China's Model of Managing the Financial System*

Markus K. Brunnermeier† Michael Sockin‡ Wei Xiong§

June 2021

Abstract

China's economic model involves regular and intensive government interventions in financial markets, while Western policymakers often refrain from substantial interventions outside crisis periods. We develop a theoretical framework to rationalize the approaches of both China and the West to managing the financial system as being optimal given the differences in their respective economies. In this framework, a government leans against trading of noise traders but at the expense of introducing policy noise to the market. Our welfare analysis shows that under certain underlying economic conditions, the optimal government policy induces a government-centric equilibrium, in which government intervention is so intensive that all investors choose to acquire private information about policy noise rather than fundamentals. This policy regime characterizes China's approach with financial stability prioritized over other policy objectives.

*We are grateful to Will Cong, Darrell Duffie, Anton Korinek, Debbie Lucas, Helene Rey, Alp Simsek, Larry Summers, Liyan Yang, Hao Zhou, Haoxiang Zhu, and participants of numerous seminars and workshops for helpful comments and suggestions. We are particularly grateful to Christian Hellwig and four referees for highly constructive comments.

†Princeton University and NBER. Email: markus@princeton.edu.

‡University of Texas at Austin. Email: Michael.Sockin@mccombs.utexas.edu.

§Princeton University and NBER. Email: wxiong@princeton.edu.

1 Introduction

Over the past four decades, China's model of "state capitalism" has lifted millions out of poverty. It is therefore not surprising that its approach has attracted significant attention from the international community. Although China has adopted many elements of Western economies since its economic reforms began in the late 1970s, it still relies heavily on frequent and intensive interventions by its government. The Chinese government's "visible hand" in a command economy consequently interacts with the "invisible hand" of laissez-faire capitalism to promote growth and stability in China. In this paper, we investigate the consequences of such intervention policies in China's financial system.[1]

A striking feature of China's financial system is how actively the government leans against short-term market fluctuations. The Chinese government does so through frequent policy changes, using a wide array of policy tools ranging from changes in interest rates and bank reserve requirements to stamp taxes on stock trading, suspensions and quota controls on IPO issuances, changes of mortgage rates and first payment requirements, and direct trading in asset markets through government-sponsored institutions. For example, during China's stock market turmoil in the summer of 2015, the Chinese government organized a "national team" of securities firms to backstop the market collapse, as documented by Huang, Miao, and Wang (2019) and Allen et al. (2020). A potential justification for such large-scale, active interventions is that China's financial markets are highly speculative[2] and largely populated by inexperienced retail investors. Its markets experience high price volatility and the highest turnover rate among major stock markets in the world.[3] By leaning against the transient market fluctuations created by these inexperienced investors, government intervention helps

[1] An intense economic tournament, for instance, motivates local government officials to drive local developments, see e.g., Xu (2011), Qian (2017), and Xiong (2019). Song and Xiong (2018) offer a review of the institutional foundations of China's financial system.

[2] Carpenter and Whitelaw (2017) review extensive literature on the so-called A-share premium puzzle that sees the prices of A-shares issued by publicly listed Chinese companies to domestic investors trading at substantial price premia and much higher turnover rates relative to B-shares and H-shares issued by the same companies to foreign investors. Mei, Scheinkman, and Xiong (2009) attribute this phenomenon to speculative trading of Chinese investors. Furthermore, Xiong and Yu (2011) document a spectacular bubble in Chinese warrants from 2005 to 2008, during which Chinese investors actively traded a set of deep out-of-money put warrants that had zero fundamental value.

[3] In 2008, the China Securities Regulatory Commission issued the China Capital Markets Development Report, which shows that, in 2007, retail accounts with a balance of less than 1 million RMB contributed to 45.9% of stock positions and 73.6% of trading volume on the Shenzhen Stock Exchange. This report highlights, in particular, the speculative behavior of these small investors and the lack of mature institutional investors as important characteristics of China's stock market. Hu, Pan, and Wang (2018) offer a detailed account of stock market volatility and turnover in China.

reduce market volatility and promote financial stability.

Despite the advantages of continual government involvement in financial markets, Western policymakers often refrain from substantial policy interventions outside of crisis periods out of concern that such intervention may distort financial markets and be more harmful than beneficial. Alan Greenspan and Ben Bernanke, for example, while chairing the U.S. Federal Reserve Board, explicitly stated their reluctance to lean against bubbles in asset markets. Such concerns raise questions as to whether China's expansive government intervention policy entails such a trade-off.

We develop a conceptual framework to analyze these questions. Our analysis focuses on government intervention through direct trading against noise traders in asset markets. We build upon the standard noisy rational expectations models of asset markets with asymmetric information, such as in Grossman and Stiglitz (1980) and Hellwig (1980), and their dynamic versions, including He and Wang (1995) and Allen, Morris, and Shin (2006). In these models, noise traders create short-term price fluctuations, and a group of rational investors, each acquiring a piece of private information, trades against these noise traders to provide liquidity and to speculate on their private information. Our setting also includes a new large player, a government, who is prepared to trade against noise traders to stabilize the market.

Noise traders in our setting reflect the inexperienced retail investors in the Chinese markets, who contribute to price volatility and instability. The government's intervention also introduces unintended noise, possibly stemming from agency problems of the government system, with the magnitude of this noise increasing with the intensity of intervention. Our model therefore features a basic trade-off faced by the government—its intervention leans against noise traders but at the expense of introducing policy noise into the market. Furthermore, each investor chooses between acquiring a private signal about either the asset fundamental or this government noise before trading. The information choice of investors provides an even more interesting channel for government intervention to impact the informativeness of the asset price.

We begin our analysis by characterizing a benchmark economy in which the asset fundamental is publicly observable. This baseline model illustrates an adverse volatility feedback loop. In the absence of government intervention, the volatility of the asset price can explode as the volatility of noise trading increases. When the volatility of noise trading is greater, short-term asset return volatility is greater and the risk premium that investors demand for trading the asset is higher. This higher risk premium further increases short-term return volatility. This adverse volatility feedback loop motivates the government to intervene by providing additional risk-bearing capacity to the market.

We next analyze an extended setting in which the asset fundamental is unobservable to investors and the government. We assume the government follows a linear strategy of trading against perceived noise trading based on the publicly available information. Depending on whether investors choose to acquire a private signal about either the fundamental or the noise in government intervention, there can be two different equilibrium outcomes, which we label "fundamental-centric" and "government-centric," respectively. In the fundamental-centric equilibrium, each investor acquires a private signal about the fundamental, and the asset price aggregates their information to partially reveal it. In contrast, when the government-centric equilibrium arises, investors all focus on learning about noise in future government intervention, and their trading, consequently, exposes the asset price to anticipated government noise in the future, rather than the fundamental. The likelihood of a government-centric equilibrium increases with the intensity of the government intervention.

Interestingly, for an intermediate range of government intervention intensity, both the fundamental-centric and government-centric equilibria can coexist as a result of the intertemporal complementarity in investors' information acquisition choices: if investors in the next period acquire fundamental information, the asset price in that next period will be more informative about the asset fundamental, which, in turn, makes it more desirable for investors to acquire information about the asset fundamental. Surprisingly, in the case when both equilibria exist, the same intervention intensity allows the government to achieve substantially lower price volatility in the government-centric equilibrium than in the fundamental-centric equilibrium. This occurs because, in the latter equilibrium, the government trades against both noise traders, to minimize their price distortion, and investors, who trade based on their private information. In the government-centric equilibrium, in contrast, all informed investors share the same information about the asset fundamental as the government; as a result, informed investors tend to trade alongside the government, which reinforces the government's effort to reduce price volatility. The government's intervention is consequently more effective in mitigating the price distortion of noise traders in the government-centric equilibrium. The downside to this heightened efficacy is that the informational efficiency of the asset price is also lower in the government-centric equilibrium because no one acquires information about the asset fundamental.

Finally, we investigate the government's optimal intervention policy based on a microfounded welfare objective. We expand our model to incorporate a real sector, in which firms make investment decisions based on the asset price, and taxpayers who are the residual claimants of the government's trading profits. Maximizing social welfare can be traced to two closely related, albeit subtly different, objectives. The first is to reduce asset return volatility, which, in turn, reduces the risk premia faced by market participants and ensures financial stability. The second is to improve market efficiency, or the informativeness of the asset price, which improves the allocative efficiency of firm investment. When investors have no information acquisition choice, government intervention accomplishes both objectives by simply leaning against noise traders. This "divine coincidence" has often motivated policymakers to treat these two objectives as being interchangeable. In practice, policymakers focus on reducing asset return volatility, as it is easier to measure than market efficiency; see, for instance, Stein and Sundarem (2018). In the presence of investors' information choice, however, our analysis shows that the government faces a trade-off between these two seemingly congruent objectives—more-intensive interventions can lead to a government-centric equilibrium with lower return volatility but worse price efficiency.

Our analysis reveals that under certain underlying economic conditions—when noise-trader risk is sufficiently high or when firms face sufficiently high idiosyncratic noise to render market efficiency less relevant to firm investment—the government's optimal policy induces a government-centric equilibrium. We believe this policy regime characterizes the Chinese government's regular and intensive intervention in its financial system, with financial stability prioritized over other policy objectives. In contrast, the fundamental-centric equilibrium is reminiscent of the attitude of Western governments who restrict the scale of their intervention policies to avoid distorting market efficiency. We can therefore rationalize both China's and the West's approach to managing the financial system within our unified framework as being optimal given the differences in their respective economies and without having to appeal to differences in welfare objectives based on political considerations. Interestingly, our analysis also predicts that China may eventually outgrow its aggressive intervention regime as investors become more experienced and firms become more dependent on asset prices for investment guidance.

Our paper builds on the literature that studies information choice in noisy rational expectations models. Hellwig and Veldkamp (2009) demonstrate that, in settings with strategic complementarity in actions, strategic complementarity also arises in information choices, leading agents to choose to learn the same information as others. Ganguli and Yang (2009) and Manzano and Vives (2011) investigate the complementarity in information choice among investors when they can choose to acquire private information either about supply noise or about fundamentals in static settings and the resulting multiplicity and stability of equilibria. Farboodi and Veldkamp (2016) examine the role of investors' acquisition of information about order flows, instead of fundamentals, in explaining the ongoing trend of increasing price informativeness and declining market liquidity in financial markets. Goldstein, Schneemeier, and Yang (2020) examine the disconnect between real and market efficiency when traders acquire information and firms are exposed to multiple sources of uncertainty. Different from the intratemporal complementarity in information choices studied by these papers, our model highlights intertemporal complementarity of investors' information choice, in a spirit similar to Froot, Scharfstein, and Stein (1992). More important, our paper builds on the complementarity in investors' information choices to analyze an important policy issue.

Our work also contributes to the literature on the financial market implications of government intervention. Bond and Goldstein (2015) study the impact on information aggregation in prices when uncertain, future government intervention influences a firm's real outcomes. Cong, Grenadier, and Hu (2017) explore the information externality of government intervention in money market mutual funds in a global games environment in which investors face strategic coordination issues and intervention changes the information publicly available to them. Angeletos, Hellwig, and Pavan (2006) and Goldstein and Huang (2016) consider information design by an informed policymaker that can send messages through its actions to coordinate the response of private agents in a global games setting. Goldstein and Yang (2019) illustrate how public disclosure by a real decision maker can harm real efficiency by making asset prices less informative. In contrast to these studies, we focus on the incentives of market participants to acquire information when there is uncertainty about the scope of government intervention in financial markets through large-scale asset purchases. Our government, by internalizing investors' information acquisition choices, faces a tension between reducing price volatility and improving price efficiency.

The paper is organized as follows. Section 2 provides institutional background. By first taking the government intervention as given, Sections 3 and 4 analyze its effects under perfect information and information frictions, respectively. Section 5 analyzes the government's optimal intervention policy. Section 6 concludes. We cover the salient features of the model under different settings in the main text while providing more detailed descriptions of the model in the Appendix. A separate Online Appendix contains all technical proofs involved in our analysis.

2 Institutional Background

Western governments usually refrain from large-scale interventions in the financial system.[4] They typically intervene only during financial crises when massive market failures threaten to damage the financial system and the economy. For example, during the 2008 financial crisis, the U.S. government instituted the $700 billion Troubled Asset Relief Program to purchase toxic assets from banks and also temporarily halted short-sales of financial stocks. In contrast, the Chinese government has been engaged in regular and intensive interventions in the financial system not just during crises but also during booms. This section summarizes the extent of government intervention in China's financial system, focusing in particular on the general strategy of the Chinese government to lean against short-term market fluctuations either through direct trading or broad policy interventions.

The national team and the 2015 stock market crash. In 2014–2015, the Chinese stock market experienced a dramatic boom-and-bust cycle, as described by Allen et al. (2020). The initial market boom precipitated a large inflow of new investors with little financial knowledge and investment experience yet substantial leverage through margin financing of their stock positions. In June 2015, when the stock market initially plunged by over 30%, many investors received margin calls, which forced them to liquidate their leveraged positions. Bian et al. (2017) provide a systematic account of the resulting margin spiral, which directly threatened the stability of the whole financial system. In response, the Chinese government organized a national team of investment firms to bail out the stock market in the period from June to September of 2015. According to Allen et al. (2020) and Huang, Miao, and Wang (2019), during this bailout period the national team invested in

[4] Policymakers in Western governments often hesitate to intervene during asset market booms. For example, both Alan Greenspan and Ben Bernanke have acknowledged the difficulty for central bankers to determine the presence of asset bubbles, which in turn makes them reluctant to lean against a potential asset bubble.

1,365 stocks, which accounted for about 50% of the total number of listed stocks and 6% of the capitalization of the Chinese stock market. Their analysis shows that, by stabilizing the market, the intervention of the national team substantially increased the value of the rescued nonfinancial firms through increased stock demand, reduced default probabilities, and improved market liquidity.

Regular policy interventions in the stock market. The China Securities Regulatory Commission (CSRC), the regulator of China's stock market, has regularly used a large set of policy tools to lean against cycles in the stock market, not just to support the market during crashes but also to slow down the market during booms. For example, the CSRC has changed the rate of transaction tax on stock trading seven times since 1994, increasing the tax rate during market booms and reducing it during market downturns; see, for example, Deng, Liu, and Wei (2018) and Cai et al. (2019). The CSRC has also used its control of the IPO issuance to lean against market cycles by increasing issuance quotas during booms and suspend issuance during busts. Since 1994, the CSRC has suspended IPO issuance nine times, usually when the stock market was distressed, and sometimes for as long as 15 months. Packer and Spiegel (2016) find a significant, positive relation between the number of IPOs and the market index return in China's stock market, confirming the CSRC's effort to use IPO issuance to lean against the market cycle. During the 2015 stock market turmoil, the CSRC also employed another measure to stabilize the market: prohibiting large shareholders from selling their shares. As discussed by Allen et al. (2020), on July 8, 2015, the CSRC imposed a lockup on shareholders owning 5% or more of their companies, initially for six months. The lockup was extended in January 2016 after the stock market declined sharply again.

Countercyclical interventions in other markets. The Chinese government has also actively intervened in other markets besides the stock market. According to Liu and Xiong (2020), the real estate market has perhaps even more systemic importance to the Chinese economy because of the substantial exposures of local governments, real estate developers, firms, and households who use real estate assets as collateral for debt financing. As a result, the Chinese government has used a wide range of policy measures to lean against real estate cycles. During booms, the government tends to increase land supply for real estate development. It also restricts purchases of investment homes in large cities by both residents and nonresidents and increases mortgage down payments and mortgage rates for purchases of both primary and investment homes. During downturns, the government tends to reverse these measures. Furthermore, the People's Bank of China (PBC) also adopts countercyclical monetary policies to assist government efforts to lean against real estate cycles.

During the past decade, the Chinese government has made great efforts to internationalize its RMB currency and liberalize its capital accounts. This process exposed the RMB exchange rate to intense market speculation and China's capital accounts to dramatic inflows and outflows. In 2013–2015, domestic enterprises took on dollar debt from the global capital markets to take advantage of the substantially lower interest rates outside China, leading to large capital inflows. The direction of capital flows reversed after late 2015 when China's economic growth slowed and intense market pressure mounted to speculate against the RMB exchange rate. In the subsequent two years, capital outflows led to China's loss of FX reserves in excess of $1 trillion. In response to these developments, the PBC has adopted a series of macroprudential regulatory measures to lean against speculative capital inflows/outflows. As detailed in the 2018 report of Hong Kong Stock Exchange, during periods of capital outflows or depreciation pressure on the RMB, the PBC adopted the following measures: 1) an increase of the FX risk reserve requirement ratio to 20%; 2) the introduction of reserve requirements on foreign financial institutions' RMB deposits in domestic financial institutions, which directly affect the supply of RMB to foreign speculators for shorting RMB; 3) the use of a countercyclical adjustment factor in the mechanism of determining the RMB's central parity rate; and 4) the imposition of unified regulations on local and foreign currencies. During periods of capital inflows or appreciation pressure on the RMB, the PBC reversed the aforementioned measures.

3 The Basic Model with Perfect Information

We develop a model to analyze China's model of government interventions in the financial markets in several steps. We first present a generic model of government interventions in an asset market with perfect information in this section and then one with information frictions in Section 4. In both of these sections, we take the government's intervention strategy as given to focus on the effects of the intervention on market dynamics. In Section 5, we further expand the model to introduce a real sector to analyze social welfare and the optimal intervention policy.

In this section, we present a baseline setting with perfect information to illustrate how government intervention helps mitigate the volatility explosion caused by the reluctance of short-term investors to trade against noise traders. Our model can be seen as a generalized version of De Long et al. (1990) with fundamental risk. Consider an infinite-horizon economy in discrete time with infinitely many periods: $t = 0, 1, 2....$ There is a risky asset, which can be viewed as stock issued by a firm that has a stream of cash flows D_t over time:

$$D_t = V_t + \sigma_D \varepsilon_t^D.$$

The component V_t is a persistent component of the fundamentals, while ε_t^D is independent and identical cash flow noise with a Gaussian distribution of $\mathcal{N}(0,1)$ and $\sigma_D > 0$ measures the volatility of cash flow noise.

As the literature has already extensively studied the direct effects of government policies on the profitability of firms,[5] we intend to analyze a different channel through which government intervention can impact market dynamics without directly affecting the firm's cash flow. Specifically, we assume that the asset's fundamental V_t follows an exogenous AR(1) process:

$$V_t = \rho_V V_{t-1} + \sigma_V \varepsilon_t^V,$$

where $\rho_V \in (0,1)$ measures the persistence of V_t, $\sigma_V > 0$ measures its volatility, and $\varepsilon_t^V \sim \mathcal{N}(0,1)$ is independently and identically distributed shock.

In this section, we assume that at time t, V_{t+1} is **observable** to all agents in the economy. This setting serves as a benchmark.[6] We will remove this assumption in the next section to make V_{t+1} unobservable to both the government and investors and then discuss how government intervention affects the investors' information acquisition.

For simplicity, suppose there is also a risk-free asset in elastic supply that pays a constant gross interest rate $R^f > 1$. In what follows, we define R_{t+1} to be the excess payoff, not the

[5] For example, if the government faces a time-varying cost in implementing such a policy, the cost of the policy can become an important factor in driving variation in a stock's cash flows and thus its price dynamics. See Pastor and Veronesi (2012, 2013) for recent studies that explore this channel. In addition, when government policies affect the cash flow of publicly traded firms, Bond and Goldstein (2015) show that such intervention feeds back into how market participants trade on their private information. This results in socially inefficient aggregation of private information about the unobservable fundamental v_t into asset prices, which can impede policymaking if the government also infers relevant information about v_t from the traded asset price in determining the scale of its intervention.

[6] We make v_{t+1}, not just v_t, observable at time t so that this benchmark is exactly the limiting case of the setting in the next section, where we allow the precision of each investor's private information about v_{t+1} to become arbitrarily large.

percentage return, to holding the risky asset:

$$R_{t+1} = D_{t+1} + P_{t+1} - R^f P_t.$$

There are three types of agents in the asset market: noise traders, investors, and the government. We describe each of them below.

3.1 Noise Traders

Motivated by the large number of inexperienced retail investors in China's stock markets, we assume that, in each period, these inexperienced investors, whom we call noise traders, submit exogenous market orders into the asset market. This way of modeling noise trading is standard in the market microstructure literature. We denote the quantity of their net *buy* orders by N_t and assume that N_t is an i.i.d. process:

$$N_t = \sigma_N \varepsilon_t^N,$$

where $\sigma_N > 0$ measures the volatility of noise trading (or noise-trader risk in this market), and $\varepsilon_t^N \sim \mathcal{N}(0,1)$ is independently and identically distributed shocks to noise traders. The presence of noise traders creates incentives for other investors to trade in the asset market.

3.2 Investors

There is a continuum of investors in the market who trade the asset on each date t. We assume that these investors are myopic. They can be thought of as living for only two periods, trading in the first and consuming in the second. That is, in each period, a group of new investors with measure 1 joins the market, replacing the group from the previous period. We index an individual investor by $i \in [0,1]$. Investor i born at date t is endowed with wealth $\bar{W}$ and has constant absolute risk aversion (CARA) preferences with coefficient of risk aversion γ over its next-period wealth W_{t+1}^i:

$$U_t^i = E\left[-\exp\left(-\gamma W_{t+1}^i\right) \mid \mathcal{F}_t\right].$$

It purchases X_t^i shares of the asset and invests the rest in the risk-free asset at a constant rate R^f, so that W_{t+1}^i is given by

$$W_{t+1}^i = R^f \bar{W} + X_t^i R_{t+1}.$$

The investors have symmetric, perfect information, and their expectations are all taken with respect to the full-information set $\mathcal{F}_t = \sigma\left(\{V_{s+1}, N_s, D_s\}_{s\leq t}\right)$ in this section. As a result of CARA preferences, an individual investor's trading behavior is insensitive to his initial wealth level.

The assumption of investor myopia follows from De Long et al. (1990) and can be motivated from agency problems faced by institutional investors; see, for example, Shleifer and Vishny (1997). In our setting, this assumption also serves to capture the short-termism of Chinese investors. This assumption is innocuous for the volatility explosion that motivates government intervention, although it is key for market breakdown when noise-trader risk becomes sufficiently large.

3.3 Equilibrium Without Government Intervention

To facilitate our discussion, we first characterize the rational expectations equilibrium without government intervention. Specifically, we derive the equilibrium price and show formally that market volatility explodes when noise-trader risk, σ_N, rises.

We first conjecture a linear rational expectations equilibrium.[7] In this equilibrium, the asset price P_t is a linear function of the fundamental V_{t+1} and the noise-trader shock N_t:

$$P_t = \frac{1}{R^f - \rho_V} V_{t+1} + p_N N_t,$$

where $\frac{1}{R^f-\rho_V} V_{t+1}$ is the expected present value of cash flows from the asset. With this conjected price function, an investor holding the asset faces, at time t, price risk from fluctuations of both V_{t+1} and N_t, as given by

$$Var\left(R_{t+1}|\mathcal{F}_t\right) = \sigma_D^2 + \left(\frac{1}{R^f - \rho_V}\right)^2 \sigma_V^2 + p_N^2 \sigma_N^2.$$

CARA utility with normally distributed payoffs implies identical asset demand X_t^i:

$$X_t^i = -\frac{1}{\gamma} \frac{p_N R^f}{\sigma_D^2 + \left(\frac{1}{R^f-\rho_V}\right)^2 \sigma_V^2 + p_N^2 \sigma_N^2} N_t,$$

which trades off expected asset return with return variance over the subsequent period.

Then, imposing market-clearing in the asset market $X_t^i = N_t$ leads to a quadratic equation that pins down the price coefficient p_N. There may be two positive roots for p_N. We focus

[7] We later verify in the proof of Proposition 2 that there cannot be any nonlinear equilibrium if we treat the economy as the infinite-horizon limit of an economy with a finite number of trading periods.

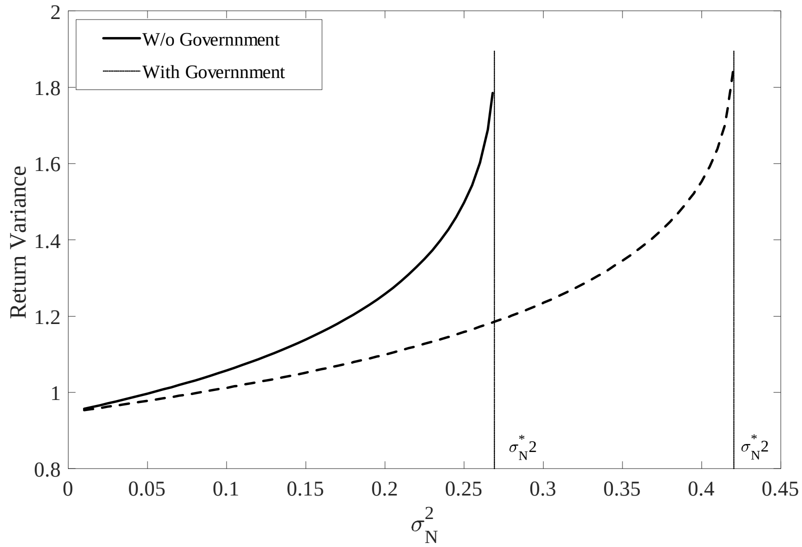

Figure 1: Asset return variance with and without government intervention with respect to the variance of noise trading σ_N^2. The solid line represents the case without government intervention, and the dashed line represents the case with government intervention at a given intensity ϑ_N, based on the following parameters: $\gamma = 1$, $R^f = 1.01$, $\rho_v = 0.75$, $\sigma_v^2 = 0.01$, $\sigma_D^2 = 0.8$, $\vartheta_N = 0.2$.

on the less positive root.[8] The following proposition shows that the asset return variance increases with noise-trader risk, σ_N, and the rate of this increase explodes as σ_N rises:

Proposition 1 *If noise-trader risk $\sigma_N \leq \sigma_N^*$, where*

$$\sigma_N^* = \frac{R^f}{2\gamma\sqrt{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2}}, \tag{1}$$

then asset return variance, $Var\left[R_{t+1}|\mathcal{F}_t\right]$, is increasing and convex in σ_N^2, with the slope rising to ∞ at $\sigma_N = \sigma_N^*$. If $\sigma_N > \sigma_N^*$, no equilibrium exists.

We provide a proof to Proposition 1 in the Online Appendix. As σ_N rises, investors demand a higher risk premium to take on a position against noise traders, that is, a more positive coefficient p_N, which, in turn, leads to higher asset return volatility.[9] Through this

[8] As $\sigma_N \to 0$ (i.e., noise-trader risk vanishes from the economy), the less positive root has the nice property that $p_N\sigma_N \to 0$ (i.e., the price impact of noise traders diminishes), while the more positive root diverges. Furthermore, if one treats the quadratic equation defining p_N as a recursion, the less positive root is backward stable while the more positive root is forward stable, and market breakdown occurs when both roots diverge ($p_N \to \infty$). The less positive root is consequently the more intuitive root since prices are determined by backward, rather than forward, induction of future payoffs.

[9] This feedback effect between future risks and current risk premia can also lead to self-fulfilling panics when investors coordinate on sunspots to select among multiple equilibria. See, for instance, Bacchetta, Tille, and Wincoop (2012).

feedback process, the asset price variance explodes as σ_N rises, as illustrated by Figure 1. The myopia of the investors makes the market dynamics even more dramatic in that no investors are willing to trade against noise traders as σ_N rises above a threshold σ_N^*.[10] The explosion of asset price volatility motivates government interventions because it presents an important externality to the whole economy, which we micro-found in an expanded model setting with risk sharing and firm investment in Section 5.1.

3.4 Equilibrium With Government Intervention

We now incorporate government intervention into the model. Specifically, we augment the baseline setting to include a government that actively intervenes in the asset market. The government follows a linear trading rule:

$$X_t^G = -\vartheta_N N_t + \sigma_N \vartheta_N G_t.$$

The first term $-\vartheta_N N_t$ captures the government's intended intervention strategy of trading against the noise traders, with the coefficient ϑ_N measuring the intensity of the intervention. We choose the convention of a negative coefficient because this term will partially offset noise trader demand when we later impose market clearing. We also include the second term $\sigma_N \vartheta_N G_t$ to capture unintended noise that arises from frictions in the government system and the intervention process, such as behavioral biases, lobbying effort, or information frictions. Specifically, $G_t = \sigma_G \varepsilon_t^G$ with $\varepsilon_t^G \sim \mathcal{N}(0, 1)$ as independently and identically distributed shocks and σ_G as a volatility parameter. The magnitude of this noise component scales up with the intended intervention intensity $\sigma_N \vartheta_N$. This specification is reasonable as it is easier for frictions to affect the government's intervention when the intervention strategy requires more-intensive trading. Furthermore, the government can neither correct nor trade against its own noise, because the noise originates from its own system. Instead, the government can internalize the amount of noise by choosing its trading intensity ϑ_N. For now, we take ϑ_N as given. We analyze the optimal intervention choice in Section 5.

