Financial and Monetary Economics

Eco529 Fall 2020

Lecture 07: Welfare – Optimal Policy

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The big Roadmap: Towards the I Theory of Money

- One sector model with idio risk - "The I Theory without I"
  (steady state focus)
  - Store of value
    - Insurance role of money *within sector*
  - Money as bubble or not
  - Fiscal Theory of the Price Level
  - Medium of Exchange Role ⇒ SDF-Liquidity multiplier ⇒ Money bubble
- 2 sector/type model with money and idio risk
  - Generic Solution procedure (compared to lecture 03)
  - Equivalence btw experts producers and intermediaries
  - Real debt vs. nominal debt/money
    - Implicit insurance role of money *across sectors*
  - I Theory
- Welfare analysis
- Optimal Monetary Policy and Macroprudential Policy
- International Monetary Model

Previous lectures

Today

Next lectures

Optimal Policy

- Finding the optimal policy is generally complicated, need
    1. precise definition of policy space
    2. analytical tools to characterize the optimum
- One side: inefficiencies / tradeoffs
    - insurance vs. investment (one sector/type)
    - allocation of assets / risk (across sectors/types)
- Other side: "large" policy space
    - controlling money growth rate
    - macroprudential tools / wealth redistribution
    - risk redistribution
- Approach
    - Start with simple model
    - Add step-by-step more model elements

Roadmap

- Expected Utility/Value function with log-utility
- One sector model with stochastic idiosyncratic volatility
- Two sector model
  - with exogenous net worth share η
  - With endogenous wealth share η
  - I theory (with two technologies)

Welfare with log utility

- The welfare for any agent $\tilde{\imath}$ of type i

$$E\left[\int_0^\infty e^{-\rho t} \log(c_t^{\tilde{\imath}})\, dt\right]$$

$$\tilde{\eta}_0^{\tilde{\imath}} = 1, \frac{d\tilde{\eta}_t^{\tilde{\imath}}}{\tilde{\eta}_t^{\tilde{\imath}}} = \tilde{\sigma}_t^{\tilde{\eta}^i} d\tilde{Z}_t^{\tilde{\imath}}$$

- Recall from general model with log utility
  - $\frac{c_t^{\tilde{\imath}}}{n_t^{\tilde{\imath}}} = \rho$
  - $c_t^{\tilde{\imath}} = \rho \eta_t^i (A(\boldsymbol{\kappa_t}) - \iota_t) K_t \, \tilde{\eta}_t^{\tilde{\imath}}$ using goods market clearing

Welfare with log utility

- The welfare of any agent $\tilde{\imath}$ is

$$E\left[\int_0^\infty e^{-\rho t}\log(c_t^{\tilde{\imath}})\,dt\right] = E\left[\int_0^\infty e^{-\rho t}\log(\eta_t^i(A(\boldsymbol{\kappa_t}) - \iota_t)K_t\,\tilde{\eta}_t^{\tilde{\imath}})\ dt\right]$$

ignoring constant $\frac{\log \rho}{\rho}$

$$= E\left[\int_0^\infty e^{-\rho t}\log \eta_t^i\,dt\right] + E\left[\int_0^\infty e^{-\rho t}\log(A(\boldsymbol{\kappa_t}) - \iota_t)\ dt\right]$$

$$+E\left[\int_0^\infty e^{-\rho t}\log K_t\,dt\right] + E\left[\int_0^\infty e^{-\rho t}\log \tilde{\eta}_t^{\tilde{\imath}}\,dt\right]$$

Welfare with log utility

- Recall

$$\log X_t - \log X_0 = \int_0^t d\log X_s$$

- Apply to Ito's lemma

$$d\log X_t = \left(\mu_t^X - \frac{1}{2}\left(\sigma_t^X\right)^2 - \frac{1}{2}\left(\tilde{\sigma}_t^X\right)^2\right)dt + \sigma_t^X dZ_t + \tilde{\sigma}_t^X d\tilde{Z}_t$$

- Plug into Expected integral

$$E\left[\int_0^\infty e^{-\rho t}\log(X_t)\ dt\right]$$
$$= \frac{1}{\rho}\log(X_0) + \frac{1}{\rho}E\left[\int_0^\infty e^{-\rho t}\left(\mu_t^X - \frac{1}{2}\left(\sigma_t^X\right)^2 - \frac{1}{2}\left(\tilde{\sigma}_t^X\right)^2\right)dt\right]$$

Welfare with log utility

- The welfare of any agent $\tilde{\imath}$ is

$$E\left[\int_0^\infty e^{-\rho t}\log(c_t^{\tilde{\imath}})\,dt\right] = E\left[\int_0^\infty e^{-\rho t}\log(\eta_t^i(A(\boldsymbol{\kappa}_t)-\iota_t)K_t\,\tilde{\eta}_t^{\tilde{\imath}})\;dt\right]$$

ignoring constant $\frac{\log\rho}{\rho}$

$$= \underbrace{E\left[\int_0^\infty e^{-\rho t}\log\eta_t^i\,dt\right]}_{\frac{\log\eta_0^i}{\rho}+E\left[\int_0^\infty e^{-\rho t}\left(\frac{\mu_t^{\eta^i}}{\rho}-\frac{\left(\sigma_t^{\eta^i}\right)^2}{2\rho}\right)dt\right]} + E\left[\int_0^\infty e^{-\rho t}\log(A(\boldsymbol{\kappa}_t)-\iota_t)\;dt\right]$$

$$\underbrace{+E\left[\int_0^\infty e^{-\rho t}\log K_t\,dt\right]}_{\frac{\log K_0}{\rho}+E\left[\int_0^\infty e^{-\rho t}\left(\frac{\Phi(\iota_t)-\delta}{\rho}-\frac{\left(\sigma_t^K\right)^2}{2\rho}\right)dt\right]} + \underbrace{E\left[\int_0^\infty e^{-\rho t}\log\tilde{\eta}_t^{\tilde{\imath}}\,dt\right]}_{-E\left[\int_0^\infty e^{-\rho t}\left(\frac{\left(\tilde{\sigma}_t^{\tilde{\eta}^i}\right)^2}{2\rho}\right)dt\right]}$$

Welfare of Intermediaries I and HH h

- Intermediaries (Pareto weight λ)

$$E\left[\int_0^\infty e^{-\rho t}\left(\log \eta_t + \log(A(\kappa) \; - \iota_t) + \frac{\Phi(\iota_t) - \delta}{\rho} - \frac{\sigma^2}{2\rho} - \frac{(1-\vartheta_t)^2}{2\rho}\frac{\kappa^2 \varphi^2 \tilde{\sigma}^2}{\eta^2}\right) dt\right]$$

- Households (Pareto weight $1 - \lambda$)

$$E\left[\int_0^\infty e^{-\rho t}\left(\log(1-\eta_t) + \log(A(\kappa) - \iota_t) + \frac{\Phi(\iota_t) - \delta}{\rho} - \frac{\sigma^2}{2\rho} - \frac{(1-\vartheta_t)^2}{2\rho}\frac{(1-\kappa)^2 \tilde{\sigma}^2}{(1-\eta)^2}\right) dt\right]$$

Roadmap

- Expected Utility/Value function with log-utility
- One sector model with stochastic idiosyncratic volatility
- Two sector model
  - with exogenous net worth share η
  - With endogenous wealth share η
  - I theory (with two technologies)

One Sector Model with Money

- Agent $\tilde{i}$'s preferences

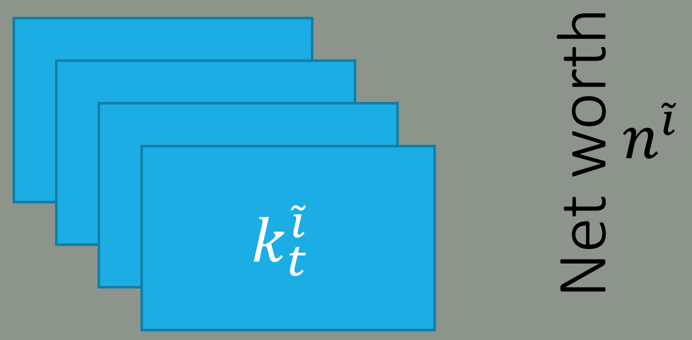
$$E\left[\int_0^\infty e^{-\rho t} \log c_t^{\tilde{i}}\, dt\right]$$

- Each agent operates one firm
  - Output

$$y_t^{\tilde{i}} = a k_t^{\tilde{i}}$$

  - Physical capital k

$$\frac{dk_t^{\tilde{i}}}{k_t^{\tilde{i}}} = \left(\Phi(\iota_t^{\tilde{i}}) - \delta\right)dt + \tilde{\sigma}_t d\tilde{Z}_t^{\tilde{i}}$$



$$\sigma = 0$$

  - Idiosyncratic risk $\tilde{\sigma}$ is stochastic (hence a state variable)

$$d\tilde{\sigma}_t = \mu(\tilde{\sigma}_t)dt + \nu(\tilde{\sigma}_t)dZ_t^\nu$$

e.g. CIR process

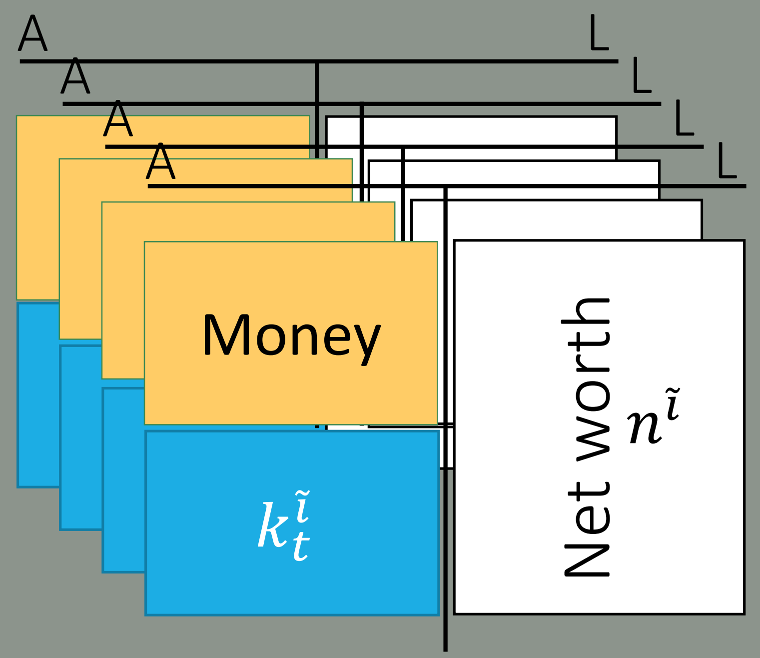
$$d\tilde{\sigma}_t = \alpha\left(\sigma^{SS} - \tilde{\sigma}_t\right)dt + \nu\sqrt{\tilde{\sigma}_t}\, dZ_t^\nu$$

