rho}$	$(1-\vartheta_t)\frac{1+\phi a}{(1-\vartheta_t)+\phi\rho}$
q_t^B	0	$\vartheta_t\frac{1+\phi a}{(1-\vartheta_t)+\phi\rho}$
ϑ_t	0	$\rho-\mu_t^\vartheta+\mu_t^B$ $=(1-\vartheta_t)^2\left(\eta_t\frac{\chi_t^2\varphi^2}{\eta_t^2}-(1-\eta_t)\frac{(1-\chi_t)^2}{(1-\eta_t)^2}\right)\tilde{\sigma}^2$
ι_t	$\frac{a-\rho}{1+\phi\rho}$	$\frac{(1-\vartheta_t)a-\rho}{(1-\vartheta_t)+\phi\rho}$

Example: Nominal Debt/Money with $\bar{\chi} = 1$

- $a = .15, \rho = .03, \sigma = .1, \phi = 2, \delta = .03, \tilde{\sigma}^e = .2, \tilde{\sigma}^h = .3,\ \varphi =.,\bar{\chi} = 1$

Blue: real debt model

Red: nominal model

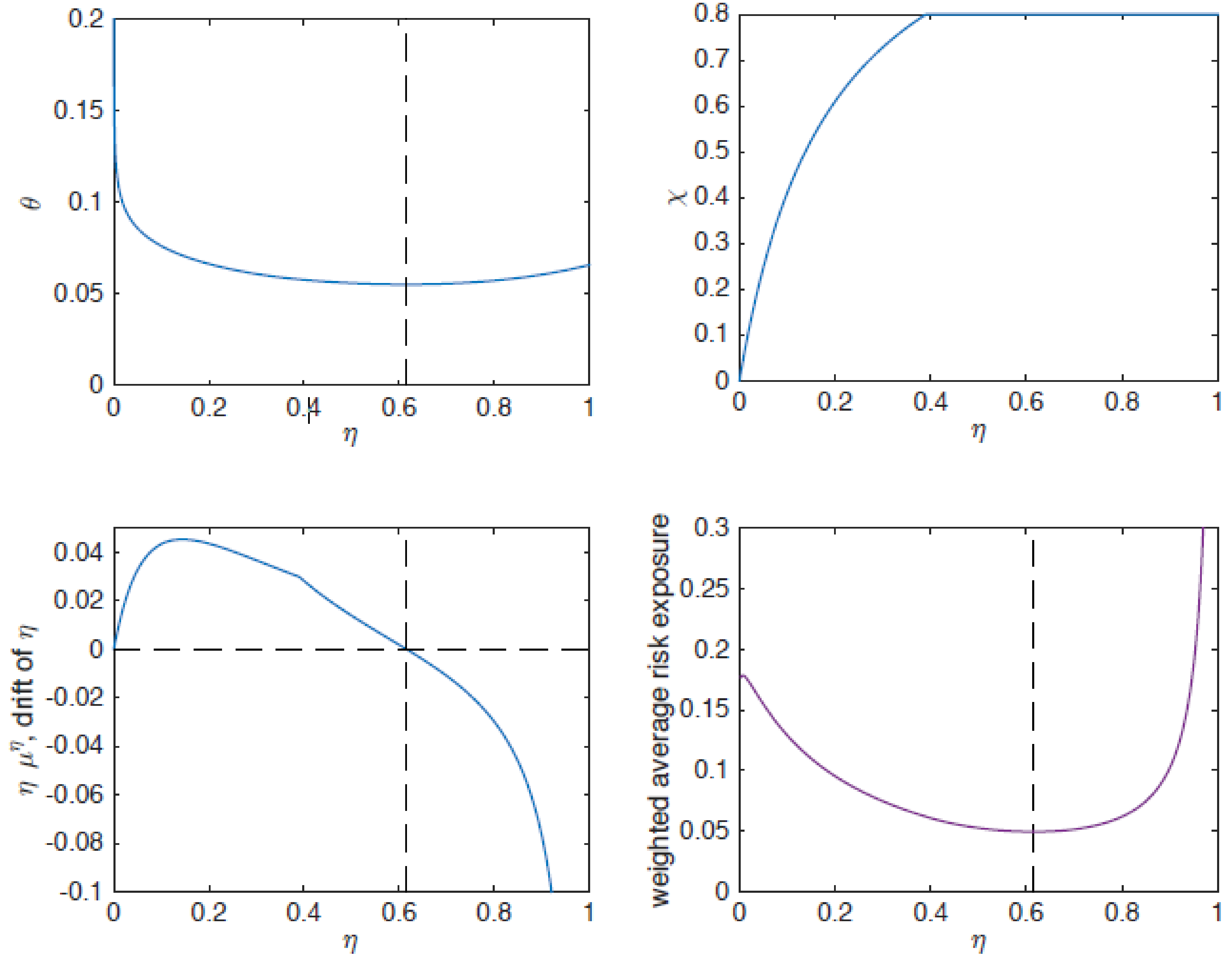


Contrasting Real with Nominal Debt

- Real debt model:
  - Changes in η are absorbed by risk-free rate moves
  - Aggregate risk
  - $\iota(\eta)$ and $q^K(\eta)$ are constant
- Nominal debt/money model
  - Inflation risk completes markets
  - Perfect aggregate risk sharing
    - Banks balance sheet is perfectly hedged!!!
  - Risk-free rate is high
  - $\iota(\eta)$ and $q^K(\eta)$ are functions of η

Example: Nominal Debt with Limit on Risk Offloading

- $\rho = .05, \kappa = 2, \tilde{\sigma} = .5, \phi = .4, \bar{\chi} = .8$



Combining Nominal & Real Debt

- Adding real debt to money model does not alter the equilibrium, since
  - Markets are complete w.r.t. to aggregate risk (perfect aggregate risk sharing)
  - Markets are incomplete w.r.t. to idiosyncratic risk only
  - Real debt is a redundant asset

- Note: Result relies on absence of price stickiness

- *Both Settings: Real Debt and Money/Nominal Debt converge in the long-run to the "I Theory without I" steady state model of Lecture 10 if* $\bar{\chi} = 1$.

