

International Monetary Theory: A Risk Perspective*

- Preliminary -

Markus K. Brunnermeier and Yuliy Sannikov[†]

June 18, 2019

Abstract

We build a two-country model, in which currency values are endogenously determined. Risk plays a key role - including idiosyncratic risk that creates precautionary savings demand for money, and aggregate risk that creates a demand for foreign assets and affects the risk profile of the local currency. In equilibrium, different currencies can coexist, but the value of local currency is non-monotonic in the idiosyncratic risk level. With idiosyncratic risk frictions, the optimal policy can differ significantly from the competitive outcome in terms of both the net foreign asset position in dollars and the value of local currency. Dollar monetary policy can have significant spillovers on the small country both in the competitive equilibrium and under optimal policy.

Keywords: Monetary Economics, Currencies, Exchange Rates, International Trade, Risk Sharing, Financial Frictions.

JEL Codes: E32, E41, E44, E51, E52, E58, G01, G11, G21.

*We are grateful to Iván Werning, Gianluca Benigno and Yuzhe Zhang for helpful comments.

[†]Brunnermeier: Department of Economics, Princeton University, markus@princeton.edu, Sannikov: Stanford GSB, sannikov@gmail.com

1 Introduction

What determines the risk-return profile of currencies and the exchange rate risk dynamics across currencies? Why do international capital flows flow primarily uphill, i.e. from South to North, instead of downhill from advanced to emerging economies? What drives sudden stops? Should we impose capital controls in emerging economies to boost their welfare and output? What drives the spillovers of advanced economies monetary policy to emerging economies?

To answer these and related questions, we develop an international macroeconomic model in which different types of currencies can coexist and their risk profiles as well as risk premia are endogenously determined. The real value of money, prices, and exchange rates are risky and driven by productivity shocks. Agents hold and borrow in various currencies to manage their real and currency risk exposure.

Specifically, we consider a model with full risk dynamics in which the value of currencies arises – similar to Bewley models – purely from financial frictions. Our modeling approach follows Brunnermeier and Sannikov (2016a). Citizens choose a portfolio consisting of physical capital, which is subject to uninsurable idiosyncratic and aggregate risk, local money, which is impacted by aggregate shocks, and international safe asset, the dollar. Since money including nominal government debt is free of idiosyncratic risk, it serves as a store of value.

Citizens in the small open economy hold physical capital to produce a single output good. Production requires holding physical capital and is plagued with uninsurable idiosyncratic risk as well as aggregate risk. Physical capital is freely tradable within a country. However, selling or buying physical asset across countries is subject to a proportional discount or surcharge. To self-insure against the idiosyncratic risk, citizens can hold the local currency. The local currency's return depends on the growth rate of the economy, the monetary policy, and aggregate shocks. It carries a risk-premium since it is exposed to aggregate shocks. In addition, local citizens can hold the global safe asset, i.e. risk-free dollar assets, say in form of US Treasury bonds. US safe assets can be used as a hedge against aggregate country-wide shocks. When the (real) dollar interest rate is sufficiently low, they borrow in dollars instead, but only up to the international resale value of their assets. In other words, borrowing in dollars is subject to an endogenous collateral constraint.

Our framework yields the following free market outcomes absent any policy intervention. First, both currencies can coexist if idiosyncratic risk is sufficiently high. Second, the local currency is subject to sudden stops, i.e. the coexistence is fragile. Indeed, the existence

and value of the local currency is non-monotonic in the degree of uninsurable idiosyncratic risk. For very low levels of idiosyncratic risk, the local currency is not sufficiently attractive and citizens prefer to hold physical capital and save in US dollars for precautionary savings reasons if the dollar interest rate is sufficiently high. If the US dollars interest rate is very low, they might borrow in dollars. For intermediate levels of idiosyncratic risk, the local currency exists and has a (positive) value. Local citizens like the local currency to save for precautionary reasons. In addition, they borrow in US dollars to lever up their physical capital holdings. In this range of idiosyncratic risk, an increase in idiosyncratic risk tilts citizens' portfolio towards local money holding – which is a bubble – and away from physical capital. A lower capital price and investment rate, lowers the growth rate of the economy and with it, the real expected interest rate. For high levels of idiosyncratic risk, when the growth rate of the economy drops below the dollar interest rate, the local currency collapses and ceases to exist. As one increases idiosyncratic risk, a sudden stop occurs. Third, there are large spillover effects from changes in U.S. monetary policy rates. An increase in the US dollar safe asset interest rate lowers citizens willingness to hold the local currency. Citizens also scale back their physical capital holdings and investments. This, in turn, lowers the growth rate of the local economy, which in turn makes the local currency even less attractive. Fourth, the international resale value of the physical capital which determines its collateral value affect the country's degree of the indebtedness but does not bind in equilibrium. Citizens are afraid to borrow all the way to the limit since they do not want to expose themselves to an additional adverse shock.

Our second set of results concern the non-optimality of laissez faire equilibrium and the characterization of the optimal outcome. First, our analysis shows that absent any idiosyncratic risk, the market equilibrium outcome is socially optimal. That is, the first welfare theorem holds.

However, in a world with uninsurable idiosyncratic risk, the market outcome is strikingly (constrained) inefficient. Citizens are far off finding the optimal trade-off between (i) hedging idiosyncratic risk with the local currency (ii) using dollars as a safe asset to hedge aggregate risk and (iii) borrowing at the low international dollar interest rate. Interestingly, with a low dollar interest rate, the social planner, who can internalize all pecuniary externalities and can impose capital controls, borrows a significantly larger amount at the international capital market at the dollar interest rate, investing the proceeds in higher return yielding physical capital at home. The socially optimal capital flow is downhill from North to South, whereas in the competitive equilibrium capital flows uphill, from the emerging to the ad-

vanced economies. The social planner would also issue a local currency to grant his citizens at home an asset to self-insure, even when the local currency does not exist in the market equilibrium outcome. When the dollar interest rate is high, he still imposes capital controls, issues the local currency at a possibly low interest rate and invests it in high yielding dollar US Treasuries. In a sense, it is socially optimal to conduct “financial repression” as long as the proceeds are rebated to the citizens in the form of transfers.

In general, there are many ways to implement the optimal policy characterized in this paper. Capital controls appear to be a key element. They are not only needed to ensure that local currency exists for a broad range of parameters but also to grant the planner enough room to choose the optimal return-risk profile (including the risk premium) of the local currency. Imposing capital control increases the effectiveness of expanding or shrinking the money supply and thereby affecting the exchange rate. The resulting seignorage revenue is of course transferred in the form of subsidies to its citizens. If the planner has to shrink the money supply or has to pay interest to dollar lenders he has to impose taxes to the local citizens.

If the planner cannot impose capital controls and adheres to free capital flows, then the net foreign asset position and local monetary policy become interlinked. It can be welfare improving to strengthen the local currency to prevent capital outflows.

Related Literature. From the beginning, the key interest of this project has been the co-existence of currencies. Is it possible to write a model in which currencies derive value from certain frictions, and in which it is possible to discuss exchange rates in a meaningful way? As Kareken and Wallace (1981), there is natural indeterminacy here: just as money may or may not have value in multiple equilibria in Bewley settings, so are the values of two currencies.¹ While multiple equilibria exist, a small perturbation selects the equilibrium in which money has value (see Sims (1994)), e.g. in the Bewley setting, if the money has a small benefit as a medium of exchange, it becomes also a financial asset and a store of value. The same is true in our setting: even though locals can save in dollar and use it for transactions, a perturbation/requirement that they use the local currency as a medium of exchange (at least sometimes) gives it value, makes it a financial asset and determines exchange rate and currency risk.

As financial assets, the two currencies are imperfect substitutes. This has implications about the range of monetary policy and spillovers, related to the old Mundell-Fleming point about the impossibility to have fixed exchange rate, independent monetary policy

¹Sturzenegger (1994) studies currency substitution in the context of a hyperinflation scenario.

and free capital flows. Obstfeldt and Rogoff (1995) present a dynamic model of exchange rates, spillovers and fiscal policy based on monopolistic competition.

In our model, imbalances follow from the assumption of incomplete markets for sharing of aggregate risks between countries. While we assume that the small country can deal with aggregate risk only by holding the dollar, this assumption is not essential: if aggregate risk could be traded with some risk premium (nonzero so risk sharing is still less than full), the small country would choose to retain some aggregate risk exposure and dynamics will be similar. Existing literature takes various perspectives. Backus and Smith (1993) and Cole and Obstfeldt (1991) assume complete markets. Baxter and Crucini (1995) study the role of market incompleteness for the propagation of business cycles. Kehoe and Perri (2003) study endogenous incompleteness based on imperfectly enforceable international loans.

Several papers explore risk in an international setting. Devereux and Sutherland (2007) study the portfolio allocation problem in an international setting, using the second-order Taylor approximation as a solution method. Berrie and Bhattarai (2013) study a two-period model that includes risk and generates home bias: holding domestic assets allows agents to hedge against government actions.

We aim to include risk and solve for full dynamics, characterizing prices, investment, exchange rates, savings and spending as functions of the foreign net asset position. We also solve for the endogenous borrowing limit in the foreign currency.

The paper is organized as follows. Section 2 presents the model, and aims to present the equilibrium equations and the formulation of the optimal policy problem as efficiently as follows. Section 3 covers certain simple benchmarks: the steady-state analysis of the model without aggregate shocks, and the result that with aggregate shocks alone, the competitive outcome is efficient. Section 4 discusses the equilibrium in the full dynamic model. Section 5 analyzes the optimal policy. Section 6 touches upon policy without capital controls.

2 The Model

In this section we model a small open economy. There are financial frictions among the country's citizens: they face uninsurable idiosyncratic risk, which gives rise to the demand for money as in the model Bewley (1980). We build on continuous-time versions of those models with idiosyncratic capital risk developed in Brunnermeier and Sannikov (2016a) and (2016b). Citizens can use money to protect themselves against idiosyncratic risk. For that, they can use both the global currency (the dollar) and the local currency. We build a model

in which the demand for both currencies stems from financial frictions. The model allows for international savings and international debt in the global currency. Throughout, we use the dollar as the numeraire.

Citizens of the small country can use capital for production, of which the aggregate amount is denoted by K_t . Total capital in the country evolves according to

$$\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta) dt - d\lambda_t + d\bar{\lambda}_t,$$

where ι_t is the rate of investment per unit of capital, $\Phi(\iota)$ is an increasing and weakly concave function that reflects investment adjustment costs, λ_t are sales of capital abroad, and $\bar{\lambda}_t$ are purchases of capital from abroad.² We assume the capital can be sold abroad at the price of \underline{q} and bought from abroad for the price of $\bar{q} > \underline{q}$.