- Financial Friction: Incomplete markets: Agents cannot share $d\tilde{Z}_t^{\tilde{i}}$

One Sector Model with Money

- Agent $\tilde{\imath}$'s preferences

$$E\left[\int_0^\infty e^{-\rho t} \log c_t^{\tilde{\imath}}\, dt\right]$$



- Each agent operates one firm
  - Output

$$y_t^{\tilde{\imath}} = a k_t^{\tilde{\imath}}$$

  - Physical capital k

$$\frac{dk_t^{\tilde{\imath}}}{k_t^{\tilde{\imath}}} = \left(\Phi\left(\iota_t^{\tilde{\imath}}\right) - \delta\right)dt + \tilde{\sigma}_t d\tilde{Z}_t^{\tilde{\imath}}$$

$$\sigma = 0$$

- Financial Friction: Incomplete markets: Agents cannot share $d\tilde{Z}_t^{\tilde{\imath}}$
- Outside money/Gov. bond

$$\frac{dM_t}{M_t} = \mu_t^M dt + \nu_t^M dZ_t^\nu.$$

State variable is $\tilde{\sigma}$: -- Monetary policy $\mu^M(\tilde{\sigma}_t), \nu^M(\tilde{\sigma}_t)$

One Sector Model with Money

- Dynamics of $\tilde{\eta}_t$:

$$d\tilde{\eta}_t/\tilde{\eta}_t = d\left(\frac{n_t^{\tilde{\imath}}}{N_t^i}\right)/d\tilde{\eta}_t = \underbrace{(1-\vartheta_t)\,\tilde{\sigma}_t}_{\tilde{\sigma}_t^{\tilde{\eta}^{\mathrm{I}}}}\, d\tilde{Z}_t^{\tilde{\imath}}$$

- Total wealth as numeraire has return ρ, $dr_t^N = \rho dt$
- Money has return

$$dr_t^{\vartheta_t/M_t} = \frac{d(\vartheta_t/M_t)}{\vartheta_t/M_t} = \underbrace{\left(\mu_t^{\vartheta} - \mu_t^M + \nu_t^M\left(\nu_t^M - \sigma_t^{\vartheta}\right)\right)}_{\mu_t^{\vartheta/M}} dt + \underbrace{\left(\sigma_t^{\vartheta} - \nu_t^M\right)}_{\sigma_t^{\vartheta/M}} d\tilde{Z}_t^{\nu}$$

- Money valuation equation

$$\rho - \mu_t^{\vartheta/M} = \left(\tilde{\sigma}_t^{\tilde{\eta}^i}\right)^2 = (1-\vartheta_t)^2\,\tilde{\sigma}_t^2$$

  - Without policy, equation

$$\rho - \mu_t^{\vartheta} = (1-\vartheta_t)^2\,\tilde{\sigma}_t^2$$

has a unique solution in $\vartheta(\tilde{\sigma}_t) \in (0,1)$ (if $\tilde{\sigma}_t$ sufficiently large)

One Sector Model with Money/Gov. Bond

Recall Equilibrium

- Price of physical capital

$$q_t^K = (1-\vartheta_t)\frac{1+\phi a}{(1-\vartheta_t)+\phi\rho}$$

- Price of nominal capital

$$q_t^M = \vartheta_t \frac{1+\phi a}{(1-\vartheta_t)+\phi\rho}$$

- Optimal investment rate

$$\iota_t = \frac{(1-\vartheta_t)a-\rho}{(1-\vartheta_t)+\phi\rho}$$

- Fraction of nominal wealth ϑ_t

$$1-\vartheta_t = \frac{\sqrt{\rho+\mu_t^M-(\sigma_t^M)^2-\mu_t^\vartheta+\sigma_t^\vartheta\sigma_t^M}}{\tilde{\sigma}_t}$$

Optimal Policy

- Welfare is

$$\frac{\log K_0}{\rho} - \frac{\delta}{\rho^2} + E\left[\int_0^\infty e^{-\rho t} \log(\underbrace{A(\boldsymbol{\kappa_t})}_{=a} - \iota_t)\ dt\right]$$

$$E\left[\int_0^\infty e^{-\rho t} \log\left(\rho \frac{a\phi + 1}{\rho\phi + 1 - \vartheta_t}\right) dt\right]$$

$$+E\left[\int_0^\infty e^{-\rho t} \frac{\Phi(\iota_t)}{\rho} dt\right] - E\left[\int_0^\infty e^{-\rho t} \frac{(1-\vartheta_t)^2 \tilde{\sigma}_t^2}{2\rho} dt\right]$$

$$\frac{1}{\rho\phi} E\left[\int_0^\infty e^{-\rho t} \log\left(\frac{(a\phi + 1)(1 - \vartheta_t)}{\rho\phi + 1 - \vartheta_t}\right) dt\right]$$

Optimal Policy

- Welfare is

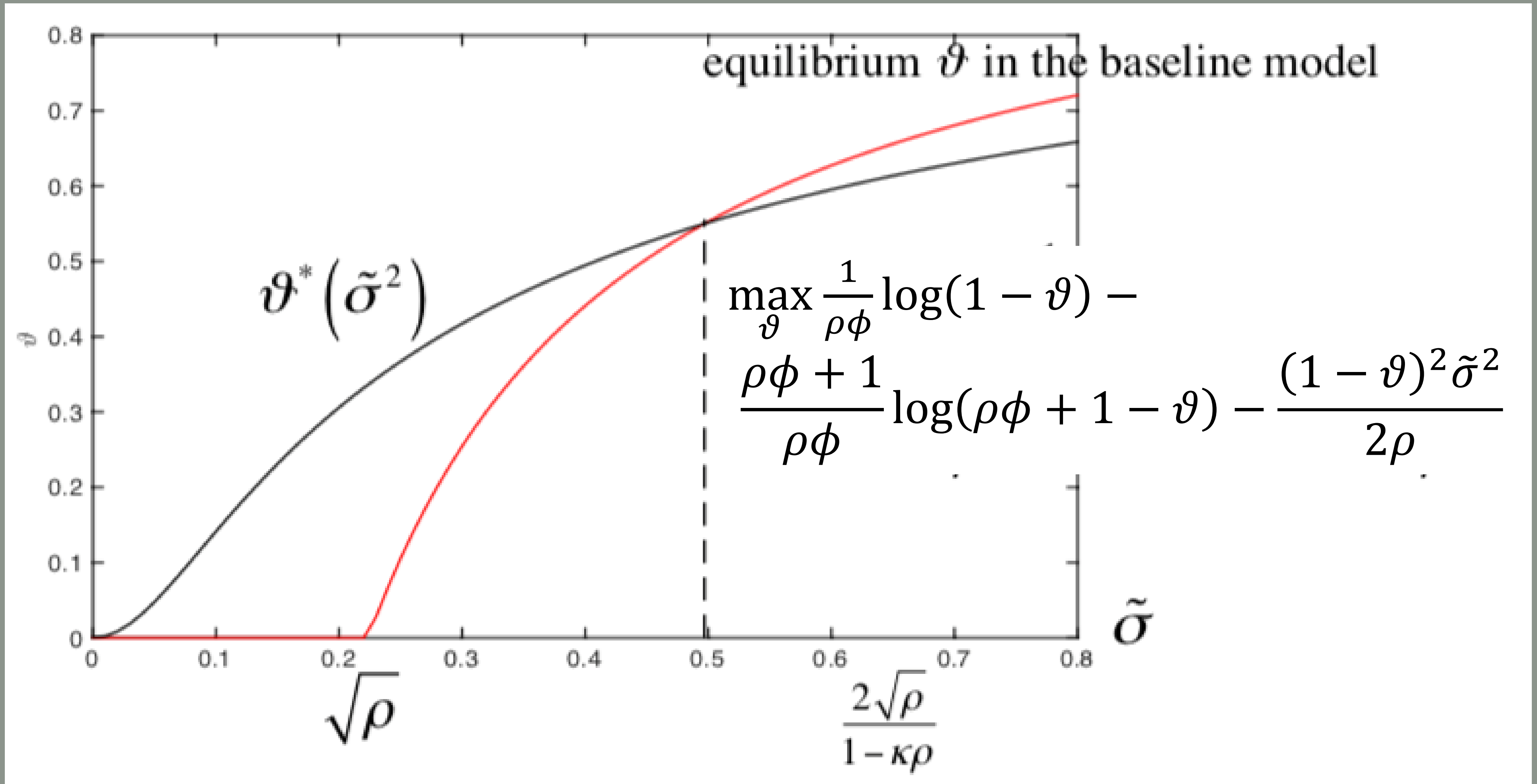
$$\frac{\log K_0}{\rho} - \frac{\delta}{\rho^2} + E\left[\int_0^\infty e^{-\rho t}\left[\log\left(\rho\frac{a\phi+1}{\rho\phi+1-\vartheta_t}\right) + \frac{1}{\rho\phi}\log\left(\frac{(a\phi+1)(1-\vartheta_t)}{\rho\phi+1-\vartheta_t}\right) - \frac{(1-\vartheta_t)^2\tilde{\sigma}_t^2}{2\rho}\right]dt\right]$$

- Lemma: Problem collapses to a static problem for each t

- Let $\vartheta^*(\tilde{\sigma}_t^2)$ be be the maximizer of welfare (optimal policy)

$$\max_{\vartheta}\frac{1}{\rho\phi}\log(1-\vartheta_t) - \frac{\rho\phi+1}{\rho\phi}\log(\rho\phi+1-\vartheta_t) - \frac{(1-\vartheta_t)^2\tilde{\sigma}_t^2}{2\rho}$$

