

# Blockchain Economics

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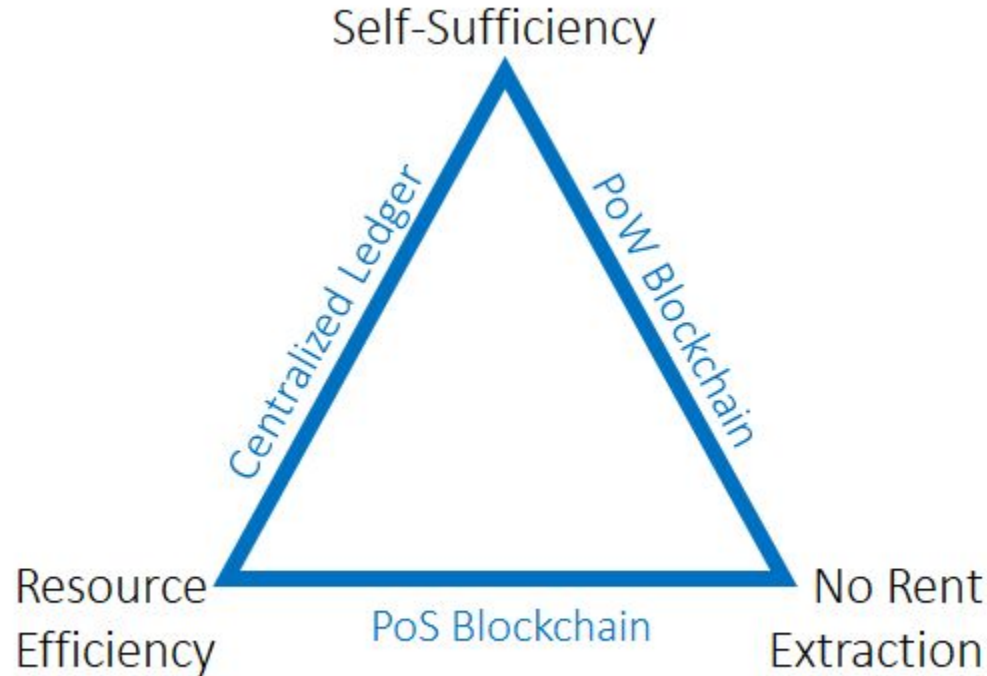
# How can we generate consensus?

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- Fundamental problem of record-keeping: Create trusted *ledger*
- What are the assumptions required to operate a trusted ledger?
  - Centralized ledger: **Rents**
  - PoS blockchain: **External trust**
  - PoW blockchain: **Resource costs**
- What are the **tradeoffs** and **constraints** in record-keeping?
  - When is PoW necessary?
  - How is PoS trust different from centralized trust?
  - Does the desired mechanism imply an optimal consensus algorithm?

# Blockchain Trilemma

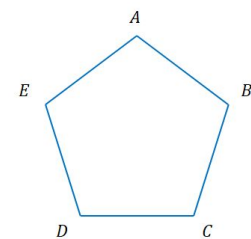
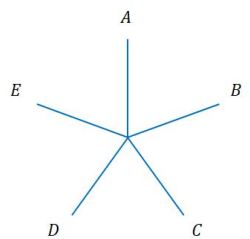
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# Self-sufficiency and external trust

- External trust: Capacity to **punish** other agents
  - a. Mutually beneficial relationships
    - Business relationships (news media, non-ledger related business)
    - Social connections (friends, colleagues)
    - Elected officials
  - b. Legal enforcement relationships
- Tradeoff: Lose social trust ⇒ System collapses
- Different from traditional centralized trust model! **Local** trust can be **scaled globally**

Centralized trust



Scaled local trust

# Summary of Trilemma

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- Economic reasoning behind trilemma?
  - Three ways of distorting consensus
    - i. Digital signatures (lose **rents**)
    - ii. Social messages (lose **external trust**)
    - iii. PoW (pay **resource cost**)
  
- Guiding framework about optimal record-keeping system
  - Small rent distortions ⇒ **Centralized/Permissioned**
  - Robust external trust ⇒ **PoS, Ripple**
  - No external trust + large rent distortions ⇒ **PoW**