ϑ Minimized at Stochastic Steady State

- Claim: $\vartheta(\eta)$ and average idiosyncratic risk exposure, $X(\eta)$, is minimized at the stochastic steady state of η.
  - Intuition: at steady state both sectors earn same risk premia + idiosyncratic seems well spread out … less desire to hold money to self-insure

- With $\sigma_t^B = 0 \; \forall t$
  - $\sigma_t^\eta = 0$, (perfect risk sharing with nominal debt)
  - $\mu_t^\eta = (\tilde{\sigma}_t^I)^2 - \eta_t(\tilde{\sigma}_t^I)^2 - (1-\eta_t)\left(\tilde{\sigma}_t^h\right)^2 = (1-\eta_t)(1-\vartheta_t)^2 \underbrace{\left(\frac{\chi_t^2\phi^2}{\eta_t^2} - \frac{(1-\chi_t)^2}{(1-\eta_t)^2}\right)\tilde{\sigma}^2}_{-dX/d\eta}$

  for steady state s.t. $\chi = \bar{\chi}$

- Money valuation equation

$$\rho - \mu_t^{\vartheta/B} = \underbrace{(1-\vartheta_t)^2 \overbrace{\left(\eta_t \frac{\chi_t^2\varphi^2}{\eta_t^2} - (1-\eta_t)\frac{(1-\chi_t)^2}{(1-\eta_t)^2}\right)\tilde{\sigma}^2}^{X(\eta):=}}_{\eta_t(\tilde{\sigma}_t^I)^2 + (1-\eta_t)(\tilde{\sigma}_t^h)^2}$$

where $\chi_t = \min\left(\frac{\eta_t}{\eta_t + (1-\eta_t)\phi^2}, \bar{\chi}\right)$

Cashless/Bondless Limit with Jump

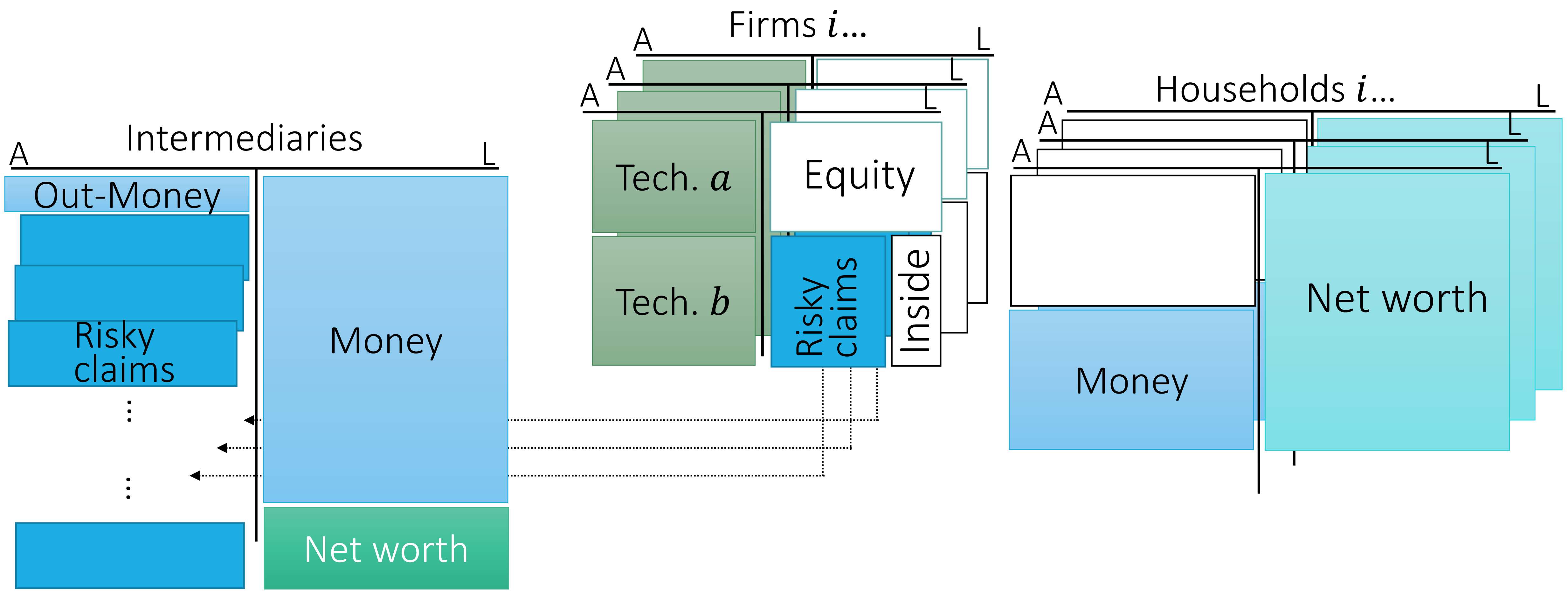
- Removing cash/nominal gov. bonds (comparative static)
  - $B > 0$ vs. $B = 0$
    - Price flexibility ⇒ Neutrality of money
  - Discontinuity at $\lim_{B \to 0}$
  - Remark:
    - Different from Woodford (2003) – medium of exchange role of money
      - CIA becomes relevant for fewer and fewer goods
- Inflation on nominal claims (bond/cash)
  - Change μ^B and subsidize capital
  - Continuous process

I Theory of Money

- Aim: intermediary sector is not perfectly hedged
- Idiosyncratic risk that HH have to bear is time-varying
- Needed: Intermediaries' aggregate risk $\neq$ aggregate risk of economy
- One way to model: 2 technologies a and b

Technology	a	b
Capital share (Leontieff)	$1-\bar{\kappa}$	$\bar{\kappa}$
Risk	$\frac{dk_t}{k_t} = (\cdot)dt + \sigma^a dZ_t + \tilde{\sigma} d\tilde{Z}_t$	$\frac{dk_t}{k_t} = (\cdot)dt + \sigma^b dZ_t + \tilde{\sigma} d\tilde{Z}_t$
Intermediaries	No	Yes, reduce $\widetilde{\sigma}$ to $\varphi\widetilde{\sigma}$
Excess risk $\sigma_t^{xK^a}, \sigma_t^{xK^b}$	$-\bar{\kappa}(\sigma^b - \sigma^a) - \frac{\sigma^\vartheta - \sigma^B}{1-\vartheta}$	$(1-\bar{\kappa})\underbrace{(\sigma^b - \sigma^a)}_{\sigma :=} - \frac{\sigma^\vartheta - \sigma^B}{1-\vartheta}$

I Theory: Balance Sheets



- Frictions:
  - Household cannot diversify idio risk
  - Limited risky claims issuance
  - Only nominal deposits

Overview Slide that Explains the Role of Each Model Ingredient

- $\bar{\chi}$ -- avoid degenerated distribution (households dying out)
- φ
  - if $\varphi = 1$ intermediaries would die out,
  - if $\varphi = 0$ don't earn risk premium (except for aggregate risk)
- $\sigma^b > \sigma^a$ – avoid perfect hedging for intermediaries
  - (except $\sigma^B \neq 0$ – for example risk-free asset is in zero net supply)
    (like AER paper/handbook chapter)
- Fraction $\bar{\kappa}$ of K has aggregate risk of $\sigma = \sigma^b - \sigma^a$,
  rest has risk of zero (it's exogenous) (allocation does not
  determine total risk in aggregate economy)
  (To keep it clean (taste choice): price-taking planner's
  choice is less involved)

- …

1b. Price-taking Planner's Allocation

- Minimize weighted average cost of financing

$$\min_{\chi_t \le \bar{\chi}} (1-\bar{\kappa})\varsigma_t^h \sigma_t^{xK^a} + \left(\varsigma_t^I \chi_t + \varsigma_t^h(\bar{\kappa}-\chi_t)\right)\sigma_t^{xK^b} + \left(\tilde{\varsigma}_t^I \varphi \chi_t + \tilde{\varsigma}_t^h (1-\chi_t)\right)\tilde{\sigma}$$