Several notable features of our setting merit discussion. First, we model government intervention as direct trading in asset markets to take advantage of the well-developed framework from the market microstructure literature. We nevertheless believe this framework is able to capture the implications for the broad-based policy interventions used by the Chinese

[10] This market breakdown hinges on the investors' myopia. When the investors have infinite trading horizons, a market equilibrium always exists but the volatility would still explode as noise-trader risk rises.

government, as summarized in Section 2. Second, in the absence of financial crises in our setting, our model does not capture government interventions aimed to bail out the market during crises. Instead, our specification of the government's linear intervention strategy is symmetric to both booms and busts and thus captures regular policy interventions adopted by the Chinese government to lean against market cycles.

As the government trades alongside investors to accommodate the trading of noise traders, the market-clearing condition $\int_0^1 X_t^i di + X_t^G + N_t = 0$ implies the following linear asset price function with the government noise as an additional factor:

$$P_t = \frac{1}{R^f - \rho_V} V_{t+1} + p_N N_t + P_g G_t.$$

The following proposition rules out other nonlinear price equilibria and characterizes this linear market equilibrium, with the proof given in the Online Appendix.

Proposition 2 *For a given intervention intensity $\vartheta_N < 1/\left(1+\sigma_G^2\right)$, government intervention stabilizes the market, with the asset return volatility decreasing in ϑ_N, the price informativeness increasing in ϑ_N, and the return volatility exploding to ∞ at a higher threshold for the noise-trader risk $\frac{1}{\sqrt{(1-\vartheta_N)^2+\vartheta_N^2\sigma_G^2}}\sigma_N^*$.*

Figure 1 depicts the effect of the government intervention in mitigating the volatility explosion relative to the case without government intervention. Also note that if $\vartheta_N > \frac{2}{1+\sigma_G^2}$, government trading actually makes the market even more volatile, as the government intervention also injects its own noise into the market. Taken together, countercyclical government interventions in asset markets help mitigate market volatility and ensure market stability. With informational frictions, however, the intervention to stabilize asset prices has additional effects on market dynamics, which we investigate in the next section.

4 An Extended Model with Information Frictions

We now extend the model to introduce realistic information frictions that investors and the government face in financial markets, while keeping the other features of the model the same as before. Specifically, we assume that the asset fundamental V_{t+1} and noise trading N_t are both unobservable at time t to all agents in the economy. For simplicity, we assume that the noise in government trading G_t is publicly observable at date t, albeit not before t. Since the government has no private information, this is equivalent to assuming that the scale of government intervention, X_t^G, is observable at date t.[11] As the government noise affects the asset price in equilibrium, investors have an incentive to acquire information about the next period's government noise. This extended model consequently allows us to analyze how government intervention interacts with both trading and information acquisition of investors, which ultimately affect the information efficiency of asset prices.

4.1 Information and Equilibrium

We first describe the information structure of the economy and the asset-market equilibrium.

Public information. All market participants observe the full history of all public information, which includes all past dividends, asset prices, and government noise:

$$\mathcal{F}_t^M = \{D_s, P_s, G_s\}_{s \leq t},$$

which we will hereafter refer to as the "market" information set. We define

$$\hat{V}_{t+1}^M = E\left[V_{t+1} \mid \mathcal{F}_t^M\right]$$

as the conditional expectation of V_{t+1} with respect to $\mathcal{F}_t^M$. The government needs to trade against noise trading based on its conditional expectation of N_t. At the risk of abusing notation, we define

$$\hat{N}_t^M = E\left[N_t \mid \mathcal{F}_t^M\right],$$

which represents expectations of the current-period N_t rather than N_{t+1}. We also define

$$\hat{G}_{t+1}^M = E\left[G_{t+1} \mid \mathcal{F}_t^M\right]$$

as the market's conditional expectations of the next-period G_{t+1}. These three belief variables, $\hat{V}_{t+1}^M$, $\hat{N}_t^M$, and $\hat{G}_{t+1}^M$, are time-t expectations of V_{t+1}, N_t, and G_{t+1}, respectively. Together with the publicly observed current-period G_t, they summarize the public information at time t regarding the aggregate state of the market. We collect these variables as a state vector:

$$\Psi_t = \left[\begin{array}{cccc} \hat{V}_{t+1}^M & \hat{N}_t^M & \hat{G}_{t+1}^M & G_t \end{array}\right].$$

[11] In an earlier draft of the paper, we analyzed the case with G_t being unobservable even after t. The results were qualitatively similar to our current setting, although the analysis is substantially more complex.

Government intervention. We assume that the government does not have any private information. Instead, at date t the government trades against noise traders based only on the publicly available information $\mathcal{F}_t^M$.[12] As before, we adopt the following intervention program, instituted to trade against the conditional market expectation $\hat{N}_t^M$:

$$X_t^G = -\vartheta_{\hat{N}} \hat{N}_t^M + \sqrt{Var\left[\vartheta_{\hat{N}} \hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]} G_t, \tag{2}$$

where $\vartheta_{\hat{N}}$ is the intensity of the government's intervention. We also extend the noise brought by the government intervention to be increasing with the conditional variance of government trading, $\sqrt{Var\left[\vartheta_{\hat{N}} \hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]}$, which is consistent with $\sigma_N \vartheta_N$ in the perfect-information case. In this section, we continue to take the government's intervention intensity $\vartheta_{\hat{N}}$ as given and focus on analyzing investors' information choice. We will analyze the government's intervention choice in the next section.

Investors' information choice. In each period, investors face uncertainty in the asset fundamental, the noise trading, and the government noise. Specifically, at date t, each investor can choose to acquire a private signal about either the next-period asset fundamental V_{t+1} or the next-period government noise G_{t+1}. We denote the investor's choice as $a_t^i \in \{0, 1\}$, with 1 representing the choice of a fundamental signal and 0 the choice of a signal about the government noise.[13] When the investor chooses $a_t^i = 1$, the fundamental signal is

$$s_t^i = V_{t+1} + 1/\sqrt{a_t^i \tau_s} \varepsilon_t^{s,i},$$

where $\varepsilon_t^{s,i} \sim \mathcal{N}(0, 1)$ is signal noise, independent of all other random variables in the setting, and τ_s represents the precision of the signal if chosen. When the investor chooses $a_t^i = 0$,

[12] In a previous draft, we adopted an alternative setting in which the government possesses private signals about the fundamental. This private information causes the government to hold different beliefs about the fundamental and noise trading from investors and, more importantly, makes the government's trading not fully observable to the investors. Through this latter channel, the noise in the government's signals endogenizes the government's intervention noise G_t. Such a structure substantially complicates the analysis by introducing a double learning problem for the investors to acquire information about the government's belief, which is itself the outcome of a learning process. It is reassuring that this more elaborate setting gives similar results as in our current setting with exogenous government intervention noise.

[13] Generally speaking, the investors may also acquire private information about noise trading rather than asset fundamentals and government noise. Introducing such a third type of private information would complicate the analysis without any particular gain in economic insight. In our current setting, each investor can indirectly infer the value of noise trading through the publicly observed asset price. See, for instance, Ganguli and Yang (2009) for a setting in which investors can learn either about the asset fundamental or noise trading.

the government signal is

$$g_t^i = G_{t+1} + 1/\sqrt{(1-a_t^i)\,\tau_g}\varepsilon_t^{g,i},$$

where $\varepsilon_t^{g,i} \sim \mathcal{N}(0,1)$ is signal noise, independent of all other random variables in the setting, and τ_g represents the precision of the signal if chosen. These signals allow the investor to better predict the next-period asset return by forming more-precise beliefs about V_{t+1} and G_{t+1}. Motivated by limited investor attention and a realistic fixed cost in information acquisition, we assume that each investor chooses one and only one of these two signals.[14]

At date t, each investor first makes his information acquisition choice a_t^i based on the public information set $\mathcal{F}_{t-1}^M$ from the previous period. After receiving his private information $a_t^i s_t^i + (1-a_t^i)\, g_t^i$ and the public information D_t, P_t, and G_t released during the period, the investor chooses his asset position X_t^i to maximize his expected utility:

$$U_t^i = \max_{a_t^i \in \{0,1\}} E\left[\max_{X_t^i} E\left[-\exp\left(-\gamma W_{t+1}^i\right) \mid \mathcal{F}_t^i\right] \middle| \mathcal{F}_{t-1}^M\right],$$

where the investor's full information set $\mathcal{F}_t^i$ is

$$\mathcal{F}_t^i = \mathcal{F}_t^M \vee \left\{a_t^i, a_t^i s_t^i + \left(1-a_t^i\right) g_t^i\right\}.$$

Noisy rational expectations equilibrium. Market clearing of the asset market requires that the net demand from the investors and the government be equal to the supply of the noise traders at each date t: $\int_0^1 X_t^i di + X_t^G + N_t = 0$. By assuming elastic supply of riskless debt, the credit market clears automatically.

We also assume that the investors and the government have an initial prior with Gaussian distributions at $t=0$: $(V_0, N_0) \sim \mathcal{N}\left(\left(\bar{V}, \bar{N}\right), \Sigma_0\right)$, where $\Sigma_0 = \begin{bmatrix} \Sigma_0^V & 0 \\ 0 & \Sigma_0^N \end{bmatrix}$. Note that the variables in both $\mathcal{F}_t^M$ and $\mathcal{F}_t^i$ all have Gaussian distributions. As a result, conditional beliefs of the investors and the government about V_t and N_t under any of the information sets are always Gaussian. Furthermore, the variances of these conditional beliefs follow deterministic dynamics over time and will converge to their respective steady-state levels at exponential rates. Throughout our analysis, we will focus on steady-state equilibria, in which the belief

[14] Instead of a discrete information acquisition choice $a \in \{0,1\}$, one could generalize our framework to allow for a continuous choice $a \in [0,1]$, which corresponds to a signal that is partially informative about both the fundamental and the government noise. We conjecture that, in such a setting, instead of having a government-centric outcome, investors would nevertheless tilt their information acquisition too much toward acquiring government information, when the government's objective is to minimize price volatility.

variances of the government and investors have reached their respective steady-state levels and their policies are time homogeneous.

At time t, a noisy rational expectations equilibrium is a list of policy functions: $a^i(\Psi_{t-1})$, and $X^i(\Psi_t, a_t^i, a_t^i s_t^i + (1-a_t^i) g_t^i, P_t)$, and a price function $P(\Psi_t, V_{t+1}, N_t, G_{t+1})$, which jointly satisfy the following:

- Investor optimization: each investor i takes as given the government's intervention strategy $\vartheta_{\hat{N}}$ to make his information acquisition choice $a_t^i = a^i(\Psi_{t-1})$ based on his ex ante information set $\mathcal{F}_{t-1}^M$ and then makes his investment choice $X^i(\Psi_t, a_t^i, a_t^i s_t^i + (1-a_t^i) g_t^i, P_t)$ based on other investors' information acquisition choices $\{a_t^{-i}\}_{-i}$ and his full information set $\mathcal{F}_t^i$.

- Market clearing:

$$\int_0^1 X^i\left(\Psi_t, a_t^i, a_t^i s_t^i + \left(1-a_t^i\right) g_t^i, P_t\right) di + X^G(\Psi_t) + N_t = 0.$$

- Consistency: investor i and the government form their expectations of V_{t+1}, G_{t+1}, and N_t based on their information sets $\mathcal{F}_t^i$ and $\mathcal{F}_t^M$, respectively, according to Bayes' Rule.

4.2 The Equilibrium

We restrict our attention to covariance-stationary linear equilibria. We analyze the equilibrium by describing its key elements to convey the key economic mechanism of the model. The complete steps of deriving the equilibrium and formulas are in Appendix A.

4.2.1 Price Conjecture and Equilibrium Beliefs

With government intervention introducing noise into the equilibrium asset price as an additional factor, each investor faces a nontrivial choice at date t in whether to acquire private information about either the next-period fundamental V_{t+1} or government noise G_{t+1}. When all investors choose to acquire information about the government noise, the asset price does not aggregate any private information about V_{t+1} but rather brings the next-period government noise G_{t+1} into the current-period asset price. To analyze the equilibrium asset price, we begin by conjecturing a linear price function:[15]

$$P_t = \frac{1}{R^f - \rho_V}\hat{V}^M_{t+1} + p_g G_t + p_{\hat{G}}\hat{G}^M_{t+1} + p_V\left(V_{t+1} - \hat{V}^M_{t+1}\right) + p_G\left(G_{t+1} - \hat{G}^M_{t+1}\right) + p_N N_t. \quad (3)$$

The first term $\frac{1}{R^f - \rho_V}\hat{V}^M_{t+1}$ is the expected asset fundamental conditional on the market information $\mathcal{F}^M_t$ at date t, the term $p_g G_t$ reflects the noise introduced by the government into the asset demand in the current period, while the term $p_{\hat{G}}\hat{G}^M_{t+1}$ reflects the market expectation of the government noise in the next period. These three pieces serve as anchors in the asset price based on the public information. The fourth term $p_V\left(V_{t+1} - \hat{V}^M_{t+1}\right)$ captures the fundamental information aggregated through the investors' trading. Following the insight from Hellwig (1980), if each investor acquires a private signal about the asset fundamental V_{t+1}, their trading aggregates their private signals and allows the equilibrium price to partially reveal V_{t+1}. If all investors choose to acquire information about the next-period government noise G_{t+1}, instead of V_{t+1}, the coefficient of this term p_V would be zero. Instead, their trading aggregates their private information about G_{t+1}, as captured by the fifth term $p_G\left(G_{t+1} - \hat{G}^M_{t+1}\right)$.[16] The final term $p_N N_t$ represents the price impact of noise trading.

Given the asset price in (3), in order to predict the asset return, an individual investor needs to infer not only the asset fundamental, V_{t+1}, but also the government noise, G_{t+1}. As each individual investor has a piece of a private signal, $a^i_t s^i_t + (1 - a^i_t)\ g^i_t$, his learning process simply requires adding this additional signal to the market beliefs. We summarize the filtering process through the updating equation as

$$\begin{aligned}\begin{bmatrix}\hat{V}^i_{t+1}\\ \hat{G}^i_{t+1}\end{bmatrix} &= \begin{bmatrix}\hat{V}^M_{t+1}\\ \hat{G}^M_{t+1}\end{bmatrix} + Cov\left\{\begin{bmatrix}V_{t+1}\\ G_{t+1}\end{bmatrix}, a^i_t s^i_t + \left(1 - a^i_t\right) g^i_t \middle|\ \mathcal{F}^M_t\right\}\\ &\quad \cdot Var\left\{a^i_t s^i_t + \left(1 - a^i_t\right) g^i_t \middle|\ \mathcal{F}^M_t\right\}^{-1}\left[a^i_t\left(s^i_t - \hat{V}^M_{t+1}\right) + \left(1 - a^i_t\right)\left(g^i_t - \hat{G}^M_{t+1}\right)\right].\end{aligned}$$

The variance and covariance in this expression depend on various endogenous objects such as the informativeness of the equilibrium asset price and the precision of the market beliefs, and are fully derived in Appendix A. This expression makes clear that the investor's private signal helps him infer the asset fundamental or the government's trading noise in the next period, both of which impact the asset return.

[15] This conjectured functional form is not unique because the market's beliefs about V_{t+1}, N_t, and G_{t+1} are correlated objects after observing the asset price. That is, $\hat{N}^M_t$ can be replaced by a linear combination of P_t, $\hat{V}^M_{t+1}$, and $\hat{G}^M_{t+1}$ and as such does not have to appear in the price function, even though $\hat{N}^M_t$ determines the government's intervention.

[16] There is no need to incorporate a term related to investors' (higher order) cross-beliefs about V_{t+1} or G_{t+1} because $\int_0^1 a^i_t s^i_t di = V_{t+1}$ and $\int_0^1 \left(1 - a^i_t\right) g^i_t di = G_{t+1}$ by the Weak Law of Large Numbers.

4.2.2 Information Choice

To analyze an individual investor's information choice, it is convenient to decompose the expected asset return based on his information set relative to the market information set. We can update $E\left[R_{t+1} \mid \mathcal{F}_t^i\right]$ from $E\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ by the Bayes' Rule according to

$$\begin{aligned} & E\left[R_{t+1} \mid \mathcal{F}_t^i\right] = E\left[R_{t+1} \mid \mathcal{F}_t^M \vee a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i\right] \\ = \; & E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \frac{CoV\left[R_{t+1}, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i \mid \mathcal{F}_t^M\right]}{Var\left[a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i \mid \mathcal{F}_t^M\right]} \cdot \left[a_t^i \left(s_t^i - \hat{V}_{t+1}^M\right) + \left(1 - a_t^i\right)\left(g_t^i - \hat{G}_{t+1}^M\right)\right]. \end{aligned}$$

The investor's private information through either s_t^i or g_t^i helps him better predict the excess asset return relative to the market information. Given the investor's myopic CARA preferences, his demand for the asset is

$$X^i = \frac{1}{\gamma} \frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}. \tag{4}$$

In choosing whether to acquire either s_t^i or g_t^i at date t, the investor maximizes his expected utility based on the ex ante market information:

$$E\left[U_t^i \mid \mathcal{F}_{t-1}^M\right] = \max_{a_t^i \in \{0,1\}} -E\left\{E\left[\exp\left(-\gamma R^f \bar{W} - \frac{1}{2} \frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]^2}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}\right) \middle| \mathcal{F}_t^M\right] \middle| \mathcal{F}_{t-1}^M\right\},$$

which has already incorporated the investor's optimal asset position in (4).

The investor's expected CARA utility in our Gaussian framework is fully determined by the second moment of the return distribution conditional on his information set $\mathcal{F}_t^i$. This nice feature allows us to simplify his information choice to

$$a_t^i = \arg \max_{a_t^i \in \{0,1\}} -Var\left[R_{t+1} | \mathcal{F}_t^M, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i, a_t^i\right]. \tag{5}$$

This objective involves only minimizing the conditional price change variance, which is stationary in the steady-state equilibria that we consider. Therefore, the information acquisition choice faced by each individual investor is time-invariant. By noting that

$$Var\left[R_{t+1} \mid \mathcal{F}_t^M, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i\right] = Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] - \frac{CoV\left[R_{t+1}, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i \mid \mathcal{F}_t^M\right]},$$

we arrive at the following proposition, which corresponds to Proposition A7 in Appendix A.

Proposition 3 *At date t, investor i chooses to acquire information about the next-period fundamental V_{t+1} if $\frac{CoV\left[R_{t+1}, g_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[g_t^i \mid \mathcal{F}_t^M\right]} < \frac{CoV\left[R_{t+1}, s_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[s_t^i \mid \mathcal{F}_t^M\right]}$ and about the next-period government noise G_{t+1} otherwise.*

The investor chooses his signal to maximize his informational advantage over the public information set when trading. Proposition 3 states that this objective is equivalent to choosing the signal that leads to a greater reduction in the conditional variance of the excess asset return. The investor may choose to acquire the signal on the government noise over the signal on the asset fundamental, because the government noise affects the asset return when the investor sells his asset holding on the next date. As a result, the more the government noise covaries with the unpredictable component of the asset return from the market information set, the more valuable the signal about the government noise is to the investor.

In models of information aggregation, such as in Grossman and Stiglitz (1980) and Hellwig (1980), investors' information choices are typically strategic substitutes. That is, all else being equal, if some investors at time t acquire private information about V_{t+1}, then the equilibrium asset price at time t will become more informative about it, and this *reduces* the incentives of other investors to acquire information about V_{t+1}. In models in which investors can acquire different sources of information, including those in Ganguli and Yang (2009), Manzano and Vives (2011), and Farboodi and Veldkamp (2016), information choices can exhibit intratemporal strategic complementarity. As some investors learn more about one asset fundamental, e,g. cash flow news or noise trading, asset prices become more informative about that fundamental and less informative about others. This reduced informativeness *strengthens* the incentive of investors to acquire information about these other fundamentals.

Interestingly, our model features intertemporal complementarity between investors' information choices and government policy across periods. For instance, similar to Froot, Scharfstein, and Stein (1992), investors have incentive to align their information choices across generations when the asset fundamental is persistent.[17] From (5), an investor will learn about whichever information provides the higher reduction in return variance, which is determined by the extent to which that information is reflected in the price in (3). If more investors at time $t+1$ acquire information about V_{t+2}, then p_V is larger at time $t+1$ and there is greater incentive for investors at time t to acquire information about V_{t+1}, as V_{t+2} partially reflects V_{t+1}. Novel to our setting, however, is that there is also intertemporal complementarity between the government's announced intervention policy at time $t+1$ and the investors' choice to learn about G_{t+1} at date t, because the government is a large trader

[17] This intertemporal complementarity does not operate through the government policy noise, G_t, because it is independent over time. If we were to relax this simplifying assumption, as we did in a previous version of the paper, the model will display even stronger complementarity in investors' information choices.

with a price impact. If the government trades more intensively at time $t+1$ (a larger $|\vartheta_{\hat{N}}|$), then p_g is larger at time $t+1$ and there is greater incentive for investors at time t to acquire information about G_{t+1}.

Importantly, the government internalizes that it can influence the investors' information choices when choosing its policy.[18] In contrast to Hellwig and Veldkamp (2009), in which intratemporal complementarity in agents' actions leads to complementarity in their information choices, here the government's future intervention policy incentivizes investors today to learn about future noise in government intervention because the government's policy materially impacts their return from trading the risky asset. This complementarity can be sufficiently strong to dominate the substitution effect in information choice across investors and to lead all of them to acquire private information about the same variable.

The choice of an individual investor to acquire information about the government noise rather than the asset fundamental introduces an externality for the overall market. When investors devote their limited attention to do so, less information about the asset fundamental is imputed into the asset price, which causes the asset price to be a poorer signal about the asset fundamental. In addition, as investors devote attention to learning about G_{t+1}, the asset price will aggregate more of the investors' private information about G_{t+1}, causing the next-period government noise to impact the current-period asset price. In this sense, the investors' speculation of government noise may exacerbate its impact on asset prices.

4.2.3 Market Equilibrium

Given the investors' optimal information and asset choices and the government's intervention strategy, we have the following market-clearing condition:

$$
\begin{aligned}
0 \;=\;& N_t - \vartheta_{\hat{N}}\hat{N}_t^M + \sqrt{Var\left[\vartheta_{\hat{N}}\hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]}G_t + \int \frac{a_t^i}{\gamma}\frac{E\left[R_{t+1} \mid \mathcal{F}_t^M, s_t^i\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^M, s_t^i\right]}di \\
&+ \int \frac{1-a_t^i}{\gamma}\frac{E\left[R_{t+1} \mid \mathcal{F}_t^M, g_t^i\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^M, g_t^i\right]}di.
\end{aligned}
$$

The Weak Law of Large Numbers implies that aggregating the investors' asset positions will partially reveal their private information about V_{t+1} if $\int a_t^i di = 1$ and G_{t+1} if $\int a_t^i di = 0$. By

[18] This is also in contrast to the literature on information aggregation with strategic traders, as in, for instance, Kyle (1989). Since the solution concept in these models is an "equilibrium in demand curves," large traders do not internalize that they can impact the learning and information decisions of other large traders. As such, these equilibria are ex post efficient up to the impact of market power.

matching the coefficients of all the terms on both sides of this equation, we obtain a set of equations to determine the coefficients of the conjectured equilibrium price function in (3).

Several types of equilibrium can exist.

- **Fundamental-centric outcome**. When all investors choose to acquire information about the asset fundamental, the asset price aggregates the investors' private information and partially reflects the asset fundamental but does not reflect the next-period government noise. As a result, the asset price takes a particular form of

$$P_t = \frac{1}{R^f - \rho_V}\hat{V}^M_{t+1} + p_g G_t + p_V\left(V_{t+1} - \hat{V}^M_{t+1}\right) + p_N N_t, \tag{6}$$

  which is different from the general asset price specification in (3) in that the terms $p_{\hat{G}}\hat{G}^M_{t+1}$ and $p_G\left(G_{t+1} - \hat{G}^M_{t+1}\right)$ do not appear.

- **Government-centric outcome**. When all investors choose to acquire information about the next-period government noise, the asset price partially reflects the next-period government noise but not the asset fundamental:

$$P_t = \frac{1}{R^f - \rho_V}\hat{V}^M_{t+1} + p_g G_t + p_{\hat{G}}\hat{G}^M_{t+1} + p_G\left(G_{t+1} - \hat{G}^M_{t+1}\right) + p_N N_t,$$

  where the term $p_V\left(V_{t+1} - \hat{V}^M_{t+1}\right)$ does not appear.

- **Mixed outcome**. It is also possible to have a mixed equilibrium with a fraction of the investors acquiring information about the asset fundamental and the others having information about the government noise. In such a mixed equilibrium, the general price function specified in (3) prevails.

Depending on the model parameters, there can be multiple equilibria as all three types of equilibrium may appear. For simplicity, we omit discussion of mixed equilibria in the sequel. In the presence of multiple equilibria, we assume the government, as a large agent, has the capacity to select the equilibrium most desirable to its objective.

In the special case that the fundamental V_t is i.i.d., or $\rho_V = 0$, the following proposition establishes a necessary and sufficient condition for the government-centric equilibrium to occur for a given government intervention intensity $\vartheta_{\hat{N}}$.

Proposition 4 *Suppose $\rho_V = 0$, and fix a government intervention intensity $\vartheta_{\hat{N}}$. A government-centric equilibrium exists under a necessary and sufficient condition:*

$$\begin{aligned} & \frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}} - \sqrt{\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{c}} \\ \geq \; & \frac{\sigma_V^2}{\sqrt{\sigma_V^2+\tau_s^{-1}}}\left(1+x\right)\sqrt{\left(\sigma_G^2+\left(1+x\right)\tau_g^{-1}\right)\left(\frac{\frac{1-\vartheta_{\hat{N}}}{\vartheta_{\hat{N}}}}{\sigma_G^2 - R^f\frac{x}{1-\vartheta_{\hat{N}}}}\right)^2}, \end{aligned} \tag{7}$$

where x is given by

$$x\left(1+x\right)^3 = \left(\frac{\vartheta_{\hat{N}}}{R^f}\sigma_G^3\right)^2,$$

and c is a nonnegative function of $\left\{\vartheta_{\hat{N}}, R^f, \sigma_G\right\}$ given in the Online Appendix. This equilibrium is more likely to exist the higher σ_N^2 and σ_D^2 are, and it always exists for σ_V^2 that is sufficiently small.

In a government-centric equilibrium, the asset price P_t aggregates only private information about the future noise in government trading, G_{t+1}. In this situation, all investors are willing to acquire information about G_{t+1} if it reduces their conditional uncertainty about the future price, P_{t+1}, which contains G_{t+1} through the government's trading, more than would learning about the fundamental, V_{t+1}. Proposition 4 reveals that this can occur for two reasons. The first is that the benefit to learning about the fundamental, as measured by its uncertainty, σ_V^2, is small. The second is that the benefit to learning about the future noise in the government's trading is large. The larger the noise in prices from noise trading, $p_N\sigma_N$ (which is the left-hand side of (7)), the less aggregated private information about G_{t+1} is revealed by the price, and the more motivated investors are to acquire private information about G_{t+1}. Since $p_N\sigma_N$ is increasing in the uncertainty about noise trading and the unlearnable part of the dividend, σ_N^2 and σ_D^2, respectively, a government-centric equilibrium is more likely to occur the larger σ_N^2 and σ_D^2 are.

4.3 Consequences of Government Intervention

This subsection analyzes how government intervention affects the market dynamics. For comparison, we also include a benchmark case without government intervention, which corresponds to the classic Hellwig (1980) equilibrium, in which each investor acquires a fundamental signal, and the equilibrium asset price follows the form in (6). Proposition A1 in the

Table I: Baseline Model Parameters

Government:	$\gamma_\sigma = 1.25,\ \gamma_V = 1,\ \sigma_G^2 = 2$
Asset Fundamental:	$\rho_V = 0.75,\ \sigma_V^2 = 0.01,\ \sigma_D^2 = .8$
Noise Trading:	$\sigma_N^2 = 0.2$
Investors:	$\gamma = 1,\ \tau_s = 500,\ \tau_g = 500,\ R^f = 1.01$

Appendix characterizes the Hellwig equilibrium and, in particular, shows that information frictions reduce the critical level of noise-trader risk so that the market is more likely to break down. Proposition A2 further shows that when an equilibrium exists, asset return volatility is higher and price efficiency is lower in the presence of information frictions.

We analyze the effects of government intervention through a series of numerical examples, based on a set of baseline parameter values listed in Table I. Figure 2 illustrates how the asset market dynamics vary with a given intensity $\vartheta_{\hat{N}}$ of the government intervention. As we will discuss in the next section, the government can choose an optimal level of intervention intensity to accomplish a certain policy objective. Panels A and B depict the conditional asset return variance $Var\left[R_t\left(\vartheta_{\hat{N}}\right)|\mathcal{F}_{t-1}^M\right]$ and the conditional asset price deviation from fundamental $Var\left[P_t\left(\vartheta_{\hat{N}}\right) - \frac{1}{R^f-\rho_V}V_{t+1}\ |\mathcal{F}_{t-1}^M\right]$, our price efficiency measures, respectively.