Capital held by an individual citizen produces output

$$dy_t = (a - \iota_t)k_t dt + \sigma k_t dZ_t + \tilde{\sigma} k_t d\tilde{Z}_t$$

for consumption and trade, where Z is the aggregate countrywide shock and \tilde{Z} is individual-specific idiosyncratic shock. Brownian motions Z and \tilde{Z} are independent and \tilde{Z} are independent across citizens and cancel out in the aggregate. The price per unit of output is one dollar.

Citizens in the small country can save dollars or borrow dollars from abroad, using capital as the collateral. They can borrow up to \underline{q} dollars per unit of capital, the value of capital if sold abroad. The net foreign asset position $\$t \geq -\underline{q}K_t$ of the small country follows

$$d\$t = r\$t - C_t dt + (a - \iota_t)K_t dt + \sigma K_t dZ_t + \underline{q}K_t d\lambda_t - \bar{q}K_t d\bar{\lambda}_t,$$

where r is the risk-free rate on the dollar and C_t is total consumption in the small country.

Citizens can also hold the local currency. In the baseline scenario in the absence of monetary policy, we assume that the nominal supply of local currency (the number of “coins” or “chips”) is fixed. As in a Bewley economy, these chips are intrinsically worthless but they can have value in equilibrium because people use them to buffer up savings against idiosyncratic shocks. Thus, while the nominal supply of chips is fixed, the real value of output or dollars that the chips can buy is determined endogenously in equilibrium. Of course, citizens can also use dollars to protect themselves against idiosyncratic shocks as

²Both λ_t and $\bar{\lambda}_t$ are nondecreasing stochastic processes.

well.

Individuals in the small country have CRRA utilities

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

with risk aversion coefficient $\gamma > 0$ and discount rate $\rho > 0$. Notice that the economy is scale-invariant in K_t and $\$t$, and the state variable $\nu_t = \$t/K_t \in [-\underline{q}, \infty)$ describes the state of the economy, up to a scaling factor. Ito's formula implies that

$$d\nu_t = \nu_t(r - \Phi(\nu_t) + \delta) dt + (a - \nu_t) dt - \zeta_t dt + \sigma dZ_t + (\underline{q} + \nu_t) d\lambda_t - (\bar{q} + \nu_t) d\bar{\lambda}_t,$$

where is $\zeta_t = C_t/K_t$ is consumption per unit of capital.

To recap, individuals can hold dollars, capital and local money. Denote by $q_t \in [\underline{q}, \bar{q}]$ or $q(\nu_t)$ the price of capital on the local market. Denote by $p_t K_t = p(\nu_t) K_t$ the value of local money. This notation captures scale invariance: that with a fixed *nominal* supply of local currency, the *real* value of local currency should be proportional to the size of the economy K_t , when everything else (including dollar holdings) scales proportionately.

Equilibrium Equations. We look for an equilibrium, which is Markov in ν_t . Individuals hold portfolios of dollars, capital and local currency with weights

$$\frac{\nu_t}{p_t + q_t + \nu_t}, \quad \frac{q_t}{p_t + q_t + \nu_t} \quad \text{and} \quad \frac{p_t}{p_t + q_t + \nu_t}, \quad (2.1)$$

and they have to constantly trade these assets to rebalance - these adjustments are necessitated by the idiosyncratic shocks. We would like to determine equilibrium prices p_t and q_t and consumption rates ζ_t such that the weights above solve individual citizens' optimal consumption and portfolio problems.

The value function of an individual who holds a portfolio with weights (2.1) that contains k_t units of capital has the form

$$f(\nu_t) \frac{k_t^{1-\gamma}}{1-\gamma}.$$

This form takes into account the effect of the scale of the portfolio on individual utility, together with the effect of the state variable ν_t . The law of motion of capital held by an individual differs from the law of motion of aggregate capital due to idiosyncratic risk. The level of idiosyncratic risk is given by $\tilde{\sigma} k_t$ for a portfolio with value $n_t = (p_t + q_t + \nu_t) k_t$. Hence, the idiosyncratic risk exposure of capital held by individuals, relative to total capital

in the country, is given by

$$\frac{dk_t}{k_t} = \frac{dK_t}{K_t} + \frac{\tilde{\sigma}}{p_t + q_t + \nu_t} d\tilde{Z}_t. \quad (2.2)$$

Marginal utility of consumption equals the marginal utility of wealth. Hence, individual consumption satisfies

$$c_t^{-\gamma} = \frac{d}{dn_t} \left(\frac{f(\nu_t)}{1 - \gamma} \frac{n_t^{1-\gamma}}{(p_t + q_t + \nu_t)^{1-\gamma}} \right) = f(\nu_t) \frac{n_t^{-\gamma}}{(p_t + q_t + \nu_t)^{1-\gamma}} = f(\nu_t) \frac{k_t^{-\gamma}}{p_t + q_t + \nu_t}.$$

Multiplied by $e^{-\rho t}$, this is the stochastic discount factor (SDF) that can price all investments available to the agent. Since $\zeta_t = c_t/k_t$, we have

$$\boxed{\zeta^{-\gamma} = f^s(\nu) \equiv \frac{f(\nu)}{p + q + \nu}.} \quad (2.3)$$

The individual solving the optimal consumption and portfolio choice problem if and only if the SDF

$$e^{-\rho t} \frac{f(\nu_t) k_t^{-\gamma}}{p_t + q_t + \nu_t}$$

prices all investments available to the agent. The valuation principle is that wealth invested in any asset, with reinvestment of dividends, times the SDF is a martingale. We have three assets, so we need to price three investment opportunities that span the asset space. First, pricing the entire portfolio (value $k_t(p_t + q_t + \nu_t)$ without dividend reinvestment), we get³

$$\underbrace{\frac{\zeta_t}{p_t + q_t + \nu_t}}_{\text{dividend yield}} + E_t [\text{growth}(e^{-\rho t} f(\nu_t) k_t^{1-\gamma})] = 0. \quad (2.4)$$

Between the boundaries $\underline{\nu}$ where capital is sold abroad and $\bar{\nu}$ where capital is bought, this equation can be transformed to

$$\boxed{\zeta^{1-\gamma} + \mathcal{L}f - \frac{\gamma(1-\gamma)\tilde{\sigma}^2}{2(p+q+\nu)^2} f(\nu) = 0,} \quad (2.5)$$

³To clarify notation, for a process $dX_t = \mu_t^X dt + \sigma_t^X dZ_t$,

$$E_t[\text{growth}(X_t)] = \mu_t^X / X_t.$$

where the differential operator $\mathcal{L}f$ is given by

$$\mathcal{L}f = f'(\nu)(\nu(r - \Phi(\iota) + \delta) + a - \iota - \zeta) + \frac{f''(\nu)}{2}\sigma^2 + ((1 - \gamma)(\Phi(\iota) - \delta) - \rho)f(\nu).$$

Second, let us price local currency, which has value $p_t K_t$ and has no dividend yield. Hence,

$$E_t [\text{growth}(e^{-\rho t} f^h(\nu_t) k_t^{-\gamma} K_t)] = 0, \quad \text{where} \quad f^h(\nu) = \frac{f(\nu_t)}{p_t + q_t + \nu_t} p_t \quad \Rightarrow \quad (2.6)$$

$$\boxed{\mathcal{L}f^h + \frac{\gamma(1 + \gamma)\tilde{\sigma}^2}{2(p + q + \nu)^2} f^h(\nu) = 0 \quad \text{for } \nu \in (\underline{\nu}, \bar{\nu}),} \quad (2.7)$$

where \mathcal{L} is the same differential operator as above.

Third, let us price global currency. A portfolio that consists of K units of global currency pays the dividend of $r - \text{growth}(K_t)$. Hence,

$$r - E_t [\text{growth}(K_t)] + E_t [\text{growth}(e^{-\rho t} f^s(\nu_t) k_t^{-\gamma} K_t)] = 0 \quad \Rightarrow \quad (2.8)$$

$$\boxed{(r - \Phi(\iota) + \delta)f^s + \mathcal{L}f^s + \frac{\gamma(1 + \gamma)\tilde{\sigma}^2}{2(p + q + \nu)^2} f^s(\nu) = 0 \quad \text{for } \nu \in (\underline{\nu}, \bar{\nu}).} \quad (2.9)$$

Next, let me address the boundary conditions where capital is sold or bought. At $\underline{\nu}$, we have $dk/k = dK/K = -d\lambda$ and $d\nu = (\underline{q} + \nu) d\lambda$. Hence, (2.4), (2.6) and (2.8) imply that at $\underline{\nu}$,

$$\boxed{(\underline{q} + \underline{\nu})f' = (1 - \gamma)f, \quad (\underline{q} + \underline{\nu})(f^h)' = (1 - \gamma)f^h \quad \text{and} \quad (\underline{q} + \underline{\nu})(f^s)' = -\gamma f^s.} \quad (2.10)$$

Likewise, at $\bar{\nu}$,

$$\boxed{(\bar{q} + \bar{\nu})f' = (1 - \gamma)f, \quad (\bar{q} + \bar{\nu})(f^h)' = (1 - \gamma)f^h \quad \text{and} \quad (\bar{q} + \bar{\nu})(f^s)' = -\gamma f^s.} \quad (2.11)$$

Lastly, investment rate ι is determined from the price of capital q by the first-order condition

$$\boxed{\Phi'(\iota)q = 1, \quad \text{where} \quad q = \frac{f - f^h}{f^s} - \nu.} \quad (2.12)$$

The system of three pricing equations (2.5), (2.7) and (2.9) together with equations (2.3) and (2.12) fully characterize the equilibrium. We use the iterative method to solve these equations - see Brunnermeier and Sannikov (2016c). Specifically, we can set reasonable

terminal conditions relatively arbitrarily and solve parabolic versions of the pricing equations backward until convergence, by adding time dimension to the operator \mathcal{L} . That is, we replace the operator \mathcal{L} with \mathcal{L}^t ,

$$\mathcal{L}^t f(\nu, t) = \frac{\partial f}{\partial t} + \mathcal{L}f.$$

We deal with capital sales and purchases from abroad as follows, we take the state space to be $[\underline{\nu}, \bar{\nu}]$, with $\underline{\nu}$ only slightly higher than $-q$, and a sufficiently large upper bound $\bar{\nu}$. We impose boundary conditions (2.10) and (2.11) at $\underline{\nu}$ and $\bar{\nu}$. The idea is to choose a sufficiently large state space that it contains endogenous boundaries $\underline{\nu}$ and $\bar{\nu}$. We find them by solving (2.5), (2.7) and (2.9) backward in time with time step ϵ , identifying $\underline{\nu}$ and $\bar{\nu}$ as levels where price $q = (f - f^h)/f^s - \nu$ reaches \underline{q} and \bar{q} . We then replacing functions f , f^h and f^s with the solutions of (2.10) and (2.11) beyond those levels.