- FOC: (equality if $\chi_t < \bar{\chi}$)

$$\varsigma_t^I \sigma_t^{xK^b} + \tilde{\varsigma}_t^I \varphi \tilde{\sigma} \le \varsigma_t^h \sigma_t^{xK^b} + \tilde{\varsigma}_t^h \tilde{\sigma}$$

  - $\sigma_t^{xK^b} = (1-\bar{\kappa})\sigma - \frac{\sigma^\vartheta - \sigma^B}{1-\vartheta}$
  - Price of risk with log-utility in total wealth numeraire:

	Intermediaries	Households
Aggregate risk:	$\varsigma_t^I = \sigma_t^\eta$	$\varsigma_t^h = -\eta_t \sigma_t^\eta/(1-\eta_t)$
Idiosyncratic risk	$\tilde{\varsigma}_t^I = (1-\vartheta_t)\frac{\chi_t}{\eta_t}\varphi\tilde{\sigma}$	$\tilde{\varsigma}_t^h = (1-\vartheta_t)\frac{(1-\chi_t)}{(1-\eta_t)}\tilde{\sigma}$

$$\sigma_t^\eta \left((1-\bar{\kappa})\sigma - \frac{\sigma_t^\vartheta - \sigma_t^B}{1-\vartheta_t}\right) + \left[(1-\vartheta_t)\frac{\chi_t}{\eta_t}\varphi\tilde{\sigma}\right]\varphi\tilde{\sigma} \le \frac{-\eta_t \sigma_t^\eta}{1-\eta_t}\left((1-\bar{\kappa})\sigma - \frac{\sigma_t^\vartheta - \sigma_t^B}{1-\vartheta_t}\right) + \left[(1-\vartheta_t)\frac{(1-\chi_t)}{(1-\eta_t)}\tilde{\sigma}\right]\tilde{\sigma}$$

1c. Money Evaluation + 2. η-Drift

- As before in money/nominal debt model

- Money evaluation

$$\rho - \mu_t^{\vartheta/B} = \eta_t \left(\left(\sigma_t^{\eta}\right)^2 + \left(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}}\right)^2 \right) + (1-\eta_t) \left(\left(\frac{\eta_t \sigma_t^{\eta}}{1-\eta_t}\right)^2 + \left(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}\right)^2 \right)$$

- η-drift

$$\mu_t^{\eta} = (1-\eta_t) \left(\left(\sigma_t^{\eta}\right)^2 + \left(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{I}}}\right)^2 - \left(\frac{\eta_t \sigma_t^{\eta}}{1-\eta_t}\right)^2 - \left(\tilde{\sigma}_t^{\tilde{\eta}^{\tilde{h}}}\right)^2 \right) - \sigma_t^{\eta} \underbrace{\sigma_t^{\vartheta/B}}_{\sigma_t^{\vartheta} - \sigma^B}$$

η-Volatility and Amplification

$\sigma_t^\eta = \quad \sigma_t^{r^B} + (1-\theta_t^I)\sigma_t^{xK^b}$

- Where portfolio share $1-\theta_t^I = \frac{\chi_t}{\eta_t}(1-\vartheta_t)$

$$\sigma_t^\eta = \sigma_t^\vartheta - \sigma^B + \frac{\chi_t(1-\vartheta_t)}{\eta_t}\left((1-\bar{\kappa})\sigma - \frac{\sigma_t^\vartheta - \sigma^B}{1-\vartheta_t}\right)$$

$\eta_t\sigma_t^\eta = (1-\vartheta_t)\chi_t(1-\bar{\kappa})\sigma$ if $\sigma^B = \sigma^\vartheta$

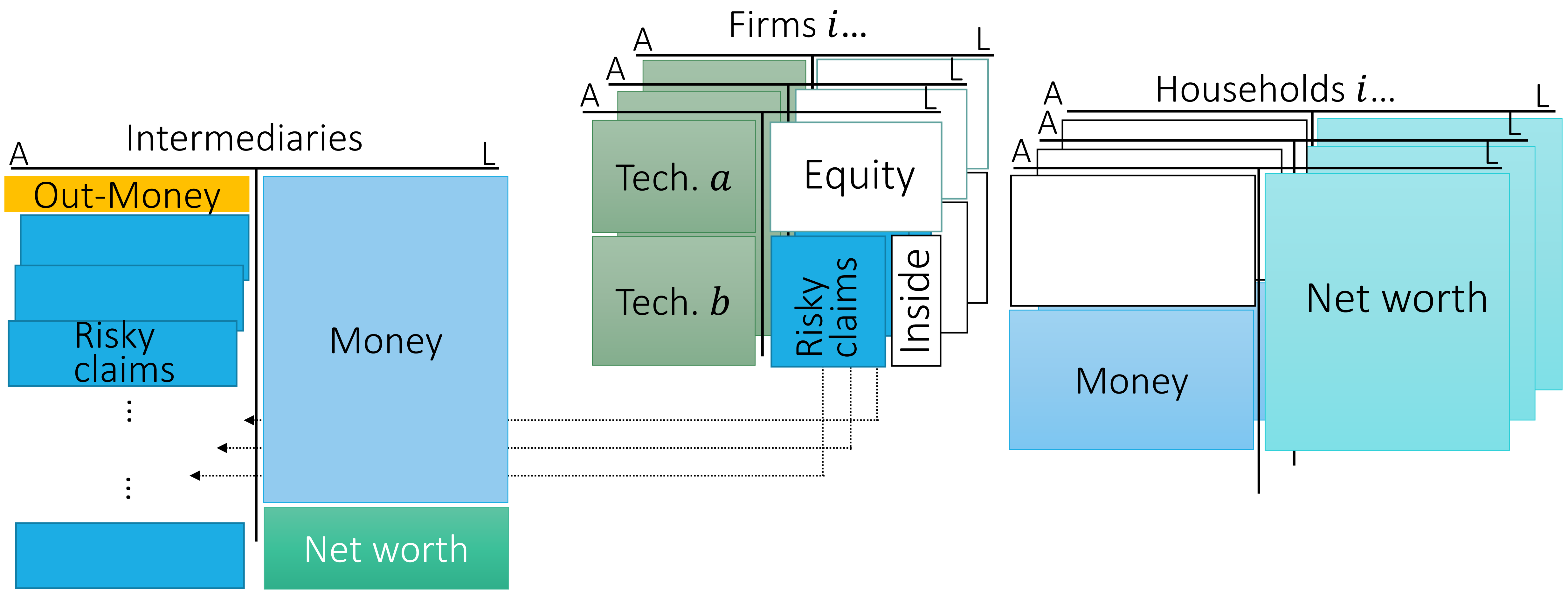
Policy removes endog. amplification

$$\Rightarrow \eta_t\sigma_t^\eta = \frac{(1-\vartheta_t)\chi_t(1-\bar{\kappa})\sigma + (\chi_t-\eta_t)\sigma^B}{1-\frac{\chi_t-\eta_t}{\eta_t}\left(\frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)}\right)}$$

- Note that $\frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)} = (1-\vartheta_t)\left(\frac{q^{K'}(\eta_t)\eta_t}{q^K(\eta_t)} + \frac{-q^{B'}(\eta_t)\eta_t}{q^B(\eta_t)}\right)$