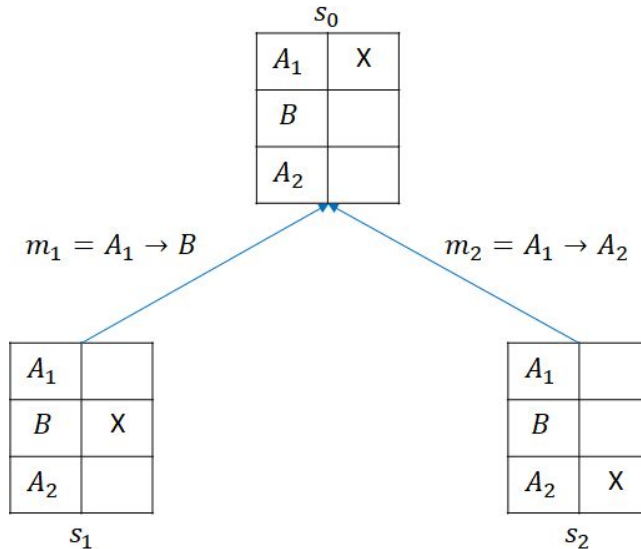
# Roadmap

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- Challenge of digital record-keeping
- Key model ingredients
- Benchmark example
  - a. Centralized
  - b. PoS blockchain
  - c. PoW blockchain
- Proof idea

# Challenge of digital record-keeping

- Key issue: No scarcity of digital “assets”
  - Unlike physical tokens
  - Ordering of messages matters



# Solution: Consensus algorithm

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- Three types: Differ in **info. requirements** to determine state
  - **Objective**: Set of messages sufficient for all users to achieve consensus
    - E.g. **PoW** “longest chain rule”
  - **Weakly subjective**: Set of messages + recent past state needed
    - Attacker votes twice ⇒ “Checkpoint” might be necessary
    - E.g. **PoS** “supermajority rule”
  - **Subjective**: Different users can come to different conclusions
    - E.g. **Centralized system, Ripple**
- Consensus guaranteed by **incentive schemes**
  - **Objective**: Cost of participation
  - **Weakly subjective**: Short-run punishments + Long-run reputation
  - **Subjective**: Long-run reputation





# Roadmap

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# Model: Users and mechanism

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- **Users:** (Large number  $N$ )
  - External **trust relationships** between users  $i, j \Rightarrow$  Bilateral utilities  $u_{ij}$ 
    - Underlying graph  $G$  of social connections
  - Users may pay **physical cost**  $\kappa W$  to produce  $w$  units of PoW
  - Two types of communication: Social messages + (pseudonymous) digital messages
- **Mechanism:**
  - State  $s$  summarized by token holdings in pseudonymously-owned addresses
  - Mechanism  $\mathcal{M}$  specifies actions  $a_i(s)$  as a function of state, address ownership
    - Implicitly defines **rents**  $r_{ij}$  extracted by user  $i$  when  $j$  is present
  - Utility of user  $i$ :

$$U_i = \underbrace{V_i(s)}_{\text{Tokens}} + \underbrace{\sum_j r_{ij}}_{\text{Rents}} + \underbrace{\sum_j u_{ij}}_{\text{Social trust}} - \underbrace{\kappa W_i(s)}_{\text{Exp. PoW}}$$

# Model: Blocks and record-keeping

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- **State  $s$ :** Allocation of tokens to addresses
  - Purpose of blockchain: Generate consensus on current state
- **Token transfer messages:** Message  $(n, n', q)$  transfers  $q$  tokens from  $n$  to  $n'$ 
  - Also incorporate seignorage/block rewards
- **Votes:** Arbitrary collection of messages  $\mathcal{V}$  used to update state
  - Two types of permissions:
    - **Digital signatures:** E.g. **PoS**: Fraction of validators who sign a checkpoint
    - **External Proof:** E.g. **PoW**: Expected quantity of work required
- **Blocks:** Tuple  $b = (m, v, p)$ 
  - $m$  Token transfer messages,
  - $v$  Votes cast on block
  - $p$  Pointer to previous block