The solution that we obtain corresponds to the equilibrium in a finite-horizon setting, with value functions and prices having a specific form that corresponds to our terminal conditions and with opportunities to buy or sell capital abroad arising only at discrete time intervals of length ϵ and at boundaries $\underline{\nu}$ and $\bar{\nu}$. As we let the time horizon go to infinity, ϵ go to 0, and for a sufficiently large state space interval, approach the stationary equilibrium of the infinite-horizon setting.

These equations characterize the equilibrium in the absence of policy and with free capital flows, in which the amount of local currency is in fixed supply (and thus the nominal interest rate on the local currency is zero). We want to compare how the free equilibrium outcome differs from that under the optimal policy. In order to make this comparison, we need a proper definition of the policy space.

Policy Space. There is more than one way to specify an economically meaningful space of policies that the local policy maker can select from. We propose one specification, which is an important benchmark, and which also has certain tractability properties that make it amenable to theoretical analysis. In particular, suppose that the policy maker can use capital controls to choose dollar inflows and outflows from the country, and can use taxation and the printing press to control the value of local currency. In this specification, pricing equations (2.7) and (2.9) are overruled by the policy, which determines the value of local currency and dollar holdings. Equation (2.5) still holds, as it evaluates welfare, i.e the value functions of individual citizens. We assume that the policy maker cannot control individual consumption and investment decisions, i.e. equations (2.3) and (2.12) continue to hold, except that the values of f^h and f^s are controls.

These observations imply that the planner's value function satisfies the HJB equation

$$\max \left(\max_{\zeta, \iota} \zeta^{1-\gamma} + \mathcal{L}f - \frac{\gamma(1-\gamma)\tilde{\sigma}^2}{2\zeta^{2\gamma}f(\nu)}, (\underline{q} + \nu)f' - (1-\gamma)f, (1-\gamma)f - (\bar{q} + \nu)f' \right) = 0. \quad (2.13)$$

This equation captures the choice of purchases and sales of capital with foreign countries.

The Two Currencies. One exciting feature of our setting - and this is our key interest - is this co-existence of currencies. For this we need two shocks, two frictions, and each currency plays a particular role in moderating these frictions. The dollar is necessary for the small country to manage the aggregate shocks. Citizens will use dollars to absorb the aggregate shocks between two boundaries, $\underline{\nu}$ where they sell capital abroad at the price of \underline{q} to pay off debt, and $\bar{\nu}$ where they spend excess dollars to buy more capital. Individuals can also use dollars to protect themselves against idiosyncratic shocks. However, under certain conditions - when idiosyncratic risk is large enough and the dollar does not have the optimal risk and return profile to protect individuals against these shocks, there is room for local currency to have value as well. Of course, the exchange rate depends very much on frictions. This is the equilibrium perspective.

The policy maker has a related but somewhat different perspective. The planner also needs dollars to protect the country from aggregate shocks, as we shall see later, the welfare perspective of using dollars versus the local currency for the insurance against idiosyncratic risk is quite different. When it comes to idiosyncratic risk, the planner's objective with respect to the quantity of local currency and the use of the local currency versus the dollar is quite different.

Before putting the two fractions together, in the next section we look at each friction individually. With respect to aggregate risk, the friction is between the small country and the rest of the world: we assume that this risk cannot be shared perfectly through the use of derivatives or by selling equity. With respect to idiosyncratic risk, the friction is amount the small country's citizens.

3 Special Cases: Shocks of only One Type.

Only Idiosyncratic Shocks, Only Local Currency. Suppose that there are only idiosyncratic shocks, i.e. $\sigma = 0$. Let us first understand what happens in the small country if it is an isolated economy, i.e. individuals cannot hold dollars. We can use existing equations

to characterize the equilibrium and the optimal policy. Individual value functions take the form

$$\frac{fk^{1-\gamma}}{1-\gamma}.$$

The equilibrium equations reduce to

$$\frac{\zeta}{\zeta^\gamma f} + (1-\gamma)(\Phi(a-\zeta) - \delta) - \rho - \frac{\gamma(1-\gamma)\tilde{\sigma}^2}{2\zeta^{2\gamma}f^2} = 0 \quad (3.1)$$

$$(1-\gamma)(\Phi(a-\zeta) - \delta) - \rho + \frac{\gamma(1+\gamma)\tilde{\sigma}^2}{2\zeta^{2\gamma}f^2} = 0. \quad (3.2)$$

There is an equilibrium in which local currency has value if and only if there is a solution (ζ, f) to this system that satisfies

$$\underbrace{1/\Phi'(a-\zeta)}_q < \underbrace{\zeta^\gamma f}_{p+q}.$$

The policy maker chooses ζ to maximize $f/(1-\gamma)$, subject to the constraint (3.1). That is, the policy maker has to respect equation (3.1), which is analogous to (2.5), but can overrule equation (3.2), analogous to (2.7), through taxes and transfers.

Individuals hold currency to protect themselves against idiosyncratic risk. Adverse shocks \tilde{Z} affect only the capital in individual portfolios, not the money. Idiosyncratic risk creates the demand for money. The attractiveness of money depends on policy, i.e. the extent to which money is backed by taxes and money printing. By making money more attractive, the policy maker reduces the price of capital, hence distorts investment and lowers growth.

Figure 1 compares the equilibrium without policy and the optimal policy outcome for different risk aversion coefficients $\gamma = 1.5$ and 4 for the investment function of the form $\Phi(\iota) = \log(\kappa\iota + 1)/\kappa$. Other parameters are $\rho = 4\%$, $a = 0.14$, $\kappa = 2$ and $\delta = 0.02$. In the absence of policy, idiosyncratic risk has to be sufficiently large for an equilibrium in which money has value to exist. With policy, it is always optimal to have some money. For low $\tilde{\sigma}$, the policy maker would like to increase money supply by backing money with taxes (on capital or wealth). For high $\tilde{\sigma}$, individuals hold too much money in the absence of policy intervention - this is the “paradox of prudence.” Optimal policy makes it less attractive for people to hold money (by printing money and making transfers to individuals). With logarithmic utility, $\gamma = 1$, this model has been analyzed in Brunnermeier and Sannikov (2016a) and (2016b).

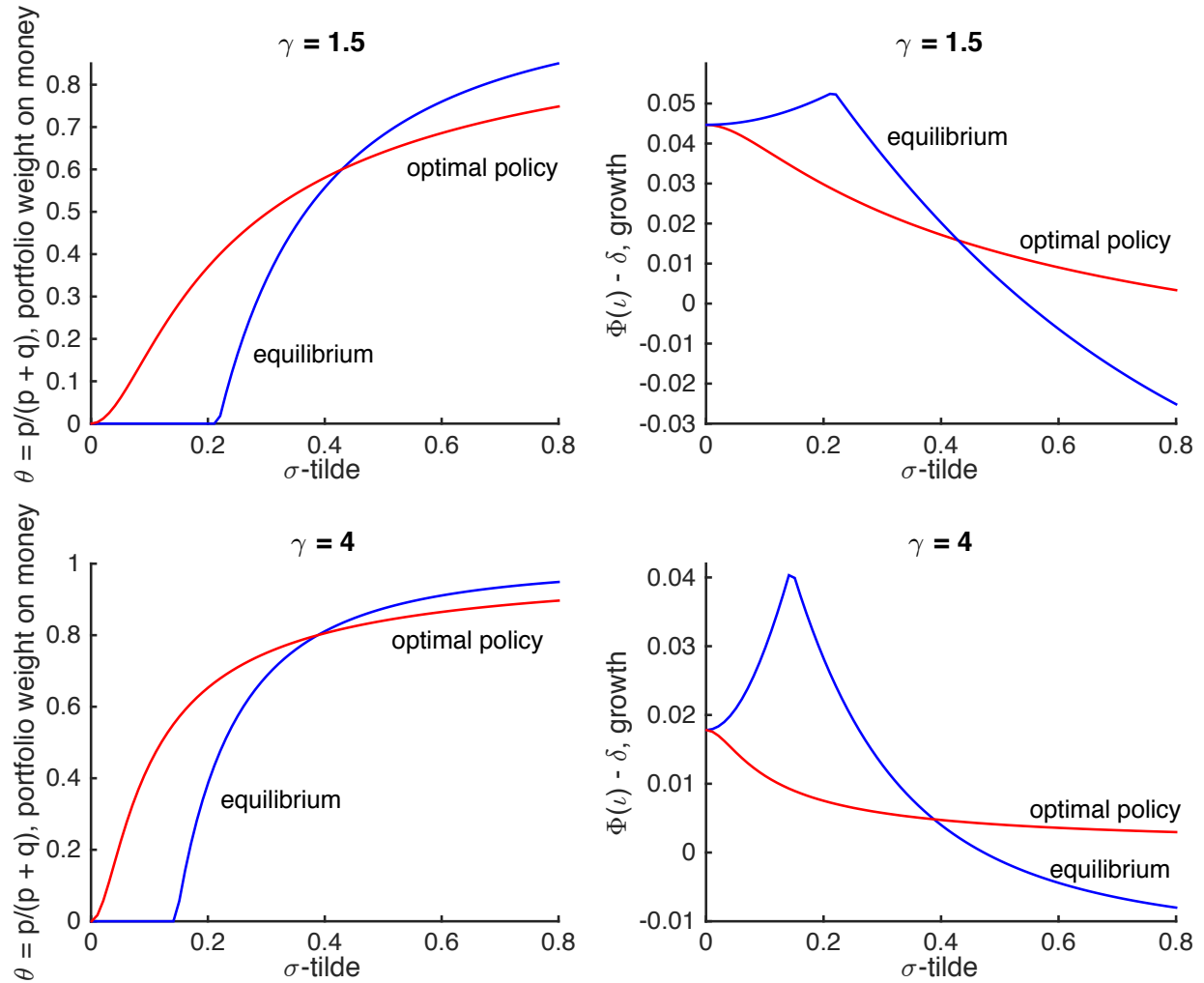


Figure 1: Equilibrium and Optimal Policy without Aggregate Risk or Dollar.

Only Idiosyncratic Shocks, Local Currency and The Dollar. Now, suppose that individuals can hold the local currency as well as dollars. Here we present two examples, one in which $\underline{q} = 0$, so the dollar borrowing limit is 0, and another with $\underline{q} > 0$.