Liquidity Spiral Disinflationary Spiral

I Theory: Balance Sheets



- Frictions:
  - Household cannot diversify idio risk
  - Limited risky claims issuance
  - Only nominal deposits

Consequences of a Shock in 4 Steps

1. **Shock:** destruction of some capital
   - % loss in intermediaries net worth > % loss in assets
   - Leverage shoots up
   - Intermediaries %-loss > Household %-losses
     - η-derivative shifts losses to intermediaries
2. **Response:** shrink balance sheet / delever
   - For given prices no impact

Paradox of Prudence

3. Asset side: asset price q^K shrinks Liquidity spiral
   - Further losses, leverage ↑, further deleveraging

4a. Liability side: money supply declines
value of money q^B rises

4b. Households' money demand rises
   - HH face more idiosyncratic risk (can't diversify)

Disinflationary spiral

Policy

- Fiscal policy
- Monetary policy without fiscal implications
- Macroprudential policy

Fiscal policy

- Includes monetary policy that has fiscal implications
- …

Monetary Policy

- No fiscal implications, no seigniorage $\tau^{i,\tilde{i}} = 0 \ \forall i, \tilde{i}$
  - Any seigniorage is paid out to government debt/money holders in form of interest
- Introducing interest rates on bond/reserves i_t.

$$\mathrm{d}r_t^B = i_t \mathrm{d}t + \frac{\mathrm{d}(1/P_t)}{1/P_t} = i_t \mathrm{d}t + \frac{\mathrm{d}(q_t^B K_t / B_t)}{q_t^B K_t / B_t}$$

$$= \left\{ i_t + \Phi(\iota_t) - \delta + \mu_t^{q^B} - \left[\mu_t^B + (\sigma_t^{q^B} - \sigma_t^B)\sigma_t^B \right] \right\} \mathrm{d}t + (\sigma_t^{q^B} - \sigma_t^B) \mathrm{d}Z_t^{\tilde{\sigma}}.$$

To study monetary policy *without* fiscal implications, we let $\sigma_t^B = 0$, so

$$\mathrm{d}r_t^B = \left\{ i_t - \mu_t^B + \Phi(\iota_t) - \delta + \mu_t^{q^B} \right\} \mathrm{d}t + \sigma_t^{q^B} \mathrm{d}Z_t^{\tilde{\sigma}}.$$

Monetary Policy: Super-neutrality

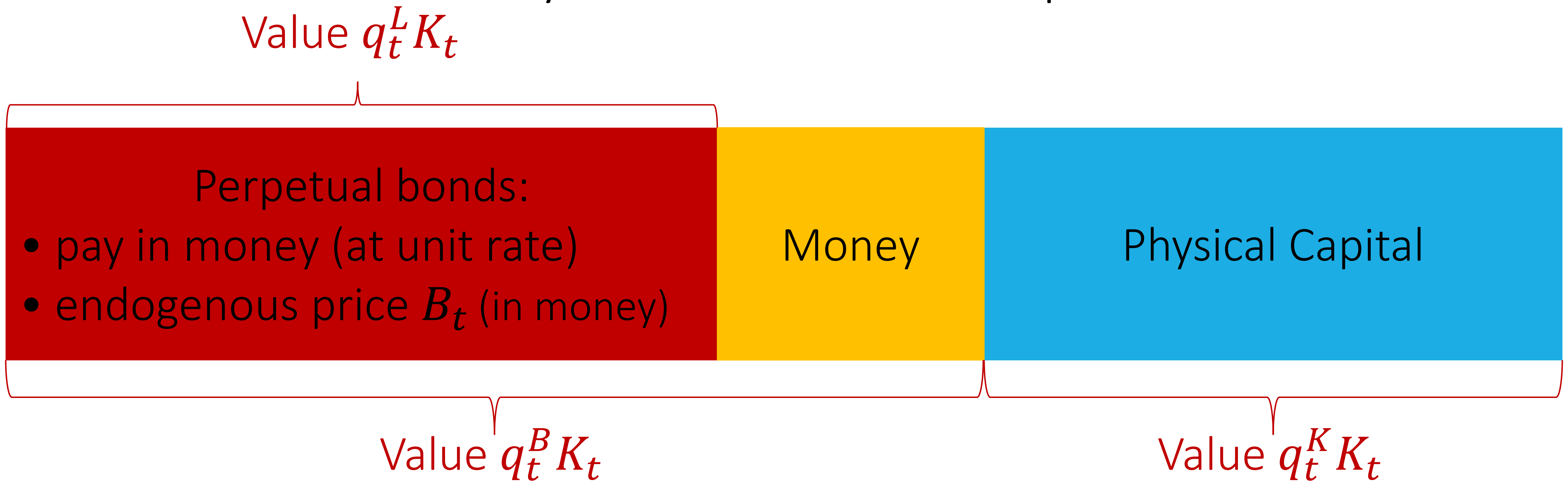
- If interest paid on bond holdings is simply financed by issuing new bonds (issuing money), then money is
  - Neutral
  - Super-neutral
  - $\frac{dB_t}{B_t} = i_t dt$

Recall $\breve{\mu}^B$

- Fisher equation
  - $dr_t^B = i_t dt - d\pi_t$

Introducing Long-term Gov. Bond

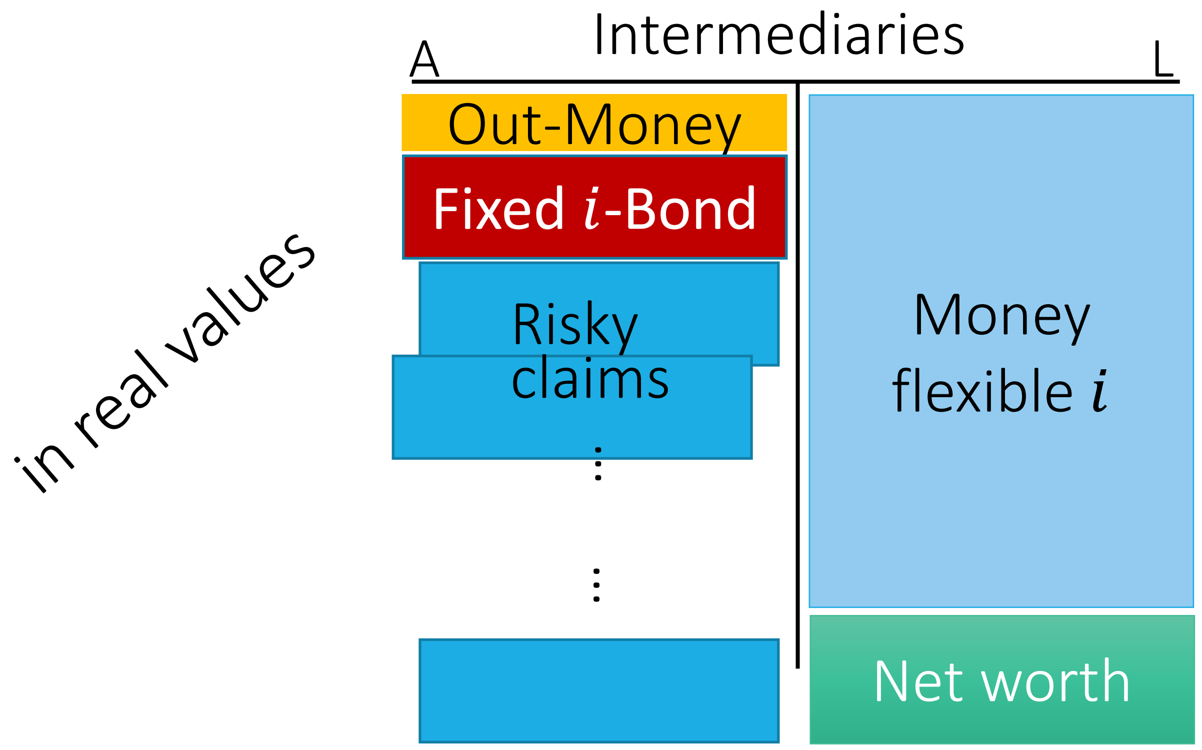
- Introduce long-term (perpetual) bond
  - No default … held by intermediaries in equilibrium