# Model: Consensus

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- **Block tree:** Partially ordered set  $B$  of blocks
  - Ordering induced by block pointers  $p$
  - Blockchain: Ordered subset  $C \subset B$
- **Consensus algorithm:** Update consensus chain given previous consensus  $C_t^*$ , blocks  $B_{t+1}$ 
  - Function  $C_{t+1}^* = g(C_t^*, B_{t+1})$
  - Previous state may be needed to determine consensus chain
- **Fundamental problem:** Desire to distort consensus
  - Three ways of distorting consensus  $\Rightarrow$  Three types of costs

$$U_i = V_i(s) + r_i + u_i - \kappa W_i(s)$$

$$\Rightarrow \Delta V_i \leq \Delta r_i + \Delta u_i - \kappa \Delta W_i$$



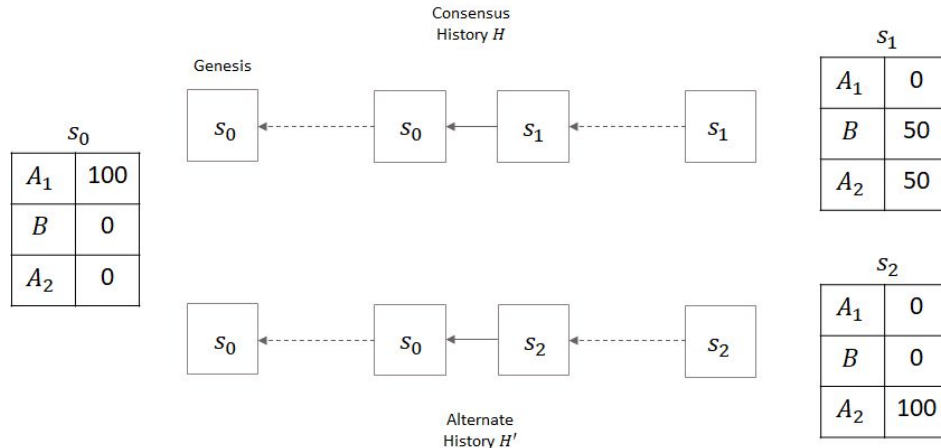
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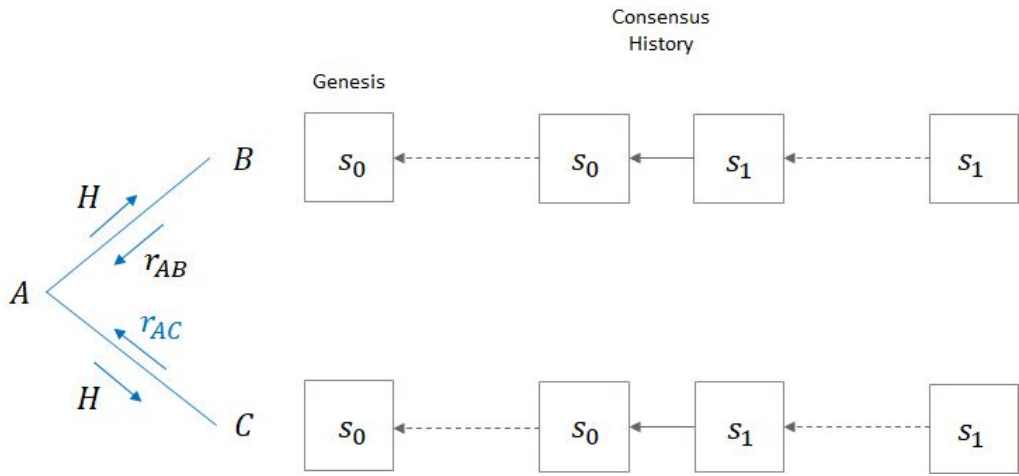
# Benchmark example

- Consensus history ( $H$ ): A sent 50 tokens to B and 50 tokens to own account
- Alternate history ( $H'$ ): A sent tokens to own account only
  - Can A convince a new user C of the alternate history?
  - Can A generate consensus on alternate history?