The top panels of Figure 2 illustrate what happens with $\underline{q} = 0$. In the absence of policy, there is no equilibrium in which local currency has value when $\tilde{\sigma}$ is small. Individuals can protect themselves only by investing more, hence growth increases in that region. Above a critical level of around 0.2, there is an equilibrium in which local currency has value, and the top left panel shows the holdings of local currency per unit of capital. At the steady state, local currency has the same return as the growth rate of the economy. As $\tilde{\sigma}$ increases, the

demand for risky capital falls, hence investment and growth fall. Once growth falls below the real return on the dollar (of $r = 2\%$ in this example), the dollar dominates the local currency. This happens at about $\tilde{\sigma} = 0.35$. Hence, the value of local currency held by the locals is non-monotone in $\tilde{\sigma}$. The blue curves of the top two panels illustrate what happens at the steady state in equilibrium. It is interesting also that above $\tilde{\sigma} = 0.35$, even away from the steady state when individuals hold no dollars, when potential demand for idiosyncratic risk insurance is high, local currency cannot have value in the absence of policy, because in the medium run, the dollar is expected to replace the local currency.

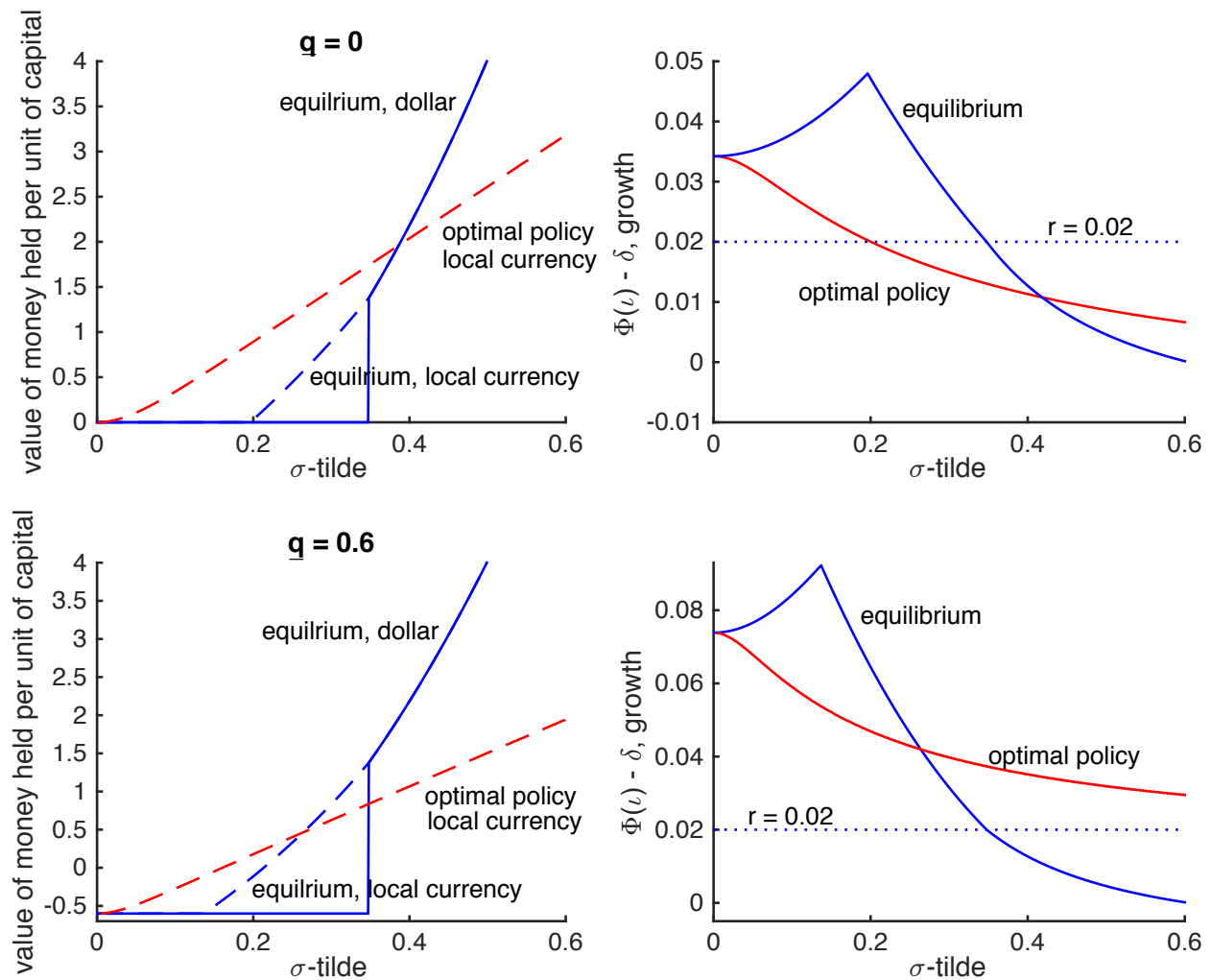


Figure 2: Equilibrium and Optimal Policy without Aggregate Risk: Two Currencies.

What would the policy maker do? Recall that the policy maker can control the amount

of dollars held in the small country, and control the value of local currency. It turns out that for high $\tilde{\sigma}$, the policy maker would prefer to spend all the dollars and only allow individuals hold the local currency. To see the intuition behind this result, let us consider replacing dollars with an equal supply of the local currency in an equilibrium for $\tilde{\sigma}$ slightly about 0.35. This would have no effect on growth or exposure to idiosyncratic risk, because total money supply remains unchanged, but this would allow the policy maker to increase consumption temporarily by spending the dollars. This is a welfare improvement. And, of course, moving from the equilibrium supply of local currency to the optimal supply of local currency is another improvement in welfare.

When locals can borrow in dollars, the situation is even more dramatic. The bottom panels of Figure 2 illustrate what happens in that case, if $\underline{q} = 0.6$. Without policy, as long as the growth is above the rate of return on the dollar r , locals borrow in dollars to the limit and hold local currency to protect themselves against idiosyncratic risk when $\tilde{\sigma}$ is high enough. Local currency has value for lower levels of $\tilde{\sigma}$ because idiosyncratic risk is amplified with dollar leverage. Once growth dips below r , local currency has no value and individuals save dollars instead of borrowing. Leverage allows the economy to grow at a faster rate, but once it goes away (when locals no longer use their own currency) the equilibrium steady states on top and bottom panels are identical.

In contrast, the policy maker borrows dollars to the limit (recall that there are no aggregate shocks to hedge) and forces locals to save only in their currency. In this particular example the policy results in significantly higher growth, above r under this optimal policy.

Only Aggregate Shocks. A version of this model with only aggregate shocks has been studied in corporate finance in the context of liquidity management. See, for example, Bolton, Chen and Wang (2013). They study how much a firm chooses to invest as a function of its cash holdings, with extreme cash shortages leading to equity issuances at the lower boundary and surpluses leading to dividends. The dynamics in our model is similar, as all agents in the small country acting identically. Our model determines not only investment but also consumption, due to the agents' risk aversion. There are other differences, but the common core is that the equilibrium outcome is determined by the HJB equation (for agents or the firm).

This observation leads to the result is that in the absence of idiosyncratic shocks, the competitive equilibrium is efficient, i.e. the policy maker cannot do better. Hence, the source of inefficiency in this model is idiosyncratic risk. It produces inefficiencies with respect to the quantity and value of local currency, and holdings of local currency vs. the dollar, as

illustrated in the examples above.

Proposition 1. With aggregate risk alone, the competitive equilibrium coincides with the outcome of optimal policy.

PROOF. Consider what would happen if the country had just one agent. Then the agent's optimization problem is identical to that of the policy-maker: to maximize the value function $f(\nu)$ by the choices of consumption, investment, and the boundaries $\underline{\nu}$ and $\bar{\nu}$ where to buy or sell capital. The agent's HJB equation is the same as that of the policy maker, (2.13). The shadow price of capital, at which the agent would be indifferent to trade a marginal unit of capital without affecting the value function $f(\nu)k^{1-\gamma}/(1-\gamma)$, satisfies

$$(q + \nu)f'(\nu) - (1 - \gamma)f(\nu) = 0. \quad (3.3)$$

Now, if a continuum of agents behave identically to this single agent/planner, then they would be in equilibrium if the market price of capital is given by (3.3), and at this price there would be no trade. Hence, the equilibrium outcome is identical to the planner solution. In this equilibrium, the local currency has no value.

Furthermore, some straightforward algebra can be used to verify that given the solution f of (2.13), function

$$f^{\$}(\nu) = \frac{f(\nu)}{q + \nu} = \frac{f'(\nu)}{1 - \gamma}$$

satisfies equation (2.9) with appropriate boundary conditions. QED

We finish this section by presenting several computed examples to better understand how aggregate shocks affect the maximal amount that the small country is willing to borrow, as well as the dollar buffer that the country tries to maintain away from that boundary. Since aggregate risk is a key determinant of dollar holdings, these examples gives us important comparison points for the equilibrium outcomes and optimal policy with idiosyncratic risk.

Figure 3 compares equilibria with aggregate risk levels $\sigma = 0.1$ and $\sigma = 0.15$. Capital can be sold abroad at the price of $\underline{q} = 0.6$, but if ν had reached $-\underline{q}$, all capital would have to be sold abroad to pay debt and the locals would get utility of minus infinity (with $\gamma = 2$). To avoid this, the locals choose to sell capital at a higher threshold $\underline{\nu}$, the level where the price of capital q reaches \underline{q} in the top left panel. The locals choose to borrow and maintain debt on average when $\sigma = 0.1$, and try to maintain some buffer of savings but occasionally go into debt when $\sigma = 0.15$, as we can see from the drift and the stationary distribution of ν in the right panels.