As the government gradually increases its intervention intensity $\vartheta_{\hat{N}}$ from zero, investors continue to acquire information about the fundamental. In this fundamental-centric equilibrium, both conditional price variance and conditional price deviation from the fundamental drop from their respective values in the Hellwig benchmark, confirming the common wisdom that, by leaning against noise traders, government intervention ensures financial stability and improves price efficiency.

More surprising, Figure 2 shows that by trading more aggressively against noise traders, ensuring financial stability and improving price efficiency are not always consistent with each other, which is a key insight of our model. Specifically, as $\vartheta_{\hat{N}}$ exceeds 0.22, a government-centric equilibrium emerges with all investors choosing to acquire information about the government noise. When the market transitions from the fundamental-centric equilibrium to the government-centric equilibrium, the asset price variance slumps downward, indicating

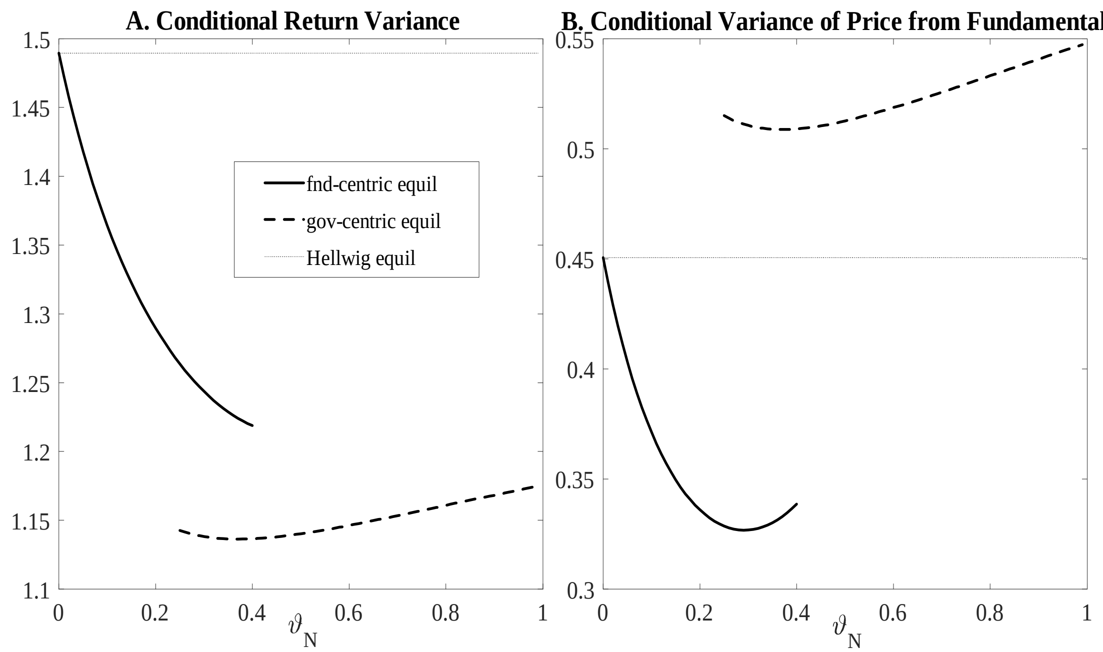

Figure 2: Equilibrium dynamics across intervention intensity $\vartheta_{\hat{N}}$. Panel A depicts the conditional return variance $Var\left[R_t\left(\vartheta_{\hat{N}}\right)|\mathcal{F}_{t-1}^M\right]$ and Panel B the conditional variance of price deviation from the fundamental $Var\left[P_t\left(\vartheta_{\hat{N}}\right) - \frac{1}{R^f-\rho_V}V_{t+1}\ |\mathcal{F}_{t-1}^M\right]$.

that government intervention is able to further mitigate the price effect of noise traders. The conditional variance of the price deviation from its fundamental value jumps up, however, suggesting that price efficiency is reduced rather than improved. This occurs because intensive government intervention makes government noise an important factor in asset returns, which, in turn, diverts investor attention from acquiring fundamental information to acquiring information about future government noise. Panel B shows that when this happens, price efficiency can become even worse than the benchmark case without government intervention. Interestingly, this tension between return volatility and price informativeness based on investor information acquisition is distinct from that in Davila and Parlatore (2019), who show that whether the two positively or negatively comove depends on how much information has already been aggregated in the asset price.

Figure 2 also shows a more subtle implication of our model: the government-centric equilibrium may allow the government to more effectively reduce the price impact of noise traders without trading more. When the intervention intensity $\vartheta_{\hat{N}}$ is in an intermediate range between 0.22 and 0.40, both the fundamental-centric and the government-centric equilibria exist[19] as a result of the aforementioned intertemporal complementarity in investors' information choices.[20] Comparing these two equilibria for a given level of intervention intensity shows that asset price volatility is substantially lower in the government-centric equilibrium without requiring more government trading. This occurs because, in the fundamental-centric equilibrium, each investor has his own private information about the asset fundamental, and the private information causes investors to hold beliefs different from each other and from the government about not only the asset fundamental but also the current-period noise trading. As a result, the government has to trade against not only noise traders but also investors. Investors' trading aggregates their private fundamental information into the asset price and improves its information efficiency, but partially offsets the government's effort to counter noise traders. In contrast, in the government-centric equilibrium, investors' private information is about the next-period government noise, and, like the government, investors all use the same public information to infer the current-period noise trading. Consequently, investors tend to trade against noise traders along the same direction as the government, thereby reinforcing the effectiveness of the government's intervention in reducing volatility. This mechanism further highlights the tension between reducing price volatility and improving price efficiency.

5 Optimal Intervention Policy

In this section, we discuss the objective of government intervention and analyze the resulting optimal intervention policy. We first expand the model setting to provide a micro-founded welfare objective for the government. We then contrast the optimal intervention policy of the Chinese government with that of Western governments based on underlying differences in the economic conditions of their respective economies. In our analysis, we assume that the government, as a large player in financial markets, has the capacity to select the equilibrium that maximizes its objective in the presence of multiple equilibria among investors.

[19] A mixing equilibrium is also possible when $\vartheta_{\hat{N}}$ is in this range. As discussed in Section 4.2.3, we omit discussion of mixed equilibria for simplicity.

[20] The presence of this strong intertemporal complementarity also implies that even if each investor is free to choose a mixed signal that is partially informative about the asset fundamental and the government noise (as discussed in footnote 14), the investor may nevertheless choose to acquire a pure signal about either the asset fundamental or the government noise.

5.1 Social Welfare

In this subsection, we provide a welfare analysis of government intervention by expanding the model setting to include four groups of agents: investors, noise traders, entrepreneurs, and taxpayers. For simplicity, we assume that these four groups do not overlap. All agents are risk-averse and have CARA utility with a common coefficient of absolute risk aversion γ. For ease of exposition, we relegate the full model setting to Appendix B and provide only a brief introduction of the four groups here:

- Investors follow directly from the main model in Section 4, and their expected utility in each period is derived in (A9).

- We microfound noise traders as discretionary liquidity traders, in a manner similar to Han, Tang, and Yang (2016), to explicitly account for their welfare from trading. These liquidity traders participate in asset-market trading to receive a hedging benefit by submitting correlated market orders of a random size in each period. We derive their expected utility in each period in (A10).

- We also introduce a group of entrepreneurs who can invest in risky projects whose payoffs are correlated with the traded asset. As a result, these entrepreneurs benefit from extracting useful information from the asset price. We show in (A11) that their expected utility is decreasing in $\Sigma^{M,VV}$, the conditional variance of the asset fundamental based on each period's public information, and σ_y^2, the variance of project-specific noise. Note that $\Sigma^{M,VV}$ is inversely related to the informativeness of the asset price. As project-specific noise, σ_y^2, rises, the usefulness of the asset price signal, that is, the impact of $\Sigma^{M,VV}$ on entrepreneurs' welfare, declines.

- Taxpayers are the residual claimants to the government's trading profits. Their expected utility from the government's trading profit in each period is given in (A12).

We assume that the government maximizes the Nash social welfare function proposed by Kaneko and Nakamura (1979), which is a monotonic transformation of the product of the utilities of all agents in the economy. As specified in (A13), this welfare function is essentially given by the sum of the logarithmic expected utilities of the four aforementioned groups. As each group has CARA utility and Gaussian-distributed payoffs, its logarithmic expected utility is the sum of its expected profit and a utility penalty for risk that is decreasing in the conditional payoff variance. As the asset-market trading is a zero-sum game among investors, liquidity traders, and taxpayers, we are able to establish the following proposition for the objective function of government intervention, which is fully determined by the second moments of market beliefs and the asset return:

Proposition 5 *The government chooses its intervention intensity $\vartheta_{\hat{N}}$ to maximize*

$$\sup_{\vartheta_{\hat{N}}} \quad \frac{\sigma_V^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV}+\sigma_y^2} - \frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} - \gamma^2\left(\sigma_N^2+\sigma_n^2+\vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)\left(\sigma_N^2-\Sigma^{M,NN}\right)\right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right], \tag{8}$$

where $\frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}$ *and* $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ *are given in Appendix B.*

The social welfare derived in Proposition 5 contains three components. The first component captures the entrepreneurs' production efficiency. Entrepreneurs use the asset price and other public information to improve their inference of the payoffs from their risky projects and thus to make more-efficient production decisions. As a result, the first component is decreasing with $\Sigma^{M,VV}$, the conditional variance of the asset fundamental based on the public information in each period. Higher informativeness of the asset price implies a lower value of $\Sigma^{M,VV}$, thus improving entrepreneurs' production efficiency. Furthermore, the first component is also decreasing with σ_y^2, the variance of project-specific noise faced by entrepreneurs. A higher value of σ_y^2 makes the fundamental information conveyed by the asset price less relevant to entrepreneurs' production decisions.

The second component in Proposition 5 represents the trading risk borne by investors. As the trading gains/losses are transfers between investors and other market participants, the expected gains of investors due to their private information do not enter social welfare and the government's objective. Trading, however, exposes investors to risk, and such risk affects investors' expected utility and thus affects social welfare. Intuitively, this component is inversely related to investors' information advantage relative to the market $Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right] / Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]$. This is because more information advantage induces investors to trade more aggressively and therefore bear more risk.

The third component in Proposition 5 represents the trading risk faced by noise traders and taxpayers. This component is decreasing in $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ (note $\Sigma^{M,NN} \leq \sigma_N^2$), the asset return volatility conditional on public information, and increasing in the conditional uncertainty about noise trading, $\Sigma^{M,NN}$, because the government trades more when there is more-precise information about noise trader demand.

The social welfare in (8) depends on a number of second moments. Proposition 6 further shows that it is ultimately determined by two sufficient statistics.

Proposition 6 *The government's welfare objective in (8) reduces to targeting two sufficient statistics based on each period's public information: 1) the conditional asset return volatility, $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$; and 2) the posterior uncertainty about the asset fundamental, $\Sigma^{M,VV}$, in a fundamental-centric equilibrium, or the posterior uncertainty about future policy noise, $\Sigma^{M,GG}$, in a government-centric equilibrium.*

These sufficient statistics are closely related to two widely recognized intervention objectives by policymakers—reducing asset return volatility and improving market efficiency. Reducing asset return volatility $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ is consistent with attenuating the risk premia required by market participants, as captured by our microfounded model through the third component of the welfare objective, and, more generally, the destabilizing effects of asset price volatility on leveraged investors and firms, as suggested by Brunnermeier and Pedersen (2009) and Geanakoplos (2010). As the asset price is part of the public information set, reducing $\Sigma^{M,VV}$ is equivalent to making asset prices more informative and consequently more efficient in guiding resource allocation in the economy, as reviewed by Bond, Edmans, and Goldstein (2012) and captured by our model through the first component of the welfare objective. Proposition 6 thus establishes that the social welfare objective in (8) is traced to two key policy targets—financial stability and economic growth, with the weights determined by the underlying parameters governing the economy.[21]

Reducing market volatility and improving market efficiency are often viewed as congruent objectives because an intervention strategy of leaning against noise trading reduces the impact of noise trading on asset prices, which should reduce both return volatility and improve asset price informativeness. Since return volatility is much easier to measure in practice

[21] In the government-centric equilibrium, the posterior uncertainty about future policy noise, $\Sigma^{M,GG}$, is determined by the government's intervention intensity $\vartheta_{\hat{N}}$ through the following implicit function:

$$\Sigma^{M,GG} = \frac{\sigma_G^2}{1 + \left(\frac{\vartheta_{\hat{N}}}{R^f} \frac{\Sigma^{M,GG}}{\Sigma^{M,GG} + \tau_g^{-1}}\right)^2 \Sigma^{M,GG}},$$

and is independent of the asset fundamental and noise-trading. As such, it regulates the level of government intervention rather than introduces a third welfare objective.

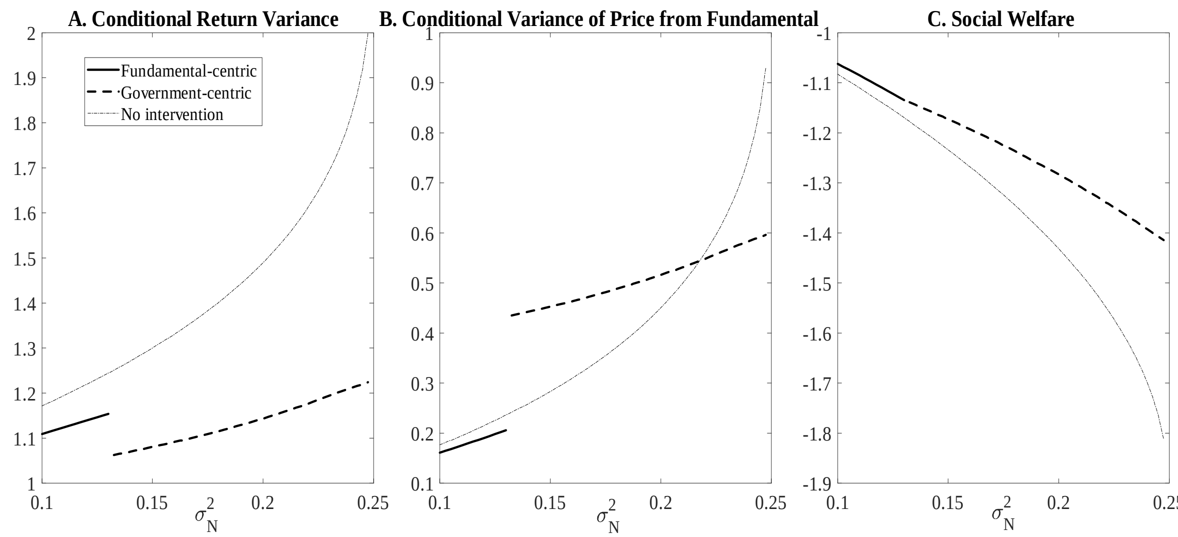

Figure 3: Equilibrium dynamics across σ_N^2 when the government maximizes social welfare. Panel A depicts the conditional return variance $Var\left[R_t\left(\vartheta_{\hat{N}}\right) \mid \mathcal{F}_{t-1}^M\right]$, Panel B the conditional variance of price deviation from the fundamental $Var\left[P_t\left(\vartheta_{\hat{N}}\right) - \frac{1}{R^f-\rho_v}v_{t+1} \mid \mathcal{F}_{t-1}^M\right]$, and Panel C the social welfare. In each panel, the dotted line represents the Hellwig equilibrium without government intervention, the solid line the fundamental-centric equilibrium, and the dashed line the government-centric equilibrium, based on the parameters in Table I and $\sigma_y^2 = 0.06$, $\sigma_n^2 = 0$.

than the market efficiency of asset prices, policymakers often view reducing price volatility as the more operational intervention objective (e.g., Stein and Sundarem (2018)). In the presence of investors' information choices, however, these two reduced-form objectives may be at variance with each other. We shall therefore examine how they relate to each other as the government chooses an intervention policy to maximize the welfare objective in (8).

In Figure 3, we illustrate how the government's optimal intervention policy varies with noise-trader risk, σ_N^2. Panel A depicts the conditional return variance $Var\left[R_t \mid \mathcal{F}_{t-1}^M\right]$, Panel B depicts the conditional variance of the asset price deviation from its fundamental value $Var\left[P_t\left(\vartheta_{\hat{N}}\right) - \frac{1}{R^f-\rho_v}v_{t+1} \mid \mathcal{F}_{t-1}^M\right]$, and Panel C depicts the social welfare. As a benchmark, we use a dotted line in each panel to represent the Hellwig equilibrium without government intervention. As σ_N^2 rises, both the asset return variance and the variance of price deviation from the fundamental rise, while social welfare deteriorates.

In the presence of government intervention, the government chooses a modest intervention policy when σ_N^2 is below a threshold level around $\sigma_N^2 = 0.13$, so that the asset market remains in a fundamental-centric equilibrium. This is represented by the solid line in each panel. In this region, government intervention reduces asset return variance and improves asset price efficiency relative to the Hellwig benchmark without government intervention. Interestingly, as σ_N^2 rises above the critical level, the government intervenes more intensively and, as a result, the asset market shifts to a government-centric equilibrium, represented by the dashed line in each panel. The intensive government intervention causes a discrete drop in the asset return variance around the critical level. There is also, however, a sharp upward jump in the conditional variance of the deviation of the asset price from its fundamental value to a level even higher than in the Hellwig benchmark. This reveals that intensive government intervention can actually worsen market efficiency and, compared to Figure 2, can represent an optimal trade-off of government policy. Despite the discontinuity in both asset return volatility and price efficiency around the critical level in σ_N^2, social welfare is continuous, reflecting the government's balancing of these two objectives in optimizing social welfare.

Taken together, the panels in Figure 3 show that reducing price volatility by targeting noise-trader risk is not equivalent to improving price efficiency in the presence of investors' information choices. When the government intervention is sufficiently intensive, it would eventually cause investors to divert their attention to government noise and away from asset fundamentals. To the extent that these two objectives are not fully congruent with each other, neither can serve as a sufficient statistic for social welfare (the ultimate policy objective).

5.2 China's Approach to Financial Market Intervention

The Chinese government has announced multiple goals for its financial policies (e.g., Amstad, Sun, and Xiong (2020)), which include the two policy objectives captured in our model—maintaining financial stability and stimulating economic growth. Although Chinese policymakers do not provide explicit weights on these policy objectives, our microfoundation of social welfare derives the optimal weights as functions of the underlying parameters that govern the economy. Figure 3 illustrates two sharply different regimes of government intervention policies, which are conveniently represented by the fundamental-centric and government-centric equilibria. The government-centric equilibrium features regular and intensive intervention to lean against noise traders, which largely mitigates asset price volatility yet diverts investor attention from asset fundamentals toward noise in government policy implementation. In contrast, the fundamental-centric equilibrium features modest intervention, which mitigates asset price volatility without diverting investor attention away from asset

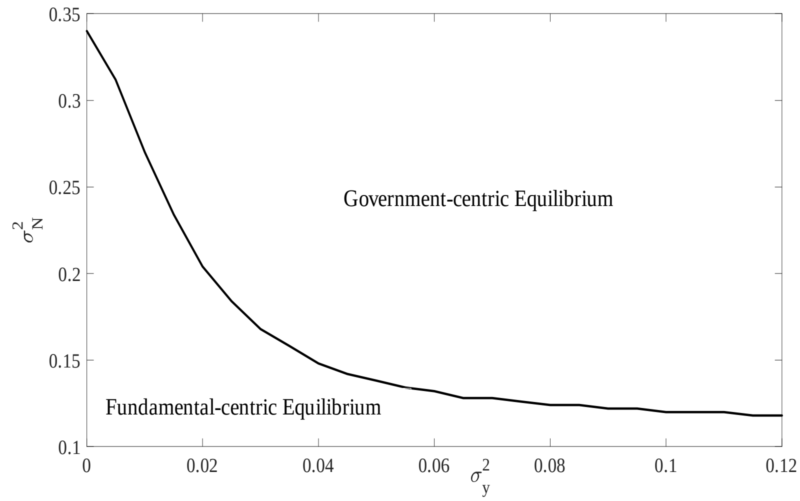

Figure 4: Boundary between fundamental-centric and government-centric equilibria under the optimal intervention policy, based on the parameters in Table I and $\sigma_y^2 = 0.06$, $\sigma_n^2 = 0$.

fundamentals.

Figure 3 highlights that if the noise-trader risk is sufficiently high, the government's optimal intervention policy may prioritize financial stability over market efficiency. Our model therefore allows us to analyze the conditions under which such a stability-dominated policy is optimal.[22] We illustrate these conditions in Figure 4 by focusing on two model parameters σ_N^2, which represent the key source of financial instability, and σ_y^2, the variance of entrepreneurs' project-specific noise that determines the relevance of market efficiency for social welfare through the first component in (8). Intuitively, as project-specific noise becomes more uncertain, the information extracted from the asset price becomes less useful to entrepreneurs. Building on the social welfare objective in (8), Figure 4 depicts the boundary between the government-centric and the fundamental-centric equilibrium on a plane of σ_y^2 and σ_N^2 with the values of other parameters given in Table I. As σ_y^2 rises, improving market efficiency becomes less relevant. Consequently, the government intervenes more aggressively

[22] In practice, governments may not have the ability to commit to an intervention strategy, and a time-consistency problem arises that reinforces the government-centric equilibrium. In this situation, the government may want to initially convince investors that it will not intervene too aggressively, in the hope of inducing them to acquire information about asset fundamentals. After investors have collected fundamental information, however, the government—even with a single objective of improving information efficiency—has incentive to change its intentions ex post and to trade more aggressively against noise traders than it initially promised. Rationally anticipating this opportunistic behavior by the government, investors would always choose to collect information about the government's future trading noise instead. In this way, the time-consistency problem may lead to the government-centric equilibrium, even when the government prefers the fundamental-centric outcome. In a related paper, Brunnermeier, Sockin, and Xiong (2017) explore this time-consistency problem in the context of China's financial reform.

and the market shifts from the fundamental-centric equilibrium to the government-centric equilibrium at a lower threshold of σ_N^2.

Proposition 7 characterizes the boundary that separates the fundamental-centric equilibrium from the government-centric equilibrium under the government's optimal intervention policy, with the proof given in the Online Appendix.

Proposition 7 *Suppose that under a set of model parameters, the government's optimal policy leads to a government-centric equilibrium. As σ_y falls below a critical level $\sigma_y^* \geq 0$, the government's optimal policy transitions to a fundamental-centric equilibrium. Under a certain sufficient condition, the critical level σ_y^* is decreasing in noise-trader risk σ_N.*

We view the government-centric equilibrium as aptly characterizing the Chinese government's regular and intensive intervention in its financial system, as summarized in Section 2. In particular, this regime reflects the prioritization of financial stability over other policy objectives, including stimulating economic growth, which is consistent with the emphasis regularly placed by Chinese policymakers on financial stability (Xu (2020)). As a consequence, China's financial markets are less volatile but also less informative about asset fundamentals and real activity than if the government pursued more moderate intervention policies. In contrast, the fundamental-centric equilibrium is reminiscent of the attitude of Western governments, which are typically more concerned that intensive intervention will distort financial markets; as a result, they intervene only in times of extreme market stress.

Our microfounded welfare objective allows us to link differences in preferences across governments for financial stability versus market efficiency to differences in the parameters underlying economic conditions in their respective economies. A government is more likely to prefer a government-centric intervention policy if its economy has higher extrinsic volatility in financial markets, as measured by noise-trader risk, σ_N^2, and if it has a lot of idiosyncratic production risk, σ_y^2, so that prices are not that helpful in shaping real allocative decisions. The dominance of inexperienced investors in China's financial markets and the lack of direct influence of asset prices in guiding firm investment suggest that it is optimal for the Chinese government to prioritize financial stability over market efficiency. Interestingly, Figure 4 also suggests that China might eventually outgrow such an aggressive intervention regime and adopt policies similar to its Western counterparts, as investors become more experienced and firms depend more on asset prices for investment guidance. During this process, there would be a trend of decreasing asset return volatility and improving price efficiency. Interestingly, our model further predicts that when the government eventually chooses to loosen its policy interventions, there might be a discrete increase in asset price volatility along with an improvement in asset price efficiency, as shown by Figure 3.

Evidence of China's Approach to Financial Market Intervention. There has been significant academic and policy discourse about the consequences of the extensive countercyclical government interventions in China. Many acknowledge that such interventions have been largely successful in reducing market fluctuations and ensuring financial stability. Consistent with our analysis, however, some commentators have also highlighted the potential adverse effects of such interventions on market efficiency. Allen et al. (2020) and Huang, Miao, and Wang (2019), for instance, argue that although the massive stock purchases by the national team during the 2015 stock market crash helped alleviate downside risk, this benefit may have come at the expense of preventing price discovery and exacerbating the disconnect between prices and their fundamental values. Indeed, Dang, Li, and Wang (2020) show that, in the cross-section of stocks, the trading of the national team is associated with reduced informativeness of stock prices.[23]

More generally, there is extensive evidence that stock prices in China's equity markets exhibit less market efficiency than those in more developed equity markets. Morck, Yeung, and Yu (2000), for instance, find strong comovement among Chinese stock prices, which could reflect a greater focus by market participants on the macroeconomy and government policy interventions compared to the fundamentals of individual stocks. This strong comovement also implies that Chinese stock prices provide less information than those in more developed economies to guide firm-level investment. Consistent with this implication, there is also extensive evidence showing the lower allocational efficiency of the Chinese economy relative to the U.S. economy. Hsieh and Klenow (2009) and David and Venkateswaran (2019) document substantially larger dispersion in the marginal product of capital among Chinese manufacturing firms relative to that among U.S. manufacturing firms, indicative of capital

[23] Furthermore, Zhu (2016) argues that the Chinese government's intensive intervention in its financial system, motivated from a desire to ensure financial and social stability, has created implicit guarantees that have incentivized risk-seeking behavior among investors who are unconcerned about the underlying risks and asset fundamentals. For example, Zhu (2016) argues that the dearth of public-firm delistings from the stock exchanges, in part related to regulators' reluctance to upset stakeholders of potentially distressed firms, has emboldened stock investors to ignore firms' fundamentals and to instead speculate on rumors and fads. In addition, the lack of public defaults by firms, mainly driven by the government's frequent bailouts of troubled borrowers, has motivated households to invest in opaque shadow-banking credit products, contributing to China's leverage boom in recent years.

misallocation across firms. David, Hopenhayn, and Venkateswaran (2016) develop a quantitative framework to show that severe informational frictions contribute significantly to the lower allocational efficiency of investment of Chinese manufacturing firms.

Although the lower market efficiency of Chinese stock prices and allocational efficiency of the Chinese economy are both consistent with our theory, there still remains the challenge of linking these inefficiencies to the unobserved information choices of Chinese investors between firms' economic fundamentals and the government's intervention policies. There is, however, a promising dataset to systematically examine this link. Financial regulation in China requires mutual fund managers to provide their outlook on financial markets, in addition to their investments, in their funds' quarterly statements. In this outlook, fund managers regularly state their expectations about macroeconomic fundamentals and government policies in determining the financial market fluctuations in the short and medium terms. By using textual analysis to quantify the fund managers' expectations for monetary policy, Ammer et al. (2020) show that fund managers act on these expectations and that correctly anticipating shifts in Chinese monetary policy improves fund performance. It is possible to build further on this disclosure of expectations by fund managers to examine explicitly the extent to which they are distracted by government policies; in conjunction with their holdings, one could also assess the extent to which the prices of the assets in which they invest reflect information about government policy versus asset fundamentals.

6 Conclusion

Our model highlights that, when adopting policies that lean against noise traders in financial markets, a government faces a tension between ensuring financial stability and improving price efficiency. We believe that this tension represents a key trade-off faced by policymakers across the world in managing their respective financial systems. Our micro-founded welfare analysis highlights the economic conditions, specifically, when noise-trader risk is sufficiently high or when firms face sufficiently high idiosyncratic noise, under which the government's optimal policy entails intensive interventions, which induce a government-centric equilibrium with all investors acquiring private information about policy noise rather than fundamentals. This policy regime characterizes China's approach of government interventions in financial markets, which prioritizes financial stability over other policy objectives. Our analysis thus rationalizes China's approach without appealing to government objectives based on political considerations.

References

Allen, Franklin, Stephen Morris, and Hyun Song Shin (2006), Beauty Contests and Iterated Expectations in Asset Markets, *Review of Financial Studies* 19, 719–752.

Allen, Franklin, Jun "QJ" Qian, Chenyu Shan, and Julie Lei Zhu (2020), The Development of the Chinese Stock Market, in Marlene Amstad, Guofeng Sun, and Wei Xiong (Eds): *The Handbook of China's Financial System*, Princeton University Press, 283–313.

Ammer, John, John H. Rogers, Gang Wang, and Yang Yu (2020), Monetary Policy Expectations, Fund Managers, and Fund Returns: Evidence from China, FRB International Finance Discussion Paper 1285.

Amstad, Marlene, Guofeng Sun, and Wei Xiong (2020), *The Handbook of China's Financial System*, Princeton University Press.

Angeletos, George Marios, Christian Hellwig, and Alessandro Pavan (2006), Signaling in a Global Game: Coordination and Policy Traps, *Journal of Political Economy* 114, 452–484.