Optimal Policy



Red: equilibrium ϑ in the baseline model

Black: optimal policy ϑ^*

Pecuniary Externality Explanation

- Money growth μ^M affects
  - Shadow risk-free rate
  - (Steady state) inflation in two ways

$$\pi = \mu^M + i - \underbrace{\left(\Phi\left(\iota(\mu^M)\right) - \delta\right)}_{g}$$

- Proposition:
  - For sufficiently large $\tilde{\sigma}$ and $\phi < \infty$ welfare maximizing $\mu^{M^*} > 0$.
    - Laissez-faire Market outcome is not even constrained Pareto efficient
    - Economic growth rate g is also higher
  - Growth maximizing $\mu^{g*} \geq \mu^{M*}$, s.t. $p^{g*} = 0$, Tobin (1965)

- Corollary: No super-neutrality of money
  - i: Super-neutrality only w.r.t. part of money growth rate that is used to pay interest on money
  - μ^M: Nominal money growth rate affects real economic growth by distorting portfolio choice if $\phi < \infty$
    - No price/wage rigidity, no monopolistic competition

Optimal Policy

- If the planner can control ϑ_t directly, she would set $\vartheta_t = \vartheta^*(\tilde{\sigma}_t^2)$

$$d\tilde{\sigma}_t = \mu^{\tilde{\sigma}}(\tilde{\sigma}_t)dt + \nu^{\tilde{\sigma}}(\tilde{\sigma}_t)dZ_t^\nu$$

$$d\vartheta_t = \mu^\vartheta(\tilde{\sigma}_t)\vartheta_t dt + \nu^\vartheta(\tilde{\sigma}_t)\vartheta_t dZ_t^\nu$$

- The planner can choose instruments $\mu^M(\tilde{\sigma}_t), \nu^M(\tilde{\sigma}_t)$ to achieve any function ϑ_t
  - How to find the instruments $\mu^M(\tilde{\sigma}_t), \nu^M(\tilde{\sigma}_t)$ that achieve $\vartheta^*(\tilde{\sigma}_t^2)$?

    -- solving money valuation equation

$$\rho - \underbrace{(\mu_t^\vartheta + \nu_t(\nu_t - \sigma_t^\vartheta))}_{=\mu_t^{\vartheta/M}} = (1 - \vartheta_t)^2\,\tilde{\sigma}_t^2$$

- Optimal policy is easier to find than equilibrium outcome
  - differentiation vs. integration (or solve PDEs)

Roadmap

- Expected Utility/Value function with log-utility
- One sector model with stochastic idiosyncratic volatility
- Two sector model
  - with exogenously fixed net worth share η
  - With endogenous wealth share η
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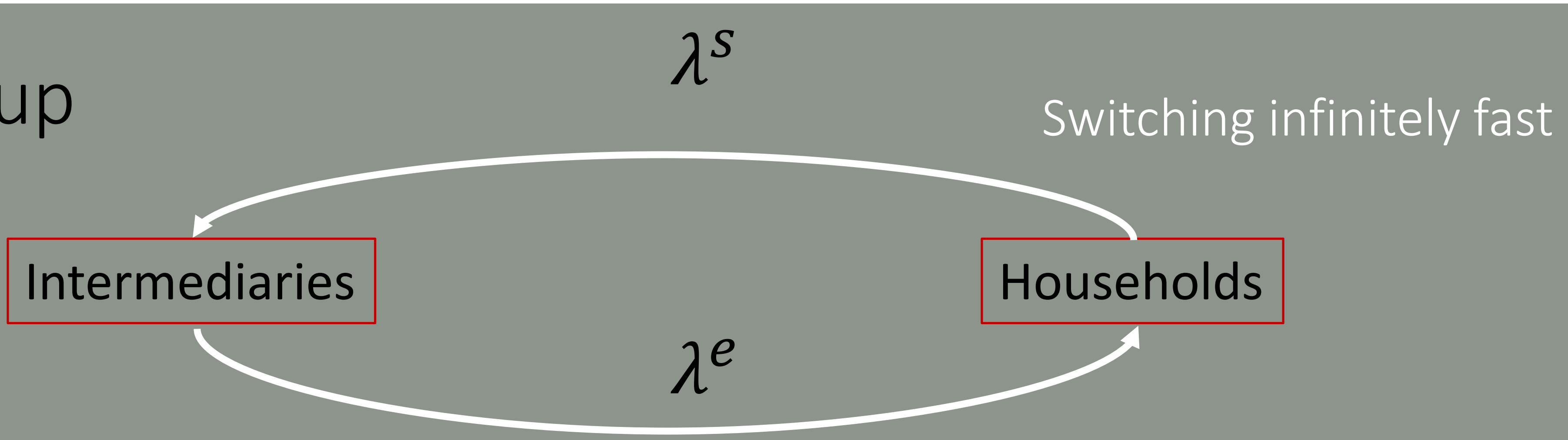
Two Switching Sector model with Exogenous wealth dist.

- Model Setup



Agents	Intermediaries	Household
Share of agents = net worth share	η	$1-\eta$
Idiosyncratic risk of capital	$\varphi\tilde{\sigma}, \varphi \in (0,1)$ diversification	$\tilde{\sigma}$
Output per unit of capital	a the same, independently of the allocation	

Policy marker can choose the money growth rate μ_t^M.

Remark

- Policy-marker cannot affect the wealth shares
- Welfare Pareto weights
  - $\lambda = \eta$ for intermediaries and
  - $1 - \lambda = 1 - \eta$ for households from the setup
- Optimal monetary
  (with or without macroprudential policy – controlling capital allocation)
  - Perfect commitment – Ramsey problem

Equilibrium capital allocation

- Fraction χ of risk (κ of capital) is held by the intermediaries ($\chi = \kappa$)
- Capital allocation must be such that

$$\underbrace{\varphi\tilde{\sigma}_t}_{idio\ risk\ of\ I}\ \underbrace{\frac{(1-\vartheta)\kappa\varphi\tilde{\sigma}_t}{\eta}}_{I's\ price\ of\ idio.risk} = \underbrace{\tilde{\sigma}_t}_{idio\ risk\ of\ h}\ \underbrace{\frac{(1-\vartheta)(1-\kappa)\tilde{\sigma}_t}{1-\eta}}_{h's\ price\ of\ idio.risk}$$

$$\Rightarrow \kappa = \frac{\eta}{\varphi^2(1-\eta)+\eta}$$

- Policy marker may try to affect κ…

Welfare of Intermediaries I and HH h

- Intermediaries (Pareto weight λ)

$$E\left[\int_0^\infty e^{-\rho t}\left(\log \eta_t + \log(a-\iota_t) + \frac{\Phi(\iota_t)-\delta}{\rho} - \frac{\sigma^2}{2\rho} - \frac{(1-\vartheta_t)^2}{2\rho}\frac{\kappa^2\varphi^2\tilde{\sigma}^2}{\eta^2}\right)dt\right]$$

- Households (Pareto weight $1-\lambda$)

$$E\left[\int_0^\infty e^{-\rho t}\left(\log(1-\eta_t) + \log(a-\iota_t) + \frac{\Phi(\iota_t)-\delta}{\rho} - \frac{\sigma^2}{2\rho} - \frac{(1-\vartheta_t)^2}{2\rho}\frac{(1-\kappa)^2\tilde{\sigma}^2}{(1-\eta)^2}\right)dt\right]$$

Welfare

- Law of large numbers: switching risk does not matter. Everyone's wealth growth averages out to $\Phi(\iota_t) - \delta$ and idiosyncratic risk exposure, to

$$\eta\left(\tilde{\sigma}^I\right)^2 + (1-\eta)\left(\tilde{\sigma}^h\right)^2 = (1-\vartheta)^2 \underbrace{\tilde{\sigma}^2\left(\lambda\frac{\kappa^2\varphi^2}{\eta^2} + (1-\lambda)\frac{(1-\kappa)^2}{(1-\eta)^2}\right)}_{(\tilde{\sigma}^{Ave})^2 :=}$$

$$\tilde{\sigma}^I = \frac{(1-\vartheta)\kappa\varphi\tilde{\sigma}}{\eta}, \tilde{\sigma}^h = \frac{(1-\vartheta)(1-\kappa)\tilde{\sigma}}{1-\eta}$$

- Welfare

$$E\left[\int_0^\infty e^{-\rho t}\log(a-\iota(\vartheta))\ dt\right] + E\left[\int_0^\infty e^{-\rho t}\frac{\Phi(\iota(\vartheta))-\delta}{\rho}dt\right] - E\left[\int_0^\infty e^{-\rho t}\frac{(1-\vartheta)^2\left(\tilde{\sigma}^{Ave}\right)^2}{2\rho}dt\right]$$

- Given $\tilde{\sigma}^A$, optimal to set $\vartheta = \vartheta^*\left(\left(\tilde{\sigma}^{Ave}\right)^2\right)$.

- Set $\lambda = \eta$ (Pareto weight is population share)

Money valuation

- Money valuation equation

$$\rho - \underbrace{\left(\mu_t^{\vartheta} - \mu_t^M + \nu_t^M\left(\nu_t^M - \sigma_t^{\vartheta}\right)\right)}_{\mu_t^{\vartheta/M}} = \underbrace{\eta\left(\tilde{\sigma}_t^I\right)^2 + (1-\eta)\left(\tilde{\sigma}_t^h\right)^2}_{(1-\vartheta_t)^2\left(\tilde{\sigma}_t{}^{Ave}\right)^2}$$

Macroprudential tools

Recall: can use χ instead of κ
(depends on model interpretation)

- Average idiosyncratic risk of capital

$$\tilde{\sigma}^2 \left(\frac{\kappa^2 \varphi^2}{\eta} + \frac{(1-\kappa)^2}{1-\eta} \right)$$

is minimized when

$$\frac{\kappa \varphi^2}{\eta} = \frac{1-\kappa}{1-\eta} \Rightarrow \kappa = \frac{\eta}{\varphi^2 (1-\eta) + \eta}$$

- This is the equilibrium allocation!