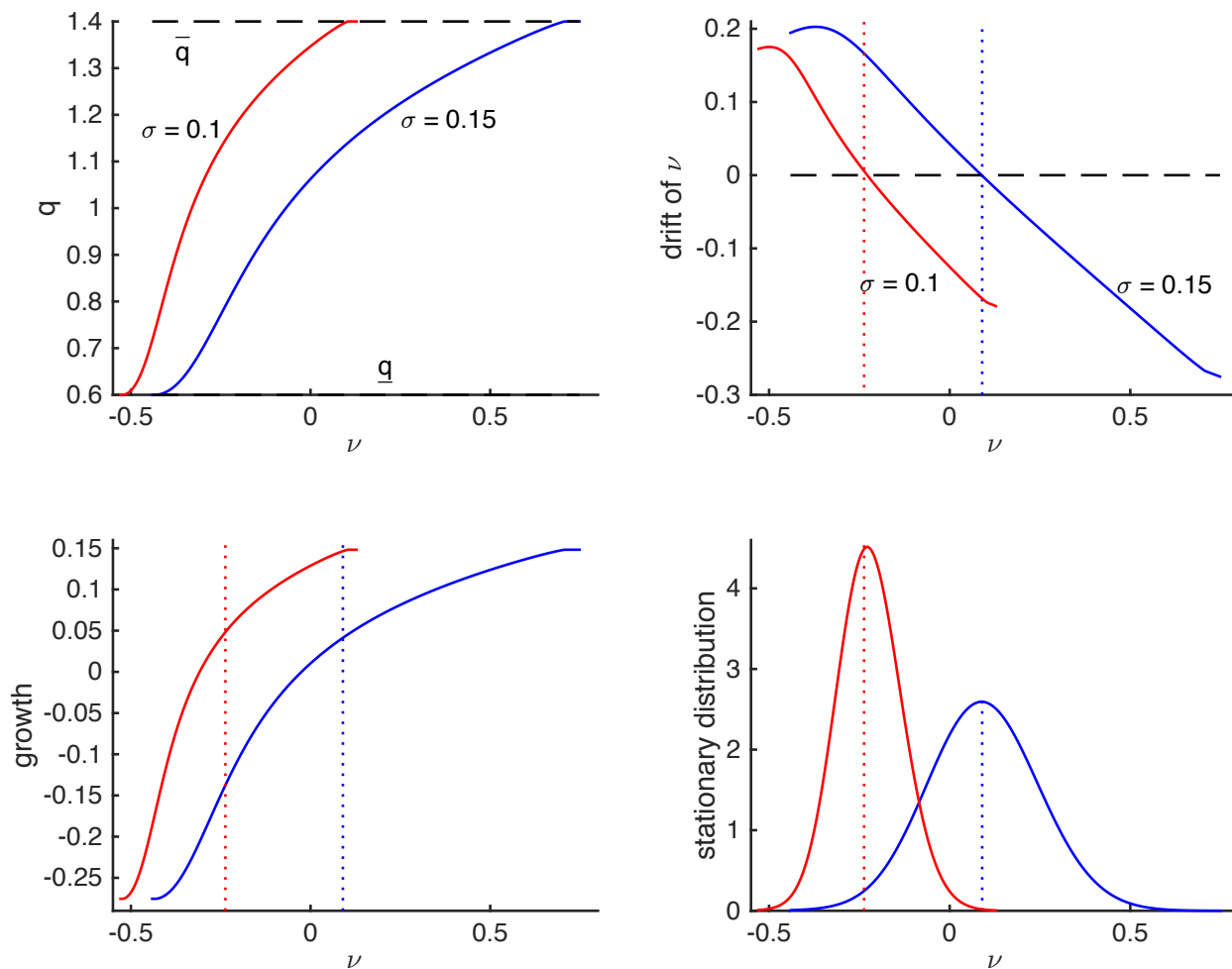


Figure 3: Equilibrium with $\rho = 4\%$, $a = 0.14$, $\kappa = 2$, $\delta = 0.02$, $\gamma = 2$, $\underline{q} = 0.6$ and $\bar{q} = 1.4$.

Figure 4 shows how the liquidation value of capital \underline{q} affects equilibrium. Higher \underline{q} increases the borrowing limit. Agents choose to borrow more and also hit the borrowing limit a lot more frequently, as seen from the stationary distribution in the lower right panel. Average growth is also higher when \underline{q} goes up: capital becomes more liquid.

Finally, Figure 5 shows how the equilibrium depends on the dollar rate r . There is a slight effect on maximal dollar debt (which is greater when r is lower), but a significant effect on average dollar borrowings. The drift and stationary distribution of ν illustrate that even without idiosyncratic shocks, the policy rate on the dollar can have significant effects on the small country, especially in reaction to changes in r .

To recap, without idiosyncratic risk, there is no role for local currency in this model.

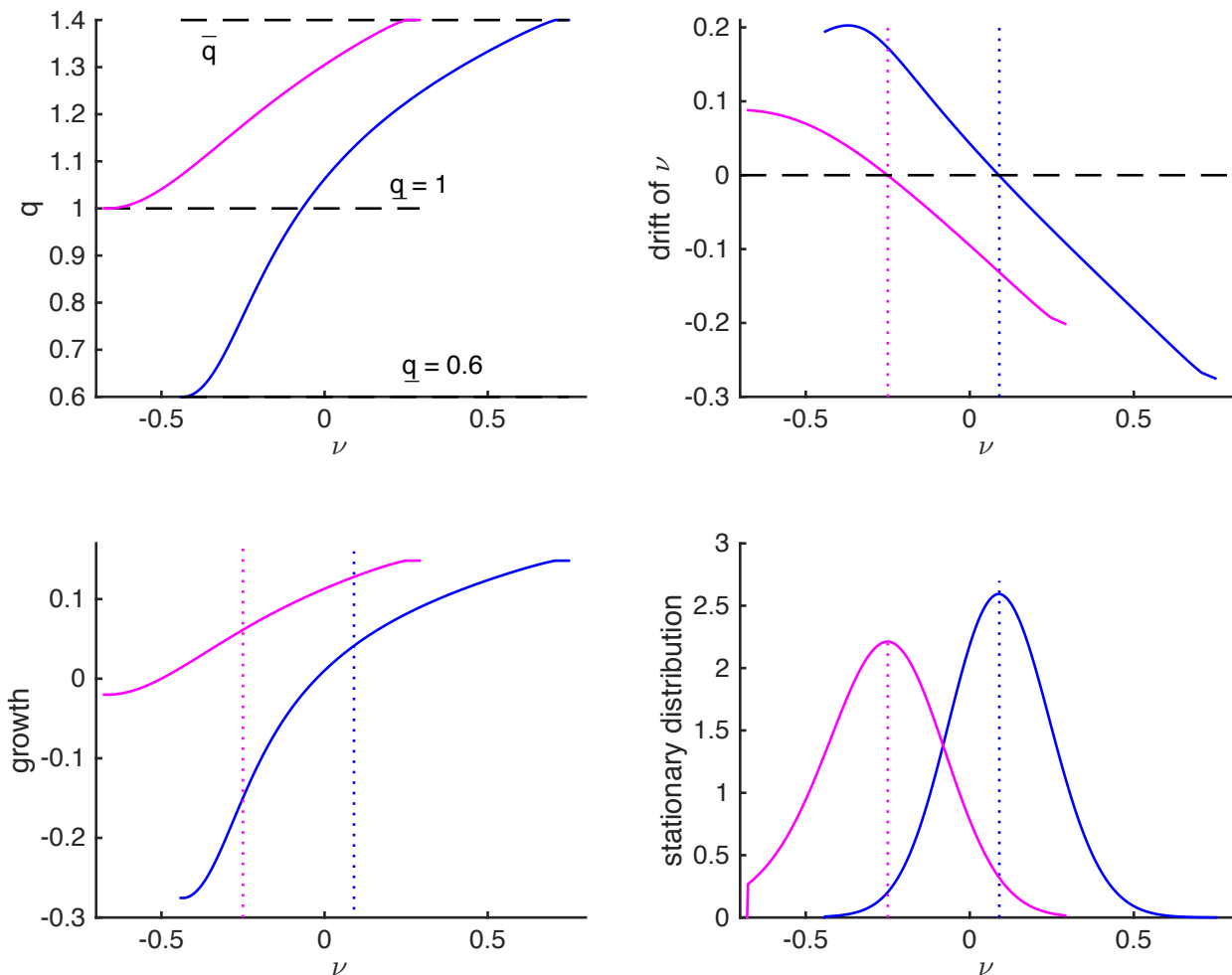


Figure 4: Equilibrium with $\sigma = 0.15$, and two values of \underline{q} , 0.6 and 1.

Without aggregate risk, there is essentially role for only one currency (local currency has value only when the small country is at the dollar borrowing limit). However, which currency it is depends on whether we look at competitive equilibrium or the optimal policy outcome. In the next section we study the equilibrium and optimal policy with both idiosyncratic and aggregate shocks.

4 Two Currencies with Two Frictions: Equilibrium.

So far, we have looked at the value of local currency only at the steady state, in a model without aggregate shocks. Now, aggregate shocks will drive the dollar balances, and this

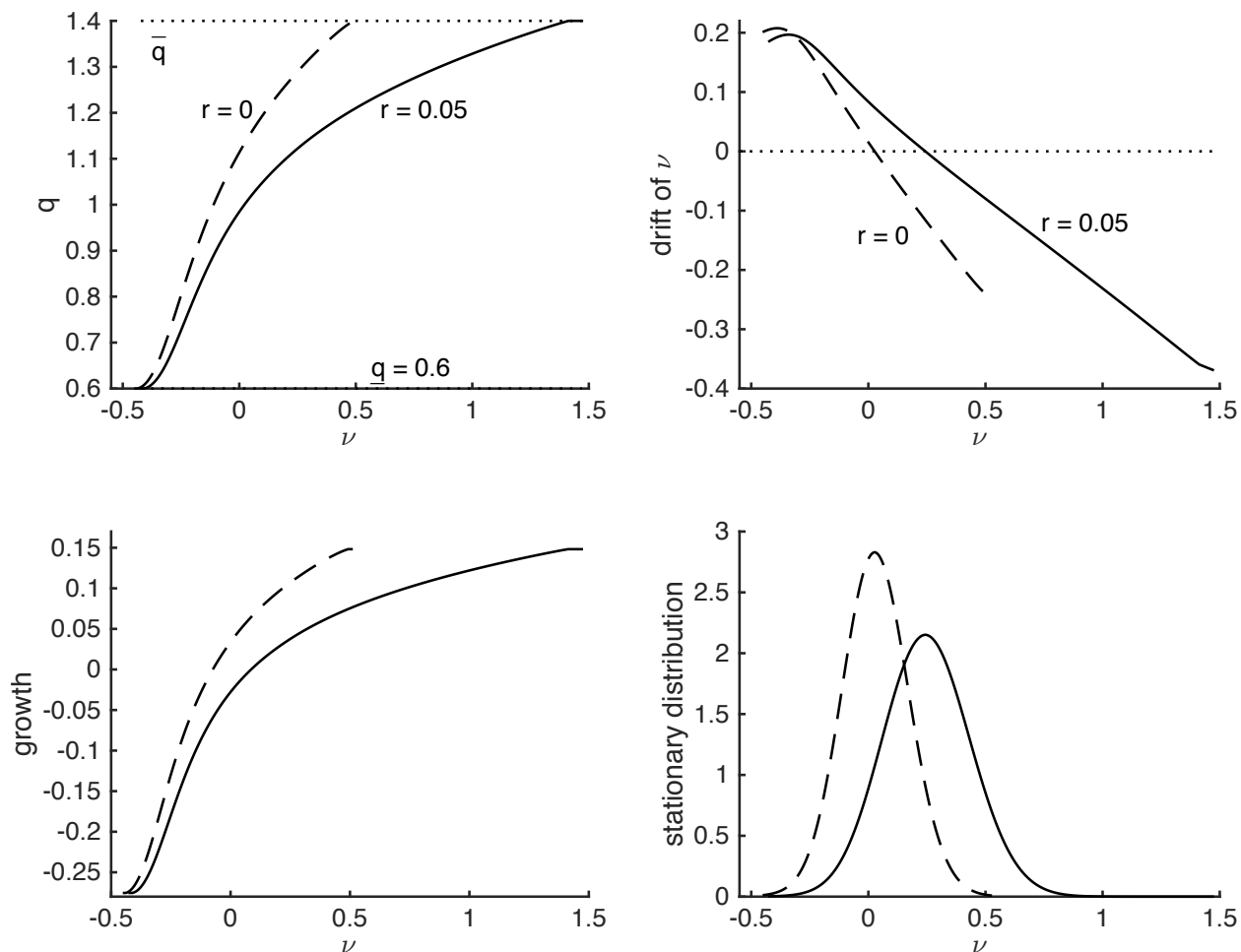


Figure 5: Equilibrium with $\sigma = 0.15$, $\underline{q} = 0.6$, and two values of r , 0 and 5%.

affects the value of local currency and the exchange rate, if the local currency has any value at all.