  - Value of long-term fixed i-bond is endogenous

$$dP_t^L / P^L{}_t = \mu_t^{P^L} dt + \sigma_t^{P^L} dZ_t$$

Redistributive MoPo: Ex-post perspective



- Adverse shock $\rightarrow$ value of risky claims drops
- Monetary policy
  - Interest rate cut $\Rightarrow$ long-term bond price ↑
  - Asset purchase $\Rightarrow$ asset price ↑
  - $\Rightarrow$ "stealth recapitalization" - redistributive
  - $\Rightarrow$ risk premia ↓
- Liquidity & Deflationary Spirals are mitigated

"stealth recapitalization"

Introducing long-term bonds

- Long-term bond
  - yields fixed coupon interest rate on face value $F^{(i,m)}$
  - Matures at random time with arrival rate $1/m$
  - Nominal price of the bond $P_t^{B(i,m)}$
  - Nominal value of all bonds outstanding of a certain maturity

$$B_t^{(m)} = P_t^{B(i,m)} F^{(i,m)}$$

  - Nominal value of all bonds $B_t = \sum_m B_t^{(m)}$
- Special bonds
  - Reserves: $B_t^{(0)}$ and note $P_t^{B(0)} = 1$
  - Consol bond: $B_t^{(\infty)}$

Debt evolution w/o fiscal implications

- $dB_t^{(0)} = i_t B_t^{(0)} dt$
$$+ \sum_{(i,m)} \left[\left(i + \frac{1}{m} \right) F_t^{(i,m)} dt - \frac{B_t^{(i,m)}}{F_t^{(i,m)}} \left(dF_t^{i,m} + \frac{1}{m} F_t^{(i,m)} dt \right) \right]$$

- Money $B_t^{(0)}$ is different since it pays floating interest rate

- If we have only reserves and consol bond, then

$$dB_t^{(0)} + \frac{B_t^{(i,\infty)}}{F_t^{(i,\infty)}} dF_t^{i,\infty} = i_t B_t^{(0)} dt + i F_t^{(i,\infty)} dt$$

$$\boxed{dM_t + P_t^L dF_t^L = i_t M_t dt + i^L F_t^L dt}$$

New notation:

$B_t^{(0)} = M_t$

$F_t^{(i,\infty)} = F_t^L$

- Define fraction of value of bonds that are not in short-term reserves
- $\vartheta_t^L = \frac{P_t^L F_t^L}{B_t}$ ← All gov. bonds together

- Let's postulate the price of a single long-term consol bond
$$\frac{dP_t^L}{P_t^L} = \mu_t^{P^L} dt + \sigma_t^{P^L} dZ_t$$
- In the total net worth numeraire the
- $E_t[dr_t^L - dr_t^M] = \sigma_t^{P^L} \sigma_t^{\eta}$ (*for now* assuming that only intermediaries find it worthwhile to hold consul bonds)
- $\sigma_t^n = \cdots (in\ net\ worth\ numeraire)$ (5.3)
- $dr_t^L = dr_t^M + \sigma_t^{P^L} \sigma_t^{\eta} dt + \sigma_t^{P^L} dZ_t$

- Return of total bond portfolio (in total net worth numeraire)
- $dr_t^B = \mu_t^\vartheta dt + \sigma_t^\vartheta dZ_t$ (since no fiscal implications)
- $dr_t^B = dr_t^M + \vartheta_t^L (dr_t^L - dr_t^M)$
- $dr_t^B = dr_t^M + \vartheta_t^L (\sigma_t^{P^L} \sigma_t^\eta dt + \sigma_t^{P^L} dZ_t)$

- Return of a single coin (reserve unit/short-term bond)
- $dr_t^M = (\mu_t^\vartheta - \vartheta_t^L \sigma_t^{P^L} \sigma_t^\eta) dt + (\sigma_t^\vartheta - \vartheta_t^L \sigma_t^{P^L}) dZ_t$
- $\vartheta_t^L \sigma_t^{P^L}$ shows importance of long-term bond price variation
  - the dZ-term is a "risk-transfer".
  - The dt-term shows that it also affects risk premia

η-Volatility and Amplification

$\sigma_t^{\eta} = \quad \sigma_t^{r^M} + (1 - \theta_t^{M,I} - \theta_t^{L,I})\sigma_t^{xK^b} + \theta_t^{L,I}(\sigma_t^{r^L} - \sigma_t^{r^M})$

Note that money is our benchmark asset (since HH cannot go short L-bond)

- Where portfolio share $1 - \theta_t^{M,I} - \theta_t^{L,I} = \frac{\chi_t}{\eta_t}(1 - \vartheta_t)$ and $\theta_t^{L,I} = \vartheta_t^L \vartheta_t / \eta_t$

$$\sigma_t^{\eta} = \sigma_t^{\vartheta} - \vartheta_t^L \sigma_t^{P^L} + \frac{\chi_t(1-\vartheta_t)}{\eta_t}\left((1-\bar{\kappa})\sigma - \frac{\sigma_t^{\vartheta}}{1-\vartheta_t} + \vartheta_t^L \sigma_t^{P^L}\right) + \frac{\vartheta_t^L \vartheta_t}{\eta_t}\sigma_t^{P^L}$$

Collect $\sigma_t^{P^L}$-terms

$$\sigma_t^{\eta} = \sigma_t^{\vartheta} + \frac{\chi_t(1-\vartheta_t)}{\eta_t}\left((1-\bar{\kappa})\sigma - \frac{\sigma_t^{\vartheta}}{1-\vartheta_t}\right) + \frac{\chi_t(1-\vartheta_t) + \vartheta_t - \eta_t}{\eta_t}\vartheta_t^L\, \sigma_t^{P^L}$$