- Lemma: Optimal not to use macroprudential tools.

assuming $\lambda = \eta$

Remarks

- Same trade-off between insurance and investment
- Equilibrium allocation is efficient,
  minimizes the cost of risk exposure
- Policy space
  - (1) money growth and
  - (1) + (2) (money growth + macroprudential tools)
    leads to the same outcome

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Endogenous law of motion of η

- Wealth distribution can change endogenously with
  - risk exposure of intermediaries and households
  - risk premia
  - consumption rates
- Consider the following model

Fixed types (no switching)

- Model Setup

Intermediaries

Households

Types fixed
(no switching)

Agents	Intermediaries	Household
Welfare weights	λ	$1-\lambda$
Wealth share	η	$1-\eta$
Aggregate risk	σ	σ
Idiosyncratic risk of capital	$\varphi\tilde{\sigma}, \varphi \in (0,1)$	$\tilde{\sigma}$
Output per unit of capital	a the same, independently of the allocation	

You have already seen this model except here $\bar{\kappa} = 1$

Two policy settings:
(1) money growth rate μ_t^M only
(1) + (2) also choose allocation (macroprudential) and transfer wealth between group (why/how?)

Welfare of Intermediaries I and HH h

- Intermediaries (Pareto weight λ)

$$E\left[\int_0^\infty e^{-\rho t}\left(\log \eta_t + \log(a-\iota_t) + \frac{\Phi(\iota_t)-\delta}{\rho} - \frac{\sigma^2}{2\rho} - \frac{(1-\vartheta_t)^2}{2\rho}\frac{\kappa^2\varphi^2\tilde{\sigma}^2}{\eta^2}\right)dt\right]$$

- Households (Pareto weight $1-\lambda$)

$$E\left[\int_0^\infty e^{-\rho t}\left(\log(1-\eta_t) + \log(a-\iota_t) + \frac{\Phi(\iota_t)-\delta}{\rho} - \frac{\sigma^2}{2\rho} - \frac{(1-\vartheta_t)^2}{2\rho}\frac{(1-\kappa)^2\tilde{\sigma}^2}{(1-\eta)^2}\right)dt\right]$$

Optimal policy: (1) MoPo + (2) MacroPru

- Planner chooses ϑ, κ and η to max discount integral of

$$\lambda \log \eta_t + (1-\lambda)\log(1-\eta_t) + \log(a - \iota(\vartheta_t)) + \frac{\Phi(\iota(\vartheta_t))-\delta}{\rho} - \frac{\sigma^2}{2\rho}$$

$$-\frac{(1-\vartheta_t)^2\tilde{\sigma}^2}{2\rho}\underbrace{\left(\lambda\frac{\kappa^2\varphi^2}{\eta^2} + (1-\lambda)\frac{(1-\kappa)^2}{(1-\eta)^2}\right)}_{\frac{\lambda(1-\lambda)\varphi^2}{\lambda\varphi^2(1-\eta)^2+(1-\lambda)\eta^2}}$$

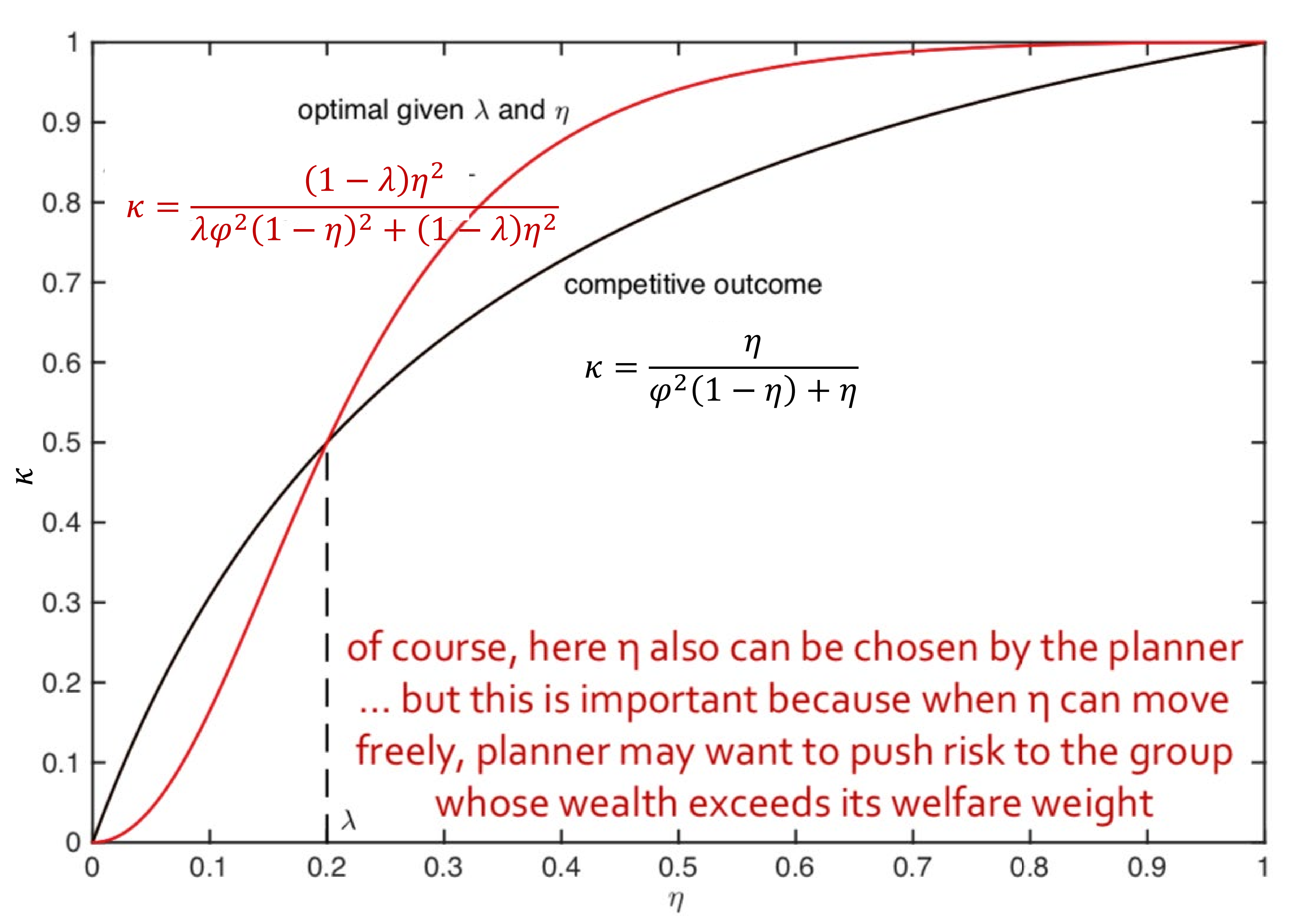
given the optimal choice of $\kappa = \frac{(1-\lambda)\eta^2}{\lambda\varphi^2(1-\eta)^2+(1-\lambda)\eta^2}$

not the competitive allocation (unless $\eta = \lambda$)

Optimal policy: (1) MoPo + (2) MacroPru

- Step 1: Solve optimal κ (or χ) for a given η and λ

Competitive κ vs. minimizing cost of risk



Optimal policy: (1) MoPo + (2) MacroPru

- Planner chooses ϑ, κ and η to max discount integral of

$$\lambda \log \eta_t + (1-\lambda)\log(1-\eta_t) + \log(a - \iota(\vartheta_t)) + \frac{\Phi(\iota(\vartheta_t))-\delta}{\rho} - \frac{\sigma^2}{2\rho}$$

$$-\frac{(1-\vartheta_t)^2\tilde{\sigma}^2}{2\rho}\underbrace{\left(\lambda\frac{\kappa^2\varphi^2}{\eta^2} + (1-\lambda)\frac{(1-\kappa)^2}{(1-\eta)^2}\right)}_{\frac{\lambda(1-\lambda)\varphi^2}{\lambda\varphi^2(1-\eta)^2+(1-\lambda)\eta^2}}$$

given the optimal choice of $\kappa = \frac{(1-\lambda)\eta^2}{\lambda\varphi^2(1-\eta)^2+(1-\lambda)\eta^2}$

not the competitive allocation (unless $\eta = \lambda$)

- Step 2: Solve $\vartheta_t = \vartheta^*(\cdot)$ (having used optimal κ_t) for each given η
- Given κ and η, optimal to set ϑ to

$$\vartheta = \vartheta^*\Big(\underbrace{\tilde{\sigma}^2\frac{\lambda(1-\lambda)\varphi^2}{\lambda\varphi^2(1-\eta)^2+(1-\lambda)\eta^2}}_{\text{welfare weighted average risk exposure}}\Big)$$