Figure 6 compares the equilibria with and without idiosyncratic risk for the same parameters as in Figure 2, $\rho = 0.04$, $r = 0.02$, $\kappa = 2$, $\delta = 0.02$, $\gamma = 2$ and $a = 0.14$, with $\sigma = 0.1$ and $\tilde{\sigma} = 0$ and 0.2 . In this example, idiosyncratic risk gives value to the local currency, but lower growth implies that the locals borrow fewer dollars. This takes away from the value of the local currency because the two currencies are imperfect substitutes in the sense that both protect the locals from idiosyncratic risk.

The top left panel shows how much local currency people hold over the dollar balance, as a function of ν . Local currency appreciates in value as ν rises (and growth increases) even

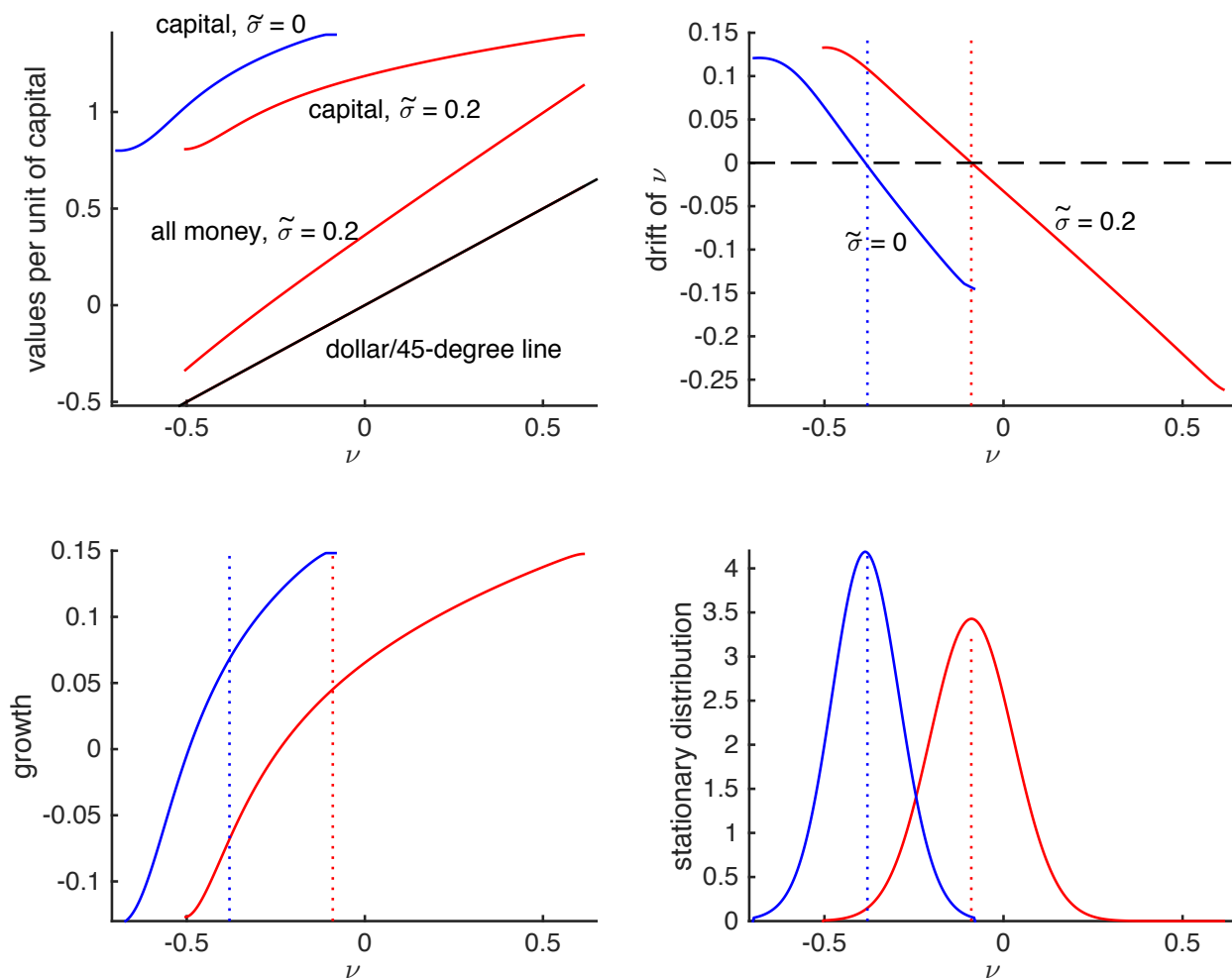


Figure 6: Equilibrium with idiosyncratic risk $\tilde{\sigma} = 0$ (blue) and $\tilde{\sigma} = 0.2$ (red).

though portfolio weight $\theta = p/(p + q + \nu)$ on the local currency falls (not shown). The top right panel shows that the locals borrow a lot less. From the bottom left panel we can see that average growth decreases with idiosyncratic risk.

Figure 7 is plotted for the same parameters, except that we reduced \bar{q} to 1.3 (to prevent $\bar{\nu}$ from becoming too large). We see that the value of local currency is non-monotonic in $\tilde{\sigma}$, reinforcing the message of Figure 2. The reason is that savings in both currencies help individuals deal with the idiosyncratic risk friction. Higher idiosyncratic risk creates demand for money, and the value of local currency rises gradually in $\tilde{\sigma}$, but then falls much faster as growth declines and the dollar dominates local currency.

Finally, Figure 8 shows that there are high-sensitivity regions, where the value of local

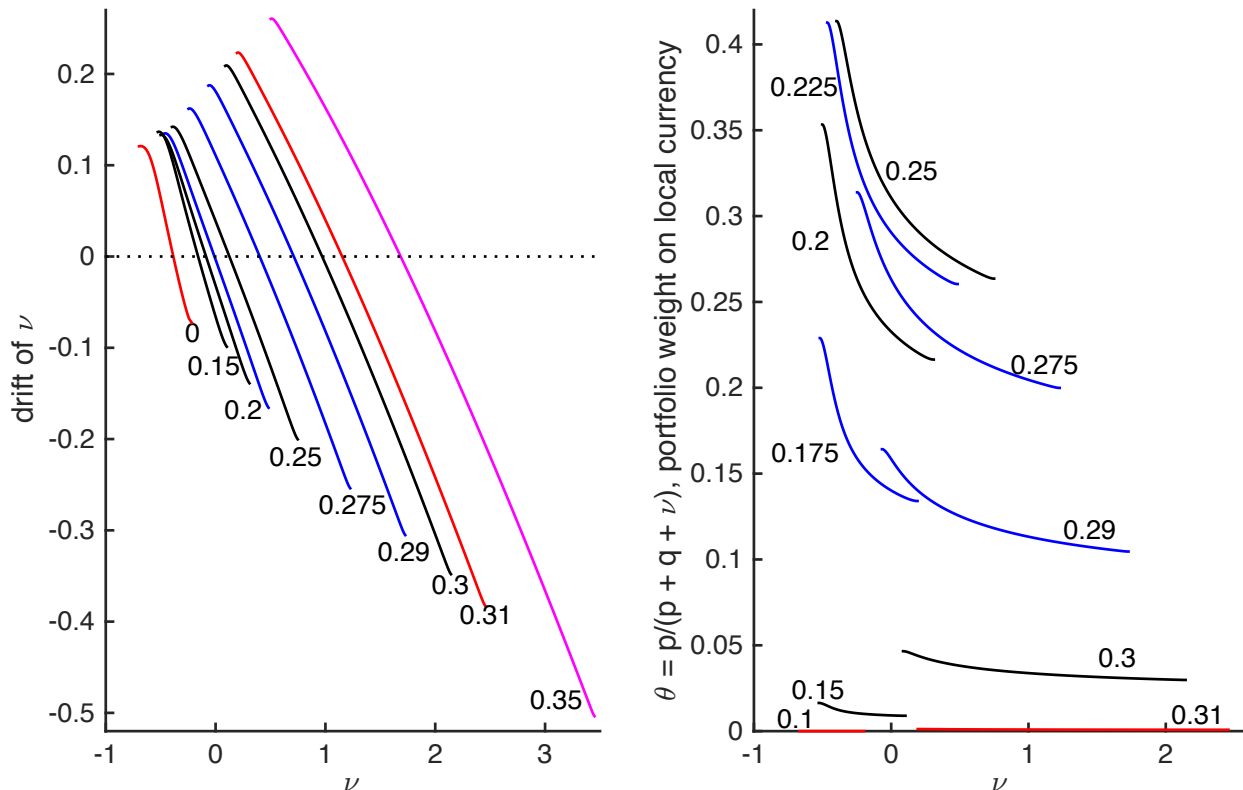


Figure 7: Equilibrium with different levels of idiosyncratic risk $\tilde{\sigma}$.

currency/exchange rate can change dramatically with small movements in dollar risk-free rate. In this example, the portfolio weight on local currency is only moderately sensitive to r when r is low. However, when r approaches the average rate of growth of the small country's economy, local currency can depreciate very fast.

5 Optimal Policy.

How does the optimal policy compare with equilibrium outcome? Figure 9 compares the equilibrium outcome (black) and optimal policy (red) for parameters $\rho = 0.04$, $r = 0.02$, $\kappa = 2$, $\delta = 0.02$, $\gamma = 2$, $a = 0.14$, $\sigma = 0.1$, $\underline{q} = 0.8$, $\bar{q} = 1.2$, for $\tilde{\sigma} = 0.25$. In equilibrium on average citizens hold a positive amount of dollars (the drift of ν becomes zero when ν is about 0.135), and they borrow to at most $\nu = -0.4$. In contrast the planner maintains an average dollar debt, with maximal debt of about $\nu = -0.65$. This is what computation gives us - how can it be justified?