Bacchetta, Philippe, Cédric Tille, and Eric Van Wincoop (2012), Self-Fulfilling Risk Panics, *American Economic Review* 102, 3674–3700.

Bian, Jiangze, Zhiguo He, Kelly Shue, and Hao Zhou (2017), Leverage-Induced Fire Sales and Stock Market Crashes, Working Paper, University of Chicago.

Bond, Philip, Alex Edmans, and Itay Goldstein (2012), The Real Effects of Financial Markets, *Annual Review of Financial Economics* 4, 339–360.

Bond, Phillip, and Itay Goldstein (2015), Government Intervention and Information Aggregation by Prices, *Journal of Finance,* 70, 2777–2811.

Brunnermeier, Markus, and Lasse Heje Pedersen (2009), Market Liquidity and Funding Liquidity, *Review of Financial Studies* 22, 2201–2238.

Brunnermeier, Markus, Michael Sockin, and Wei Xiong (2017), China's Gradualistic Economic Approach and Financial Markets, *American Economic Review Papers and Proceedings* 107(5): 608–613.

Cai, Jinghan, Jibao He, Wenxi Jiang, and Wei Xiong (2019), The Whack-A-Mole Game: Tobin Taxes and Trading Frenzy, *Review of Financial Studies*, forthcoming.

Carpenter, Jennifer, and Robert Whitelaw (2017), The Development of China's Stock Market and Stakes for the Global Economy, *Annual Review of Financial Economics* 9, 233–257.

Compte, Olivier, and Philippe Jehiel (2010), The Coalitional Nash Bargaining Solution, *Econometrica* 78, 1593–1623.

Cong, Lin William, Steven R. Grenadier, and Yunzhi Hu (2017), Intervention Policy in a Dynamic Environment: Coordination and Learning, *Journal of Financial Economics*, forthcoming.

Dang, Tri Vi, Wei Li, and Yongqin Wang (2020), Managing China's Stock Markets: The Economics of the National Team, Working Paper, Columbia University and Fudan University.

David, Joel M., Hugo A. Hopenhayn, and Venky Venkateswaran (2016), Information, Misallocation, and Aggregate Productivity, *Quarterly Journal of Economics* 131, 943–1005.

David, Joel M., and Venky Venkateswaran (2019), The Sources of Capital Misallocation, *American Economic Review* 109, 2531–67.

Davila, Eduardo and Cecille Parlatore (2019), Volatility and Informativeness, mimeo Yale and NYU Stern.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann (1990), Noise Trader Risk in Financial Markets, *Journal of Political Economy,* 98, 703–738.

Deng, Yongheng, Xin Liu, and Shang-Jin Wei (2018), One Fundamental and Two Taxes: When Does a Tobin Tax Reduce Financial Price Volatility? *Journal of Financial Economics* 130, 663–692.

Farboodi, Maryam, and Laura Veldkamp (2016), Long-Run Growth of Financial Technology, Working Paper, Princeton University and NYU Stern.

Froot, Kenneth A., David S. Scharfstein, and Jeremy Stein (1992), Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation, *Journal of Finance* 47, 1461–1484.

Ganguli, Jayant Vivek, and Liyan Yang (2009), Complementarities, Multiplicity, and Supply Information, *Journal of the European Economic Association*, 7, 90–115.

Geanakoplos, John (2010), The Leverage Cycle, *NBER Macroeconomics Annual* 24, 1–66.

Goldstein, Itay, and Chong Huang (2016), Bayesian Persuasion in Coordination Games, *American Economic Review: Papers and Proceedings* 106, 592–596.

Goldstein, Itay, Jan Schneemeier, and Liyan Yang (2020), Market Feedback: Who Learns What? mimeo Wharton, Kelley School of Business, and Rotman School of Management.

Goldstein, Itay and Liyan Yang (2019), Good Disclosure, Bad Disclosure, *Journal of Financial Economics* 131, 118–138.

Grossman, Sanford, and Joseph Stiglitz (1980), On the Impossibility of Informationally Efficient Markets, *American Economic Review* 70, 393–408.

Han, Bing, Ya Tang, and Liyan Yang (2016), Public Information and Uninformed Trading: Implications for Market Liquidity and Price Efficiency, *Journal of Economic Theory* 163, 604–643.

He, Hua, and Jiang Wang (1995), Differential Information and Dynamic Behavior of Stock Trading Volume, *The Review of Financial Studies* 8, 919–972.

Hellwig, Martin (1980), On the Aggregation of Information in Competitive Markets, *Journal of Economic Theory* 22, 477-498.

Hellwig, Christian, and Laura Veldkamp (2009), Knowing What Others Know: Coordination Motives in Information Acquisition, *Review of Economic Studies* 76, 223–251.

Hong Kong Exchange (2018), Macro-Prudential Management of Cross-Border Capital Flows and the Opening Up of China's Bond Market, Chief China Economist's Office.

Hsieh, Chang-Tai, and Peter J. Klenow (2009), Misallocation and Manufacturing TFP in China and India, *Quarterly Journal of Economics* 124, 1403–1448.

Hu, Xing, Jun Pan, and Jiang Wang (2018), Chinese Capital Market: An Empirical Overview, Working Paper, MIT Sloan.

Huang, Yi, Jianjun Miao, and Pengfei Wang (2019), Saving China's Stock Market, *IMF Economic Review* 67, 349–394.

Kaneko, Mamoru, and Kenjiro Nakamura (1979), The Nash Social Welfare Function, *Econometrica* 47, 423–435.

Kyle, Albert S. (1989), Informed Speculation with Imperfect Competition, *Review of Economics Studies* 56, 317–355.

Liu, Chang, and Wei Xiong (2020), China's Real Estate Market, in Marlene Amstad, Guofeng Sun, and Wei Xiong (Eds): *The Handbook of China's Financial System,* Princeton University Press, 181–207.

Manzano, Carolina, and Xavier Vives (2011), Public and Private Learning from Prices, Strategic Substitutability and Complementarity, and Equilibrium Multiplicity, *Journal of Mathematical Economics*, 47, 346–369.

Mei, Jianping, José Scheinkman, and Wei Xiong (2009), Speculative Trading and Stock Prices: Evidence from Chinese A-B Shares Premia, *Annals of Economics and Finance* 10, 225–255.

Morck, Randall, Bernard Yeung, and Wayne Yu (2000), The Information Content of Stock Markets: Why Do Emerging Markets Have Synchronous Stock Price Movements?, *Journal of Financial Economics* 58, 215–260.

Moulin, Hervé (2004), *Fair Division and Collective Welfare*, MIT Press.

Packer, Frank, and Mark Spiegel (2016), China's IPO Activity and Equity Market Volatility, *FRBSF Economic Letter* 18, 1–5.

Pastor, Lubos, and Pietro Veronesi (2012), Uncertainty about Government Policy and Stock Prices, *Journal of Finance* 67, 1219–1264.

Pastor, Lubos, and Pietro Veronesi (2013), Political Uncertainty and Risk Premia, *Journal of Financial Economy* 110, 520–545.

Qian, Yingyi (2017), *How Reform Worked in China*, MIT Press.

Shleifer, Andrei, and Robert Vishny (1997), The Limits of Arbitrage, *Journal of Finance* 52, 35–55.

Song, Zheng, and Wei Xiong (2018), Risks in China's Financial System, *Annual Review of Financial Economics* 10, 261–286.

Stein, Jeremy, and Adi Sundarem (2018), The Fed, the Bond Market, and Gradualism in Monetary Policy, *Journal of Finance* 73 (3), 1015–1060.

Xiong, Wei, and Jialin Yu (2011), The Chinese Warrants Bubble, *American Economic Review* 101, 2723–2753.

Xiong, Wei (2019), The Mandarin Model of Growth, Working Paper, Princeton University.

Xu, Chenggang (2011), The Fundamental Institutions of China's Reforms and Development, *Journal of Economic Literature* 49(4), 1076–1151.

Xu, Zhong (2020), *Keep Reforming: China's Strategic Economic Transformation*, Springer.

Zhu, Ning (2016), *China's Guaranteed Bubble*, McGraw Hill.

Appendix A Deriving Equilibrium with Information Frictions and Government Intervention

In this Appendix, we derive the equilibrium with information frictions and government intervention in several steps. We assume that the economy is initialized from its stationary equilibrium, in which all conditional variances from learning have reached their deterministic steady state and the coefficients in prices and policies are time homogeneous.

We first consider the case without government intervention. We begin, as in the main text, by conjecturing a linear equilibrium price function:

$$P_t = p_{\hat{V}} \hat{V}_{t+1}^M + p_V \left(V_{t+1} - \hat{V}_{t+1}^M \right) + p_N N_t.$$

Importantly, we recognize that it must be the case that $p_{\hat{V}} = \frac{1}{R^f - \rho_V}$, since a unit shift in V_t must raise the discounted present value of future cash flows by $\frac{1}{R^f - \rho_V}$.

We first state several properties of the linear equilibrium without government intervention. We defer the derivation of the noisy rational expectations equilibrium to the case with government intervention, which is the more general case.

Proposition A1 *In the presence of informational frictions, the coefficient on the fundamental V, p_V, is less than $p_{\hat{V}}$, and the coefficient on noise trading, p_N, is more positive. In addition, market breakdown occurs at a lower value of σ_N, σ_N^{**}, such that $\sigma_N^{**} \geq \sigma_N^*$, where σ_N^* is given in (1).*

In the presence of informational frictions, investors systematically underreact to information about the fundamental in prices (since $p_V < p_{\hat{V}}$) and overreact to noise. In addition, market breakdown occurs at lower levels of noise-trading variance than with perfect information. Since informational frictions introduce additional return volatility, investors require a higher risk premium to accommodate noise traders for the same level of noise-trader risk, σ_N^2. As a result, the critical value at which investors demand too high a risk premium to accommodate noise traders occurs at a smaller σ_N^2.

In the special case in which the fundamental, V, is i.i.d. ($\rho_V = 0$), we can express the condition for breakdown implicitly as

$$R^f < 2\gamma\sigma_N\sqrt{\sigma_D^2 + \left(\frac{1}{R^f}\right)^2\left(\sigma_V^2 + \left((R^f)^2 - \frac{\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}\right)\frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}\right)},$$

which reveals that uncertainty about V, parameterized through the posterior conditional variance of beliefs, $\Sigma^{M,VV}$, effectively raises the volatility of the fundamental from σ_V^2 to $\sigma_V^2 + \left(\left(R^f\right)^2 - \frac{\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}\right)\frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}$. There is both a direct effect that, for a fixed $\Sigma^{M,VV}$, the critical σ_N that leads to market breakdown falls, and an indirect effect that an increase in σ_N also increases $\Sigma^{M,VV}$.

We can also establish that price informativeness is lower and that price volatility is higher with informational frictions.

Proposition A2 *Price informativeness is lower and return volatility is higher in the presence of informational frictions.*

Having characterized the noisy rational expectations equilibrium without the government, we now consider the case with government intervention. We again conjecture a linear equilibrium price function:

$$P_t = p_{\hat{V}}\hat{V}_{t+1}^M + p_{\hat{G}}\hat{G}_{t+1}^M + p_V\left(V_{t+1} - \hat{V}_{t+1}^M\right) + p_G\left(G_{t+1} - \hat{G}_{t+1}^M\right) + p_g G_t + p_N N_t.$$

Importantly, we recognize that it must be the case that $p_{\hat{V}} = \frac{1}{R^f - \rho_V}$, since a unit shift in V_t must raise the discounted present value of future cash flows by $\frac{1}{R^f-\rho_V}$.

We now construct the equilibrium in several steps. We first solve for the learning processes of the government and investors, which begin with an intermediate step of deriving the beliefs from the perspective of the market that has access only to public information. Given the market's beliefs, which we can define recursively with the Kalman filter, we can construct the conditional posterior beliefs of the government and the posterior beliefs of each investor by applying Bayes' Rule to the market's beliefs given the private signal of each investor. We then solve for the optimal trading and information acquisition policies of the investors. Imposing market clearing, we can then express the government's objective in terms of the equilibrium objects we derive from learning.

Appendix A.1 Equilibrium Beliefs

In this subsection, we characterize the learning processes of the government and the investors. As we will see, it will be convenient to first derive the market's posterior beliefs about V_{t+1}, N_t, and G_{t+1}, respectively, which are Gaussian with conditional mean $\left(\hat{V}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M\right) = E\left[(V_{t+1}, N_t, G_{t+1}) \mid \mathcal{F}_t^M\right]$ and conditional variance $\Sigma_t^M = Var\left[\begin{bmatrix} V_{t+1} \\ N_t \\ G_{t+1}\end{bmatrix} \mid \mathcal{F}_t^M\right]$. Importantly, the market faces strategic uncertainty over the government's action as a result of the noise in the government's trading. As such, one must form expectations about this noise both for extracting information from prices and for understanding price dynamics and portfolio choice.

To solve for the market beliefs, we first construct the innovation process η_t^M for the asset price from the perspective of the market:

$$\begin{aligned} \eta_t^M &= P_t - (p_{\hat{V}} - p_V)\,\hat{V}_{t+1}^M - (p_{\hat{G}} - p_G)\,\hat{G}_{t+1}^M - p_g G_t \\ &= p_V V_{t+1} + p_G G_{t+1} + p_N N_t. \end{aligned}$$

Given that the investors and the government do not observe G_{t+1} (the next-period government noise), they must account for it in their learning.

Importantly, the asset price P_t and the innovation process η_t^M contain the same information, such that $\mathcal{F}_t^M = \sigma\left(\left\{D_s, \eta_s^M, G_t\right\}_{s\leq t}\right)$. Since the market's posterior about V_{t+1} will be Gaussian, we need only specify the laws of motion for the conditional expectation $\left(\hat{V}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M\right)$ and the conditional variance Σ_t^M. As is standard with a Gaussian information structure, these estimates are governed by the Kalman filter. As a result of learning from prices, the beliefs of the market about V_{t+1}, N_t, and G_{t+1} will be correlated ex post after observing the asset price. We summarize this result in the following proposition.

Proposition A3 *Given the normal prior* $(V_0, N_0) \sim \mathcal{N}\left(\left(V, \bar{N}\right), \Sigma_0\right)$ *and* $G_0 \sim \mathcal{N}\left(0, \sigma_G^2\right)$, *the posterior market beliefs are Gaussian* $(V_{t+1}, N_t, G_{t+1}) \mid \mathcal{F}_t^M \sim \mathcal{N}\left(\left(\hat{V}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M\right), \Sigma_{t+1}^M\right)$, *where the filtered estimates* $\left(\hat{V}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M\right)$ *follow the stochastic difference equations*

$$\begin{bmatrix} \hat{V}_{t+1}^M \\ \hat{N}_t^M \\ \hat{G}_{t+1}^M \\ G_t \end{bmatrix} = \begin{bmatrix} \rho_V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{V}_t^M \\ \hat{N}_{t-1}^M \\ \hat{G}_{t|t-1}^M \\ G_{t-1} \end{bmatrix} + \mathbf{K}_t^M \begin{bmatrix} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \\ G_t - \hat{G}_{t|t-1}^M \end{bmatrix},$$

and the conditional variance

$$\Sigma_t^M = \begin{bmatrix} \Sigma_t^{M,VV} & \Sigma_t^{M,VN} & \Sigma_t^{M,VG} & 0 \\ \Sigma_t^{M,VN} & \Sigma_t^{M,NN} & \Sigma_t^{M,NG} & 0 \\ \Sigma_t^{M,VG} & \Sigma_t^{M,NG} & \Sigma_t^{M,GG} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{A1}$$

follows a deterministic induction equation. The market's posterior expectations of V_{t+1}, N_t, *and* G_{t+1} *are related through*

$$p_V V_{t+1} + p_G G_{t+1} + p_N N_t = p_V \hat{V}_{t+1}^M + p_G \hat{G}_{t+1}^M + p_N \hat{N}_t^M.$$

Importantly, when the market tries to extract information from the price, market participants realize that the price innovations η_t^M contain the government trading noise G_{t+1}. As such, they must take into account the information content in the government noise when learning from the price and must form expectations about G_{t+1}. Through this channel, the path dependence of the government noise feeds into the market's beliefs, and the market has incentives to forecast the future noise in the government's trading.

Since investors learn through Bayesian updating, we can update their beliefs sequentially by beginning with the market beliefs, based on the coarser information set $\mathcal{F}_t^M$, and then updating the market beliefs with the private signals of investor i (s_t^i, g_t^i). Given that the market posterior beliefs and investor private signals are Gaussian, this second updating process again takes the form of a linear updating rule. We summarize these steps in the following proposition.

Proposition A4 *Given the market beliefs, the conditional beliefs of investor i are also Gaussian $(V_{t+1}, N_t, G_{t+1}) \mid \mathcal{F}_t^i \sim \mathcal{N}\left(\left(\hat{V}_{t+1}^i, \hat{N}_t^i, \hat{G}_{t+1|t}^i\right), \Sigma_t^s(i)\right)$, where*

$$\begin{bmatrix} \hat{V}_{t+1}^i \\ \hat{N}_t^i \\ \hat{G}_{t+1}^i \end{bmatrix} = \begin{bmatrix} \hat{V}_{t+1}^M \\ \hat{N}_t^M \\ \hat{G}_{t+1}^M \end{bmatrix} + \Gamma_t' \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix},$$

and $\Sigma_t^s(i)$ is related to Σ_t^M through a linear updating rule.

Since the government does not observe any private information, its conditional posterior beliefs align with those of the market. In what follows, we focus on the covariance-stationary limit of the Kalman filter, after initial conditions have diminished and the conditional variances of beliefs have converged to their deterministic, steady state. The following corollary establishes that such a steady state exists.

Proposition A5 *There exists a covariance-stationary equilibrium, in which the conditional variance of the market beliefs has a deterministic steady state. Given this steady state, the beliefs of investors are also covariance-stationary.*

Having characterized learning by investors and the government in this economy, we now turn to the optimal policies of investors.

Appendix A.2 Investment and Information Acquisition Policies

We now examine the optimal policies of an individual investor i at time t who takes the intervention policy of the government as given. Given the CARA-normal structure of each investor's problem, the separation principle applies and we can separate the investor's learning process about (V_{t+1}, N_t, G_{t+1}) from his optimal trading policy. To derive the optimal investment policy, it is convenient to decompose the excess asset return as

$$R_{t+1} = E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \phi' \varepsilon_{t+1}^M = \varsigma \Psi_t + \phi' \varepsilon_{t+1}^M,$$

where

$$\varepsilon_{t+1}^M = \begin{bmatrix} D_{t+1} - \hat{V}_{t+1}^M \\ \eta_{t+1}^M - p_V \rho_V \hat{V}_{t+1}^M - p_g \hat{G}_{t+1}^M \\ G_{t+1} - \hat{G}_{t+1}^M \end{bmatrix},$$

and $\varepsilon_{t+1}^{M} \sim N\left(\mathbf{0}_{3\times 1}, \Omega^{M}\right)$ from Proposition A3. We can then decompose the excess return based on the information set of the investor:

$$R_{t+1} = E\left[R_{t+1} \mid \mathcal{F}_t^i\right] + \phi' \varepsilon_{t+1}^{S,i},$$

where we can update $E\left[R_{t+1} \mid \mathcal{F}_t^i\right]$ from $E\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ by the Bayes' Rule according to

$$\begin{aligned}
& E\left[R_{t+1} \mid \mathcal{F}_t^M, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i\right] \\
= \; & E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + CoV\left[R_{t+1}, \begin{bmatrix} s_t^i - E\left[s_t^i \mid \mathcal{F}_t^M\right] \\ g_t^i - E\left[g_t^i \mid \mathcal{F}_t^M\right] \end{bmatrix}' \mid \mathcal{F}_t^M\right] \\
& \cdot Var\left[\begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix} \mid \mathcal{F}_t^M\right]^{-1} \begin{bmatrix} s_t^i - E\left[s_t^i \mid \mathcal{F}_t^M\right] \\ g_t^i - E\left[g_t^i \mid \mathcal{F}_t^M\right] \end{bmatrix} \\
= \; & \varsigma \Psi_t + \frac{\phi' \omega \begin{bmatrix} \Sigma^{M,GG} + \left[\left(1 - a^i\right) \tau_g\right]^{-1} & -\Sigma^{M,VG} \\ -\Sigma^{M,VG} & \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} \end{bmatrix}}{\left(\Sigma^{M,VV} + \left(a\tau_s\right)^{-1}\right)\left(\Sigma^{M,GG} + \left[\left(1-a\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG}\right)^2} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}.
\end{aligned}$$

This expression shows that the investor's private information in either s_t^i or g_t^i can help him better predict the excess asset return relative to the market-based information. Since the investor is myopic, his optimal trading strategy is to acquire a mean-variance efficient portfolio based on his beliefs. This is summarized in the following proposition.

Proposition A6 *Given the state vector $\Psi_t = \left[\hat{V}_{t+1}^M, \hat{N}_t^M, G_t, \hat{G}_{t+1}^M\right]$ and investor i's signals s_t^i and g_t^i, investor i's optimal investment policy X_t^i takes the following form:*

$$X_t^i = \frac{1}{\gamma} \frac{\varsigma \Psi_t + \frac{\phi' \omega \begin{bmatrix} \Sigma^{M,GG} + \left[\left(1 - a^i\right) \tau_g\right]^{-1} & -\Sigma^{M,VG} \\ -\Sigma^{M,VG} & \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} \end{bmatrix} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}}{\left(\Sigma^{M,VV} + \left(a\tau_s\right)^{-1}\right)\left(\Sigma^{M,GG} + \left[\left(1-a\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG}\right)^2}}{\phi' \Omega^M \phi - \frac{\phi' \omega \begin{bmatrix} \Sigma^{M,GG} + \left[\left(1 - a^i\right) \tau_g\right]^{-1} & -\Sigma^{M,VG} \\ -\Sigma^{M,VG} & \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} \end{bmatrix} \omega' \phi}{\left(\Sigma^{M,VV} + \left(a\tau_s\right)^{-1}\right)\left(\Sigma^{M,GG} + \left[\left(1-a\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG}\right)^2}},$$

with the coefficients ς, ϕ, and ω given in the Online Appendix.

This proposition shows that both signals s_t^i and g_t^i help the investor predict the asset return over the public information because they can be used to form better predictions of V_{t+1} and G_{t+1}, which determine the asset return in the subsequent period. The investor needs to choose between acquiring either s_t^i or g_t^i based on the ex ante market information:

$$\begin{aligned}
E\left[U_t^i \mid \mathcal{F}_{t-1}^M\right] &= \sup_{a_t^i \in \{0,1\}} -E\left\{E\left[\exp\left(-\gamma R^f \bar{W} - \frac{1}{2} \frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]^2}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}\right) \middle| \mathcal{F}_t^M\right] \middle| \mathcal{F}_{t-1}^M\right\} \\
&= \sup_{a_t^i \in \{0,1\}} -\sqrt{\frac{\phi'\left(\Omega^M - M\left(a_t^i\right)\right)\phi}{\phi' \Omega^M \phi}} E\left\{\exp\left(-\gamma R^f \bar{W} - \frac{\frac{1}{2}\left(\varsigma \Psi_t\right)^2}{\phi' \Omega^M \phi}\right) \middle| \mathcal{F}_{t-1}^M\right\} \\
&= \sup_{a_t^i \in \{0,1\}} -\sqrt{\frac{\phi'\left(\Omega^M - M\left(a_t^i\right)\right)\phi}{\phi' \Omega^M \phi + \varsigma \mathbf{K}^M \Omega^M \mathbf{K}^{M\prime} \varsigma'}} \exp\left(-\gamma R^f \bar{W} - \frac{\frac{1}{2}\left(\varsigma \varrho \Psi_{t-1}\right)^2}{\phi' \Omega^M \phi + \varsigma \mathbf{K}^M \Omega^M \mathbf{K}^{M\prime} \varsigma'}\right),
\end{aligned}$$

by the properties of the moment-generating function of noncentral chi-squared random variables, where

$$\begin{aligned} M\left(a^i\right) &= \frac{\omega \begin{bmatrix} \Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG} \\ -\Sigma^{M,VG} & \Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1} \end{bmatrix} \omega'}{\left(\Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1}\right)\left(\Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG}\right)^2} \\ &= \omega \begin{bmatrix} \frac{1}{\Sigma^{M,VV} + (a\tau_s)^{-1}} & 0 \\ 0 & \frac{1}{\Sigma^{M,GG} + [(1-a)\tau_g]^{-1}} \end{bmatrix} \omega'. \end{aligned}$$

Since $p_{\hat{V}} = \frac{1}{R-\rho_V}$, $\varsigma = \begin{bmatrix} 0 & -R^f p_N & p_g - R^f p_{\hat{G}} & -R^f p_g \end{bmatrix}$:

$$\varsigma \varrho \Psi_{t-1} = -R^f p_g G_{t-1},$$

and therefore

$$E\left[U_t^i \mid \mathcal{F}_{t-1}^M\right] = \sup_{a_t^i \in \{0,1\}} \sqrt{\frac{\phi'\left(\Omega^M - M\left(a_t^i\right)\right)\phi}{\phi'\Omega^M\phi + \varsigma\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\varsigma'}} \exp\left(-\gamma R^f \bar{W} - \frac{\frac{1}{2}\left(R^f p_g G_{t-1}\right)^2}{\phi'\Omega^M\phi + \varsigma\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\varsigma'}\right),$$

where $\phi'\Omega^M\phi + \varsigma\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\varsigma' = E\left[Var\left(R_{t+1}\right) \mid \mathcal{F}_{t-1}^M\right] + Var\left(E\left[R_{t+1} \mid \mathcal{F}_t^M\right] \mid \mathcal{F}_{t-1}^M\right)$. By the Law of Total Variance, this implies

$$E\left[Var\left(R_{t+1}\right) \mid \mathcal{F}_{t-1}^M\right] + Var\left(E\left[R_{t+1} \mid \mathcal{F}_t^M\right] \mid \mathcal{F}_{t-1}^M\right) = Var\left(R_{t+1} \mid \mathcal{F}_{t-1}^M\right).$$

Consequently, since $\phi'\left(\Omega^M - M\left(a_t^i\right)\right)\phi = Var\left(R_{t+1} \mid \mathcal{F}_t^i\right)$ and a_t^i is a binary choice,

$$E\left[U_t^i \mid \mathcal{F}_{t-1}^M\right] = -\max_{a_t^i \in \{0,1\}} \sqrt{\frac{Var\left(R_{t+1} \mid \mathcal{F}_t^i\right)}{Var\left(R_{t+1} \mid \mathcal{F}_{t-1}^M\right)}} \exp\left(-\gamma R^f \bar{W} - \frac{1}{2}\frac{\left(R^f p_g \hat{G}_t^M\right)^2}{Var\left(R_{t+1} \mid \mathcal{F}_{t-1}^M\right)}\right).$$

This is the expected utility of investor i based on the public information from the previous period. Importantly, we recognize that the investor's information acquisition choice is independent of the expectation with respect to $\mathcal{F}_{t-1}^M$. Intuitively, second moments are deterministic in a Gaussian framework, so the investor can perfectly anticipate the level of uncertainty he will face without knowing the specific realization of the common knowledge information vector Ψ_t tomorrow. We can further reduce this objective to

$$a^i = \arg\max_{a^i \in \{0,1\}} -\log\left\{\phi'\left[\Omega^M - M\left(a^i\right)\right]\phi\right\}, \tag{A2}$$

or, since log is a monotonic function and $\phi'\left[\Omega^M - M\left(a^i\right)\right]\phi = Var\left(R_{t+1} \mid \mathcal{F}_t^i\right)$,

$$a^i = \arg\sup_{a^i \in \{0,1\}} -Var\left(R_{t+1} \mid \mathcal{F}_t^M, a_t^i s_t^i + \left(1-a_t^i\right)g_t^i, a_t^i\right).$$

Since the optimization objective involves only variances, which are covariance-stationary, the signal choice faced by the investors is time-invariant. Intuitively, given the Gaussian price distribution and exponential utility for the investors, the benefit of more-precise private information lies with the reduction in uncertainty over the excess asset return.

By substituting $M\left(a^i\right)$ into the optimization objective, we arrive at the following result.

Proposition A7 *Investor i chooses to acquire information about the asset fundamental V_{t+1} (i.e., $a^i = 1$) with probability λ:*

$$\lambda = \begin{cases} 1, & \text{if } Q < 0 \\ (0,1), & \text{if } Q = 0 \\ 0, & \text{if } Q > 0, \end{cases},$$

where

$$Q = \frac{CoV\left[R_{t+1}, G_{t+1} \mid \mathcal{F}_t^M\right]^2}{\Sigma^{M,GG} + \tau_g^{-1}} - \frac{CoV\left[R_{t+1}, V_{t+1} \mid \mathcal{F}_t^M\right]^2}{\Sigma^{M,VV} + \tau_s^{-1}}$$

is given explicitly in the Appendix, and $\lambda \in (0,1)$ is the mixing probability when the investor is indifferent between acquiring information about the asset fundamental or the government trading noise.