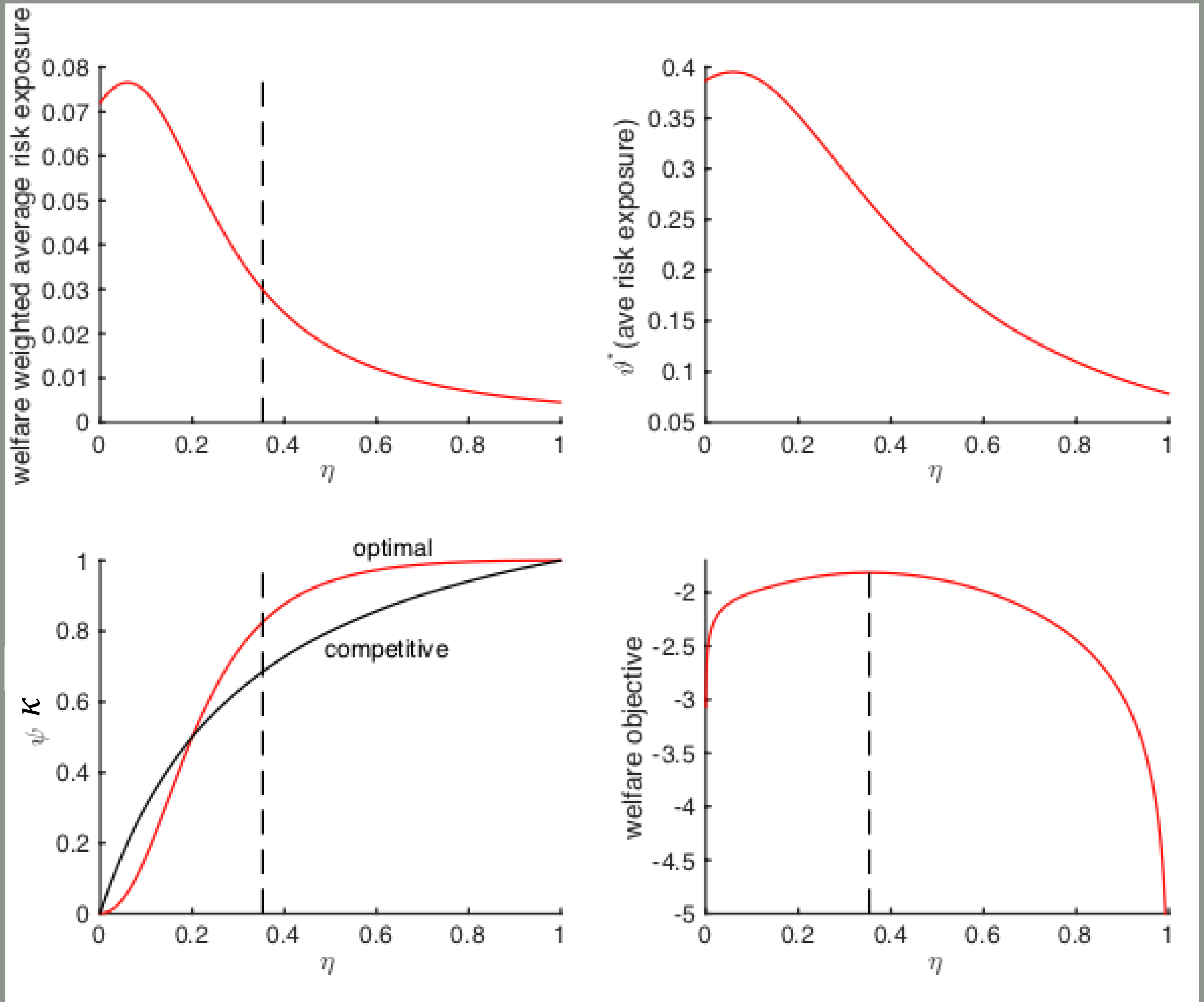
Optimal policy: (1) MoPo + (2) MacroPru

- Step 3: Optimal η (given ϑ)
- let's look at terms containing η
- Given κ and η,

$$\max_{\eta} \quad \underbrace{\lambda \log \eta_t + (1-\lambda)\log(1-\eta_t)}_{\text{concave, max at } \eta=\lambda\text{, goes to } -\infty \text{ at } 0\ \&\ 1} - \frac{(1-\vartheta_t)^2\tilde{\sigma}^2}{2\rho} \underbrace{\frac{\lambda(1-\lambda)\varphi^2}{\lambda\varphi^2(1-\eta)^2 + (1-\lambda)\eta^2}}_{\text{concave, max at } \frac{\lambda\varphi^2}{\lambda\varphi^2+1-\lambda}<\lambda}$$

- hence it is optimal to set $\eta > \lambda$
  (unfortunately no closed-form expression for the optimal η)
- push more risk onto intermediaries than they'd take under competitive outcome
- relative to previous infinite switching model
  - it is optimal to give intermediaries more wealth, because they are more efficient at absorbing risk
  - overall risk is reduced and the value of money is lower (more intermediation)

Optimizing over η



$\rho = .05, \phi = 2, \tilde{\sigma} = .3, \varphi = .5, \lambda = .2$

Optimal policy: (1) MoPo only

- Planner cannot alter competitive alloc. $\kappa_t = \frac{\eta_t}{\varphi^2(1-\eta_t)+\eta_t}$
- Welfare is the discount integral of

$$\lambda \log \eta_t + (1-\lambda)\log(1-\eta_t) + \log(a - \iota(\vartheta_t)) + \frac{\Phi(\iota(\vartheta_t)) - \delta}{\rho} - \frac{\sigma^2}{2\rho}$$

$$- \frac{(1-\vartheta_t)^2\tilde{\sigma}^2}{2\rho} \underbrace{\left(\lambda \frac{\kappa_t^2\varphi^2}{\eta_t^2} + (1-\lambda)\frac{(1-\kappa_t)^2}{(1-\eta_t)^2}\right)}_{\frac{\lambda\varphi^2+(1-\lambda)\varphi^4}{(\varphi^2(1-\eta_t)+\eta_t)^2}}$$

s.t.

$$\frac{d\eta_t}{\eta_t} = (1-\eta_t)\left(\left(\tilde{\sigma}_t^I\right)^2 - \left(\tilde{\sigma}_t^h\right)^2\right)dt = (1-\eta_t)\frac{(1-\vartheta_t)^2\tilde{\sigma}^2\varphi^2\left(1-\varphi^2\right)}{(\varphi^2(1-\eta_t)+\eta_t)^2}dt$$

- Planner can not choose κ_t or η_t but has some control over μ_t^η
- Now, fully dynamic problem!

Optimal policy: (1) MoPo only

- Payoff flow: $f(\eta_t, \vartheta_t) = \lambda \log \eta_t + (1-\lambda)\log(1-\eta_t) + \frac{\log(1-\vartheta_t)}{\rho\phi}$
  $-\frac{\rho\phi+1}{\rho\phi}\log(\rho\phi+1-\vartheta_t) - \frac{(1-\vartheta_t)^2\tilde{\sigma}^2}{2\rho}\left(\lambda\frac{\kappa_t^2\varphi^2}{\eta_t^2} + (1-\lambda)\frac{(1-\kappa_t)^2}{(1-\eta_t)^2}\right),$
  - with $\kappa = \frac{\eta}{\varphi^2(1-\eta)+\eta}$

- HJB equation

$$\rho V(\eta) = \max_{\vartheta} f(\eta, \vartheta) + V'(\eta)\mu^\eta\eta + \frac{1}{2}V''(\eta)(\sigma^\eta\eta)^2$$

- Law of motion of η

$$\frac{d\eta}{\eta} = (1-\eta)\frac{(1-\vartheta)^2\tilde{\sigma}^2\varphi^2(1-\varphi^2)}{(\varphi^2(1-\eta)+\eta)^2}dt + 0dZ$$

Optimal policy: (1) MoPo only

- Payoff flow: $f(\eta_t, \vartheta_t) = \lambda \log \eta_t + (1-\lambda)\log(1-\eta_t) + \frac{\log(1-\vartheta_t)}{\rho\phi}$

$$-\frac{\rho\phi+1}{\rho\phi}\log(\rho\phi+1-\vartheta_t) - \frac{(1-\vartheta_t)^2\tilde{\sigma}^2}{2\rho}\left(\lambda\frac{\kappa_t^2\varphi^2}{\eta_t^2} + (1-\lambda)\frac{(1-\kappa_t)^2}{(1-\eta_t)^2}\right),$$

  - with $\kappa = \frac{\eta}{\varphi^2(1-\eta)+\eta}$

- HJB equation

$$\rho V(\eta) = \max_{\vartheta} f(\eta,\vartheta) + V'(\eta)\mu^\eta\eta + \frac{1}{2}V''(\eta)(\sigma^\eta\eta)^2$$

- Law of motion of η

$$\frac{d\eta}{\eta} = (1-\eta)\frac{(1-\vartheta)^2\tilde{\sigma}^2\varphi^2(1-\varphi^2)}{(\varphi^2(1-\eta)+\eta)^2}dt + 0dZ$$

Optimal policy: (1) MoPo only

- Optimal ϑ^*
- HJB equation

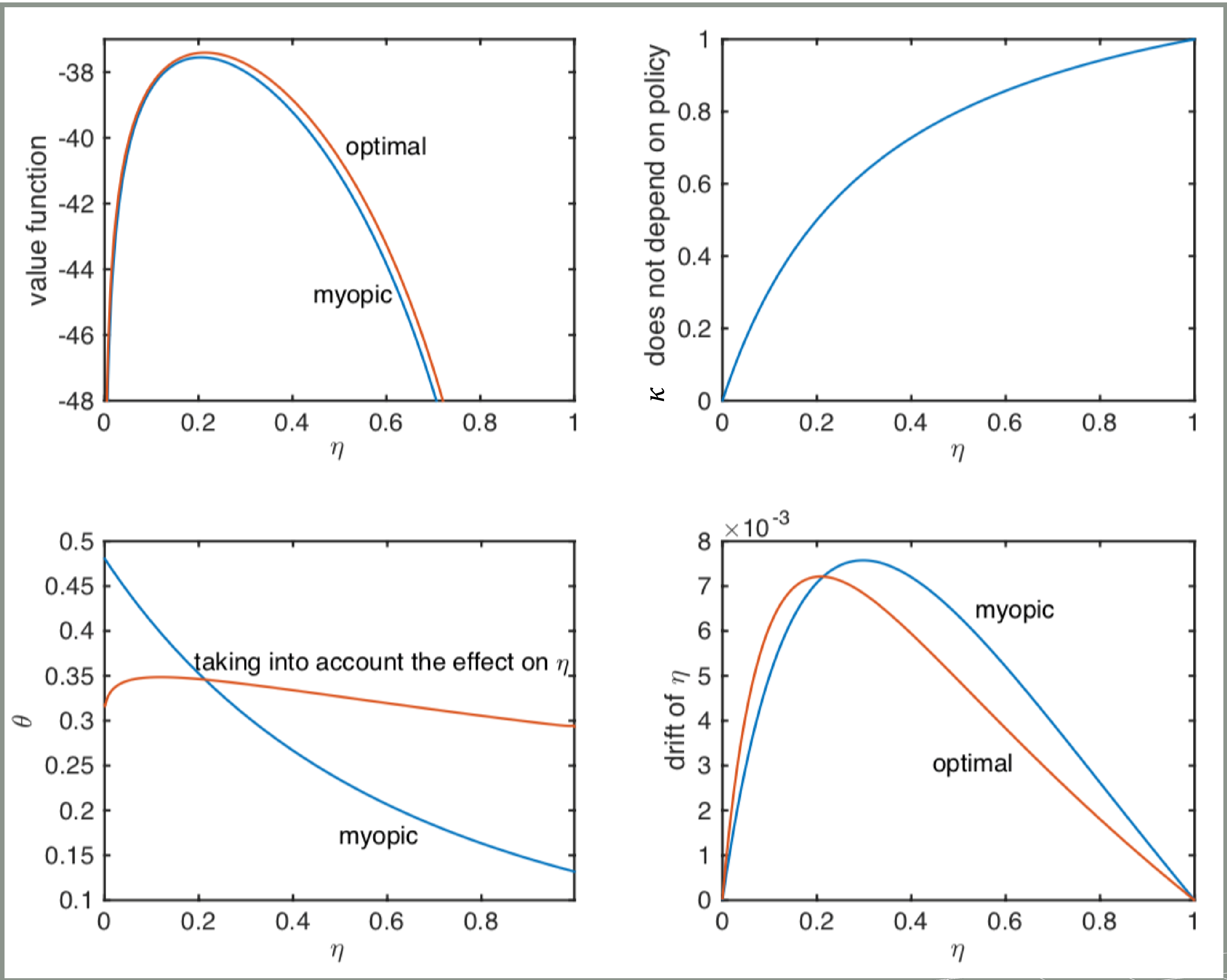
$$\max_{\vartheta} \frac{\log(1-\vartheta)}{\rho\phi} - \frac{\rho\phi+1}{\rho\phi}\log(\rho\phi+1-\vartheta) - \frac{(1-\vartheta_t)^2\tilde{\sigma}^2}{2\rho}\left(\lambda\frac{\kappa^2\varphi^2}{\eta^2} + (1-\lambda)\frac{(1-\kappa)^2}{(1-\eta)^2}\right)$$
$$+ V'(\eta)(1-\vartheta_t)^2\frac{\eta(1-\eta)\tilde{\sigma}^2\varphi^2(1-\varphi^2)}{(\varphi^2(1-\eta)+\eta)^2}$$