Replace $\sigma_t^{\vartheta} = \frac{\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)}\sigma_t^{\eta}$ and $\sigma_t^{P^L} = \frac{P^{L'}(\eta_t)\eta_t}{P^L(\eta_t)}\sigma_t^{\eta}$

$$\Rightarrow \eta_t \sigma_t^{\eta} = \frac{(1-\vartheta_t)\chi_t(1-\bar{\kappa})\sigma}{1 - \frac{\chi_t - \eta_t}{\eta_t}\left(\frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)}\right) + \vartheta_t^L\left(\frac{P^{L'}(\eta_t)\eta_t}{P^L(\eta_t)}\right)\frac{\chi_t(1-\vartheta_t)+\vartheta_t-\eta_t}{\eta_t}}$$

- Recall that $\frac{-\vartheta'(\eta_t)\eta_t}{\vartheta(\eta_t)} = (1-\vartheta_t)\left(\underbrace{\frac{q^{K'}(\eta_t)\eta_t}{q^K(\eta_t)}}_{\text{Liquidity Spiral}} + \underbrace{\frac{-q^{B'}(\eta_t)\eta_t}{q^B(\eta_t)}}_{\text{Disinflationary Spiral}}\right)$ … and is the **mitigation term** due to policy

Derive μ_t^η

- Same steps as before

Monetary Policy: Ex-post perspective

- Money view Friedman-Schwartz
  - Restore money supply
    - Replace missing inside money with outside money
  - Aim: Reduce deflationary spiral
    - … but banks extent less credit & diversify less idiosyncratic risk away
    - … as households have to hold more idiosyncratic risk, money demand rises
    - Undershoots inflation target

- Credit view Tobin
  - Restore credit
  - Aim: Switch off deflationary spiral & liquidity spiral

- I Theory: "Stealth" recapitalization of impaired sector
  - Interest policy and OMO affect asset prices

MoPo Benchmark 1: Removing endogenous Risk

- The policy that removes endogenous risk, $\sigma_t^B = \sigma_t^{\vartheta}$
- FOC gives (in closed form)

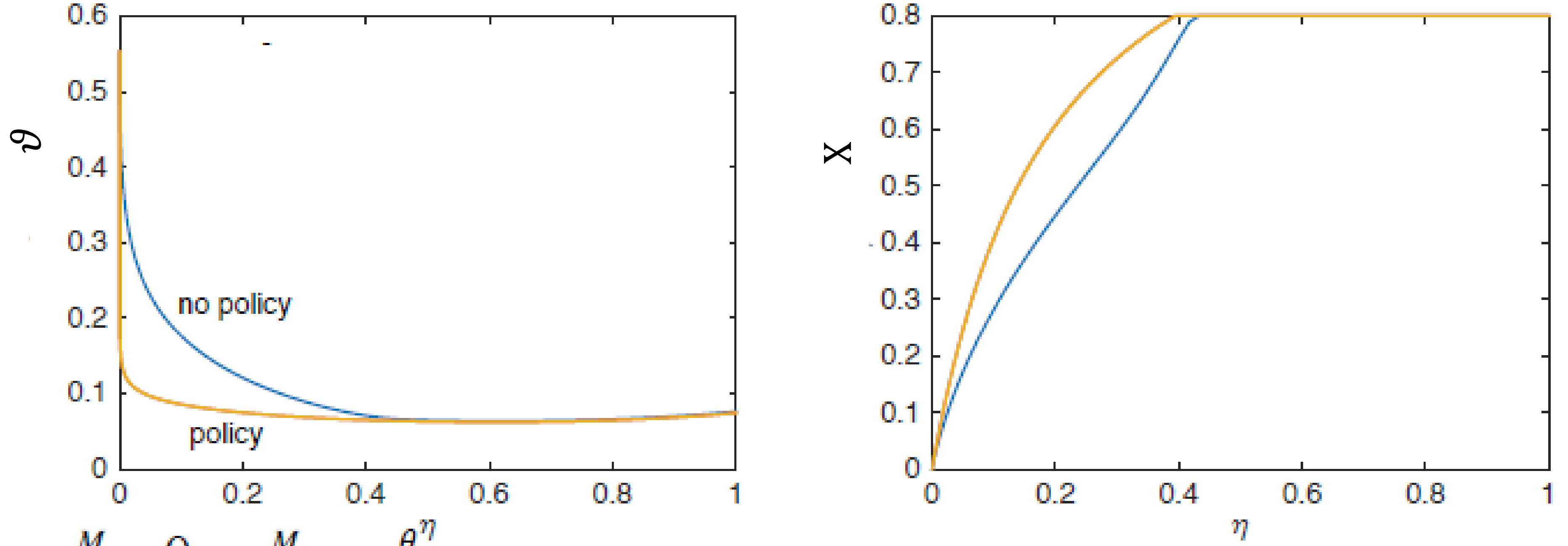
$$\chi_t = \min\left(\frac{\eta_t}{\eta_t + (1-\eta_t)\phi^2 + (1-\bar{\psi})^2(\sigma^b)^2/\tilde{\sigma}^2}, \bar{\psi}\right)$$

- η-Evolution
  - $\sigma^\eta = (1-\vartheta_t)\frac{\chi_t}{\eta_t}(1-\bar{\psi})\sigma^b$
  - $\eta_t\mu_t^\eta = \eta_t(1-\eta_t)(1-\vartheta_t)^2\left(\frac{1-2\eta_t}{(1-\eta_t)^2}\frac{\chi_t^2}{\eta_t^2}(1-\bar{\psi})^2\left(\sigma^b\right)^2 + \frac{\chi_t^2\phi^2\tilde{\sigma}^2}{\eta_t^2} - \frac{(1-\chi_t)^2\phi^2\tilde{\sigma}^2}{\eta_t^2}\right)$

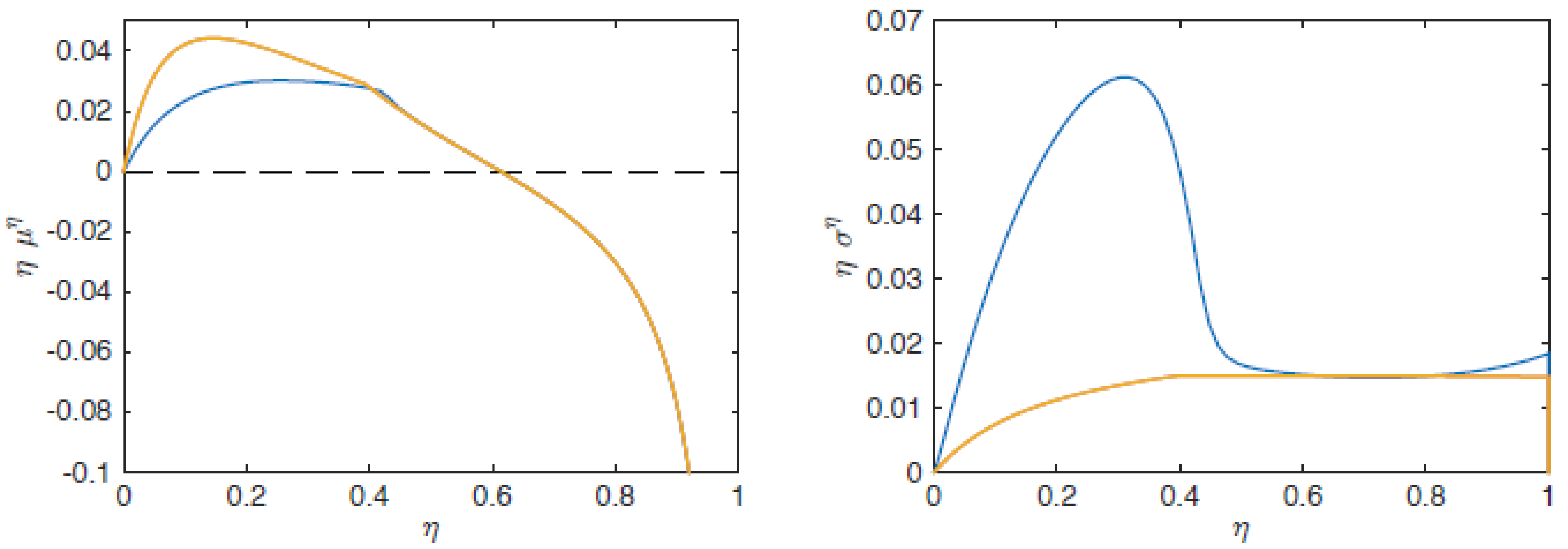
Closed form up to ϑ_t (which is choice of planner)