This proposition states that the investor chooses his signal to maximize his informational advantage over the market beliefs, based on the extent to which the signal reduces the conditional variance of the excess asset return. Importantly, this need not imply a preference for learning about V_{t+1} directly, since the government's future noise G_{t+1} also contributes to the overall variance of the excess asset return. The more the government's noise covaries with the unpredictable component of the asset return from the market's perspective, the more valuable this information is to the investors.[24] This is the partial equilibrium decision of each investor taking prices as given.

Appendix A.3 Market Clearing

Given the optimal policy for each investor from Proposition A7 and the government's trading policy in (2), imposing market clearing in the asset market leads to

$$\begin{aligned} 0 \;=\; & N_t + \lambda \frac{\varsigma\Psi_t + \frac{\phi'\omega}{\Sigma^{M,VV}+\tau_s^{-1}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(V_{t+1} - \hat{V}_{t+1}^M\right)}{\gamma\phi'\left(\Omega^M - \omega \begin{bmatrix} \frac{1}{\Sigma^{M,VV}+\tau_s^{-1}} & 0 \\ 0 & 0 \end{bmatrix} \omega'\right)\phi} \\ & + (1-\lambda) \frac{\varsigma\Psi_t + \frac{\phi'\omega}{\Sigma^{M,GG}+\tau_g^{-1}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(G_{t+1} - \hat{G}_{t+1}^M\right)}{\gamma\phi'\left(\Omega^M - \omega \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{\Sigma^{M,GG}+\tau_g^{-1}} \end{bmatrix} \omega'\right)\phi} - \vartheta_{\hat{N}}\hat{N}_t^M + \sqrt{\vartheta'\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\vartheta}\,G_t, \end{aligned} \tag{A3}$$

where $\vartheta = \begin{bmatrix} 0 & \vartheta_{\hat{N}} & 0 & 0 \end{bmatrix}'$ and we have applied the Weak Law of Large Numbers (WLLN) such that $\int_\chi s_t^i di = V_{t+1}$ and $\int_\chi g_t^i di = G_{t+1}$ over the arbitrary subset of the unit interval χ.

[24] Since higher signal precision will reduce the conditional variance of the excess asset return but impact the expected return symmetrically because the signal is unbiased, the channel through which information acquisition affects portfolio returns is through reduction in uncertainty. Given that investors can take long or short positions without limit, the direction of the news surprise does not impact the information acquisition decision.

In addition, we have recognized that $Var\left[\vartheta_{\hat{N}}\hat{N}^M \mid \mathcal{F}_{t-1}^M, \{a_t^i\}_i\right] = \vartheta'\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\vartheta$. Following the insights of He and Wang (1995), we can express the market-clearing condition with a smaller, auxiliary state space given that expectations about V_{t+1} and N_t are linked through the stock price P_t. We now recognize that

$$\hat{N}_t^M = N_t + \frac{p_V}{p_N}\left(V_{t+1} - \hat{V}_{t+1}^M\right) + \frac{p_G}{p_N}\left(G_{t+1} - \hat{G}_{t+1}^M\right), \tag{A4}$$

from Proposition A3. This allows us to rewrite Ψ_t as the state vector $\tilde{\Psi}_t = [\hat{V}_{t+1}^M, \hat{G}_{t+1}^M, V_{t+1}, N_t, G_t, G_{t+1}]$.

Matching coefficients with our conjectured price function pins down the coefficients and confirms the linear equilibrium. Importantly, the coefficients are matched to the basis $\left\{\hat{V}_{t+1}^M, V_{t+1} - \hat{V}_{t+1}^M, \hat{G}_{t+1}^M, G_{t+1} - \hat{G}_{t+1}^M, G_t, N_t\right\}$ in accordance with our conjecture on the functional form of the asset price. This yields three conditions:

$$\begin{aligned} 0 &= -\frac{1 + p_{\hat{V}}\left(\rho_V - R^f\right)}{\gamma Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}, \\ p_N &= \frac{1 - \vartheta_{\hat{N}}}{R^f}\gamma Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] \\ p_{\hat{G}} &= \frac{1}{R^f}p_g, \end{aligned} \tag{A5}$$

where

$$\begin{aligned} Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] &= \lambda\phi'\left(\Omega^M - \omega\begin{bmatrix} \frac{1}{\Sigma^{M,VV} + \tau_s^{-1}} & 0 \\ 0 & 0 \end{bmatrix}\omega'\right)\phi \\ &\quad + (1-\lambda)\phi'\left(\Omega^M - \omega\begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{\Sigma^{M,GG} + \tau_g^{-1}} \end{bmatrix}\omega'\right)\phi. \end{aligned}$$

These conditions pin down the relationship between the government's trading policy and the price coefficients, and

$$\begin{aligned} p_g &= -\frac{\gamma}{R^f}Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]\sqrt{\vartheta'\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\vartheta} \qquad \text{(A6)} \\ &= -\frac{\gamma}{R^f}\left|\vartheta_{\hat{N}}\right| Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]\sqrt{\sigma_N^2 - \Sigma^{M,NN}} \\ p_V &= \frac{1 - \vartheta_{\hat{N}}}{R^f}\frac{\phi'\omega}{\Sigma^{M,VV} + \tau_s^{-1}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \text{(A7)} \\ &= \frac{1 - \vartheta_{\hat{N}}}{R^f}\frac{\Sigma^{M,VV}}{\Sigma^{M,VV} + \tau_s^{-1}}\left(\frac{R^f}{R^f - \rho_V} - \frac{\left(\frac{1}{R^f - \rho_V} - p_V\right)\rho_V\sigma_D^2}{p_V^2\left(\frac{R^f}{1 - \vartheta_{\hat{N}}}\right)^2\frac{\left(\rho_V^2\Sigma^{M,VV} + \sigma_V^2\right)\sigma_D^2 + \sigma_V^2\Sigma^{M,VV}}{\gamma^2\sigma_N^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]^2} + \Sigma^{M,VV} + \sigma_D^2}\right) \\ p_G &= \frac{1 - \vartheta_{\hat{N}}}{R^f}\frac{\phi'\omega}{\Sigma^{M,GG} + \tau_g^{-1}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \text{(A8)} \\ &= \frac{1 - \vartheta_{\hat{N}}}{R^f}\frac{\Sigma^{M,GG}}{\Sigma^{M,GG} + \tau_g^{-1}}p_g, \end{aligned}$$

which pin down p_g, p_V, and p_G and, consequently, the informativeness of the asset price given the loading on the noise-trading p_N. As one can see above, since the investors always take a neutral position on $\hat{V}_{t+1}^M$ (as it is common knowledge), the government also takes a neutral position by market clearing. The market-clearing condition (A6) reflects that the investors take an offsetting position to the noise G_t in the government's trading.

Since the investors determine the extent to which their private information about V_{t+1} and G_{t+1} is aggregated into the asset price, the government is limited in how it can impact price informativeness. This is reflected in the last two market-clearing conditions, (A7) and (A8). The second terms in these conditions are the intensities with which the investors trade on their private information about V_{t+1} and G_{t+1}, respectively. The first terms, $\frac{p_V}{p_N}$ and $\frac{p_G}{p_N}$, are the correlations of V_{t+1} and G_{t+1} with the perceived level of noise-trading $\hat{N}_t^M$, as can be seen from (A4). Since the government trades based on $\hat{N}_t^M$, it cannot completely separate its impact on the true level of noise-trading N_t in prices from its impact on V_{t+1} and G_{t+1}.

Given that the government internalizes its impact on prices when choosing its trading strategy $\vartheta_{\hat{N}}$, we can view its optimization problem as being over the choice of price coefficients $\{p_g, p_V, p_G, p_N\}$ in the price functional $P_t = p\left(\tilde{\Psi}_t\right)$, subject to the market-clearing conditions.

Appendix A.4 Computation of the Equilibrium

To compute equilibrium numerically, we follow the Kalman filter algorithm for the market beliefs outlined in Proposition A3 to find the stationary equilibrium. We then solve for the portfolio choice of each investor, impose the market-clearing conditions, and optimize the government's objective in choosing $\vartheta_{\hat{N}}$. Finally, we check each investor's information acquisition decision by computing the Q statistic to verify that the conjectured equilibrium is an equilibrium. We perform this optimization to search for both fundamental-centric ($\lambda = 1$) and government-centric ($\lambda = 0$) equilibria, as well as mixing equilibria ($\lambda \in (0,1)$), with the same equilibrium played at each date as consistent with covariance stationarity.

Appendix B Welfare Analysis

In this Appendix, we further expand the model setting to analyze the welfare consequences of government intervention. The government is concerned with the welfare of four different types of agents in the economy: investors, noise traders, entrepreneurs, and taxpayers. For simplicity, we assume that these four groups are exclusive. All agents are risk-averse and have CARA utility with common coefficient of absolute risk aversion γ. To minimize notation, we assume that asset markets are in a covariance-stationary equilibrium and, consequently, the government follows a stationary policy.

Investors. The first group, investors, follows directly from the main model in Section 4. At date t, they each take a position t in financial markets and, from our earlier analysis, garner expected utility:

$$U_t^i = -\exp\left(-\gamma R^f \bar{W} - \gamma E\left[X_t^i R_{t+1} \mid \mathcal{F}_t^i\right] + \frac{\gamma^2}{2}\left(X_t^i\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]\right), \tag{A9}$$

where X_t^i can be decomposed as

$$X_t^i = X_t + \frac{1}{\gamma Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}\phi'\omega \begin{bmatrix} \frac{\left[a_t^i \tau_s\right]^{-1/2}}{\Sigma^{M,VV} + (a\tau_s)^{-1}} \varepsilon_t^{s,i} \\ \frac{\left[\left(1-a_t^i\right)\tau_g\right]^{-1/2}}{\Sigma^{M,GG} + \left[(1-a)\tau_g\right]^{-1}} \varepsilon_t^{g,i} \end{bmatrix},$$

and X_t is the aggregate position of informed investors and, by market clearing, equals $N_t - X_t^G$.

Noise traders. We next microfound noise traders as discretionary liquidity traders to incorporate their welfare from trading in the asset market. Similarly to Han, Tang, and Yang (2016), we assume that a continuum of liquidity traders needs to decide at date $t-2$ whether to join trading in the asset market at date t to receive a hedging benefit $B > 0$ in certainty equivalent utility. If liquidity trader j chooses to join the market, he needs to submit a market order at date t, which is given by

$$n_t^j = N_t + \sigma_n \varepsilon_t^j, \quad \varepsilon_t^j \sim iid\, \mathcal{N}(0,1),$$

where $\int_{\mathcal{D}} n_t^j dj = N_t$ by the WLLN on any measurable subset $\mathcal{D} \subseteq [0,1]$. If a trader chooses not to join the market, he earns a reservation utility, which we normalize to -1. At date $t-1$, liquidity trader j solves his expected utility from joining the market by

$$E\left[V_t^j \mid \mathcal{F}_{t-2}^M\right] = \max\left\{E\left[-\exp\left(-\gamma\left(B + n_t^i R_{t+1}\right)\right) \mid \mathcal{F}_{t-2}^M\right], -1\right\}.$$

In the Online Appendix for proof of Proposition 5, we show that we can express the excess payoff of the asset as

$$R_{t+1} = \tilde{R}_{t+1} - R^f p_g \hat{G}_{t|t-1}^M - c_N p_N N_t,$$

where $\tilde{R}_{t+1} \mid \mathcal{F}_{t-1}^M \sim \mathcal{N}\left(0, \sigma_{\tilde{R}}^2\right)$ is independent of N_t and $\hat{G}_{t|t-1}^M$ and c_N is given in the Online Appendix.

It follows, by the Law of Iterated Expectations, conditioning on $N_t = \sigma_N \varepsilon_t^N$ and ε_{t+1}^j, that

$$E\left[-\exp\left(-\gamma\left(B + n_t^i R_{t+1}\right)\right) \mid \mathcal{F}_{t-2}^M\right] = -\exp(-\gamma B)\, E\left[\exp\left(\frac{1}{2}\begin{bmatrix} \varepsilon_t^N \\ \varepsilon_t^j \end{bmatrix}' A \begin{bmatrix} \varepsilon_t^N \\ \varepsilon_t^j \end{bmatrix}\right) \mid \mathcal{F}_{t-2}^M\right],$$

where A is the symmetric matrix

$$A = \gamma^2\left(\sigma_{\tilde{R}}^2 + \left(R^f p_g\right)^2 \Sigma^{M,GG}\right)\begin{bmatrix} \sigma_N^2 & \sigma_n \sigma_N \\ \sigma_N \sigma_n & \sigma_n^2 \end{bmatrix} + \gamma c_N p_N \begin{bmatrix} 2\sigma_N^2 & \sigma_N \sigma_n \\ \sigma_N \sigma_n & 0 \end{bmatrix}.$$

By applying the properties of the the moment-generating function of the centered chi-square distribution, this implies that

$$E\left[V_t^j \mid \mathcal{F}_{t-2}^M\right] = \max\left\{-\exp\left(-\frac{1}{2}\log\left|Id_2 - A\right| - \gamma B\right), -1\right\},$$

provided that $|Id_2 - A|$ is positive semi-definite.

Consequently, a liquidity trader at date $t-2$ will participate at date t if

$$B \geq -\frac{1}{2\gamma}\log\left|Id_2 - A\right|,$$

Thus, for B sufficiently large, all liquidity traders will choose at $t-2$ to participate in the asset market at date t. Furthermore, since the asset price is covariance-stationary, the full measure of liquidity traders will participate at all dates.

For the government's welfare accounting, the expected utility of each liquidity trader at date t is

$$\begin{aligned} V_t^j &= E\left[-\exp\left(-\gamma\left(B + n_t^j R_{t+1}\right)\right) \mid \mathcal{F}_t^M\right] \\ &= -\exp\left(\gamma B - \gamma n_t^j E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \frac{\gamma^2}{2}\left(n_t^j\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]\right). \end{aligned} \tag{A10}$$

Entrepreneurs. We now introduce a third group, entrepreneurs, who make investment decisions based on information extracted from the asset price. At date t, a continuum of ex ante identical, risk-averse entrepreneurs can invest in a risky project whose quality is positively correlated with $\varepsilon_t^V = V_t - \rho_V V_{t-1}$, the innovation in the fundamental of the traded asset. By investing in capital K_t at date t, the project provides a net profit at date $t+1$ of

$$Y_{t+1}^l = \beta\left(\varepsilon_{t+1}^V + \sigma_y \varepsilon_{t+1}^l\right) K_t,$$

where $\varepsilon_{t+1}^l \sim \mathcal{N}(0,1)$ is project-specific noise that is independent across entrepreneurs and σ_y^2 is the variance of the project-specific noise. As ε_{t+1}^V is not observable to entrepreneurs at t, they rely on the history of asset prices and dividends $\{D_s, P_s\}_{s\leq t}$ contained in the public information set $\mathcal{F}_t^M$ to infer the value of V_{t+1} and ε_{t+1}^V.

An entrepreneur l chooses K_t at date t to maximize its expected utility Q_t^l :

$$\begin{aligned} Q_t^l &= \sup_{K_t} E\left[-\exp\left(-\gamma Y_{t+1}^l\right) \middle| \mathcal{F}_t^M\right] \\ &= \sup_{K_t} -\exp\left(-\gamma\beta E\left[\varepsilon_{t+1}^V + \varepsilon_{t+1}^l \mid \mathcal{F}_t^M\right] K_t + \frac{\gamma^2\beta^2}{2} Var\left[\varepsilon_{t+1}^V + \varepsilon_{t+1}^l \mid \mathcal{F}_t^M\right] K_t^2\right). \end{aligned}$$

Given its posterior $V_{t+1} \mid \mathcal{F}_t^M \sim \mathcal{N}\left(\hat{V}_{t+1}^M, \Sigma^{M,VV}\right)$, its posterior for ε_{t+1}^V is

$$\varepsilon_{t+1}^V \mid \mathcal{F}_t^M \sim \mathcal{N}\left(\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M, \left(1 - \rho_V^2\right)\Sigma^{M,VV}\right).$$

It follows that all entrepreneurs choose the same optimal level of investment:

$$K_t = \frac{E\left[\varepsilon_{t+1}^V + \varepsilon_{t+1}^l \mid \mathcal{F}_t^M\right]}{\gamma\beta Var\left[\varepsilon_{t+1}^V + \varepsilon_{t+1}^l \mid \mathcal{F}_t^M\right]} = \frac{1}{\gamma\beta}\frac{\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M}{\left(1-\rho_V^2\right)\Sigma^{M,VV} + \sigma_y^2}.$$

Then, the realized output Y_{t+1}^k is given by

$$Y_{t+1}^l = \frac{1}{\gamma}\frac{\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M}{\left(1-\rho_V^2\right)\Sigma^{M,VV} + \sigma_y^2}\left(\varepsilon_{t+1}^V + \varepsilon_{t+1}^l\right),$$

and the entrepreneur's expected utility is

$$Q_t^l = -\exp\left(-\frac{1}{2}\frac{\left(\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M\right)^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV} + \sigma_y^2}\right). \tag{A11}$$

Taxpayers. Finally, we include the fourth group, taxpayers, who are the residual claimants of the government and consequently receive its trading profit each period. At each date t, a new generation of taxpayers receives the profit from the government's trading at date t. Their expected utility as a group is

$$\begin{aligned} H_t &= E\left[-\exp\left(-\gamma X_t^G R_{t+1}\right) \mid \mathcal{F}_t^M\right] \\ &= -\exp\left(-\gamma X_t^G E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \frac{\gamma^2}{2}\left(X_t^G\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]\right). \end{aligned} \tag{A12}$$

Welfare function. This is the proof of Proposition 5. We assume that the government adopts a variant of the Nash social welfare function, as in Kaneko and Nakamura (1979):

$$U_t^G\left(\vartheta_{\hat{N}}\right) = -\int_0^1 \log\left(-U_t^i\right) di - \int_0^1 \log\left(-V_t^j\right) dj - \int_0^1 \log\left(-Q_t^l\right) dl - \log\left(-H_t\right). \tag{A13}$$

This criterion is a monotonic transformation of the product of the utilities of all agents in the economy. It is an extension of the objective in the Nash bargaining solution for two players and the coalition Nash bargaining for N agents (Compte and Jehiel (2010)) to social choice theory. Similar to utilitarian welfare, this welfare criterion satisfies several desirable properties: Pareto optimality, independence of irrelevant alternatives, anonymity, and continuity (Kaneko and Nakamura (1979)), as well as independence of a common scale and a preference for equity (Moulin (2004)).

Substituting for U_t^i, V_t^j, and Q_t^k, we arrive at

$$\begin{aligned} U_t^G\left(\vartheta_{\hat{N}}\right) &= \gamma\int_0^1 X_t^i E\left[R_{t+1} \mid \mathcal{F}_t^i\right] di - \frac{\gamma^2}{2}\int_0^1 \left(X_t^i\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] di - \gamma E\left[N_t R_{t+1} \mid \mathcal{F}_t^M\right] \\ &\quad -\frac{\gamma^2}{2}\left(N_t^2 + \sigma_n^2\right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \gamma E\left[X_t^G R_{t+1} \mid \mathcal{F}_t^M\right] \\ &\quad -\frac{\gamma^2}{2}\left(X_t^G\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \frac{1}{2}\frac{\left(\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M\right)^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV} + \sigma_y^2} + \gamma R^f \bar{W} - \gamma B, \end{aligned}$$

by noting that $\int_0^1 n_t^j dj = N_t$ and $\int_0^1 \left(n_t^j\right)^2 dj = N_t^2 + \sigma_n^2$ by the WLLN.

We assume that the government determines its intervention intensity $\vartheta_{\hat{N}}$ two periods ahead. That is, it chooses $\vartheta_{\hat{N}}$ for date t at date $t-2$. This timing reflects that the government cannot quickly adjust its intervention strategy in response to market conditions. The government has the public information set and chooses $\vartheta_{\hat{N}}$ to maximize its objective, by taking as given the information acquisition decision of informed investors. Since asset markets are covariance-stationary, the optimal information acquisition choice of informed investors at date $t-1$ who trade at date t is known to the government at date $t-2$.

By imposing the Law of Iterated Expectations, $\int_0^1 X_t^i di = X_t$, and market clearing, we recognize that

$$\begin{aligned} & E\left[\int_0^1 E\left[X_t^i R_{t+1} \mid \mathcal{F}_t^i\right] di - E\left[N_t R_{t+1} \mid \mathcal{F}_t^M\right] \mid \mathcal{F}_{t-2}^M\right] + E\left[X_t^G R_{t+1} \mid \mathcal{F}_{t-2}^M\right] \\ = \;& E\left[\left(\int_0^1 X_t^i di - N_t + X_t^G\right) R_{t+1} \mid \mathcal{F}_{t-2}^M\right] = 0, \end{aligned}$$

which simply indicates that trading is a zero-sum game between investors, noise traders, and the government. As a result, the social welfare is not affected by any group's expected trading gain, but rather by the second-moment terms:

$$\begin{aligned} E\left[U_t^G\left(\vartheta_{\hat{N}}\right) \mid \mathcal{F}_{t-2}^M\right] = \;& -\frac{\gamma^2}{2}\vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)\left(\sigma_N^2 - \Sigma^{M,NN}\right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] \\ & -\frac{1}{2}\frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} - \frac{\gamma^2}{2}\left(\sigma_N^2 + \sigma_n^2\right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] \\ & +\frac{1}{2}\frac{\sigma_V^2 + \sigma_y^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV} + \sigma_y^2} + \gamma R^f \bar{W} - \gamma B, \end{aligned}$$

where the conditional variance $\Sigma^{M,NN}$ is defined in (A1). From our earlier derivation of X_t^i, we have

$$E\left[\gamma^2\left(X_t^i\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] \mid \mathcal{F}_{t-2}^M\right] = E\left[\frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]^2}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} \mid \mathcal{F}_{t-2}^M\right] = \frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} - 1,$$

and, in addition

$$E\left[\frac{\left(\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M\right)^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV} + \sigma_y^2} \mid \mathcal{F}_{t-2}^M\right] = \frac{\sigma_V^2 + \sigma_y^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV} + \sigma_y^2} - 1.$$

Since $X_t^G = -\vartheta_{\hat{N}}\hat{N}_t^M + \sqrt{Var\left[\vartheta_{\hat{N}}\hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]}G_t$ and G_t is observable at date t, we have

$$E\left[\left(X_t^G\right)^2 \mid \mathcal{F}_{t-2}^M\right] = \vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right) Var\left[\hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right] = \vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)\left(\sigma_N^2 - \Sigma^{M,NN}\right).$$

As a result, we obtain the government's intervention objective, as stated in Proposition 5:

$$
\begin{aligned}
\sup_{\vartheta_{\hat{N}}} \quad & \frac{\sigma_V^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV}+\sigma_y^2} - \frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} \\
& \quad - \gamma^2\left(\sigma_N^2+\sigma_n^2+\vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)\left(\sigma_N^2-\Sigma^{M,NN}\right)\right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right],
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \ \mathcal{F}_t^i\right]} &= \frac{\phi'\Omega^M\phi+\varsigma\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\varsigma'+\left(R^f p_g\right)^2\left(\sigma_G^2-\Sigma^{M,GG}\right)}{\phi'\left(\Omega^M-\omega\begin{bmatrix}\frac{1}{\Sigma^{M,VV}+\left[a_i\tau_s\right]^{-1}} & 0 \\ 0 & \frac{1}{\Sigma^{M,GG}+\left[(1-a_i)\tau_g\right]^{-1}}\end{bmatrix}\omega'\right)\phi}, \\
Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] &= \phi'\Omega^M\phi.
\end{aligned}
$$

China's Model of Managing the Financial System

Markus K. Brunnermeier, Michael Sockin, Wei Xiong

Online Appendix

This online appendix present proofs of the propositions in the main paper.

Proof of Proposition 1

Note from the variance of the excess asset payoff that

$$Var\left[R_{t+1} \mid \mathcal{F}_t\right] = \sigma_D^2 + \left(\frac{1}{R^f - \rho_V}\right)^2 \sigma_V^2 + p_N^2 \sigma_N^2,$$

and thus the excess volatility is driven by the $p_N^2 \sigma_N^2$ term. Consider now the expression for the less positive root of p_N from (IA.1) in the proof of Proposition 2 in the special case in which there is an absence of government intervention:

$$p_N = \frac{1}{2\sigma_N^2} A - \sqrt{\left(\frac{1}{2\sigma_N^2} A\right)^2 - \frac{1}{\sigma_N^2} B},$$

where

$$\begin{aligned} A &= \frac{R^f}{\gamma}, \\ B &= \sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2, \end{aligned}$$

to simplify notation. Given this expression, it follows that

$$p_N^2 \sigma_N^2 = \frac{A^2}{2\sigma_N^2} - A\sqrt{\left(\frac{A}{2\sigma_N^2}\right)^2 - \frac{1}{\sigma_N^2} B} - B.$$

Differentiating with respect to σ_N^2, we find with some manipulation that

$$\frac{\partial p_N^2 \sigma_N^2}{\partial \sigma_N^2} = \frac{A}{2\sigma_N^2} \frac{2\left(\frac{A}{2\sigma_N^2}\right)^2 - \frac{1}{\sigma_N^2} B - \frac{A}{\sigma_N^2}\sqrt{\left(\frac{A}{2\sigma_N^2}\right)^2 - \frac{1}{\sigma_N^2} B}}{\sqrt{\left(\frac{A}{2\sigma_N^2}\right)^2 - \frac{1}{\sigma_N^2} B}},$$

which we can factorize as

$$\frac{\partial p_N^2 \sigma_N^2}{\partial \sigma_N^2} = \left(\frac{A}{2\sigma_N^2}\right)^2 \frac{\left(1 - \sqrt{1 - \frac{4B\sigma_N^2}{A^2}}\right)^2}{\sqrt{1 - \frac{4B\sigma_N^2}{A^2}}} \geq 0,$$

and, since $P_t = \frac{1}{R^f - \rho_V} V_{t+1} + p_N N_t$ with V_{t+1} and N_t independent of each other, $\frac{\partial Var[R_{t+1} \mid \mathcal{F}_t]}{\partial \sigma_N^2} = \frac{\partial p_N^2 \sigma_N^2}{\partial \sigma_N^2}$.

Since $\frac{\partial p_N^2 \sigma_N^2}{\partial \sigma_N^2} \geq 0$, return volatility are highest close to market breakdown, when $\left(\frac{R^f}{\gamma \sigma_N^2}\right)^2 - 4\left(\frac{\sigma_D^2}{\sigma_N^2} + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 \frac{1}{\sigma_N^2}\right) = \varepsilon$ for ε arbitrarily small. Market breakdown occurs when $\varepsilon = 0$, or:

$$\sigma_N = \frac{R^f}{2\gamma \sqrt{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2}}.$$

Furthermore, as $\varepsilon \to 0$, and $\sigma_N \to \frac{R^f}{2\gamma \sqrt{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2}}$, then

$$p_N^2 \sigma_N^2 \to \sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2.$$

Consequently, the maximum conditional excess payoff variance before breakdown occurs is $Var\left[R_{t+1} \mid \mathcal{F}_t\right] \to 2\left(\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2\right)$.

Finally, rewriting $\frac{\partial Var[R_{t+1} \mid \mathcal{F}_t]}{\partial \sigma_N^2}$ as

$$\frac{\partial Var\left[R_{t+1} \mid \mathcal{F}_t\right]}{\partial \sigma_N^2} = \frac{1}{\sqrt{1 - \frac{4B\sigma_N^2}{A^2}}} \left(\frac{p_N^2 \sigma_N^2 + B}{A}\right)^2,$$

it is straightforward to see that $\frac{\partial^2 Var[R_{t+1} \mid \mathcal{F}_t]}{\partial \left(\sigma_N^2\right)^2} \geq 0$ because $\frac{\partial Var[R_{t+1} \mid \mathcal{F}_t]}{\partial \sigma_N^2}$, $p_N^2 \sigma_N^2 + B \geq 0$. Furthermore, by L'Hospital's Rule,

$$\lim_{\sigma_N^2 \to 0} \frac{\partial Var\left[R_{t+1} \mid \mathcal{F}_t\right]}{\partial \sigma_N^2} = \left(\frac{A}{2}\right)^2 \lim_{\sigma_N^2 \to 0} \left(\frac{1 - \sqrt{1 - \frac{4B\sigma_N^2}{A^2}}}{\sigma_N^2}\right)^2 = \left(\frac{B}{A}\right)^2,$$

while for the critical σ_N^2 at which breakdown occurs, $\sigma_N^2 = \frac{A^2}{4B}$,

$$\lim_{\sigma_N^2 \to 0} \frac{\partial Var\left[R_{t+1} \mid \mathcal{F}_t\right]}{\partial \sigma_N^2} = \infty.$$

Consequently, return variance is convex in σ_N^2 and increasing from $\left(\frac{B}{A}\right)^2$ to ∞ as σ_N^2 increases from 0 to $\frac{A^2}{4B}$.