- ϑ affects the drift of η, it is optimal to choose

$$\vartheta^*\left(\tilde{\sigma}^2\left(\lambda\frac{\kappa^2\varphi^2}{\eta^2} + (1-\lambda)\frac{(1-\kappa)^2}{(1-\eta)^2}\right) - 2\rho V'(\eta)\frac{\eta(1-\eta)\tilde{\sigma}^2\varphi^2(1-\varphi^2)}{(\varphi^2(1-\eta)+\eta)^2}\right)$$

- Speed up η when $V' > 0$, slow down when $V' < 0$.

Example: using ϑ to push η



$\rho = .05, \phi = 2, \tilde{\sigma} = .3, \varphi = .5, \lambda = .2$

Optimal policy: (1) MoPo only

- Using MoPo ϑ to push η (to recapitalize banks via risk premia)
- Using screwdriver as hammer

Roadmap

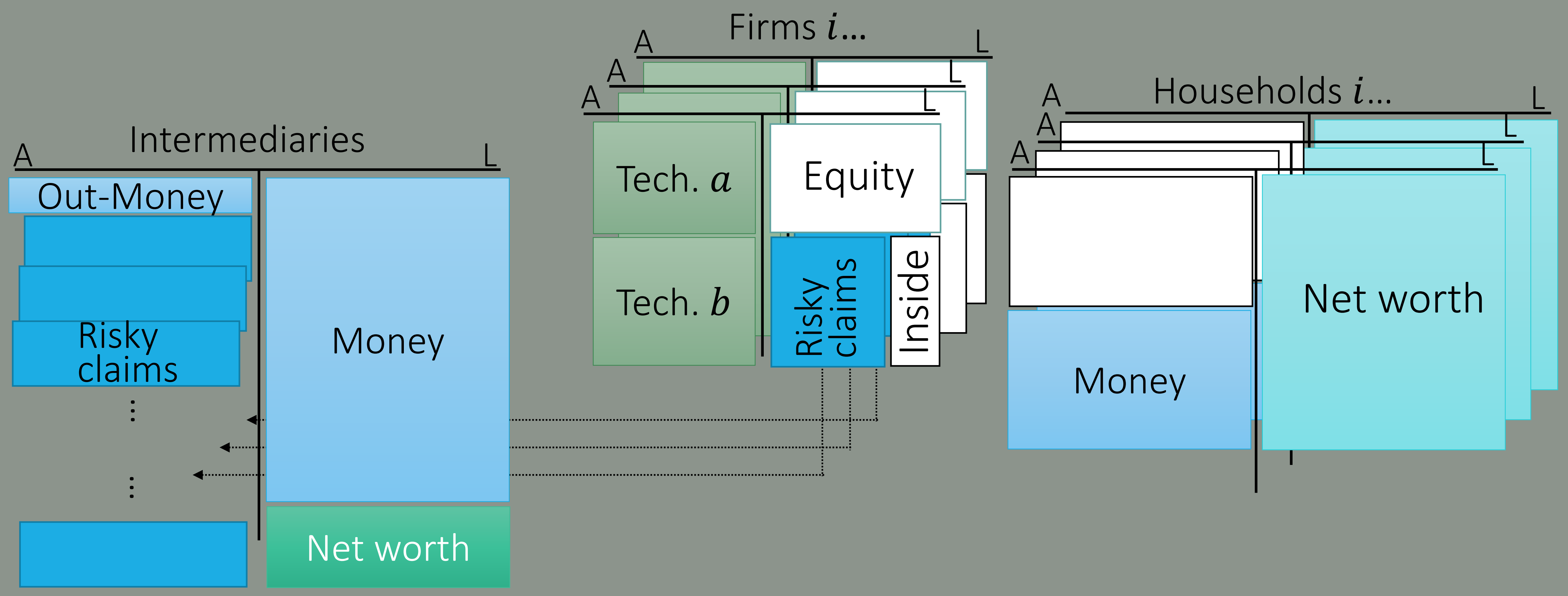
- Expected Utility/Value function with log-utility
- One sector model with stochastic idiosyncratic volatility
- Two sector model
  - With exogenously fixed net worth share η
  - With endogenous wealth share η
  - I theory (with two technologies)

I Theory of Money

- Aim: intermediary sector is not perfectly hedged
- Idiosyncratic risk that HH have to bear is time-varying
- Needed: Intermediaries' aggregate risk $\neq$ aggregate risk of economy
  - One way to model: 2 technologies a and b

Technology	a	b
Capital share (Leontieff)	$1-\bar{\kappa}$	$\bar{\kappa}$
Risk	$\frac{dk_t}{k_t} = (\cdot)dt + \sigma^a dZ_t + \tilde{\sigma} d\tilde{Z}_t$	$\frac{dk_t}{k_t} = (\cdot)dt + \sigma^b dZ_t + \tilde{\sigma} d\tilde{Z}_t$
Intermediaries	No	Yes, reduce to $\varphi\tilde{\sigma}$
Excess risk	$-\bar{\kappa}\sigma - \frac{\sigma^{\vartheta} - \sigma^M}{1-\vartheta}$	$(1-\bar{\kappa})\sigma - \frac{\sigma^{\vartheta} - \sigma^M}{1-\vartheta}$

I Theory: Balance Sheets



- Frictions:
  - Household cannot diversify idio risk
  - Limited risky claims issuance
  - Only nominal deposits

Model with Intermediaries – new policy

- Model Setup

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta)\,dt + \underbrace{\sigma\,dZ_t}_{\text{aggregate}} + \underbrace{\tilde{\sigma}\,d\tilde{Z}_t}_{\text{idiosyncratic}}$$

  - Intermediaries can hold equality share up to $\bar{\kappa}$
  - can diversify some idiosyncratic risk, reduce it to $\varphi\tilde{\sigma}$
  - Intermediaries' wealth share $\eta_t = N_t / \big((p_t + q_t)K_t\big)$
  - Welfare weights λ on intermediaries, $1 - \lambda$ on HH

Two policy settings:
(1) money growth rate μ_t^M only
(1) + (2) also choose allocation (macroprudential) and transfer wealth between group (why/how?)

Optimal policy: (1) MoPo + (2) MacroPru

- Same steps as above

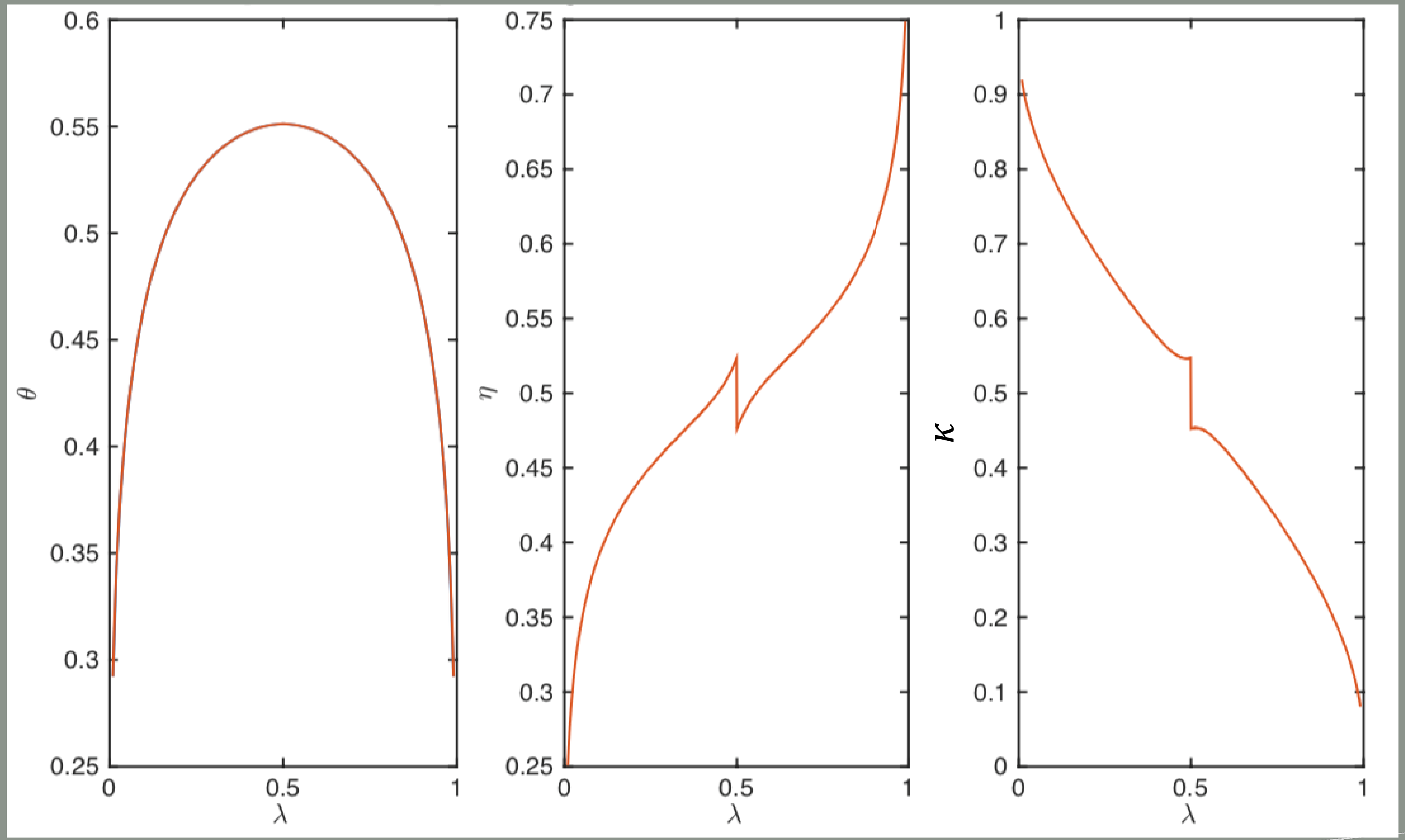
- Step 1: Optimal $\kappa = \min\left(\frac{(1-\lambda)\eta^2}{\lambda\varphi^2(1-\eta)^2+(1-\lambda)\eta^2}, \bar{\kappa}\right)$ given η
- Step 2: Optimal $\vartheta = \vartheta^*\Big(\underbrace{\tilde{\sigma}^2 \frac{\lambda(1-\lambda)\varphi^2}{\lambda\varphi^2(1-\eta)^2+(1-\lambda)\eta^2}}_{\text{welfare weighted average risk exposure}}\Big)$