Numerical Example

- $\rho = .05, \phi = 2, \tilde{\sigma} = .5, \varphi = .4, \bar{\chi} = .8, \sigma^a = 0, \sigma^b = .1$

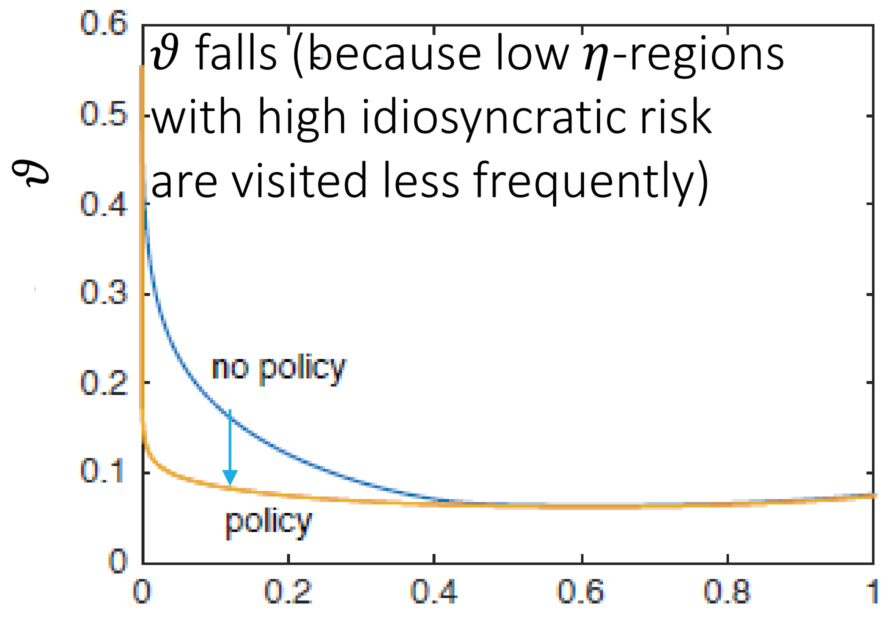


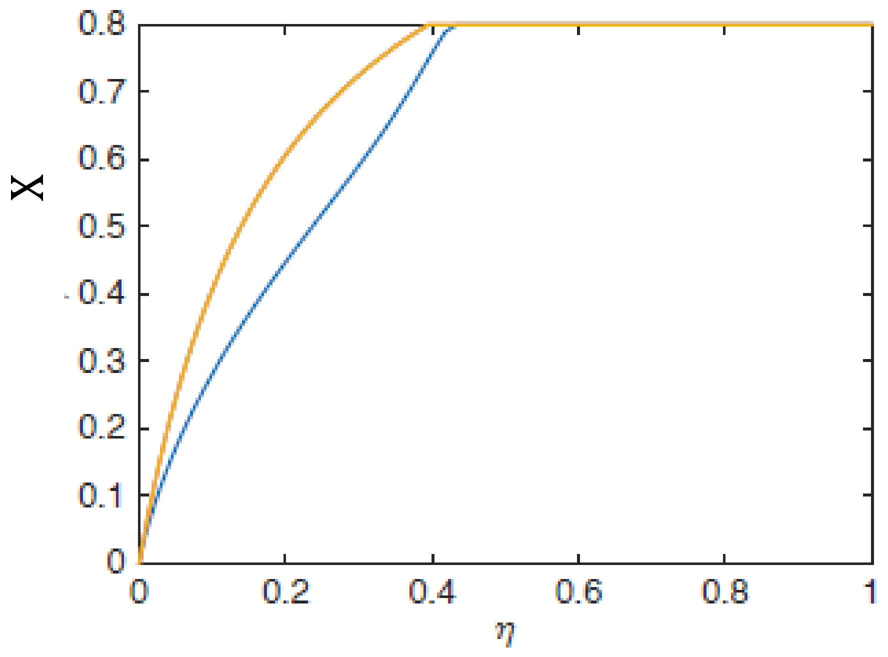
policy: $\mu^M = 0,\ \sigma^M = \sigma^{\theta^\eta}$



