

# Macro, Money and (International) Finance – Problem Set 4

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Problem set prepared by Sebastian Merkel (smerkel@princeton.edu). Please let me know, if any tasks are unclear or you find mistakes in the problem descriptions.

**The submission deadline is Friday, October 9 (end of day Princeton time). Please submit your group's solution via email to Fernando Mendo (fmendolopez@gmail.com).**

## 1 (Sunspot) Price Fluctuations in the One-sector Money Model

Consider the money model of Lecture 5 with log utility and without policy ( $\mu^B = i = \sigma^B = \mathbf{g} = 0$ ). In this problem, we want to go beyond the steady-state solutions characterized in the lecture and explore the possibility of stochastic equilibria. Specifically, let us work with the assumption that price dynamics follow Ito processes,

$$dq_t^B = q_t^B \mu_t^{q^B} dt + q_t^B \sigma_t^{q^B} dZ_t, \quad dq_t^K = q_t^K \mu_t^{q^K} dt + q_t^K \sigma_t^{q^K} dZ_t. \quad (1)$$

While we have started out with the same assumptions in the lecture, we have always set the volatility loadings to zero at some point. The goal of this problem is not to characterize all equilibria, but to find some examples of stochastic equilibria. Assume throughout that  $\bar{\sigma} > \sqrt{\bar{\rho}}$ , so that money can exist as a pure bubble. If it helps your arguments in the following, you may also assume that  $\phi \rightarrow \infty$ , so that there is no  $\iota$  choice and the growth rate of capital becomes a constant (you can call it  $g$ ).

1. Define the concept of a (non-stationary) competitive equilibrium. Derive all equilibrium conditions that characterize a competitive equilibrium according to your definition, i.e. a set of stochastic processes is an equilibrium if and only if it satisfies these conditions.

*Hint:* one of these equations is a stochastic differential equation for  $\vartheta = \frac{q^B}{q^B + q^K}$ .

2. Explain why the conditions in part 1 do not uniquely pin down the coefficients in the equilibrium price evolution (1) (2-4 sentences).
3. Suppose capital price volatility is a function of the current price,  $\sigma_t^{q^K} = \sigma^{q^K}(q_t^K)$ , let  $\underline{q}^K$  and  $\bar{q}^K$  denote the two steady state values for  $q^K$  in the monetary and the moneyless equilibrium. (verify that these are indeed valid equilibria in the sense of your definition 1). Show that there is a choice of the  $\sigma^{q^K}$  function, such that  $\sigma^{q^K}(q^K) > 0$  for all  $q^K \in (\underline{q}^K, \bar{q}^K)$  and  $\sigma^{q^K}(q^K) = 0$  otherwise. Do this by giving a specific example of such a function and showing that it is possible to satisfy all

equilibrium conditions in part 1 and the process  $q^K$  so defined never leaves the set  $[\underline{q}^K, \bar{q}^K]$  with positive probability.

Note: you do not need to know and apply results about stochastic differential equations. When you cannot make an argument formally precise, argue based on your intuition (but try not to become too vague).

4. Choose model parameters, take your example function  $\sigma^{q^K}$  from part 3 and simulate some equilibrium paths for  $q^B$  and  $q^K$ . For the simulation, you can use a simple explicit Euler discretization. If you have an arbitrary Ito process

$$dX_t = a_t dt + b_t dZ_t,$$

this naturally leads to the discretization scheme<sup>1</sup>

$$X_{t+\Delta t} \approx X_t + a_t \Delta t + b_t (Z_{t+\Delta t} - Z_t). \quad (2)$$

We know from the basic properties of Brownian motion that  $Z_{t+\Delta t} - Z_t$  is normally distributed with mean 0 and variance  $\Delta t$ . Use this knowledge to draw shocks  $Z_{t+\Delta t} - Z_t$  to simulate the evolution described by (2).

5. Let  $\sigma^{q^K}$  be your volatility function from part 3, define for all positive integers  $m$  the volatility function  $\sigma^{q^K, m}(q^K) = m\sigma^{q^K}(q^K)$ . Show that for all  $m$  there is an equilibrium with price volatility  $\sigma_t^{q^K} = \sigma^{q, K}(q_t^K)$  (it is sufficient to argue why your argument from part 3 goes through in this case). Conclude that this model can generate arbitrarily large (asset) price volatility.

## 2 Bank Money Creation

Consider the money model of Lecture 5 with a cash-in-advance constraint in production as in Problem 3 of Problem Set 3. As a reminder, this model is like in Lecture 5, except that we write  $M$  (money) instead of  $B$  (bonds), government policy is absent ( $\sigma^M = \mu^M = i$ ) and households face an additional constraint

$$\alpha \cdot a \frac{\theta_t^{k, \tilde{i}} n_t^{\tilde{i}}}{q_t^K} \leq \theta_t^{m, \tilde{i}} n_t^{\tilde{i}}, \quad (3)$$

where  $\alpha > 0$  is a model parameter. Suppose that  $\tilde{\sigma} > \sqrt{\rho}$ , so that there is a monetary steady state equilibrium even if the cash-in-advance constraint is not binding ( $\alpha \rightarrow 0$ ). In this problem, we want to add banks to the model that can create inside money for households. We do this in a very simple way:

In addition to households, there is a continuum of bankers who have also logarithmic utility with identical time preference rate  $\rho$  as households. Bankers cannot invest into capital, but only into outside money (return  $dr_t^m$ ) and nominal debt claims issued by households (return  $dr_t^\ell$ ).<sup>2</sup> We can interpret these debt claims as bank loans made to households. For most of this question, bank loans are assumed to be nominally risk-free, but in the last part of this question we allow for idiosyncratic Poisson default

<sup>1</sup>This discretization scheme has the disadvantage that it can become negative even if  $X_t$  has theoretical properties that ensure that it always remains positive. While there are other ways to deal with this, you can simply discard simulated paths that lead to values that are theoretically impossible and simulate instead new ones.