Proof of Proposition 2

We derive the perfect information equilibrium with trading by the government. We first conjecture that, when V_{t+1} and N_t are observable to the government and investors, the stock price takes the linear form:

$$P_t = p_V V_{t+1} + p_N N_t + p_g G_t.$$

Given that dividends are $D_t = V_t + \sigma_D \varepsilon_t^D$, the stock price must react to a deterministic unit shift in V_{t+1} by the present value of dividends deriving from that shock, $\frac{1}{R^f - \rho_V}$, it follows that $p_V = \frac{1}{R^f - \rho_V}$. The innovations to V_{t+1} and N_t are the only source of risk and, from the perspective of all economic agents, the conditional expectation and variance of R_{t+1} are

$$\begin{aligned} E\left[R_{t+1} \mid \mathcal{F}_t\right] &= -p_N R^f N_t - R^f p_g G_t, \\ Var\left[R_{t+1} \mid \mathcal{F}_t\right] &= \sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2 \sigma_N^2 + p_g^2 \sigma_G^2. \end{aligned}$$

Since all investors are identical when V_t and N_t are observable, it follows that in the CARA-Normal environment all investors have an identical mean-variance demand for the risky asset:

$$X_t^S = \frac{1}{\gamma} \frac{E\left[R_{t+1} \mid \mathcal{F}_t\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t\right]} = -\frac{1}{\gamma} \frac{p_N R^f N_t + R^f p_g G_t}{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2 \sigma_N^2 + p_g^2 \sigma_G^2}.$$

In the government's intervention rule,

$$X_t^G = -\vartheta_N N_t + \vartheta_N \sigma_N G_t.$$

Finally, by imposing market-clearing, we arrive at

$$\begin{aligned} N &= \frac{1}{\gamma} \frac{p_N R^f N}{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2 \sigma_N^2 + p_g^2 \sigma_G^2} + \vartheta_N N, \\ \vartheta_N \sigma_N^2 G_t &= \frac{1}{\gamma} \frac{R^f p_g}{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2 \sigma_N^2 + p_g^2 \sigma_G^2} G_t, \end{aligned}$$

which, by matching coefficients, reveals that

$$\begin{aligned} \frac{1}{\gamma} \frac{p_N R^f}{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + p_N^2 \sigma_N^2 + p_g^2 \sigma_G^2} + \vartheta_N &= 1, \\ p_N \frac{\vartheta_N}{1 - \vartheta_N} \sigma_N &= p_g. \end{aligned}$$

This confirms the conjectured equilibrium.

Rearranging this equation for p_N, and substituting for p_g, we arrive at the quadratic equation for p_N:

$$\left(1 + \left(\frac{\vartheta_N}{1 - \vartheta_N}\right)^2 \sigma_G^2\right) p_N^2 - \frac{R^f}{\gamma \sigma_N^2 \left(1 - \vartheta_N\right)} p_N + \frac{\sigma_D^2}{\sigma_N^2} + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 \frac{1}{\sigma_N^2} = 0, \qquad \text{(IA.1)}$$

from which follows that p_N has two roots:

$$p_N\left(\vartheta_N\right) = \frac{1}{2} \frac{\frac{R^f}{\gamma \sigma_N^2 (1 - \vartheta_N)} \pm \sqrt{\left(\frac{R^f}{\gamma \sigma_N^2 (1 - \vartheta_N)}\right)^2 - 4\left(1 + \left(\frac{\vartheta_N}{1 - \vartheta_N}\right)^2 \sigma_G^2\right)\left(\frac{\sigma_D^2}{\sigma_N^2} + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 \frac{1}{\sigma_N^2}\right)}}{1 + \left(\frac{\vartheta_N}{1 - \vartheta_N}\right)^2 \sigma_G^2}.$$

Recognizing that two positive solutions for p_N exist if the expression under the square root is nonnegative, it follows that the market breaks down occurs whenever

$$R^f < 2(1-\vartheta_N)\gamma\sqrt{\left(1+\left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2\right)\left(\sigma_D^2\sigma_N^2+\left(\frac{\sigma_V}{R^f-\rho_V}\right)^2\sigma_N^2\right)}.$$

Consequently, market breakdown occurs when σ_N is sufficiently large.

Given that

$$\begin{aligned} Var(\Delta P_t \mid \mathcal{F}_{t-1}) &= Var\left(P_{t+1}-\frac{1}{R^f-\rho_V}V_{t+1} \mid \mathcal{F}_{t-1}\right) \\ &= \left(\frac{\sigma_V}{R^f-\rho_V}\right)^2 + p_N^2\sigma_N^2 + p_g^2\sigma_G^2 \\ &= \left(1+\left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2\right)p_N^2\sigma_N^2, \end{aligned}$$

substituting for p_g, it follows that regardless of whether the government is concerned with price volatility or price informativeness, reducing the price variance from noise trading, $p_N^2\sigma_N^2$, would accomplish both objectives, since

$$p_N^2\sigma_N^2 = -\frac{-\frac{R^f}{\gamma(1-\vartheta_N)}p_N+\sigma_D^2+\left(\frac{\sigma_V}{R^f-\rho_V}\right)^2}{1+\left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2},$$

is increasing in σ_N^2 through p_N.

To establish that the linear equilibrium is the unique, symmetric equilibrium, we express each investor's optimization problem as

$$U_t = \max_{X_t} E\left[e^{-\gamma\left(R\bar{W}+X_t\left(V_{t+1}+\sigma_D\varepsilon_{t+1}^D+P_{t+1}-RP_t\right)\right)} \mid \mathcal{F}_t\right].$$

For an arbitary price function P_t, the FOC for the investor's holding of the risky asset X_t is

$$E\left[\left(V_{t+1}+\sigma_D\varepsilon_{t+1}^D+P_{t+1}-R^fP_t\right)e^{-\gamma X_t\left(V_{t+1}+\sigma_D\varepsilon_{t+1}^D+P_{t+1}-RP_t\right)} \mid \mathcal{F}_t\right] = 0.$$

Substituting this with the market-clearing condition:

$$X_t = -(1-\vartheta_N)N_t - \vartheta_N\sigma_N G_t,$$

we arrive at

$$E\left[\left(V_{t+1}+\sigma_D\varepsilon_{t+1}^D+P_{t+1}-R^fP_t\right)e^{\gamma((1-\vartheta_N)N_t+\vartheta_N\sigma_N G_t,)\left(V_{t+1}+\sigma_D\varepsilon_{t+1}^D+P_{t+1}-RP_t\right)} \mid \mathcal{F}_t\right] = 0.$$

Since P_{t+1} cannot be a function of ε_{t+1}^D, as P_{t+1} is forward-looking for the new generation of investors at time $t+1$, the above can be rewritten as

$$\begin{aligned} P_t &= \frac{1}{R^f}V_{t+1}+\frac{\gamma}{R^f}\sigma_D^2\left((1-\vartheta_N)N_t+\vartheta_N\sigma_N G_t\right) \\ &\quad +\frac{1}{R^f}E\left[P_{t+1}\frac{e^{\gamma((1-\vartheta_N)N_t+\vartheta_N\sigma_N G_t)P_{t+1}}}{E\left[e^{\gamma((1-\vartheta_N)N_t+\vartheta_N\sigma_N G_t)P_{t+1}} \mid \mathcal{F}_t\right]} \mid \mathcal{F}_t\right], \end{aligned} \tag{IA.2}$$

where we have used the properties of log-normal random variables to complete the square in the pdf and solve explicitly for the ε_{t+1}^{D} term. This defines a functional equation, whose fixed point is the price functional P_t. To see that the linear equilibrium we derived above solves this functional equation, we rewrite equation (IA.2) as

$$P_t = \frac{1}{R^f}V_{t+1} + \frac{\gamma}{R^f}\sigma_D^2\left((1-\vartheta_N)N_t + \vartheta_N\sigma_N G_t\right) + \frac{1}{R^f}\left|\partial_u \log E\left[e^{uP_{t+1}} \mid \mathcal{F}_t\right]\right|_{u=-\gamma((1-\vartheta_N)N_t+\vartheta_N\sigma_N G_t)},$$

and conjecture that $P_t = \frac{1}{R^f-\rho_V}V_{t+1} + p_N N_t + p_g G_t$, from which follows that p_N satisfies the recursion:

$$p_{N,t} = \frac{\gamma(1-\vartheta_N)}{R^f}\left(\sigma_D^2 + \left(\frac{\sigma_V}{R^f-\rho_V}\right)^2 + \left(1+\left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2\right)\sigma_N^2 p_{N,t+1}^2\right).$$

Suppose there is some final date $T >> 0$. On this final date, $P_T = 0$ since there is no salvage value to the asset. Then, as time goes backward, this recursion converges after a sufficiently long period of time to

$$p_{N,t} \rightarrow_{t\rightarrow 0} \frac{1}{2}\frac{\frac{R^f}{\gamma\sigma_N^2(1-\vartheta_N)} - \sqrt{\left(\frac{R^f}{\gamma\sigma_N^2(1-\vartheta_N)}\right)^2 - 4\left(1+\left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2\right)\left(\frac{\sigma_D^2}{\sigma_N^2} + \left(\frac{\sigma_V}{R^f-\rho_V}\right)^2\frac{1}{\sigma_N^2}\right)}}{1+\left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2},$$

which is the more stable of the two positive roots from the infinite horizon problem if

$$R^f < 2(1-\vartheta_N)\gamma\sqrt{\left(1+\left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2\right)\left(\sigma_D^2\sigma_N^2 + \left(\frac{\sigma_V}{R^f-\rho_V}\right)^2\sigma_N^2\right)},$$

and

$$p_{N,t} \rightarrow_{t\rightarrow 0} \infty,$$

otherwise. Consequently, we can interpret market breakdown as an unstable backward recursion in which illiquidity is growing each period as volatility diverges. Interestingly, we obtain the more positive root for the fixed point for p_N from (IA.1) from the forward recursion:

$$p_{N,t+1} = \frac{\gamma(1-\vartheta_N)}{R^f}\left(\sigma_D^2 + \left(\frac{\sigma_V}{R^f-\rho_V}\right)^2 + \left(1+\left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2\right)\sigma_N^2 p_{N,t}^2\right).$$

Consequently, the more positive root is forward stable, but backward unstable.

Finally, notice from the recursion (IA.2) that, if the price at date $t+1$, P_{t+1}, is linear in $\{V_{t+1}, N_{t+1}, G_{t+1}\}$, and therefore normally distributed, then $\log E\left[e^{uP_{t+1}} \mid \mathcal{F}_t\right] = uE\left[P_{t+1} \mid \mathcal{F}_t\right] + \frac{1}{2}u^2 Var\left[P_{t+1} \mid \mathcal{F}_t\right]$, or the moment-generating function for the normally distributed price. It then follows that the only solution is

$$\begin{aligned} P_t &= \frac{1}{R^f}V_{t+1} + \frac{\gamma}{R^f}\sigma_D^2\left((1-\vartheta_N)N_t + \vartheta_N\sigma_N G_t\right) + E\left[P_{t+1} \mid \mathcal{F}_t\right] \\ &\quad + \frac{\gamma}{R^f}\left((1-\vartheta_N)N_t + \vartheta_N\sigma_N G_t\right)Var\left[P_{t+1} \mid \mathcal{F}_t\right], \end{aligned}$$

and it follows that P_t is linear. Consequently, the linear equilibrium with the less positive p_N root of (IA.1) is the unique, backward stable equilibrium as the limit of the finite horizon problem.

Proof of Proposition A1

Based on Proposition A3 and Proposition A5, in the special case of no government intervention, the steady-state conditional means of the Kalman Filter, $\left(\hat{V}_{t+1}^M, \hat{N}_t^M\right)$, have a law of motion that satisfies

$$\left[\begin{array}{c} \hat{V}_{t+1}^M \\ \hat{N}_t^M \end{array}\right] = \left[\begin{array}{cc} \rho_V & 0 \\ 0 & 0 \end{array}\right] \left[\begin{array}{c} \hat{V}_t^M \\ \hat{N}_{t-1}^M \end{array}\right] + \mathbf{k}_t^M \left[\begin{array}{c} D_t - \hat{V}_t^M \\ \eta_t^H - p_V \rho_V \hat{V}_t^M \end{array}\right],$$

where

$$\mathbf{k}^M = \left[\begin{array}{cc} \rho_V \Sigma^{M,VV} & p_V \left(\rho_V^2 \Sigma^{M,VV} + \sigma_V^2\right) \\ 0 & p_N \sigma_N^2 \end{array}\right] \left(\Omega^M\right)^{-1}$$

is the Kalman Gain, and the conditional variance Σ^M satisfies the Ricatti Equation:

$$\begin{aligned} \Sigma^M &= \left[\begin{array}{cc} \rho_V & 0 \\ 0 & 0 \end{array}\right] \Sigma^M \left[\begin{array}{cc} \rho_V & 0 \\ 0 & 0 \end{array}\right] + \left[\begin{array}{cc} \sigma_V^2 & 0 \\ 0 & \sigma_N^2 \end{array}\right] \\ & \quad -\mathbf{k}^M \left[\begin{array}{cc} \rho_V \Sigma^{M,VV} & p_V \left(\rho_V \Sigma^{M,VV} + \sigma_V^2\right) \\ 0 & p_N \sigma_N^2 \end{array}\right]', \end{aligned}$$

and that

$$\begin{aligned} \Omega^M &= Var\left[\left[\begin{array}{c} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \end{array}\right] \mid \mathcal{F}_{t-1}^M\right] \\ &= \left[\begin{array}{cc} \Sigma^{M,VV} + \sigma_D^2 & p_V \rho_V \Sigma^{M,VV} \\ p_V \rho_V \Sigma^{M,VV} & p_V^2 \left(\rho_V^2 \Sigma^{M,VV} + \sigma_V^2\right) + p_N^2 \sigma_N^2 \end{array}\right]. \end{aligned}$$

From Proposition A4, we further recognize that:

$$\begin{aligned} \Sigma^{M,VN} &= -\frac{p_V}{p_N} \Sigma^{M,VV}, \\ \Sigma^{M,NN} &= \left(\frac{p_V}{p_N}\right)^2 \Sigma^{M,VV}. \end{aligned}$$

Consequently, recognizing that all four implied coefficient equations for Σ^M are degenerate, the equation that identifies $\Sigma^{M,VV}$ reduces to

$$\frac{\Sigma^{M,VV}}{\sigma_V^2} = \frac{\left(\frac{p_N \sigma_N}{p_V \sigma_V}\right)^2 \left(\left(1 + \rho_V^2 \left(\frac{\sigma_D}{\sigma_V}\right)^2\right) \frac{\Sigma^{M,VV}}{\sigma_V^2} + \left(\frac{\sigma_D}{\sigma_V}\right)^2\right)}{\left(\frac{p_N \sigma_N}{p_V \sigma_V}\right)^2 \left(\frac{\Sigma^{M,VV}}{\sigma_V^2} + \left(\frac{\sigma_D}{\sigma_V}\right)^2\right) + \left(1 + \rho_V^2 \left(\frac{\sigma_D}{\sigma_V}\right)^2\right) \frac{\Sigma^{M,VV}}{\sigma_V^2} + \left(\frac{\sigma_D}{\sigma_V}\right)^2}.$$

From Proposition A4, investors update their beliefs from the market beliefs by Bayes' Law in accordance with a linear updating rule. The posterior of investor i is $N\left(\hat{V}_{t+1}^i, \Sigma_t^s(i)\right)$, where $\hat{V}_{t+1}^i = E\left[V_{t+1} \mid \mathcal{F}_t^i\right]$ and $\Sigma^s(i) = E\left[\left(V_{t+1} - \hat{V}_{t+1}^i\right)^2 \mid \mathcal{F}_t^i\right]$ are given by

$$\hat{V}_{t+1}^i = \hat{V}_{t+1}^M + \frac{\Sigma^{M,VV}}{\Sigma^{M,VV} + \tau_s^{-1}} \left(s_t^i - \hat{V}_{t+1}^M\right),$$

and

$$\Sigma^{s}(i)^{-1} = \left(\Sigma^{M,VV}\right)^{-1} + \tau_s.$$

This characterizes the beliefs of investors given the market beliefs.

Since the government does not trade in this benchmark, investors have no incentive to learn about the government's behavior, and therefore the information acquisition decision is trivial. Given that investors each acquire a private signal s_t^i, standard results for CARA utility with normally distributed prices and payoffs establish that the optimal trading policy of investor i, X_t^i, is given by

$$\begin{aligned} X_t^i &= \frac{E\left[D_{t+1} + P_{t+1} - R^f P_t \mid \mathcal{F}_t^i\right]}{\gamma Var\left[D_{t+1} + P_{t+1} \mid \mathcal{F}_t^i\right]} \\ &= \frac{\left(\begin{array}{c} \left(1 + p_V\left(\rho_V - R^f\right)\right)\left(\hat{V}_{t+1}^i - \hat{V}_{t+1}^M\right) - p_N R^f \hat{N}_t^i \\ + \left[\begin{array}{c} p_{\hat{V}} - p_V \\ 0 \end{array}\right]' \mathbf{k}^M \left[\begin{array}{c} \hat{V}_{t+1}^i - \hat{V}_{t+1}^M \\ p_V \rho_V \left(\hat{V}_{t+1}^i - \hat{V}_{t+1}^M\right) \end{array}\right] \end{array}\right)}{\gamma \varphi' \Omega^S \varphi}, \end{aligned}$$

where

$$\varphi = \left[\begin{array}{c} 1 \\ 1 \end{array}\right] + \mathbf{k}^{M\prime} \left[\begin{array}{c} p_{\hat{V}} - p_V \\ 0 \end{array}\right],$$

and

$$\Omega^S = \Omega^M - \left[\begin{array}{c} 1 \\ p_V \rho_V \end{array}\right] \frac{\left(\Sigma^{M,VV}\right)^2}{\Sigma^{M,VV} + \tau_s^{-1}} \left[\begin{array}{c} 1 \\ p_V \rho_V \end{array}\right]',$$

is the conditional variance of D_{t+1} and P_{t+1} with respect to $\mathcal{F}_t^i$. we can rewrite the above as

$$X_t^i = \frac{\left(1 + p_V\left(\rho_V - R^f\right) + \left[\begin{array}{c} p_{\hat{V}} - p_V \\ 0 \end{array}\right]' \mathbf{k}^M \left[\begin{array}{c} 1 \\ p_V \rho_V \end{array}\right]\right)\left(\hat{V}_{t+1}^i - \hat{V}_{t+1}^M\right) - p_N R^f \hat{N}_t^i}{\gamma \varphi' \Omega^S \varphi}.$$

Substituting for $\hat{V}_{t+1}^i$, and recognizing from above that

$$\hat{N}_t^M = N_t + \frac{p_V}{p_N}\left(V_{t+1} - \hat{V}_{t+1}^M\right),$$

and therefore

$$\hat{N}_t^i = \hat{N}_t^M - \frac{p_V}{p_N}\left(\hat{V}_{t+1}^i - \hat{V}_{t+1}^M\right) = N_t + \frac{p_V}{p_N}\left(V_{t+1} - \hat{V}_{t+1}^M\right) - \frac{p_V}{p_N}\left(\hat{V}_{t+1}^i - \hat{V}_{t+1}^M\right),$$

we arrive at

$$X_t^i = \frac{\left(\varphi' \left[\begin{array}{c} 1 \\ p_V \rho_V \end{array}\right] \frac{\Sigma^{M,VV}}{\Sigma^{M,VV} + \tau_s^{-1}} \left(s_t^i - \hat{V}_{t+1}^M\right) - R^f p_N N_t - R^f p_V \left(V_{t+1} - \hat{V}_{t+1}^M\right)\right)}{\gamma \varphi' \Omega^S \varphi}.$$

Aggregating over the demand of investors and imposing market-clearing, we arrive at the two equations for p_V and p_N:

$$\begin{aligned} \varphi' \begin{bmatrix} 1 \\ p_V \rho_V \end{bmatrix} \frac{\Sigma^{M,VV}}{\Sigma^{M,VV} + \tau_s^{-1}} - R^f p_V &= 0, \\ \frac{R^f p_N}{\gamma \varphi' \Omega^S \varphi} &= 1. \end{aligned}$$

This completes our characterization of the linear equilibrium.

We now recognize from the market-clearing condition for p_V that we can express p_V as

$$p_V = \left(\frac{1}{R^f - \rho_V} + \frac{p_{\hat{V}} - p_V}{R^f} \frac{\rho_V^2 (1-\rho_V) \frac{\Sigma^{M,VV}}{\sigma_V^2} \left(\frac{\Sigma^{M,VV}}{\sigma_V^2} + \left(\frac{\sigma_D}{\sigma_V}\right)^2 \right) - \left(\frac{p_N \sigma_N}{p_V \sigma_V}\right)^2 \left(\frac{\sigma_D}{\sigma_V}\right)^2}{\rho_V^2 \frac{\Sigma^{M,VV}}{\sigma_V^2} \left(\frac{\sigma_D}{\sigma_V}\right)^2 + \left(1 + \left(\frac{p_N \sigma_N}{p_V \sigma_V}\right)^2\right) \left(\frac{\Sigma^{M,VV}}{\sigma_V^2} + \left(\frac{\sigma_D}{\sigma_V}\right)^2 \right)} \right) \frac{\Sigma^{M,VV}}{\Sigma^{M,VV} + \tau_s^{-1}}.$$

From our implicit equation for $\Sigma^{M,VV}$ above, we can verify that the second term in parentheses is negative (assuming $p_{\hat{V}} > p_V$), from which follows that

$$p_V \leq \frac{1}{R^f - \rho_V} \frac{\Sigma^{M,VV}}{\Sigma^{M,VV} + \tau_s^{-1}} \leq \frac{1}{R^f - \rho_V} = p_{\hat{V}},$$

confirming the assumption.

Finally, recognizing that return volatility from the market perspective satisfies

$$\begin{aligned} \varphi' \Omega^M \varphi &= \left(1 + 2 p_{\hat{V}} \rho_V + p_V^2 \rho_V^2 + 2 \left(p_{\hat{V}} - p_V\right) p_V \rho_V - \left(p_{\hat{V}} - p_V\right)^2 \left(1 - \rho_V^2\right)\right) \Sigma^{M,VV} \\ &\quad + \sigma_D^2 + p_N^2 \sigma_N^2 + p_{\hat{V}}^2 \sigma_V^2, \end{aligned}$$

which makes use of the relation by the Law of Total Variance

$$\mathbf{k}^M \Omega^M \mathbf{k}^{M\prime} = \begin{bmatrix} \rho_V^2 \Sigma^{M,VV} + \sigma_V^2 & 0 \\ 0 & \sigma_N^2 \end{bmatrix} - \Sigma^M.$$

We can rewrite the market-clearing condition for p_N, substituting with that of p_V, as

$$\begin{aligned} \frac{R^f p_N}{\gamma} &= \left(1 + 2 \left(p_{\hat{V}} + p_V p_{\hat{V}} - p_V^2\right) \rho_V + p_V^2 \rho_V^2 - \left(p_{\hat{V}} - p_V\right)^2 - \frac{\Sigma^{M,VV} + \tau_s^{-1}}{\Sigma^{M,VV}} \left(R^f\right)^2 p_V^2 \right) \Sigma^{M,VV} \\ &\quad + \sigma_D^2 + p_N^2 \sigma_N^2 + p_{\hat{V}}^2 \sigma_V^2. \end{aligned}$$

Notice in the special case that $\rho_V = 0$ that

$$p_V = \frac{\Sigma^{M,VV}}{\Sigma^{M,VV} + \tau_s^{-1}} \frac{1}{R^f},$$

and the above condition for p_N reduces to

$$\frac{R^f p_N}{\gamma} = \left(1 - \left(\frac{1}{R^f}\right)^2 \frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right) \frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} + \sigma_D^2 + \left(\frac{1}{R^f}\right)^2 \sigma_V^2 + p_N^2 \sigma_N^2,$$

since $p_{\hat{V}} = \frac{1}{R^f}$. Comparing this condition for p_N to the perfect information case:

$$\frac{1}{\gamma} p_N R^f = \sigma_D^2 + \left(\frac{1}{R^f}\right)^2 \sigma_V^2 + p_N^2 \sigma_N^2,$$

we recognize since $\frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} > 0$ that the additional term from uncertainty exacerbates the market breakdown problem. To see this, we fix $\Sigma^{M,VV}$ and express the solution to p_N as

$$p_N = \frac{1}{2\sigma_N^2} A - \sqrt{\left(\frac{1}{2\sigma_N^2} A\right)^2 - \frac{1}{\sigma_N^2} B},$$

where

$$\begin{aligned} A &= \frac{R^f}{\gamma}, \\ B &= \sigma_D^2 + \left(\frac{1}{R^f}\right)^2 \sigma_V^2 + \left(1 - \left(\frac{1}{R^f}\right)^2 \frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right) \frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}. \end{aligned}$$

Since $\left(1 - \frac{1}{R^2} \frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right) \frac{\Sigma^{M,VV} \tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} > 0$, regardless of the equilibrium value of p_N, it follows nonexistence, which occurs when

$$\left(\frac{1}{2\sigma_N^2} A\right)^2 - \frac{1}{\sigma_N^2} B < 0,$$

must now occur at a positive value of p_N, and that p_N is higher in the presence of informational frictions when a solution exists (by shrinking the $\sqrt{\left(\frac{1}{2\sigma_N^2} A\right)^2 - \frac{1}{\sigma_N^2} B}$ term in the expression for p_N). From the condition for existence in Proposition 1, it follows that market-breakdown must occur at a lower value of σ_N, $\sigma_N^{**} \geq \sigma_N^*$.

Consider the other extreme of $\rho_V = 1$, then the coefficient on $\Sigma^{M,VV}$ in the expression for p_N reduces to

$$1 + 2p_{\hat{V}} + (2p_{\hat{V}} - p_V) p_V - \frac{\Sigma^{M,VV} + \tau_s^{-1}}{\Sigma^{M,VV}} \left(R^f\right)^2 p_V^2.$$

Since $p_V \leq \frac{\Sigma^{M,VV}}{\Sigma^{M,VV} + \tau_s^{-1}} p_{\hat{V}}$ and with $p_{\hat{V}} = \frac{1}{R^f - 1}$, it follows that

$$\begin{aligned} & 1 + 2p_{\hat{V}} + (2p_{\hat{V}} - p_V) p_V - \frac{\Sigma^{M,VV} + \tau_s^{-1}}{\Sigma^{M,VV}} \left(R^f\right)^2 p_V^2 \\ > & \; 1 + 2p_{\hat{V}} + (p_{\hat{V}} - p_V) p_V - p_V \left(1 + R^f\right) \\ = & \; 1 - p_V \left(R^f - 1\right) + (p_{\hat{V}} - p_V)(2 + p_V) \\ > & \; \frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}} + (p_{\hat{V}} - p_V)(2 + p_V) \\ > & \; 0, \end{aligned}$$

and consequently similar arguments establish that market breakdown occurs sooner, and p_N is more positive with informational frictions. The intermediate cases ($\rho_V \in (0,1)$) follow by similar arguments.

Proof of Proposition A2

For the first part of the claim, one can express the deviation of the price from its fundamental value as

$$\begin{aligned}
& Var\left[P_{t+1}-p_{\hat{V}}V_{t+2}\mid \mathcal{F}_t^M\right] \\
= \; & Var\left[\left(p_V-p_{\hat{V}}\right)\left(V_{t+2}-\hat{V}_{t+2}^M\right)+p_N N_t \mid \mathcal{F}_t^M\right] \\
= \; & \left(p_V-p_{\hat{V}}\right)^2 Var\left[V_{t+2}-\hat{V}_{t+2}^M \mid \mathcal{F}_t^M\right]+p_N^2\sigma_N^2-2\left(p_V-p_{\hat{V}}\right)p_N\begin{bmatrix}1\\0\end{bmatrix}'\mathbf{k}^M\begin{bmatrix}0\\p_N\sigma_N^2\end{bmatrix} \\
= \; & \left(p_V-p_{\hat{V}}\right)^2 Var\left[V_{t+2}-\hat{V}_{t+2}^M \mid \mathcal{F}_t^M\right]+p_N^2\sigma_N^2 \\
& +\frac{2\left(p_{\hat{V}}-p_V\right)p_V p_N^2\sigma_N^2\left(\rho_V^2\Sigma^{M,VV}\sigma_D^2+\sigma_V^2\left(\Sigma^{M,VV}+\sigma_D^2\right)\right)}{\left(\Sigma^{M,VV}+\sigma_D^2\right)\left(p_V^2\left(\rho_V^2\Sigma^{M,VV}+\sigma_V^2\right)+p_N^2\sigma_N^2\right)-\left(p_V\rho_V\Sigma^{M,VV}\right)^2} \\
\geq \; & \left(p_V-p_{\hat{V}}\right)^2 Var\left[V_{t+2}-\hat{V}_{t+2}^M \mid \mathcal{F}_t^M\right]+p_N^2\sigma_N^2 \\
\geq \; & p_N^2\sigma_N^2,
\end{aligned}$$

because $p_V \leq p_{\hat{V}}$. Notice that its perfect information counterpart, $Var\left[P_{t+1}-p_{\hat{V}}V_{t+2}\mid \mathcal{F}_t\right]$, is $p_N^2\sigma_N^2$, but p_N is smaller than in the presence of informational frictions. Consequently, the conditional deviation of the price from its fundamental value, and therefore price informativeness, is lower in the presence of informational frictions.