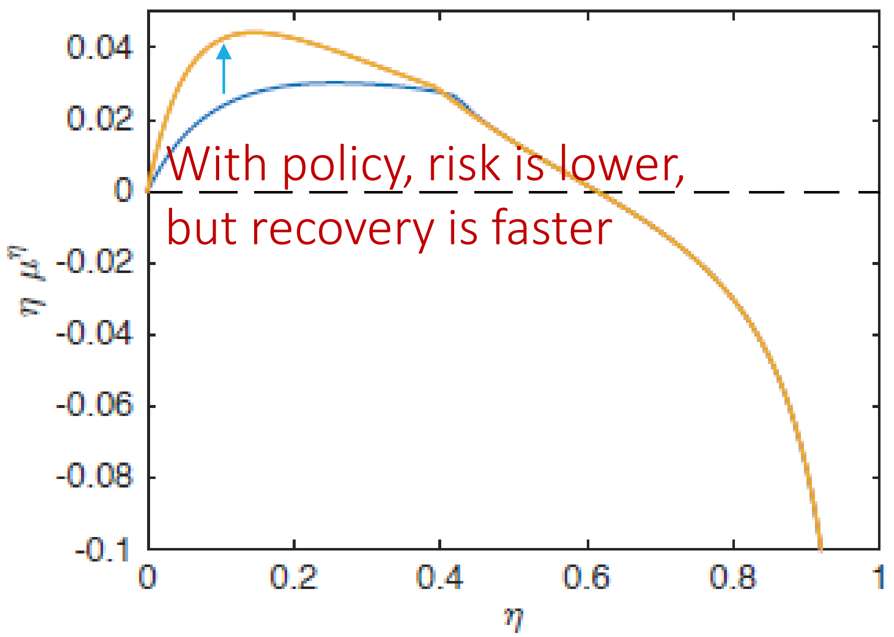
Numerical Example

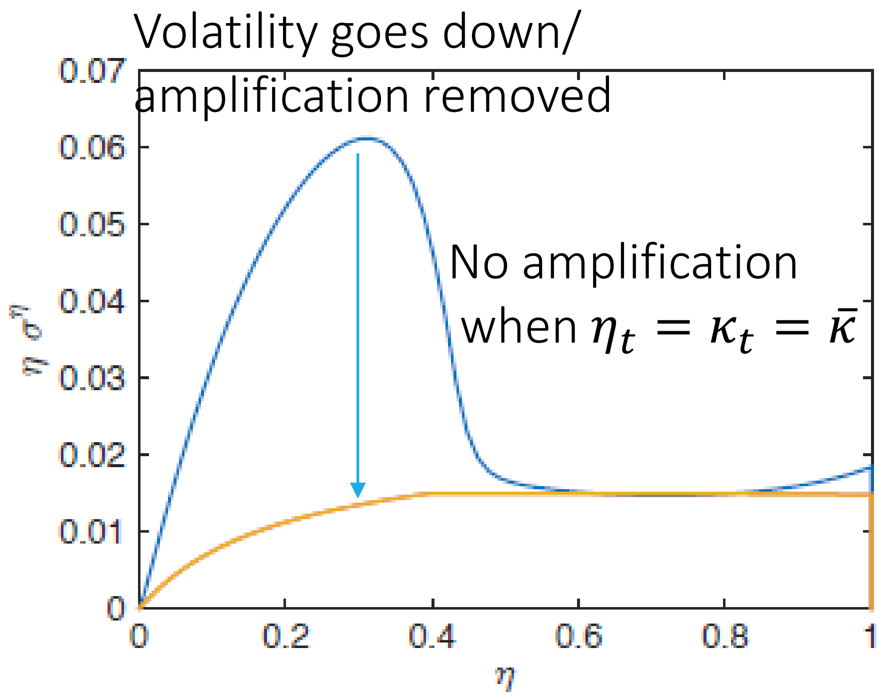
- $\rho = .05, \phi = 2, \tilde{\sigma} = .5, \varphi = .4, \bar{\chi} = .8, \sigma^a = 0, \sigma^b = .1$





policy: $\mu^M = 0,\ \sigma^M = \sigma^{\theta\eta}$





Optimal Policy

- Next lecture after we have covered welfare analysis

Recall

- Unified macro "Money and Banking" model to analyze
  - Financial stability - Liquidity spiral
  - Monetary stability - Fisher disinflation spiral

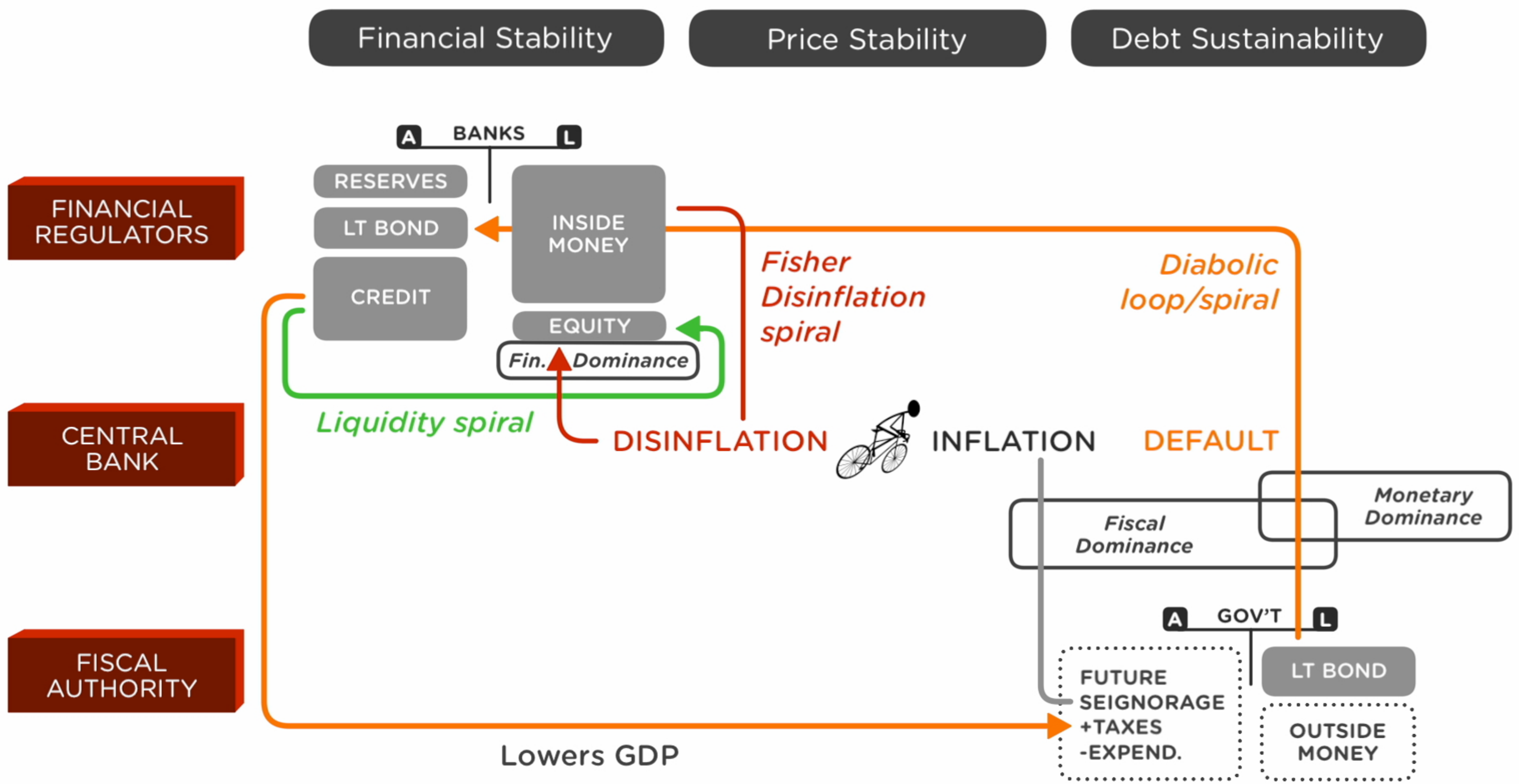
- Exogenous risk &
  - Sector specific
  - idiosyncratic
- Endogenous risk
  - Time varying risk premia – flight to safety
  - Capitalization of intermediaries is key state variable

"Paradox of Prudence"

- Monetary policy rule
  - Risk transfer to undercapitalized critical sectors
  - Income/wealth effects are crucial instead of substitution effect
  - Reduces endogenous risk – better aggregate risk sharing
    - Self-defeating in equilibrium – excessive idiosyncratic risk taking

Flipped Classroom Experience

Series of 4 YouTube videos, each about 10 minutes



Thank you!

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