<sup>2</sup>This means that this model is very similar to a monetary version of the Basak-Cuoco limited participation model from Lecture 2, only that here the bankers are the non-experts and households take on the role of the experts.

risk. Simultaneously, bankers can issue nominally risk-free debt claims in the form of bank deposits to households (return  $dr_t^d$ ).

Assume that bank deposits can be used for transactions so that the new cash-in-advance constraint is now

$$\alpha \cdot a \frac{\theta_t^{k,h,\tilde{i}} n_t^{h,\tilde{i}}}{q_t^K} \leq \left( \theta_t^{m,h,\tilde{i}} + \theta_t^{d,h,\tilde{i}} \right) n_t^{h,\tilde{i}} \quad (4)$$

instead of inequality (3). Here,  $\theta_t^{k,h,\tilde{i}}$ ,  $\theta_t^{m,h,\tilde{i}}$  and  $\theta_t^{d,h,\tilde{i}}$  are the fractions of wealth that household  $\tilde{i}$  invests into capital, money and deposits, respectively.

1. Translate the verbal description above into a formal one. Specifically:
  - (a) Write down the return processes for money ( $dr_t^m$ ), bank loans ( $dr_t^\ell$ ) and deposits ( $dr_t^d$ )
  - (b) Write down the net worth evolution for bankers and households
  - (c) Specify the decision problems for both agent types
2. Suppose that there is no monetary friction and bank loans are nominally risk-free (no default risk).
  - (a) Show that the gross quantity of deposits and bank loans are not uniquely determined in equilibrium, but their difference always is.
  - (b) Derive the money valuation equation and the evolution of the state variable  $\eta$ .
  - (c) Show that the system drifts over time to a steady state at  $\eta = 0$  and that there, model aggregates and asset prices are as in the Lecture 5 model (without bankers).
3. Suppose bank loans are nominally risk-free and households face a cash-in-advance constraint (4).
  - (a) Show that for any state  $\eta$ , bankers simultaneously make loans and issue deposits in order to create sufficient inside money that the monetary friction is not binding.
  - (b) Determine the minimum size of bank balance sheets and bank leverage for all  $\eta$ .
  - (c) Show that the allocation and all prices are otherwise as in part 2
4. We arguably do not see that banks produce arbitrary amounts of inside money to satisfy transaction demands. One reason may be that in most economies, banks cannot create an infinite amount of money because there are reserve requirements. So let's impose such a requirement on banks. Bankers must keep  $\psi \in (0, 1]$  units of outside money for each unit of deposit they issue to households.
  - (a) When does the reserve requirement have an effect in equilibrium? Derive a condition on model parameters for this to be the case.
  - (b) Determine the steady-state values for  $\eta$  and  $\vartheta$ .
  - (c) Show that under the condition derived in part (a), there is a constant money multiplier of  $1/\psi$ :  $\left( \theta_t^{m,h} + \theta_t^{d,h} \right) N_t^h = \frac{1}{\psi} q_t^M K_t$
5. Another reason why banks do not create arbitrary amounts of inside money is because loans are not risk free. Let's assume that there are no reserve requirements, but instead loans have default risk. For the sake of simplicity, we introduce default in a stylized way that is not very realistic.

Suppose that each household receives infrequent opportunities to walk away from its debt that arrive at jump times of an idiosyncratic Poisson process  $d\tilde{J}_t$  with intensity  $\tilde{\lambda} > 0$ . When such an opportunity arrives, the household's debt is simply reduced by a fixed fraction  $1 - \beta$  without any adverse consequences for the household going forward (in particular, the household can take out new bank loans after a default). As a consequence, the banker making the loan only recovers a fraction  $\beta$  from that loan. We assume further, that individual bankers hold concentrated loan portfolios in a way that prevents diversification across different  $d\tilde{J}_t$ , so that whenever one bank client receives a  $d\tilde{J}_t$  shock, all of them do and the banker loses a fraction  $1 - \beta$  on all loans simultaneously.<sup>3</sup>

- (a) Derive the money valuation equation and  $\eta$  evolution for this augmented model.
- (b) Show that there is an interior steady state  $\eta > 0$  and find algebraic equations that characterize it (most likely, you won't be able to solve them in closed form).
- (c) Solve for the steady state numerically and show how it changes as a function of  $\beta$  (plot  $\eta$ , the money multiplier,  $q^K$  and  $q^M$ ).

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<sup>3</sup>Formally, we could assume that there is a continuum of locations,  $d\tilde{J}$ -shocks are location-specific and each banker can only make loans locally to households. Making this idea formally explicit is not hard, but technically slightly involved. Special care has to be taken to make sure that all local markets clear at the same prices, e.g. by allowing households or bankers (or both) to migrate across locations.