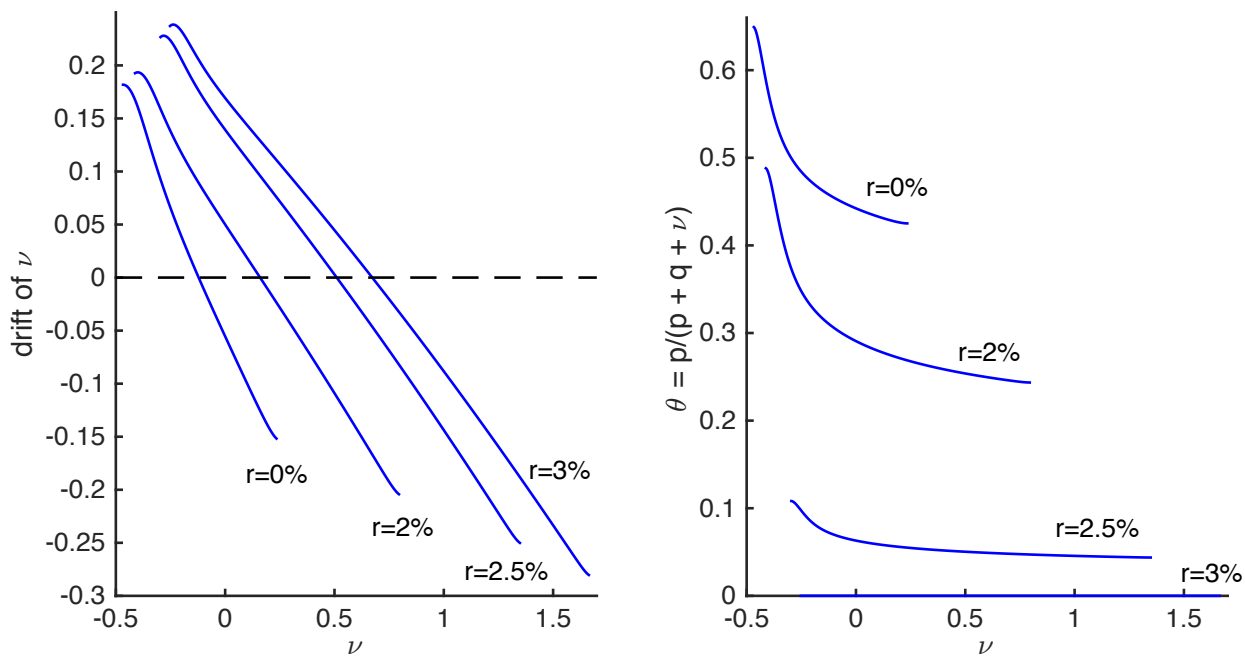


Figure 8: Equilibrium with different levels r , given $\sigma = 0.1$ and $\tilde{\sigma} = 0.25$.

What happens is that the planner replaces some dollars that citizens hold to protect themselves against idiosyncratic risk with local currency (but not in equal amount). We see in the right panel that the portfolio weight of individuals on local currency under the optimal policy is significantly higher than in equilibrium. This allows the planner to spend the dollars and generate a temporary welfare benefit. Average growth is also slightly higher under the optimal policy than in equilibrium.

Maybe this makes sense, but what is happening on a deeper level? Why does it make sense for the planner to borrow in dollars? Given aggregate risk, it seems imprudent.

To understand some of the key forces even better, recall the result of Section 3 that without idiosyncratic risk, the equilibrium and the planner's solution coincide. Then how does the equilibrium diverge from the planner's solution as $\tilde{\sigma}$ rises? And, how significant is the effect of welfare on policy quantitatively? We do this comparison in Figure 10, where we plot the equilibrium and optimal policy outcomes for $\tilde{\sigma} = 0, 0.2$ and 0.3 . Here we observe something very striking!

It turns out that the drift of ν is not very sensitive to $\tilde{\sigma}$ under the optimal policy. When $\tilde{\sigma}$ goes up, the planner chooses to accumulate only slightly more dollars and borrow (in dollars) only slightly less. It is the level of aggregate risk that is the key determinant of

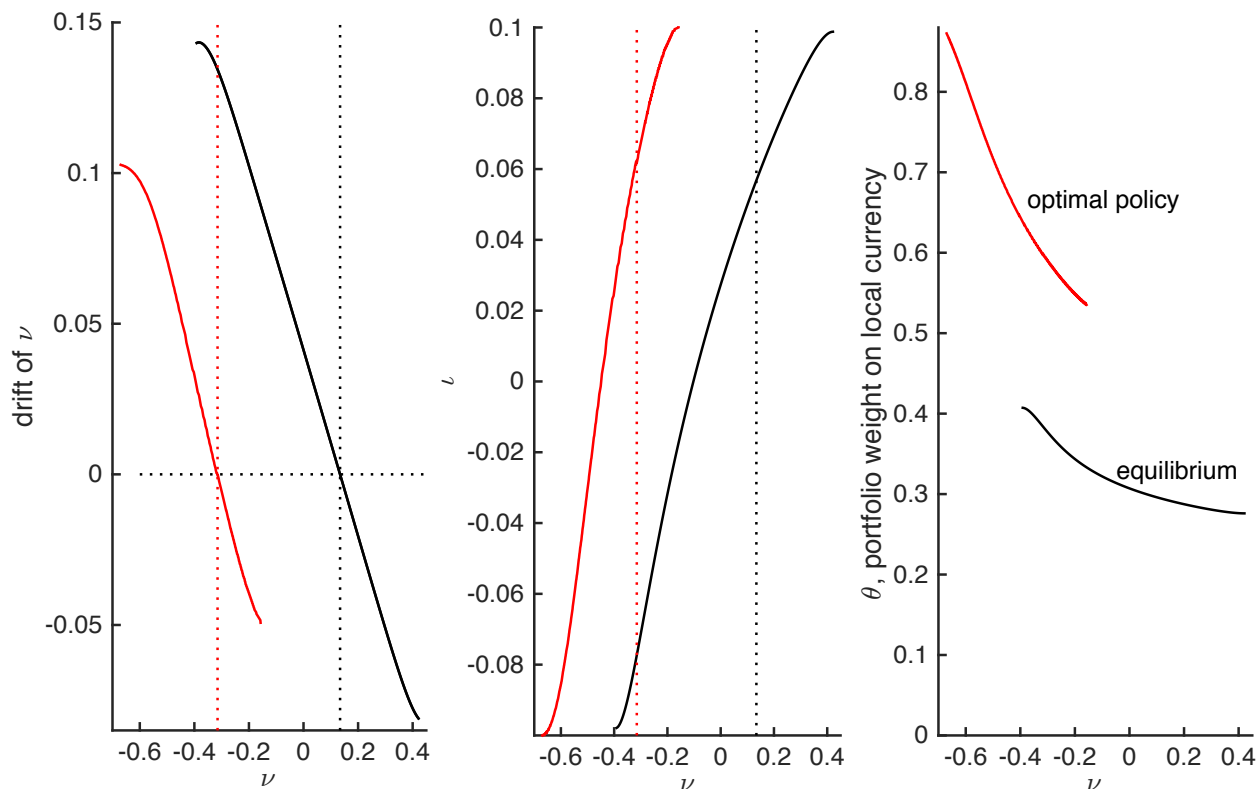


Figure 9: Equilibrium vs. the optimal policy, $\tilde{\sigma} = 0.25$.

dollar holdings. When idiosyncratic risk rises, the planner chooses to accommodate it by expanding the local currency, as seen on the bottom left panel of Figure 10. In contrast, in equilibrium dollar holdings respond much more significantly to idiosyncratic risk, as seen in the top right panel.

Welfare falls with idiosyncratic risk (both in equilibrium and under the optimal policy) as seen on the top left panel. In equilibrium, the welfare is significantly lower (the relevant comparison here is the welfare function $f(\nu)/(1 - \gamma)$ at each level of ν). Our example is not a serious calibration by any means, but for $\tilde{\sigma} = 0.3$, the welfare function falls by a factor of over 2. This is equivalent to a drop in consumption by a factor of over 2 in perpetuity.

6 Policy without Capital Controls.

In this section, we touch upon the subject of the space of possible policies when locals are free to hold as many dollars as they wish. That is, the policy maker can back the local

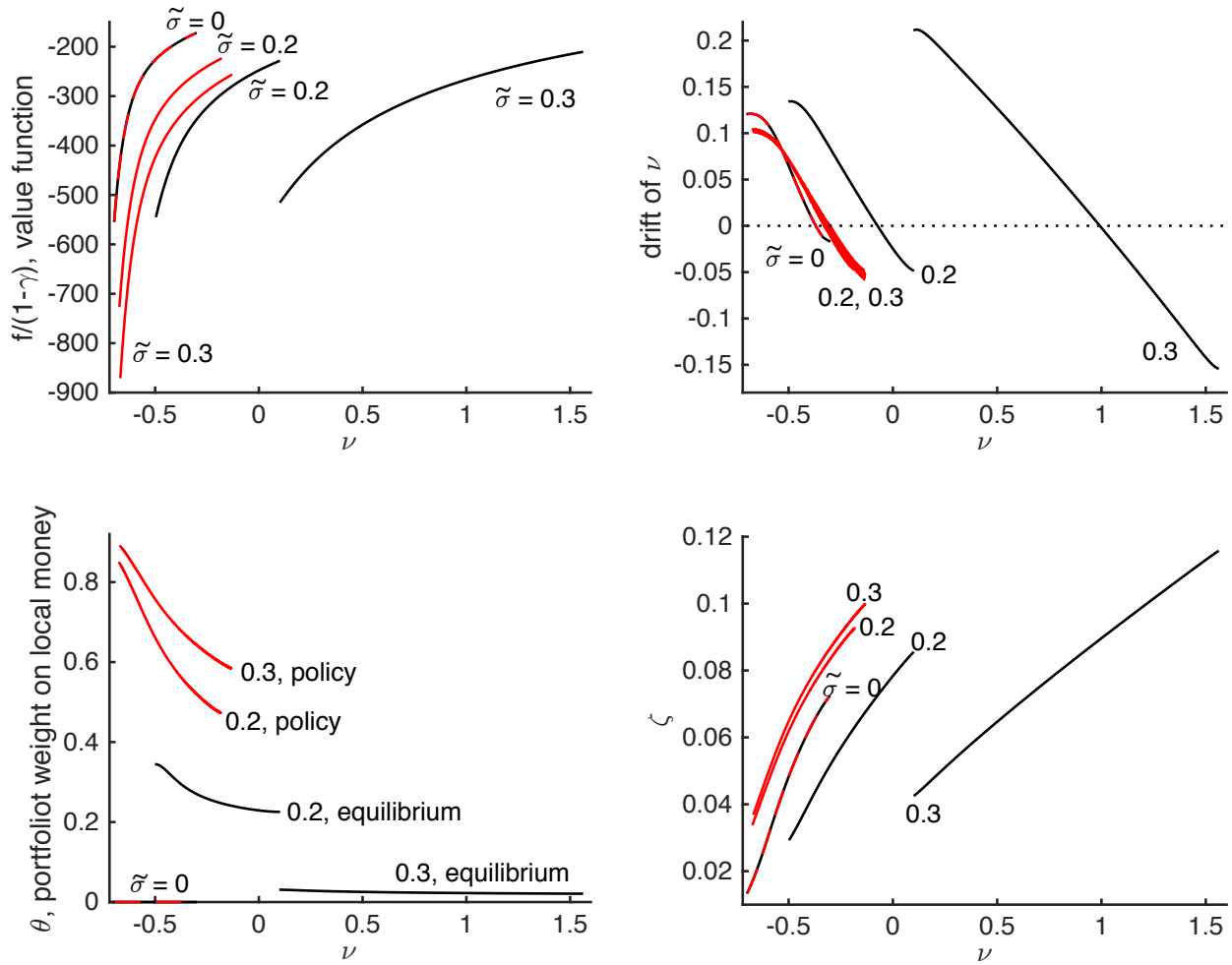


Figure 10: Equilibrium vs. the optimal policy, for $\tilde{\sigma} = 0, 0.2$ and 0.3 .

currency through taxes on capital, or print money to make transfers that are proportional to individual holdings of capital. The policy maker cannot observe or control individual dollar holdings. Then the valuation equations (2.5) and (2.9) have to hold, but (2.7) does not have to hold.