- Step 3: Optimal η (given ϑ) as a function of Pareto weight λ

Optimal policy: (1) MoPo + (2) MacroPru

- Step 3: Optimal η (given ϑ) - let's look at terms containing η

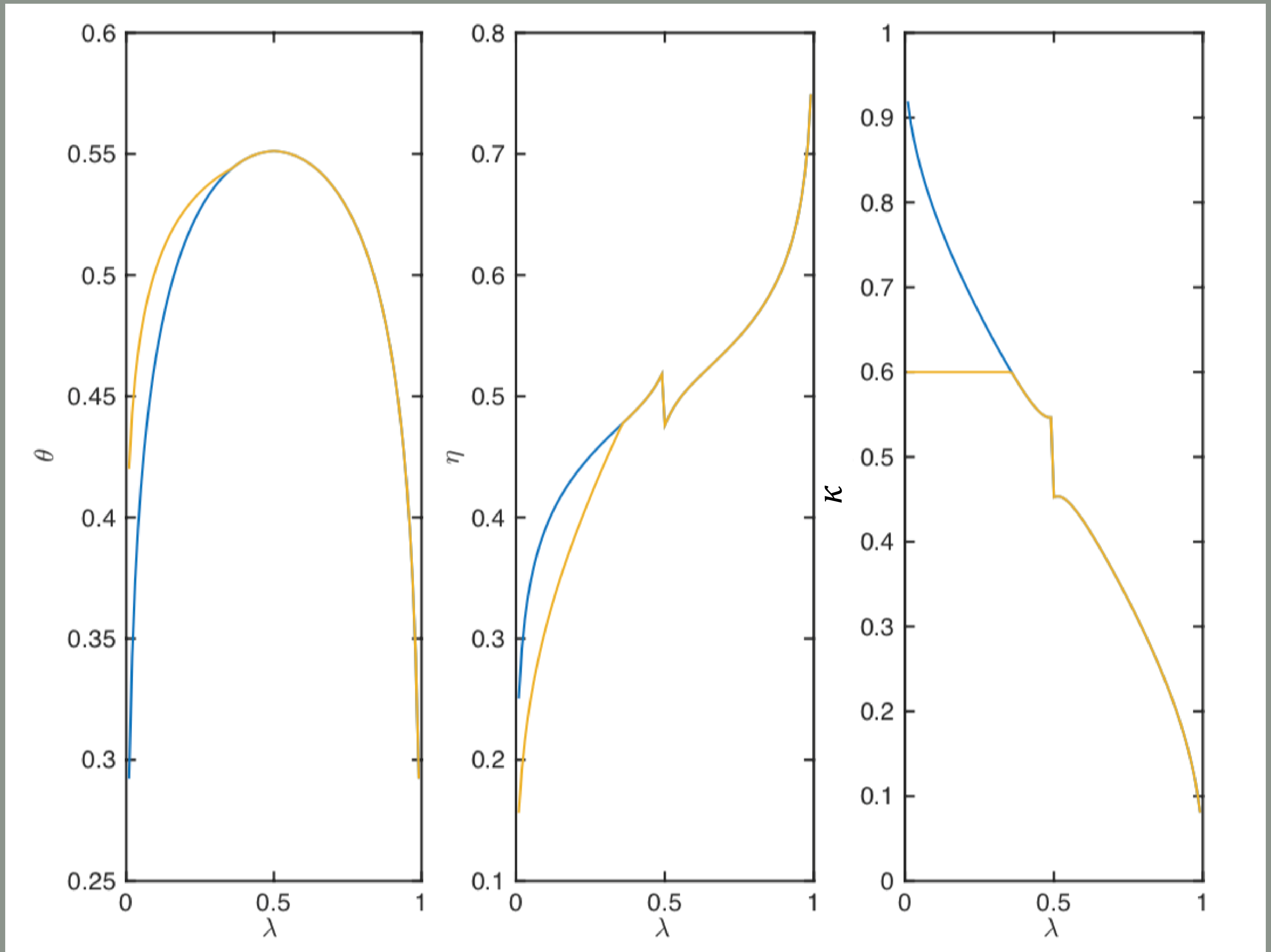
$$\max_{\eta} \underbrace{\lambda \log \eta_t + (1-\lambda)\log(1-\eta_t)}_{\text{concave, max at } \eta=\lambda, \text{ goes to } -\infty \text{ at } 0\ \&\ 1} - \frac{(1-\vartheta_t)^2\tilde{\sigma}^2}{2\rho}\frac{\lambda(1-\lambda)}{\lambda\varphi^2(1-\eta)^2+(1-\lambda)\eta^2}$$



For $\varphi = 1$, the optimal policy as a function of λ is

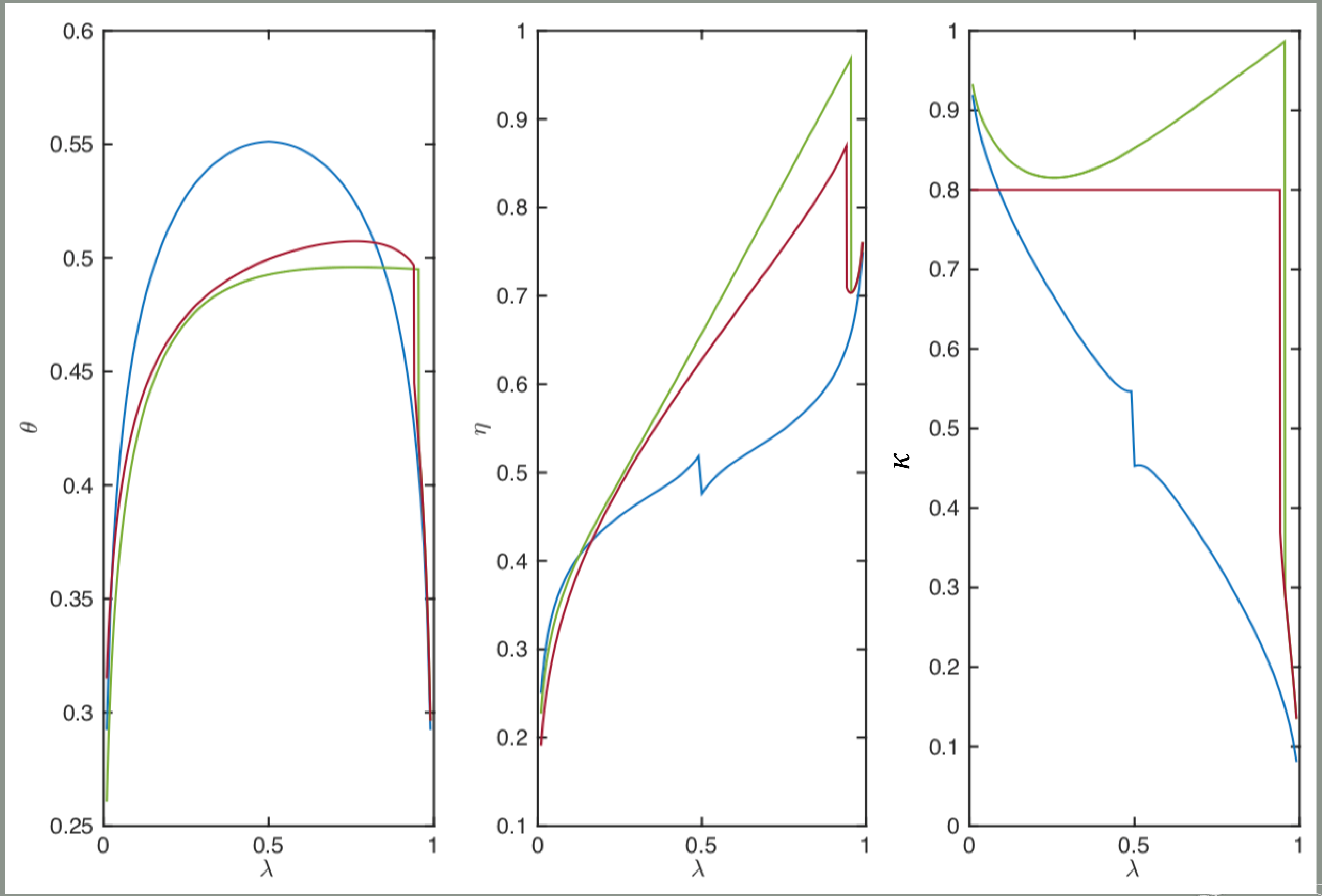
Optimal policy: (1) MoPo + (2) MacroPru

- For $\varphi = 1$, and $\bar{\kappa} = 0.6$ (intermediaries' risk taking is constrained)