For the second part of the claim, consider now conditional return volatility

$$\begin{aligned}
Var\left[R_{t+1}\mid \mathcal{F}_t^M\right] &= \varphi'\Omega^M\varphi \\
&= \left(1+2p_{\hat{V}}\rho_V+p_V^2\rho_V^2+2\left(p_{\hat{V}}-p_V\right)p_V\rho_V-\left(p_{\hat{V}}-p_V\right)^2\left(1-\rho_V^2\right)\right)\Sigma^{M,VV} \\
&\quad +\sigma_D^2+p_{\hat{V}}^2\sigma_V^2+p_N^2\sigma_N^2.
\end{aligned}$$

Define

$$J\left(\rho_V\right)=1+2p_{\hat{V}}\rho_V+p_V^2\rho_V^2+2\left(p_{\hat{V}}-p_V\right)p_V\rho_V-\left(p_{\hat{V}}-p_V\right)^2\left(1-\rho_V^2\right).$$

We next recognize that

$$J(1)=1+2p_{\hat{V}}+\left(2p_{\hat{V}}-p_V\right)p_V>0,$$

and

$$J(0)=1-\left(p_{\hat{V}}-p_V\right)^2=1-\left(\frac{1}{R^f}\frac{\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}\right)^2>0,$$

because $p_V=\frac{\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}}\frac{1}{R^f}$ and $p_{\hat{V}}=\frac{1}{R^f}$ in this special case, with $J(1)>1>J(0)$. Notice also that we can establish that

$$\begin{aligned}
\frac{1}{2}\frac{dJ\left(\rho_V\right)}{d\rho_V} &= p_{\hat{V}}+p_V^2\rho_V+\left(p_{\hat{V}}-p_V\right)p_V+\left(p_{\hat{V}}-p_V\right)^2\rho_V \\
&\quad +\left(\rho_V+p_V\rho_V-\left(p_{\hat{V}}-p_V\right)\left(1-\rho_V^2\right)\right)p_{\hat{V}}^2 \\
&\quad +\left(\left(p_{\hat{V}}-p_V\right)\left(1+\rho_V-\rho_V^2\right)-p_V\rho_V\left(1-\rho_V\right)\right)\frac{dp_V}{d\rho_V},
\end{aligned}$$

whereby it follows that

$$\frac{R^f}{2}\frac{dJ\left(\rho_V\right)}{d\rho_V}|_{\rho_V=0} = 1-\frac{\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}\left(\frac{1}{R^f}\right)^2+\frac{\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}\left(p_V+2\frac{dp_V}{d\rho_V}\right)>0,$$

and

$$\frac{1}{2}\frac{dJ\left(\rho_V\right)}{d\rho_V}|_{\rho_V=1} = p_{\hat{V}}+\left(p_{\hat{V}}-p_V\right)p_V+\left(1+p_V\right)p_{\hat{V}}^2+2\left(p_{\hat{V}}-p_V\right)\frac{dp_V}{d\rho_V}>0$$

because, intuitively, the coefficient on the asset fundamental, p_V, (weakly) increases with its persistence, or $\frac{dp_V}{d\rho_V}>0$. Since $J\left(\rho_V\right)$ is continuous in ρ_V, as are $p_{\hat{V}}$ and p_V, we conclude more broadly that $J\left(\rho_V\right)\geq 0$. This implies that

$$Var\left[R_{t+1} \mid \ \mathcal{F}_t^M\right] \geq \sigma_D^2+p_{\hat{V}}^2\sigma_V^2+p_N^2\sigma_N^2.$$

which is higher than its perfect information counterpart, $Var\left[R_{t+1} \mid \ \mathcal{F}_t\right]=\sigma_D^2+p_{\hat{V}}^2\sigma_V^2+p_N^2\sigma_N^2$, because p_N is larger with informational frictions. Return volatility is therefore higher in the presence of informational frictions.

Proof of Proposition A3

To arrive at the beliefs of investors and the government, we first characterize the market beliefs based on the public information set $\mathcal{F}_t^M$. To derive the market beliefs, we proceed in several steps. First, we assume the market posterior belief of (V_{t+1}, N_t, G_{t+1}) is jointly Gaussian, $(V_{t+1}, N_t, G_{t+1}) \sim \mathcal{N}\left(\left(\hat{V}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M\right), \Sigma_t^M\right)$, where

$$\begin{bmatrix} \hat{V}_{t+1}^M \\ \hat{N}_t^M \\ \hat{G}_{t+1}^M \\ G_t \end{bmatrix} = E\left[\begin{bmatrix} V_{t+1} \\ N_t \\ G_{t+1} \\ G_t \end{bmatrix} \mid \mathcal{F}_t^M\right],$$

$$\Sigma_t^M = \begin{bmatrix} \Sigma_t^{M,VV} & \Sigma_t^{M,VN} & \Sigma_t^{M,VG} & 0 \\ \Sigma_t^{M,VN} & \Sigma_t^{M,NN} & \Sigma_t^{M,NG} & 0 \\ \Sigma_t^{M,VG} & \Sigma_t^{M,NG} & \Sigma_t^{M,GG} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Standard results for the Kalman Filter then establish that the law of motion of the conditional expectation of the market's posterior beliefs $\left(\hat{V}_{t+1}^M, \hat{N}_t^M\right)$ is

$$\begin{bmatrix} \hat{V}_{t+1}^M \\ \hat{N}_t^M \\ \hat{G}_{t+1}^M \\ G_t \end{bmatrix} = \begin{bmatrix} \rho_V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \hat{V}_t^M \\ \hat{N}_{t-1}^M \\ \hat{G}_{t|t-1}^M \\ G_{t-1} \end{bmatrix} + \mathbf{K}_t^M \begin{bmatrix} D_t - \hat{V}_t^M \\ \eta_t^M - p_V\rho_V\hat{V}_t^M \\ G_t - \hat{G}_{t|t-1}^M \end{bmatrix},$$

where

$$\mathbf{K}_t^M = Cov\left[\left[\begin{array}{c} V_{t+1} \\ N_t \\ G_{t+1} \\ G_t \end{array}\right], \left[\begin{array}{c} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_{t-1}^M \\ G_t - \hat{G}_{t|t-1}^M \end{array}\right] \mid \mathcal{F}_{t-1}^M\right] \times Var\left[\left[\begin{array}{c} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \\ G_t - G_{t|t-1}^M \end{array}\right] \mid \mathcal{F}_{t-1}^M\right]^{-1},$$

is the Kalman Gain, and that the conditional variance Σ_t^M evolves deterministically according to:

$$\Sigma_t^M = \left[\begin{array}{cccc} \rho_V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \Sigma_{t-1}^M \left[\begin{array}{cccc} \rho_V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] + \left[\begin{array}{cccc} \sigma_V^2 & 0 & 0 & 0 \\ 0 & \sigma_N^2 & 0 & 0 \\ 0 & 0 & \sigma_G^2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] - \mathbf{K}_t^M Cov\left[\left[\begin{array}{c} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \\ G_t - \hat{G}_{t|t-1}^M \end{array}\right], \left[\begin{array}{c} V_{t+1} \\ N_t \\ G_{t+1} \\ G_t \end{array}\right] \mid \mathcal{F}_{t-1}^M\right].$$

It is straightforward to compute that

$$Cov\left[\left[\begin{array}{c} V_{t+1} \\ N_t \\ G_{t+1} \\ G_t \end{array}\right], \left[\begin{array}{c} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \\ G_t - \hat{G}_{t|t-1}^M \end{array}\right] \mid \mathcal{F}_{t-1}^M\right] = \left[\begin{array}{ccc} \rho_V \Sigma_{t-1}^{M,VV} & p_V \left(\rho_V^2 \Sigma_{t-1}^{M,VV} + \sigma_V^2\right) & \rho_V \Sigma_{t-1}^{M,VG} \\ 0 & p_N \sigma_N^2 & 0 \\ 0 & p_G \sigma_G^2 & 0 \\ \Sigma_{t-1}^{M,VG} & p_V \rho_V \Sigma_{t-1}^{M,VG} & \Sigma_{t-1}^{M,GG} \end{array}\right],$$

and that

$$\Omega_{t-1}^M = Var\left[\left[\begin{array}{c} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \\ G_t - \hat{G}_{t|t-1}^M \end{array}\right] \mid \mathcal{F}_{t-1}^M\right] = \left[\begin{array}{ccc} \Sigma_{t-1}^{M,VV} + \sigma_D^2 & p_V \rho_V \Sigma_{t-1}^{M,VV} & \Sigma_{t-1}^{M,VG} \\ p_V \rho_V \Sigma_{t-1}^{M,VV} & p_V^2 \left(\rho_V^2 \Sigma_{t-1}^{M,VV} + \sigma_V^2\right) + p_N^2 \sigma_N^2 + p_G^2 \sigma_G^2 & p_V \rho_V \Sigma_{t-1}^{M,VG} \\ \Sigma_{t-1}^{M,VG} & p_V \rho_V \Sigma_{t-1}^{M,VG} & \Sigma_{t-1}^{M,GG} \end{array}\right].$$

Since $\eta_t^M \in \mathcal{F}_t^M \subseteq \mathcal{F}_t$, I can express η_t^M as

$$\eta_t^M = p_V V_{t+1} + p_N N_t + p_G G_{t+1} = p_V \hat{V}_{t+1}^M + p_N \hat{N}_t^M + p_G \hat{G}_{t+1}^M,$$

from which follows that

$$p_V\left(V_{t+1}-\hat{V}_{t+1}^M\right)+p_N\left(N_t-\hat{N}_t^M\right)+p_G\left(G_{t+1}-\hat{G}_{t+1}^M\right)=0.$$

As a consequence, it must be that the market beliefs about V_t and N_t are ex-post correlated after observing the stock price innovation process η_t^M, such that we have the three identities by taking its variance and its covariance with $V_{t+1}-\hat{V}_{t+1}^M$ and $N_t-\hat{N}_t^M$:

$$\begin{aligned}\Sigma_t^{M,VN} &= -\frac{p_V}{p_N}\Sigma_t^{M,VV}-\frac{p_G}{p_N}\Sigma_t^{M,VG},\\ \Sigma_t^{M,NN} &= -\frac{p_V}{p_N}\Sigma_t^{M,VN}-\frac{p_G}{p_N}\Sigma_t^{M,NG},\\ \Sigma_t^{M,NG} &= -\frac{p_V}{p_N}\Sigma_t^{M,VG}-\frac{p_G}{p_N}\Sigma_t^{M,GG}.\end{aligned}$$

This completes our characterization of the market's beliefs.

Proof of Proposition A4

Updating the market beliefs to each investor's private beliefs can be done in a manner similar to that in He and Wang (1995). Note that the market beliefs act as the prior for investor i who observes the normally distributed private signal s_t^i. The posterior of investor i is $N\left(\left(\hat{V}_{t+1}^i,\hat{N}_t^i,\hat{G}_{t+1|t}^i\right),\Sigma_t^i\right)$, where $\left(\hat{V}_{t+1}^i,\hat{N}_t^i,\hat{G}_{t+1|t}^i\right)=E\left[\left(V_t,N_t,G_{t+1}\right)\mid\mathcal{F}_t^i\right]$ and $\Sigma_t^s(i)=$ $E\left[\begin{bmatrix}V_{t+1}-\hat{V}_{t+1}^i\\ N_t-\hat{N}_t^i\\ G_{t+1}-\hat{G}_{t+1|t}^i\end{bmatrix}\begin{bmatrix}V_{t+1}-\hat{V}_{t+1}^i\\ N_t-\hat{N}_t^i\\ G_{t+1}-\hat{G}_{t+1|t}^i\end{bmatrix}'\mid\mathcal{F}_t^i\right]$ are given by

$$\begin{bmatrix}\hat{V}_{t+1}^i\\ \hat{N}_t^i\\ \hat{G}_{t+1}^i\end{bmatrix}=\begin{bmatrix}\hat{V}_{t+1}^M\\ \hat{N}_t^M\\ \hat{G}_{t+1}^M\end{bmatrix}+\Gamma_t'\begin{bmatrix}s_t^i-\hat{V}_{t+1}^M\\ g_t^i-\hat{G}_{t+1}^M\end{bmatrix},$$

where

$$\begin{aligned}\Gamma_t' &= CoV\left[\begin{bmatrix}V_{t+1}\\ N_t\\ G_{t+1}\end{bmatrix},\begin{bmatrix}s_t-\hat{V}_{t+1}^M\\ g_t^i-\hat{G}_{t+1}^M\end{bmatrix}\mid\mathcal{F}_{t-1}^M\right]Var\left[\begin{bmatrix}s_t^i-\hat{V}_{t+1}^M\\ g_t^i-\hat{G}_{t+1}^M\end{bmatrix}\mid\mathcal{F}_t^M\right]^{-1}\\ &= \begin{bmatrix}\Sigma_t^{M,VV} & \Sigma_t^{M,VG}\\ \Sigma_t^{M,VN} & \Sigma_t^{M,NG}\\ \Sigma_t^{M,VG} & \Sigma_t^{M,GG}\end{bmatrix}\begin{bmatrix}\Sigma_t^{M,VV}+\left(a^i\tau_s\right)^{-1} & \Sigma_t^{M,VG}\\ \Sigma_t^{M,VG} & \Sigma_t^{M,GG}+\left[\left(1-a^i\right)\tau_g\right]^{-1}\end{bmatrix}^{-1},\end{aligned}$$

and

$$\Sigma_t^s(i)=\Sigma_t^M-\Gamma_t^{i\prime}\begin{bmatrix}\Sigma_t^{M,VV} & \Sigma_t^{M,VG}\\ \Sigma_t^{M,VN} & \Sigma_t^{M,NG}\\ \Sigma_t^{M,VG} & \Sigma_t^{M,GG}\end{bmatrix}'.$$

Since G_t is publicly revealed, it is common knowledge and speculators need not update their beliefs about it with their private information. This characterizes the beliefs of investors given the market's beliefs.

Proof of Proposition A5

After the system has run for a sufficiently long time, initial conditions will diminish and the conditional variance of the Kalman Filter for the market beliefs Σ_t^M will settle down to its deterministic, covariance-stationary steady-state. To see this, let us conjecture that $\Sigma_t^M \to \Sigma^M$. In this proposed steady-state, $\Gamma_t \to \Gamma$, where Γ is given by

$$\Gamma = \begin{bmatrix} \Sigma^{M,VV} & \Sigma^{M,VG} \\ \Sigma^{M,VN} & \Sigma^{M,NG} \\ \Sigma^{M,VG} & \Sigma^{M,GG} \end{bmatrix} \begin{bmatrix} \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} & \Sigma^{M,VG} \\ \Sigma^{M,VG} & \Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1} \end{bmatrix}^{-1}.$$

Consequently, since Γ is indeed constant, so is $\Sigma^{M,VV}$. Furthermore, the steady-state Kalman Gain $\mathbf{K}^M$ is given by

$$\mathbf{K}^M = \begin{bmatrix} \rho_V \Sigma^{M,VV} & p_V\left(\rho_V^2 \Sigma^{M,VV} + \sigma_V^2\right) & \rho_V \Sigma^{M,VG} \\ 0 & p_N \sigma_N^2 & 0 \\ 0 & p_G \sigma_G^2 & 0 \\ \Sigma^{M,VG} & p_V \rho_V \Sigma^{M,VG} & \Sigma^{M,GG} \end{bmatrix} \Omega^{M-1},$$

where

$$\Omega^M = \begin{bmatrix} \Sigma^{M,VV} + \sigma_D^2 & p_V \rho_V \Sigma^{M,VV} & \Sigma^{M,VG} \\ p_V \rho_V \Sigma^{M,VV} & p_V^2\left(\rho_V^2 \Sigma^{M,VV} + \sigma_V^2\right) + p_N^2 \sigma_N^2 + p_G^2 \sigma_G^2 & p_V \rho_V \Sigma^{M,VG} \\ \Sigma^{M,VG} & p_V \rho_V \Sigma^{M,VG} & \Sigma^{M,GG} \end{bmatrix}.$$

Consequently, since we have constructed a steady-state for the Kalman Filter for the market beliefs, such a steady-state exists.

Proof of Proposition A6

Similar to the problem for the government, it is convenient to define the state vector $\Psi_t = \left[\hat{V}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M, G_t\right]$ with law of motion:

$$\Psi_{t+1} = \begin{bmatrix} \rho_V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Psi_t + \mathbf{K}^M \varepsilon_{t+1}^M,$$

and $\varepsilon_{t+1}^M \mid \mathcal{F}_t^M \sim N\left(\mathbf{0}_{3\times 1}, \Omega^M\right)$ is given by

$$\varepsilon_{t+1}^M = \begin{bmatrix} D_{t+1} - \hat{V}_{t+1}^M \\ \eta_{t+1}^M - p_V \rho_V \hat{V}_{t+1}^M \\ G_{t+1} - \hat{G}_{t+1}^M \end{bmatrix},$$

with Ω^M given in the proof of Corollary 1.

Given that excess payoffs are normally distributed, we can decompose R_{t+1} as

$$\begin{aligned}
R_{t+1} &= E\left[R_{t+1} \mid \mathcal{F}_t^i\right] + \phi' \varepsilon_{t+1}^{S,i} \\
&= \varsigma \Psi_t + \phi' \omega \begin{bmatrix} \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} & \Sigma^{M,VG} \\ \Sigma^{M,VG} & \Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1} \end{bmatrix}^{-1} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix} + \phi' \varepsilon_{t+1}^{S,i} \\
&= \varsigma \Psi_t + \frac{\phi' \omega \begin{bmatrix} \Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG} \\ -\Sigma^{M,VG} & \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} \end{bmatrix} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}}{\left(\Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1}\right)\left(\Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG}\right)^2} + \phi' \varepsilon_{t+1}^{S,i},
\end{aligned}$$

where

$$\varepsilon_{t+1}^{S,i} = \begin{bmatrix} D_{t+1} - \hat{V}_{t+1}^i \\ \eta_{t+1}^M - p_V \rho_V \hat{V}_{t+1}^i \\ G_{t+1} - \hat{G}_{t+1}^i \end{bmatrix},$$

and

$$\begin{aligned}
\varsigma &= \begin{bmatrix} 1 + p_{\hat{V}}\left(\rho_V - R^f\right) & -p_N R^f & p_g - R^f p_{\hat{G}} & -R^f p_g \end{bmatrix}, \\
\phi &= \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \mathbf{K}^{M\prime} \begin{bmatrix} p_{\hat{V}} - p_V \\ 0 \\ p_{\hat{G}} - p_G \\ p_g \end{bmatrix}.
\end{aligned}$$

In this decomposition, we have updated the investor's beliefs sequentially from the market beliefs following Bayes' Rule as

$$\begin{aligned}
& E\left[R_{t+1} \mid \mathcal{F}_t^i\right] \\
&= E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \phi' \omega \begin{bmatrix} \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} & \Sigma^{M,VG} \\ \Sigma^{M,VG} & \Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1} \end{bmatrix}^{-1} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix} \\
&= \varsigma \Psi_t + \frac{\phi' \omega \begin{bmatrix} \Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG} \\ -\Sigma^{M,VG} & \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} \end{bmatrix} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}}{\left(\Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1}\right)\left(\Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG}\right)^2},
\end{aligned}$$

where, as in Proposition 4:

$$\begin{aligned}
\omega &= Cov\left[\varepsilon_{t+1}^M, \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}' \mid \mathcal{F}_t^M\right] \\
&= \begin{bmatrix} \Sigma^{M,VV} & \Sigma^{M,VG} \\ p_V \rho_V \Sigma^{M,VV} & p_V \rho_V \Sigma^{M,VG} \\ \rho_V \Sigma^{M,VG} & \Sigma^{M,GG} \end{bmatrix}.
\end{aligned}$$

Similarly, by Bayes' Rule, $\varepsilon_{t+1}^S \mid \mathcal{F}_t^i \sim \mathcal{N}\left(\mathbf{0}_{2\times 1}, \Omega^S\right)$, where

$$\Omega^S = \Omega^M - \frac{\omega \begin{bmatrix} \Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG} \\ -\Sigma^{M,VG} & \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} \end{bmatrix} \omega'}{\left(\Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1}\right)\left(\Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG}\right)^2}.$$

Standard results establish that the investor's problem is equivalent to the mean-variance optimization program:

$$\sup_{X_t(i)} \left\{ R^f \bar{W} + X_t^i E\left[R_{t+1} \mid \mathcal{F}_t^i\right] - \frac{\gamma}{2} X_t^{i2} Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] \right\}.$$

Importantly, since the investors have to form conditional expectations about excess payoffs at $t+1$, they must form conditional expectations about the government's future trading $E\left[G_{t+1} \mid \mathcal{F}_t^i\right]$. Given that the investors are price-takers, from the FOC we see that the optimal investment of investor i in the risky asset is given by

$$\begin{aligned} X_t^i &= \frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]}{\gamma Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} \\ &= \frac{1}{\gamma} \frac{\varsigma \Psi_t + \frac{\phi' \omega \begin{bmatrix} \Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG} \\ -\Sigma^{M,VG} & \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} \end{bmatrix} \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}}{\left(\Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1}\right)\left(\Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG}\right)^2}}{\phi' \Omega^M \phi - \frac{\phi' \omega \begin{bmatrix} \Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG} \\ -\Sigma^{M,VG} & \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} \end{bmatrix} \omega' \phi}{\left(\Sigma^{M,VV} + \left(a^i\tau_s\right)^{-1}\right)\left(\Sigma^{M,GG} + \left[\left(1-a^i\right)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG}\right)^2}}. \end{aligned}$$

This completes our characterization of the optimal trading policy of the investors.

Proof of Proposition A7

Each investor faces the optimization problem (A1) given in the main paper. It then follows that investor i will choose to learn about the payoff fundamental V_t (i.e, $a_t^i = 1$) with probability λ:

$$\lambda = \begin{cases} 1, & Q < 0 \\ (0,1), & Q = 0 \\ 0, & Q > 0, \end{cases},$$

where

$$Q = \phi' \left(M(0) - M(1)\right) \phi = \phi' \omega \begin{bmatrix} -\frac{1}{\Sigma^{M,VV} + \tau_s^{-1}} & 0 \\ 0 & \frac{1}{\Sigma^{M,GG} + \tau_g^{-1}} \end{bmatrix} \omega' \phi.$$

Given ω, we can expand out this condition to arrive at

$$\begin{aligned} Q &= \frac{\begin{pmatrix} \left(1 + \left(p_{\hat{V}} - p_V\right)\mathbf{K}_{1,1}^M + \left(p_{\hat{g}} - p_g\right)\mathbf{K}_{3,1}^M + \left(p_{\hat{G}} - p_G\right)\mathbf{K}_{4,1}^M\right)\Sigma^{M,VG} \\ + \left(1 + \left(p_{\hat{V}} - p_V\right)\mathbf{K}_{1,2}^M + \left(p_{\hat{g}} - p_g\right)\mathbf{K}_{3,2}^M + \left(p_{\hat{G}} - p_G\right)\mathbf{K}_{4,2}^M\right) \\ \times \left(p_V \rho_V \Sigma^{M,VG} + p_g \Sigma^{M,GG}\right) \end{pmatrix}^2}{\Sigma^{M,GG} + \tau_g^{-1}} \\ &\quad - \frac{\begin{pmatrix} \left(1 + \left(p_{\hat{V}} - p_V\right)\mathbf{K}_{1,1}^M + \left(p_{\hat{g}} - p_g\right)\mathbf{K}_{3,1}^M + \left(p_{\hat{G}} - p_G\right)\mathbf{K}_{4,1}^M\right)\Sigma^{M,VV} \\ + \left(1 + \left(p_{\hat{V}} - p_V\right)\mathbf{K}_{1,2}^M + \left(p_{\hat{g}} - p_g\right)\mathbf{K}_{3,2}^M + \left(p_{\hat{G}} - p_G\right)\mathbf{K}_{4,2}^M\right) \\ \times \left(p_V \rho_V \Sigma^{M,VV} + p_g \Sigma^{M,VG}\right) \end{pmatrix}^2}{\Sigma^{M,VV} + \tau_s^{-1}}. \end{aligned}$$

Recognizing that $\phi'\omega = CoV\left[R_{t+1}, \begin{bmatrix} V_{t+1} \\ G_{t+1} \end{bmatrix} \mid \mathcal{F}_t^M\right]$, we can rewrite the above more generally as:

$$Q = \frac{CoV\left[R_{t+1}, G_{t+1} \mid \mathcal{F}_t^M\right]^2}{\Sigma^{M,GG} + \tau_g^{-1}} - \frac{CoV\left[R_{t+1}, V_{t+1} \mid \mathcal{F}_t^M\right]^2}{\Sigma^{M,VV} + \tau_s^{-1}}.$$

Proof of Proposition 4

Suppose we focus on either fundamental-centric or a government-centric equilibrium. In the special case that $\rho_V = 0$, it follows that the Kalman Gain, the steady-state market beliefs, and the $Q-$statistic for information acquisition satisfy

$$\mathbf{K}^M = \begin{bmatrix} 0 & \frac{p_V \sigma_V^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} & 0 \\ 0 & \frac{p_N \sigma_N^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} & 0 \\ 0 & \frac{p_G \sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

and

$$\Sigma^M = \begin{bmatrix} \frac{p_N^2\sigma_N^2 + p_G^2\sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2}\sigma_V^2 & -\frac{p_V\sigma_V^2 p_N \sigma_N^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} & -\frac{p_V\sigma_V^2 p_G \sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} & 0 \\ -\frac{p_V\sigma_V^2 p_N \sigma_N^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} & \frac{p_V^2\sigma_V^2 + p_G^2\sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2}\sigma_N^2 & -\frac{p_N\sigma_N^2 p_G \sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} & 0 \\ -\frac{p_V\sigma_V^2 p_G \sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} & -\frac{p_N\sigma_N^2 p_G \sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} & \frac{p_V^2\sigma_V^2 + p_N^2\sigma_N^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2}\sigma_G^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$Q = \left(\frac{\sigma_V^2\sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2}\right)^2 \left[\frac{\left(\frac{p_N^2\sigma_N^2 + p_G^2\sigma_G^2 + \frac{1}{R^f}p_V\sigma_V^2 - p_g p_G \sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} p_g\left(p_V^2 + p_N^2\frac{\sigma_N^2}{\sigma_V^2}\right) - p_V p_G\right)^2}{\frac{p_V^2\sigma_V^2 + p_N^2\sigma_N^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2}\sigma_G^2 + \tau_g^{-1}} - \frac{\left(p_N^2\frac{\sigma_N^2}{\sigma_G^2} + p_G^2 - \frac{p_N^2\sigma_N^2 + p_G^2\sigma_G^2 + \frac{1}{R^f}p_V\sigma_V^2 - p_g p_G \sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2} p_g p_V p_G\right)^2}{\frac{p_N^2\sigma_N^2 + p_G^2\sigma_G^2}{p_V^2\sigma_V^2 + p_N^2\sigma_N^2 + p_G^2\sigma_G^2}\sigma_V^2 + \tau_s^{-1}}\right],$$

respectively.