# Example: Centralized ledger

- Monopolist A communicates history to users (**subjective**)
  - Old user B: Knows state transitioned from  $s_0$  to  $s_1$
  - New user C: Can be fooled by fraudulent report

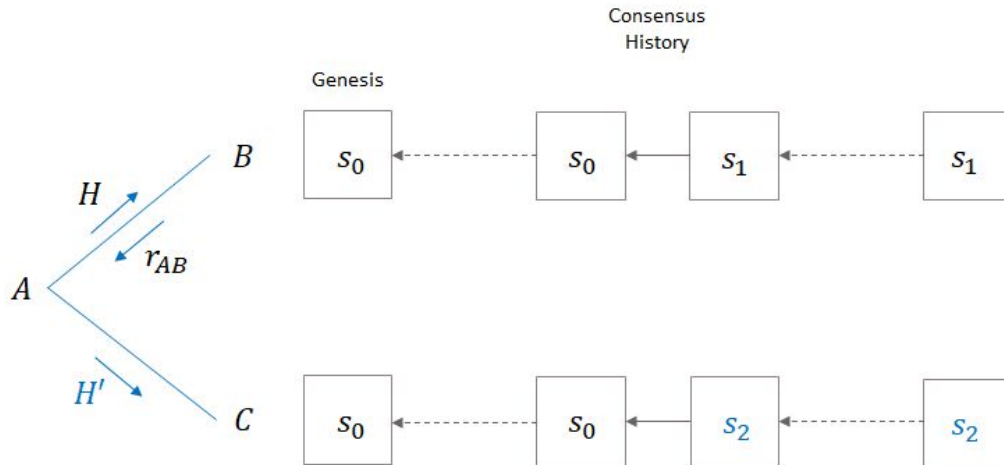


Honest reporting: A extracts rents from B and C

$$U_A = V(s_1) + r_{AB} + r_{AC}$$

# Example: Centralized ledger

- Dishonest reporting: Send **entirely different ledger** to C
  - C is fooled by A initially but stops using the system afterwards