Proposition 2. If the policy maker can impose taxes or transfers that are proportionate to holdings of capital and local currency, then the problem of finding the optimal policy is a control problem with two state variables ν and f^s , objective $f/(1-\gamma)$ and a set of controls that includes f^h .

PROOF. To be completed.

Proposition 2 lays out the methodology for solving for the optimal policy without capital

controls. Instead of solving the full problem, in this section we will look at some simple policies with constant money growth to get intuition. By following the derivation of equation (2.7), we can show that with a constant money growth rate of π the equation changes to

$$\mathcal{L}f^h - \pi f^h + \frac{\gamma(1+\gamma)\tilde{\sigma}^2}{2(p+q+\nu)^2}f^h(\nu) = 0.$$

A negative value of π corresponds to fiscal backing of local currency, i.e. taxing capital to prop up the currency's value.

We take our example from Figure 9 to compare the equilibrium without policy, the optimal policy, and simple policies with constant π . Because in equilibrium individuals overinvest in dollars, as both currencies help individuals insure idiosyncratic risk, the policy maker can improve welfare by fiscal backing of the local currency to encourage locals to use to to save against idiosyncratic risk. This is a fairly general intuition. It is interesting to note that in some cases, this policy is the opposite of what the policy maker would like to do with capital controls: we know from Section 3 that for large $\tilde{\sigma}$, it would be optimal for the policy maker to print money to stimulate investment.

We see that the simple policy and without capital controls the policy maker can improve welfare significantly. This computation suggests that the optimal fiscal backing of the local currency is nonmonotonic: the welfare functions in the left panel cross. In particular, more fiscal backing of the local currency is optimal when ν is low, but when ν is high this slows down growth.

7 Conclusions

(to be completed)

8 Bibliography

Backus, D. K., and G. W. Smith (1993) "Consumption and Real Exchange Rates in Dynamic Economies with Non-traded Goods," *Journal of International Economics*, 35, 297-316.

Baxter. M. and M. J. Crucini (1995) "Business Cycles and the Asset Structure of Foreign Trade," *International Economic Review*, Vol. 36, No. 4, pp. 821-854

Berriel, T. and S. Bhattarai (2013) "Hedging Against the Government: A Solution to

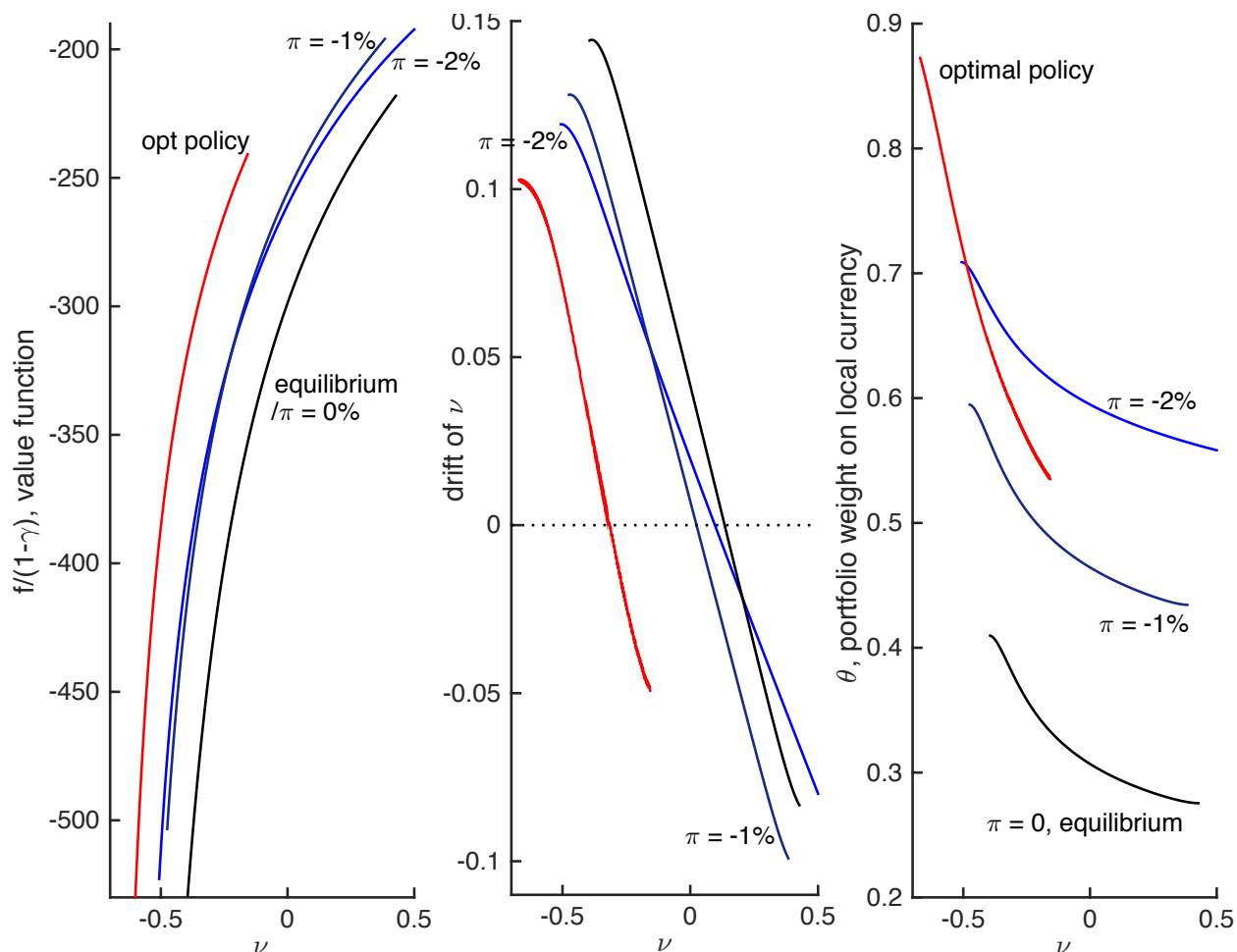


Figure 11: Several policies without capital controls.

the Home Asset Bias Puzzle,” *American Economic Journal: Macroeconomics*, Vol. 5, No. 1, pp.102-134

Bewley, T. (1980) “The Optimum Quantity of Money”, in *Models of Monetary Economics*, eds. J. Kareken and N. Wallace. Minneapolis, Minnesota: Federal Reserve Bank.

Bolton, P., H. Chen and N. Wang (2013) “Market Timing, Investment, and Risk Management,” *Journal of Financial Economics*, 109, 40-62.

Brunnermeier, M. K. and Y. Sannikov (2016a) “The I Theory of Money”, working paper, Princeton University.

Brunnermeier, M. K. and Y. Sannikov (2016b) “On the Optimal Inflation Rate,” *American Economic Review Papers and Proceedings*, vol 106(5), pages 484-489

Brunnermeier, M. K. and Y. Sannikov (2016c) “Macro, Money and Finance: A Continuous-Time Approach”, in *Handbook of Macroeconomics*, Vol. 2B, John B. Taylor and Harold Uhlig, eds., North Holland.

Cole, Harold L., and Maurice Obstfeld (1991) “Commodity trade and international risk sharing: How much do financial markets matter??” *Journal of Monetary Economics* 28 (1), 3-24.

DeMarzo, P., M. Fishman, Z. He and N. Wang, (2012) “Dynamic Agency and q Theory of Investment,” *Journal of Finance* 67, pp. 2295-2340.

Devereux, M. and A. Sutherland (2007) “Solving for Country Portfolios in Open Economy Macro Models,” working paper, IMF

Di Tella, S. (2016) “Uncertainty Shocks and Balance Sheet Recessions.” Forthcoming at the *Journal of Political Economy*.

Fleming, J. Marcus (1962) “Domestic financial policies under fixed and floating exchange rates.” *IMF Staff Papers*, 9, 369-379. Reprinted in Cooper, Richard N., ed. (1969) *International Finance*. New York: Penguin Books.

He, Z., and A. Krishnamurthy (2013) “Intermediary Asset Prices”, *American Economic Review*, 103(2), 732-770.

Kareken, J. and N. Wallace (1981) “On the Indeterminacy of Equilibrium Exchange Rates,” *The Quarterly Journal of Economics*, Vol. 96, No. 2, pp. 207-222

Kehoe, P. and F. Perri (2002) “International Business Cycles with Endogenous Incomplete Markets,” *Econometrica*, 70(3), 907-928.

Mundell, Robert A. (1963) “Capital mobility and stabilization policy under fixed and flexible exchange rates.” *Canadian Journal of Economic and Political Science*. 29 (4), 475-485. Reprinted in Mundell, Robert A. (1968) *International Economics*. New York: Macmillan.

Obstfeld, M. and K. Rogoff (1995) “Exchange Rate Dynamics Redux,” *Journal of Political Economy*, 102, pp 624-660.

Samuelson, P. A. (1958) “An exact consumption loan model of interest, with or without the social contrivance of money.” *Journal of Political Economy*, 66, 467-82.

Sims, C. (1994) “A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy,” *Economic Theory*, 4, 381-399.

Sturzenegger, F. (1994) “Hyperinflation with Currency Substitution: Introducing an Indexed Currency,” *Journal of Money, Credit and Banking*, Vol. 26, No. 3(1), pp. 377-395

A Computing Equilibria: Numerical Details

B Proofs