In a government-centric equilibrium, $p_V = 0$, and, from the market-clearing conditions, p_g and p_G satisfy

$$p_g = \frac{p_N\sigma_N}{1 - \vartheta_{\hat{N}}}\sqrt{\frac{p_N^2\sigma_N^2}{p_N^2\sigma_N^2 + p_G^2\sigma_G^2}\vartheta_{\hat{N}}^2},$$

$$p_G = \frac{1}{R^f}\left(1 - \vartheta_{\hat{N}}\right)\frac{p_g p_N^2 \sigma_N^2}{p_N^2\sigma_N^2 + p_G^2\sigma_G^2}\sigma_G^2,$$

from which follows that p_G is given by , $p_G^2 = x p_N^2 \frac{\sigma_N^2}{\sigma_G^2}$ where x satisfies

$$x\left(1+x\right)^3 = \left(\frac{\vartheta_{\hat{N}}}{R^f}\sigma_G^3\right)^2,$$

where x is increasing in $\frac{\vartheta_{\hat{N}}}{R^f}\sigma_G^3$. It then follows that Q reduces to

$$Q = \left(\frac{\left(\sigma_G^2 - R^f \frac{x}{1-\vartheta_{\hat{N}}}\right)^2}{\sigma_G^2 + (1+x)\tau_g^{-1}}\left(\frac{\vartheta_{\hat{N}}}{1-\vartheta_{\hat{N}}}\right)^2 p_N^2\sigma_N^2 - \frac{\sigma_V^4}{\sigma_V^2+\tau_s^{-1}}(1+x)^2\right)\left(\frac{\sigma_G^2}{1+x}\right)^2,$$

which suggests that, for $Q \geq 0$, it must be the case that

$$p_N^2 > \bar{p}_N^2 = \frac{\sigma_V^4}{\sigma_N^2}\frac{\sigma_G^2+(1+x)\tau_g^{-1}}{\sigma_V^2+\tau_s^{-1}}\left(\frac{1-\vartheta_{\hat{N}}}{\vartheta_{\hat{N}}}\right)^2\left(\frac{1+x}{\sigma_G^2 - R^f\frac{x}{1-\vartheta_{\hat{N}}}}\right)^2.$$

Furthermore, it is straightforward to compute that

$$\phi'\Omega^M\phi = \sigma_V^2 + \sigma_D^2 + \sigma_G^2\left(\frac{1}{1+x}\frac{\vartheta_{\hat{N}}}{1-\vartheta_{\hat{N}}}\right)^2 p_N^2\sigma_N^2 + \frac{\left(1+\frac{1+x}{1-\vartheta_{\hat{N}}}\frac{1}{\sigma_G^2}x\right)^2}{1+x}p_N^2\sigma_N^2,$$

and therefore, from market-clearing, that p_N also satisfies

$$\begin{aligned} 0 \;=\; & \left(\sigma_G^2\frac{\sigma_G^2+2(1+x)\tau_g^{-1}}{\sigma_G^2+(1+x)\tau_g^{-1}}\left(\frac{1}{1+x}\frac{\vartheta_{\hat{N}}}{1-\vartheta_{\hat{N}}}\right)^2 + \frac{\left(1+\frac{1+x}{1-\vartheta_{\hat{N}}}\frac{1}{\sigma_G^2}x\right)^2}{1+x}\right)\sigma_N^2 p_N^2 \\ & - \frac{R^f}{1-\vartheta_{\hat{N}}}\frac{1+x}{\sigma_G^2+(1+x)\tau_g^{-1}}p_N + \sigma_V^2 + \sigma_D^2. \end{aligned}$$

It follows that p_N is given by the two roots of the above quadratic form:

$$p_N = \frac{1}{2\sigma_N^2 c}\frac{R^f}{1-\vartheta_{\hat{N}}} \pm \sqrt{\left(\frac{1}{2\sigma_N^2 c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{\sigma_N^2 c}},$$

where

$$c = \sigma_G^2\frac{\sigma_G^2+2(1+x)\tau_g^{-1}}{\sigma_G^2+(1+x)\tau_g^{-1}}\left(\frac{1}{1+x}\frac{\vartheta_{\hat{N}}}{1-\vartheta_{\hat{N}}}\right)^2 + \frac{\left(1+\frac{1}{\sigma_G^2}\frac{1+x}{1-\vartheta_{\hat{N}}}x\right)^2}{1+x} \geq 0,$$

and $c = c\left(\vartheta_{\hat{N}}, R^f, \sigma_G\right)$. When p_N exists, one consequently has that $p_N > 0$. Selecting the less positive root, and recognizing that $Q \geq 0$ whenever $p_N \geq \bar{p}_N$, we can express this condition as

$$\begin{aligned} & \frac{\sqrt{\sigma_V^2+\tau_s^{-1}}}{\sigma_V^2}\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}} - \sqrt{\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{c}}\right) \\ \geq \;& (1+x)\sqrt{\left(\sigma_G^2+(1+x)\tau_g^{-1}\right)\left(\frac{\frac{1-\vartheta_{\hat{N}}}{\vartheta_{\hat{N}}}}{\sigma_G^2 - R^f\frac{x}{1-\vartheta_{\hat{N}}}}\right)^2}. \end{aligned} \quad \text{(IA.3)}$$

Notice that the LHS of equation (IA.3) is always nonnegative, since it is $\frac{\sqrt{\sigma_V^2+\tau_s^{-1}}}{\sigma_V^2} p_N \sigma_N$, and that c and the RHS of equation (IA.3) is independent of $\{\sigma_N, \sigma_V, \sigma_D\}$ since $x = x\left(\vartheta_{\hat{N}}, R^f, \sigma_G\right)$.

Since it is straightforward to compute that

$$
\begin{aligned}
\frac{dp_N \sigma_N}{d\sigma_N} &= -\frac{1}{\sigma_N}\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}} \frac{\sqrt{\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{c}} - \frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}}{\sqrt{\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{c}}} > 0, \\
\frac{dp_N \sigma_N}{d\sigma_D} &= \frac{\sigma_D}{c\sqrt{\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{c}}} > 0,
\end{aligned}
$$

it follows that the LHS is increasing in σ_N and σ_D. Consequently, the existence condition for a government-centric equilibrium relaxes as σ_N and σ_D increase, and therefore a government-centric equilibrium is more likely to exist the higher are σ_N and σ_D.

Finally, with respect to σ_V, we recognize that, as $\sigma_V \to 0$, $\frac{\sqrt{\sigma_V^2+\tau_s^{-1}}}{\sigma_V^2} p_N \sigma_N \to \infty$, and consequently the LHS exceeds the RHS and $Q > 0$. Since $\frac{\sqrt{\sigma_V^2+\tau_s^{-1}}}{\sigma_V^2} p_N \sigma_N$ is continuous in V, it follows that a government-centric equilibrium exists within a neighborhood of $\sigma_V = 0$, and consequently exists for σ_V sufficiently small.

Proof of Proposition 5

The proof of this proposition is already provided in Appendix B of the main text. Here, we provide the asset excess return decomposition used for computing noise trader welfare, which is used in the proof in Appendix B. We start by writing the excess return of the asset as

$$
\begin{aligned}
R_{t+1} &= D_{t+1} - V_{t+1} + \frac{1}{R^f - \rho_V}\left(V_{t+2} - \rho_V V_{t+1}\right) + p_N N_{t+1} + R^f p_g \hat{G}_{t+2}^M + p_G\left(G_{t+2} - \hat{G}_{t+2}^M\right) \\
&\quad + \left(p_g - R^f p_G\right) G_{t+1} - R^f p_g G_t + \left(p_V\left(\rho_V - R^f\right) - 1\right)\left(V_{t+1} - \rho_V \hat{V}_t^M\right) \\
&\quad - R^f p_N N_t + \begin{bmatrix} 1 - p_V\left(R^f - \rho_V\right) \\ 0 \\ R^f p_G - p_g \\ 0 \end{bmatrix}' \mathbf{K}^M \varepsilon_{t+1}^M \\
&= \sigma_D \varepsilon_{t+1}^D + \frac{\sigma_V}{R^f - \rho_V}\varepsilon_{t+2}^V + \left(p_V\left(\rho_V - R^f\right) - 1\right)\sigma_V \varepsilon_{t+1}^V + p_N N_{t+1} + \left(R^f p_g - p_G\right)\hat{G}_{t+2}^M \\
&\quad + p_G G_{t+2} + \left(p_V\left(\rho_V - R^f\right) - 1\right)\rho_V\left(V_t - \hat{V}_t^M\right) + \left(p_g - R^f p_G\right) G_{t+1} - R^f p_g G_t \\
&\quad - R^f p_N N_t + \begin{bmatrix} 1 - p_V\left(R^f - \rho_V\right) \\ 0 \\ R^f p_G - p_g \\ 0 \end{bmatrix}' \mathbf{K}^M \varepsilon_{t+1}^M,
\end{aligned}
$$

where $\varepsilon_{t+1}^M = \begin{bmatrix} D_t - \hat{V}_t^M & \eta_t^M - p_V \rho_V \hat{V}_t^M & G_t - \hat{G}_{t|t-1}^M \end{bmatrix}$. With some manipulation, we can establish that the excess return satisfies the decomposition

$$R_{t+1} = \tilde{R}_{t+1} - R^f p_g \hat{G}_{t|t-1}^M - c_N p_N N_t,$$

where

$$\begin{aligned} \tilde{R}_{t+1} &= r_{D1}\sigma_D \varepsilon_{t+1}^D + r_D \sigma_D \varepsilon_t^D + r_{V2}\sigma_V \varepsilon_{t+2}^V + r_{V1}\sigma_V \varepsilon_{t+1}^V + r_{N1} N_{t+1} + r_{G2} G_{t+2} + r_{G1} G_{t+1} \\ &\quad + r_{V0}\left(V_t - \hat{V}_t^M\right) + r_G \left(G_t - \hat{G}_{t|t-1}^M\right), \end{aligned}$$

is independent of N_t and $\hat{G}_{t|t-1}^M$, and has the conditional distribution:

$$\tilde{R}_{t+1} \mid \mathcal{F}_{t-1} \sim \mathcal{N}\left(0, \sigma_{\tilde{R}}^2\right),$$

where

$$\begin{aligned} \sigma_X^2 &= \left(r_{D1}^2 + r_D^2\right)\sigma_D^2 + \left(r_{V1}^2 + r_{V2}^2\right)\sigma_V^2 + r_{N1}^2 \sigma_N^2 + \left(r_{G1}^2 + r_{G2}^2\right)\sigma_G^2 \\ &\quad + r_{V0}^2 \Sigma^{M,VV} + r_G^2 \Sigma^{M,GG} + 2 r_{V0} r_G \Sigma^{M,VG}. \end{aligned}$$

In this decomposition, the coefficients on the terms in X_{t+1} are

$$\begin{aligned} r_D &= c_V \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_G \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \\ r_{D1} &= 1 + \left(R^f p_g - p_G\right) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \\ r_{V2} &= \frac{1}{R^f - \rho_V} + p_V \left(R^f p_g - p_G\right) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \end{aligned}$$

and

$$r_{N1} = \left(1 + \left(R^f p_g - p_G\right) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) p_N,$$

$$r_{G2} = \left(1 + \left(R^f p_g - p_G\right) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) p_G,$$

$$\begin{aligned} r_{V1} &= p_V \left(\rho_V - R^f\right) - 1 + \left(R^f p_g - p_G\right) p_V \rho_V \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \\ &\quad \left(+c_V \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}' + c_G \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}'\right) \mathbf{K}^M \begin{bmatrix} 0 \\ p_V \\ 0 \end{bmatrix}, \end{aligned}$$

and

$$\begin{aligned} r_{V0} &= \left(\left(p_V \left(\rho_V - R^f\right) - 1\right) + \left(R^f p_g - p_G\right) p_V \rho_V \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) \rho_V \\ &\quad + \left(c_V \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}' + c_G \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}'\right) \mathbf{K}^M \begin{bmatrix} 1 \\ p_V \rho_V \\ 0 \end{bmatrix}, \\ r_{G1} &= \left(p_g - R^f p_G\right) + \left(R^f p_g - p_G\right) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ &\quad + p_G \left(c_V \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}' + c_G \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}'\right) \mathbf{K}^M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \\ r_G &= \left(c_V \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}' + c_G \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}'\right) \mathbf{K}^M \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - R^f p_g. \end{aligned}$$

In the above, the coefficients, c_N, c_V, and c_G are given by

$$\begin{aligned} c_N &= R^f - \left(c_V \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}' + c_G \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \right) \mathbf{K}^M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \\ c_V &= 1 - p_V \left(R^f - \rho_V \right) + \left(p_G - R^f p_g \right) p_V \rho_V \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \\ c_G &= \left(R^f p_G - p_g \right) + \left(p_G - R^f p_g \right) \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \end{aligned}$$

Proof of Proposition 6

In what follows, we consider the parameter space to be the variance of noise trading, σ_N^2, and the project-specific variance of entrepreneurs, σ_y^2. We hold fixed the remaining parameters. We express the government's welfare objective as

$$\begin{aligned} \sup_{\vartheta_{\hat{N}}} \quad & \underbrace{\frac{\sigma_V^2}{(1-\rho_V^2)\Sigma^{M,VV} + \sigma_y^2}}_{Entrepreneur\ Production\ Efficiency} \underbrace{- \frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}}_{Investor\ Trading\ Risk} \\ & \underbrace{-\gamma^2 \left(\sigma_N^2 + \sigma_n^2 + \vartheta_{\hat{N}}^2 \left(1 + \sigma_G^2\right) \left(\sigma_N^2 - \Sigma^{M,NN} \right) \right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]}_{Noise\ Trader\ and\ Taxpayer\ Trading\ Risk}, \end{aligned} \tag{IA.4}$$

where

$$\begin{aligned} Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right] &= \phi' \Omega^M \phi + \varsigma \mathbf{K}^M \Omega^M \mathbf{K}^{M\prime} \varsigma' + \left(R^f p_g\right)^2 \left(\sigma_G^2 - \Sigma^{M,GG}\right), \\ Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] &= \phi' \Omega^M \phi - \phi' \omega \begin{bmatrix} \frac{1}{\Sigma^{M,VV} + [a\tau_s]^{-1}} & 0 \\ 0 & \frac{1}{\Sigma^{M,GG} + [(1-a)\tau_g]^{-1}} \end{bmatrix} \omega' \phi, \\ Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] &= \phi' \Omega^M \phi. \end{aligned}$$

Consider now the government-centric equilibrium. With some manipulation, and recognizing that

$$\Sigma^{M,NN} = \left(\frac{p_G}{p_N}\right)^2 \Sigma^{M,GG} = \frac{\left(\frac{\vartheta_{\hat{N}}}{R^f} \frac{\Sigma^{M,GG}}{\Sigma^{M,GG} + \tau_g^{-1}}\right)^2 \sigma_N^2 \Sigma^{M,GG}}{1 + \left(\frac{\vartheta_{\hat{N}}}{R^f} \frac{\Sigma^{M,GG}}{\Sigma^{M,GG} + \tau_g^{-1}}\right)^2 \Sigma^{M,GG}},$$

we can express the conditional beliefs about future government policy, $\Sigma^{M,GG}$, from Kalman Filter for public beliefs as

$$\Sigma^{M,GG} = \frac{\sigma_G^2}{1 + \left(\frac{\vartheta_{\hat{N}}}{R^f} \frac{\Sigma^{M,GG}}{\Sigma^{M,GG} + \tau_g^{-1}}\right)^2 \Sigma^{M,GG}}. \tag{IA.5}$$

With some manipulation, and substituting with (IA.5), (A6), (A8), (A5), we can rewrite the Investor Trading Risk motive as

$$\begin{aligned}
& -\frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} \\
= \quad & -1 - \frac{p_g^2 \Sigma^{M,GG} \frac{\Sigma^{M,GG}}{\Sigma^{M,GG} + \tau_g^{-1}} + \left(R^f p_N \sigma_N\right)^2 \frac{p_N^2 \sigma_N^2}{p_N^2 \sigma_N^2 + p_G^2 \sigma_G^2} + \left(R^f p_g\right)^2 \sigma_G^2}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} \\
= \quad & -1 - \gamma^2 \sigma_N^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] \left[\left(\frac{1}{\Sigma^{M,GG}} - \frac{1}{\sigma_G^2}\right) \left(\Sigma^{M,GG} + \tau_g^{-1}\right) + \vartheta_{\hat{N}}^2 \Sigma^{M,GG} \right. \\
& \left. + \left(1 - \vartheta_{\hat{N}}\right)^2 \frac{\Sigma^{M,GG}}{\sigma_G^2}\right],
\end{aligned} \tag{IA.6}$$

which is a decreasing function of $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ and $\Sigma^{M,GG}$ with

$$\begin{aligned}
& Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] \\
= \quad & \frac{1}{2} \frac{\sqrt{1 + 4\gamma^2 \sigma_N^2 \left(\frac{1}{\Sigma^{M,GG}} - \frac{1}{\sigma_G^2}\right) \left(\Sigma^{M,GG} + \tau_g^{-1}\right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]} - 1}{\gamma^2 \sigma_N^2 \left(\frac{1}{\Sigma^{M,GG}} - \frac{1}{\sigma_G^2}\right) \left(\Sigma^{M,GG} + \tau_g^{-1}\right)}.
\end{aligned} \tag{IA.7}$$

Similarly, we can rewrite the Noise Trader and Taxpayer Trading Risk motive as

$$\begin{aligned}
& -\gamma^2 \left(\sigma_N^2 + \sigma_n^2 + \vartheta_{\hat{N}}^2 \left(1 + \sigma_G^2\right) \left(\sigma_N^2 - \Sigma^{M,NN}\right)\right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] \\
= \quad & -\gamma^2 \left(\sigma_N^2 + \sigma_n^2 + \vartheta_{\hat{N}}^2 \sigma_N^2 \frac{1 + \sigma_G^2}{\sigma_G^2} \Sigma^{M,GG}\right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right],
\end{aligned} \tag{IA.8}$$

which is decreasing function of $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ and an increasing function of $\Sigma^{M,GG}$.

Consequently, the objective of the government in the government-centric equilibrium balances minimizing return volatility based on public information, $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$, with its impact on the conditional uncertainty about future government policy noise, $\Sigma^{M,GG}$.

Consider now the fundamental-centric equilibrium. With some manipulation, and substituting with (A6), (A7), (A5),

$$\Sigma^{M,NN} = \left(\frac{p_V}{p_N}\right)^2 \Sigma^{M,VV} = y\Sigma^{M,VV},$$

where $y = \frac{p_V}{p_N}$ is (in)decreasing in (σ_N^2) $\Sigma^{M,VV}$ and satisfies[25]

$$
\begin{aligned}
y &= \sqrt{\left(\frac{1}{2}\frac{\rho_V^2 \Sigma^{M,VV}\sigma_D^2 + \sigma_V^2\left(\Sigma^{M,VV}+\sigma_D^2\right)}{\sigma_N^2\left(\Sigma^{M,VV}+\sigma_D^2\right)}\right)^2 + \frac{\left(\rho_V^2\Sigma^{M,VV}+\sigma_V^2\right)\sigma_D^2 + \sigma_V^2\Sigma^{M,VV}}{\left(\Sigma^{M,VV}+\sigma_D^2\right)\Sigma^{M,VV}}} \\
&\quad -\frac{1}{2}\frac{\rho_V^2\Sigma^{M,VV}\sigma_D^2 + \sigma_V^2\left(\Sigma^{M,VV}+\sigma_D^2\right)}{\sigma_N^2\left(\Sigma^{M,VV}+\sigma_D^2\right)},
\end{aligned}
$$

and

$$
Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] = \frac{\sqrt{1+4Var\left[R_{t+1}\mid\mathcal{F}_t^M\right]}-1}{2\gamma^2 y^2\left(\Sigma^{M,VV}+\tau_s^{-1}\right)},
$$

we can rewrite the Investor Trading Risk motive as

$$
\begin{aligned}
& -\frac{Var\left[R_{t+1}\mid\mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1}\mid\mathcal{F}_t^i\right]} \qquad \text{(IA.9)}\\
&= -1-\frac{\left(\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 p_V^2\left(\Sigma^{M,VV}+\tau_s^{-1}\right)+\left(p_N R^f\right)^2\left(\sigma_N^2-\Sigma^{M,NN}\right)+\left(R^f p_g\right)^2\sigma_G^2}{Var\left[R_{t+1}\mid\mathcal{F}_t^i\right]}\\
&= -1-\frac{1}{2}\left(1+\frac{\sigma_N^2\Sigma^{M,VV}}{\Sigma^{M,VV}+\tau_s^{-1}}\frac{\left(1-\vartheta_{\hat{N}}\right)^2+\vartheta_{\hat{N}}^2\sigma_G^2}{\rho_V^2\frac{\Sigma^{M,VV}\sigma_D^2}{\Sigma^{M,VV}+\sigma_D^2}+\sigma_V^2}\right)\left(\sqrt{1+4Var\left[R_{t+1}\mid\mathcal{F}_t^M\right]}-1\right),
\end{aligned}
$$

which is a decreasing function of $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ and $\Sigma^{M,VV}$.

Similarly, we can rewrite the Noise Trader and Taxpayer Trading Risk motive as

$$
\begin{aligned}
& -\gamma^2\left(\sigma_N^2+\sigma_n^2+\vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)\left(\sigma_N^2-\Sigma^{M,NN}\right)\right)Var\left[R_{t+1}\mid\mathcal{F}_t^M\right]\\
&= -\gamma^2\left(\sigma_N^2+\sigma_n^2+\vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)\left(\sigma_N^2-y\Sigma^{M,VV}\right)\right)Var\left[R_{t+1}\mid\mathcal{F}_t^M\right]. \qquad \text{(IA.10)}
\end{aligned}
$$

which is decreasing function of $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ and an increasing function of $\Sigma^{M,VV}$.

Consequently, the objective of the government in the fundamental-centric equilibrium also balances minimizing conditional asset return volatility based on public information, $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$, with its impact on the conditional uncertainty about the asset fundamental, $\Sigma^{M,VV}$. The government now, however, also internalizes that it can impact the efficiency of entrepreneurial production through the additional $\frac{\sigma_V^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV}+\sigma_y^2}$ term in social welfare; the strength of this additional motive is decreasing continuously in σ_y^2, which does not enter into the welfare objective through any other terms.

Examining of our expressions for the various components of social welfare in the two equilibria, we conclude that the two key statistics that impact social welfare are the conditional asset return volatility based on public information, $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$, and the conditional uncertainty about either the asset fundamental, $\Sigma^{M,VV}$, or future policy noise, $\Sigma^{M,GG}$, depending on whether the equilibrium is fundamental-centric or government-centric, respectively.

[25] We recover y from the Kalman Filter recursion for the steady-state posterior variances by equating expressions for $\Sigma^{M,NN}$ with $\Sigma^{M,VV}$ according to the above relation.

Proof of Proposition 7

Note from the welfare objective (IA.4) that the first term, Entrepreneural Production Efficiency, is independent of the government's intervention intensity $\vartheta_{\hat{N}}$ in the government-centric equilibrium, because investors all acquire information about the future policy noise and no one acquires information about the fundamental. As a result, the government has additional incentive in the fundamental-centric equilibrium to make the asset price more informative about the asset fundamental, V, to maximize Entrepreneural Production Efficiency. Also note that this term is strictly decreasing with σ_y^2 and that σ_y^2 does not affect the other two terms in the welfare objective. Thus, as σ_y^2 decreases, the first term gives the government a greater incentive to improve the asset price efficiency. It then follows that there exists a critical σ_y^* (which may be zero or infinite) such that the government prefers a fundamental-centric equilibrium if $\sigma_y \leq \sigma_y^*$, and the government-centric equilibrium otherwise. Trivially, across the critical value, $\Sigma^{M,VV}$ in the fundamental-centric equilibrium when σ_y is right below the critical value must be lower than that in the neighboring government-centric equilibrium when σ_y is right above the critical value, as any information about V in the asset price reduces $\Sigma^{M,VV}$.

Next, we consider how the critical value σ_y^* varies with noise-trader risk σ_N^2. We begin by examining the second and third terms in the welfare objective—the Investor Trading Risk and Noise Trader and Taxpayer Trading Risk motives, as derived in (IA.9) and (IA.10), respectively. We first note that we can express the conditional asset return variance, $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$, with some manipulation and the substitution of (IA.5), (A6), (A8), and (A5), as

$$Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] = a + b\gamma^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]^2 \sigma_N^2, \tag{IA.11}$$

where

$$\begin{aligned} a &= \frac{\left(R^f\right)^2 - 1}{\left(R^f - \rho_V\right)^2}\Sigma^{M,VV} + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2 + \sigma_D^2 > 0, \\ b &= \left(\frac{1-\vartheta_{\hat{N}}}{R^f}\right)^2 + \left(\frac{\vartheta_{\hat{N}}}{R^f}\frac{1}{R^f}\right)^2 \Sigma^{M,GG}\left(1 - \left(1 - \left(1-\vartheta_{\hat{N}}\right)\frac{\Sigma^{M,GG}}{\Sigma^{M,GG} + \tau_g^{-1}}\right)^2 \frac{\Sigma^{M,GG}}{\sigma_G^2}\right) > 0. \end{aligned}$$

Substituting (IA.11) into (IA.7), along with (IA.5), we can express $Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]$ as the second-order polynomial equation

$$\left(\left(\frac{\vartheta_{\hat{N}}}{R^f}\right)^2 \frac{\left(\Sigma^{M,GG}\right)^2}{\Sigma^{M,GG} + \tau_g^{-1}}\frac{\Sigma^{M,GG}}{\sigma_G^2} - b\right)\gamma^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]^2 \sigma_N^2 + Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] - a = 0, \tag{IA.12}$$

from which follows by the Implicit Function Theorem, with substitution of (IA.12):

$$\frac{dVar\left[R_{t+1} \mid \mathcal{F}_t^i\right]}{d\sigma_N^2} = \frac{\gamma^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}{2a - Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}\left(b - \left(\frac{\vartheta_{\hat{N}}}{R^f}\right)^2 \frac{\left(\Sigma^{M,GG}\right)^2}{\Sigma^{M,GG} + \tau_g^{-1}}\frac{\Sigma^{M,GG}}{\sigma_G^2}\right). \tag{IA.13}$$

By direct calculation of $Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]$ from (IA.12), we obtain

$$Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] = \frac{1}{2}\frac{\sqrt{1+4a\left(\left(\frac{\vartheta_{\hat{N}}}{R^f}\right)^2 \frac{\left(\Sigma^{M,GG}\right)^2}{\Sigma^{M,GG}+\tau_g^{-1}}\frac{\Sigma^{M,GG}}{\sigma_G^2}-b\right)\gamma^2\sigma_N^2}-1}{\left(\left(\frac{\vartheta_{\hat{N}}}{R^f}\right)^2 \frac{\left(\Sigma^{M,GG}\right)^2}{\Sigma^{M,GG}+\tau_g^{-1}}\frac{\Sigma^{M,GG}}{\sigma_G^2}-b\right)\gamma^2\sigma_N^2}.$$

It follows that

$$\frac{2a-Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}{\sqrt{1+4a\left(\left(\frac{\vartheta_{\hat{N}}}{R^f}\right)^2 \frac{\left(\Sigma^{M,GG}\right)^2}{\Sigma^{M,GG}+\tau_g^{-1}}\frac{\Sigma^{M,GG}}{\sigma_G^2}-b\right)\gamma^2\sigma_N^2}} = \frac{1}{2}\frac{\sqrt{1+4a\left(\left(\frac{\vartheta_{\hat{N}}}{R^f}\right)^2 \frac{\left(\Sigma^{M,GG}\right)^2}{\Sigma^{M,GG}+\tau_g^{-1}}\frac{\Sigma^{M,GG}}{\sigma_G^2}-b\right)\gamma^2\sigma_N^2}}{\left(\left(\frac{\vartheta_{\hat{N}}}{R^f}\right)^2 \frac{\left(\Sigma^{M,GG}\right)^2}{\Sigma^{M,GG}+\tau_g^{-1}}\frac{\Sigma^{M,GG}}{\sigma_G^2}-b\right)\gamma^2\sigma_N^2} > 0.$$

Notice further that

$$\begin{aligned} b-\left(\frac{\vartheta_{\hat{N}}}{R^f}\right)^2 \frac{\left(\Sigma^{M,GG}\right)^2}{\Sigma^{M,GG}+\tau_g^{-1}}\frac{\Sigma^{M,GG}}{\sigma_G^2} &= \left(\frac{1-\vartheta_{\hat{N}}}{R^f}\right)^2 + \left(\frac{\vartheta_{\hat{N}}}{R^f}\right)^2 \Sigma^{M,GG} \\ &\cdot\left[\left(\frac{1}{R^f}\right)^2 - \left(\left(\frac{1}{R^f}\frac{\vartheta_{\hat{N}}\Sigma^{M,GG}+\tau_g^{-1}}{\Sigma^{M,GG}+\tau_g^{-1}}\right)^2 + \frac{\Sigma^{M,GG}}{\Sigma^{M,GG}+\tau_g^{-1}}\right)\frac{\Sigma^{M,GG}}{\sigma_G^2}\right], \end{aligned}$$

which depends only on R^f, σ_G^2, and $\vartheta_{\hat{N}}$, because $\Sigma^{M,GG} \leq \sigma_G^2$ is a function of these three variables. Suppose σ_G^2 is sufficiently small so that $\Sigma^{M,GG}$ is sufficiently small to ensure that

$$b-\left(\frac{\vartheta_{\hat{N}}}{R^f}\right)^2 \frac{\left(\Sigma^{M,GG}\right)^2}{\Sigma^{M,GG}+\tau_g^{-1}}\frac{\Sigma^{M,GG}}{\sigma_G^2} > 0. \tag{IA.14}$$

Then, from (IA.13),

$$\frac{dVar\left[R_{t+1} \mid \mathcal{F}_t^i\right]}{d\sigma_N^2} > 0,$$

which also implies from (IA.11) that

$$\frac{dVar\left[R_{t+1} \mid \mathcal{F}_t^M\right]}{d\sigma_N^2} > 0.$$

It is then straightforward to see from (IA.9) and (IA.10), by the Envelope Condition at the optimal policy for $\vartheta_{\hat{N}}$, that both trading motives are monotonically decreasing in σ_N^2.

Consider an economy in which the government optimally chooses a government-centric equilibrium. A decrease in σ_y^2 raises the incentive of the government to switch to a fundamental-centric equilibrium, while an increase in σ_N^2 raises the incentive to minimize return volatility because the asset fundamental's uncertainty, $\Sigma^{M,VV}$, and its impact on return volatility (the a term in (IA.11)) are independent of σ_N^2 in the government-centric equilibrium.

Then, by revealed preference, if the government switches to a fundamental-centric equilibrium as σ_y^2 increases, it is to improve the asset price efficiency. Since an increase in σ_N^2

worsens the welfare effects of return volatility in the second and third welfare terms, it follows that σ_y needs to be even lower so that the first term is sufficiently large to make the price efficiency effect more relevant to overcome the return volatility effects. Thus, σ_y^* is decreasing in σ_N^2.