Optimal policy: (1) MoPo + (2) MacroPru

- For $\varphi = 0.8, \bar{\kappa} = 1$, and $\varphi = 0.8, \bar{\kappa} = 0.8$
  Intermediaries given a lot more risk when they can diversify it



Optimal policy: (1) MoPo + (2) MacroPru

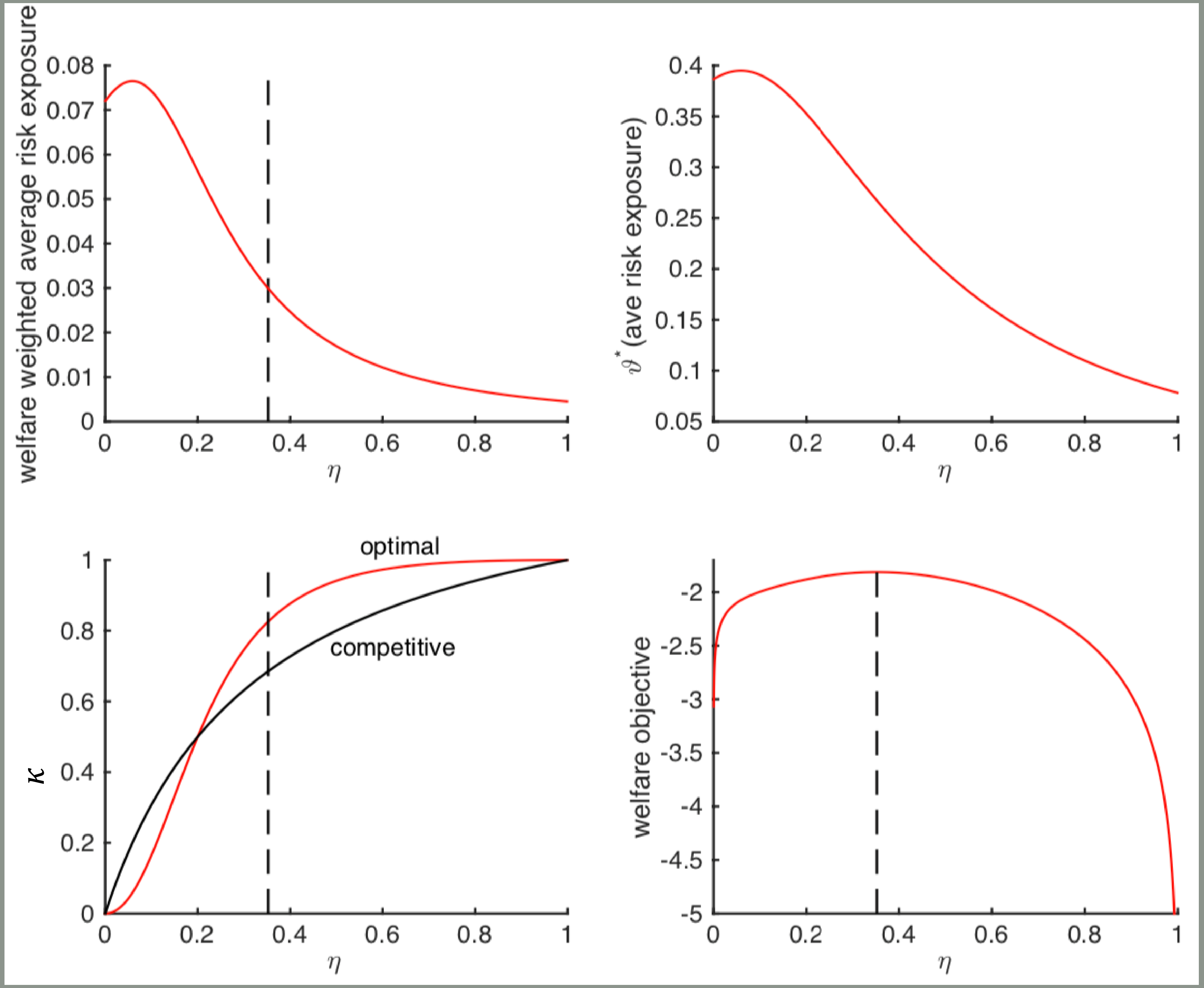
- Step 3: Optimal η (given ϑ) - let's look at terms containing η

- Same as above

- Given κ and η,
  $$\operatorname{optimax}_{\eta} \quad \underbrace{\lambda \log \eta_t + (1-\lambda)\log(1-\eta_t)}_{\text{concave, max at }\eta=\lambda\text{, goes to }-\infty\text{ at }0\ \&\ 1} - \frac{(1-\vartheta_t)^2\tilde{\sigma}^2}{2\rho} \underbrace{\frac{\lambda(1-\lambda)\varphi^2}{\lambda\varphi^2(1-\eta)^2+(1-\lambda)\eta^2}}_{\text{concave also, max at }\frac{\lambda\varphi^2}{\lambda\varphi^2+1-\lambda}<\lambda}$$

- Assuming FOC holds uniquely, it is optimal to set $\eta > \lambda$
- push more risk to intermediaries and they'd take under competitive outcome
- relative to previous infinite switching model
  - it is optimal to give intermediaries more wealth, because they are more efficient at absorbing risk
  - overall risk is reduced and the value of money is lower (more intermediation)

Optimizing over η

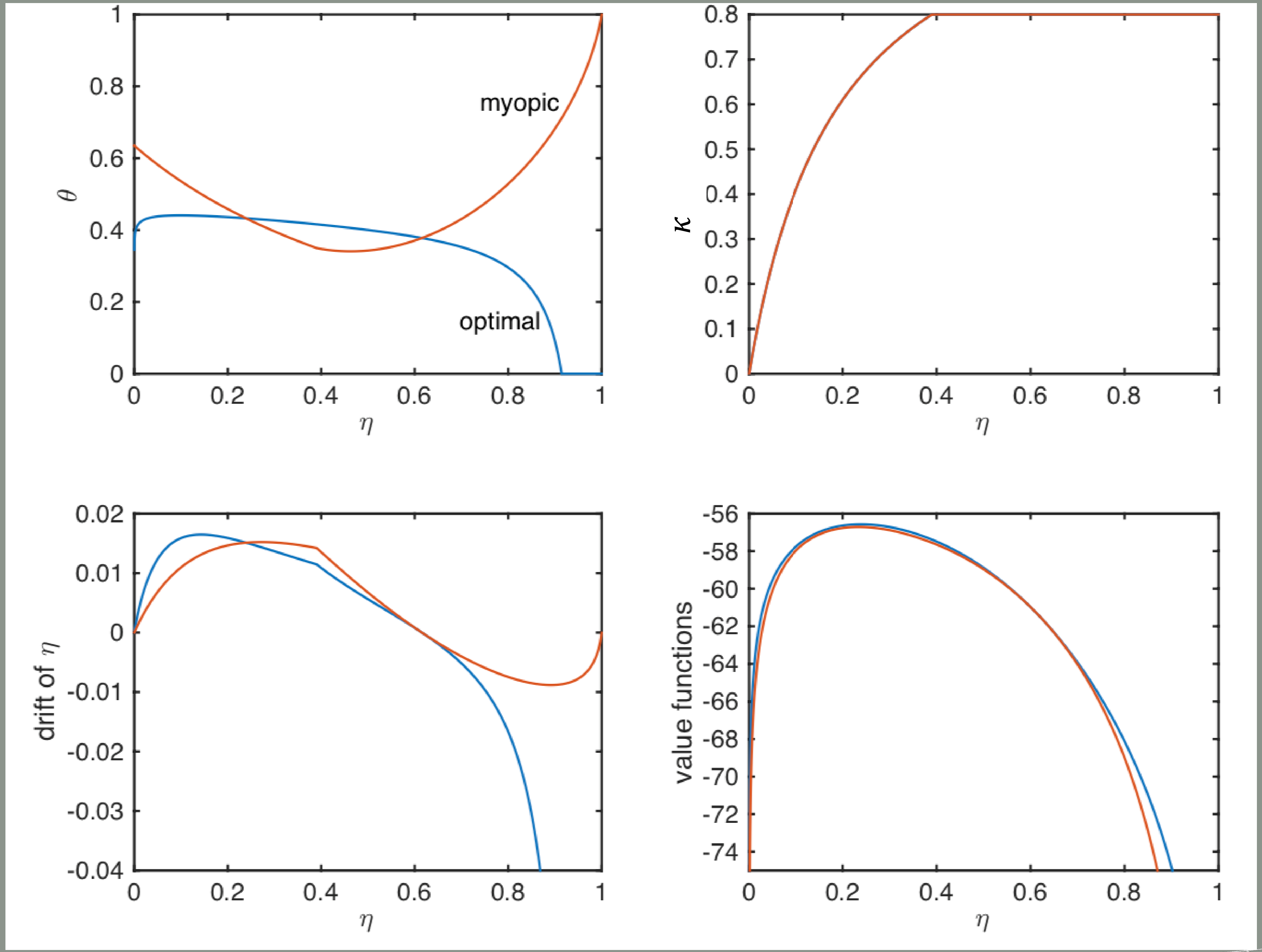


$\rho = .05, \phi = 2, \tilde{\sigma} = .3, \varphi = .5, \lambda = .2$

Optimal policy, (1) MoPo only

- Using ϑ to push η - Same analytical steps as before

$\rho = .05, \phi = 2, \tilde{\sigma} = .3, \varphi = .5, \lambda = .2$



Take-aways of Optimal Policy

- Baseline (one-sector) model
  - Trade-off: insurance vs. investment (growth)
- Multi-sector model
  - Allocation of risk/assets
- Money is not super-neutral
  - since it affect portfolio choice, risk allocation
  - Price of risk (risk premia), η-drift
- (1) MoPo + (2) MacroPru
  - Static problem – 3 steps maximization
    - Always $\vartheta^*(\cdot)$-function
- (1) MoPo only
  - Using screwdriver as hammer to push η

Thank you!

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