Dishonest reporting: Get value from attack, lose rents from C

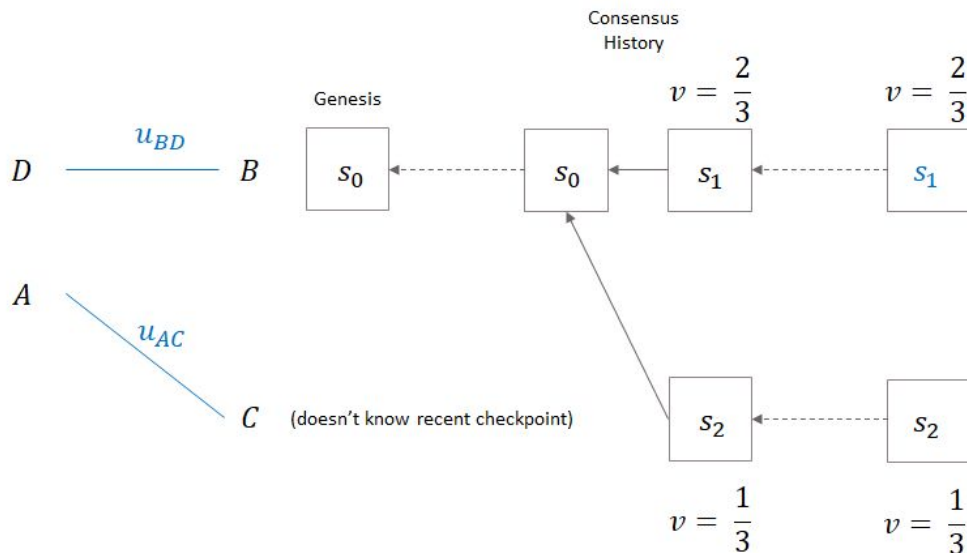
$$U_A = V(s_1) + r_{AB} + V_A$$

$$\Rightarrow \text{IC: } V_A \leq r_{AC}$$



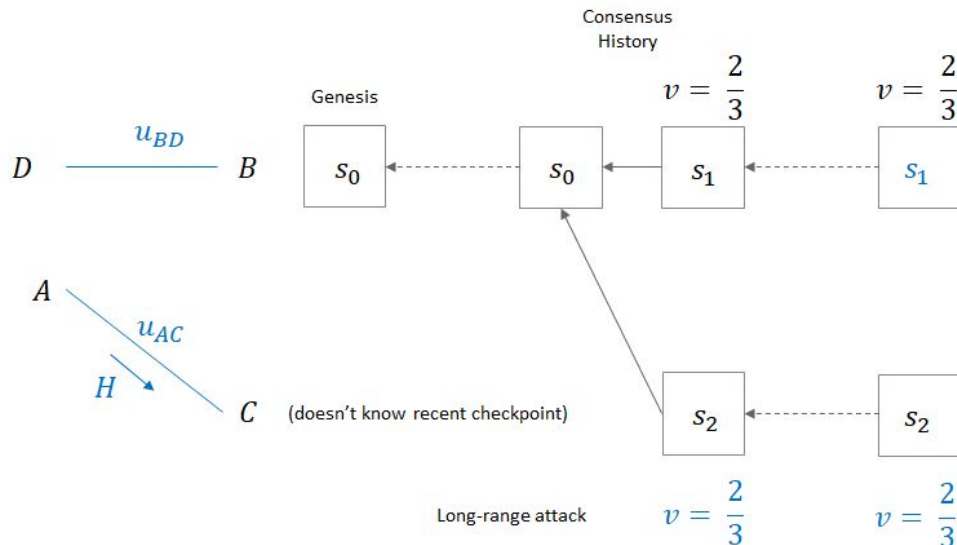
# Example: PoS blockchain

- PoS consensus algorithm: Supermajority rule (**weakly subjective**)
  - Old user *B*: Knows state transitioned from  $s_0$  to  $s_1$
  - New user *C*: Concludes state is  $s_1$  by supermajority rule



# Example: PoS blockchain

- PoS consensus algorithm: Supermajority rule (**weakly subjective**)
  - Old user *B*: Knows state transitioned from  $s_0$  to  $s_1$
  - New user *C*: Needs input from trusted connection *A*

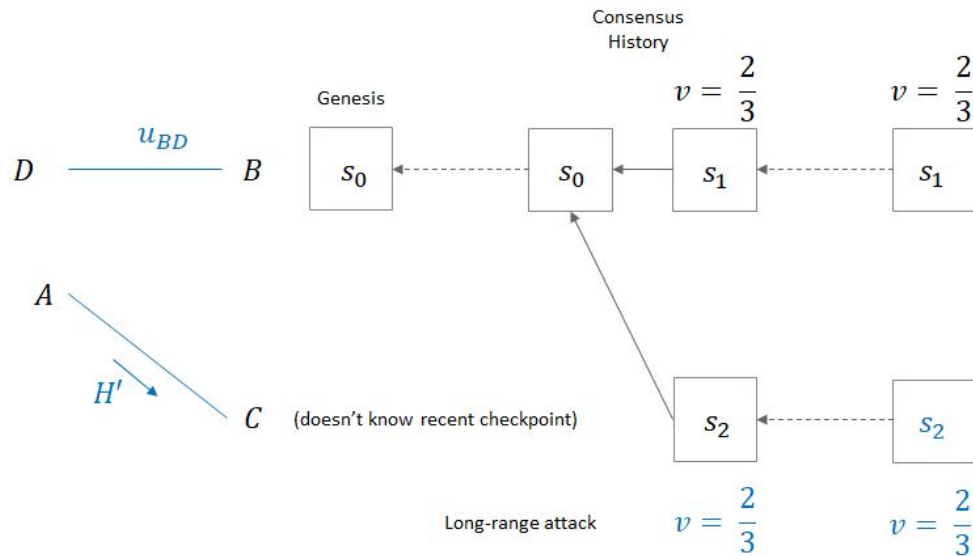


Honest reporting: *A* benefits from trust relationship with *C*

$$U_A = V(s_1) + u_{AB} + u_{AC}$$

# Example: PoS blockchain

- PoS consensus algorithm: Supermajority rule (**weakly subjective**)
  - Old user *B*: Knows state transitioned from  $s_0$  to  $s_1$
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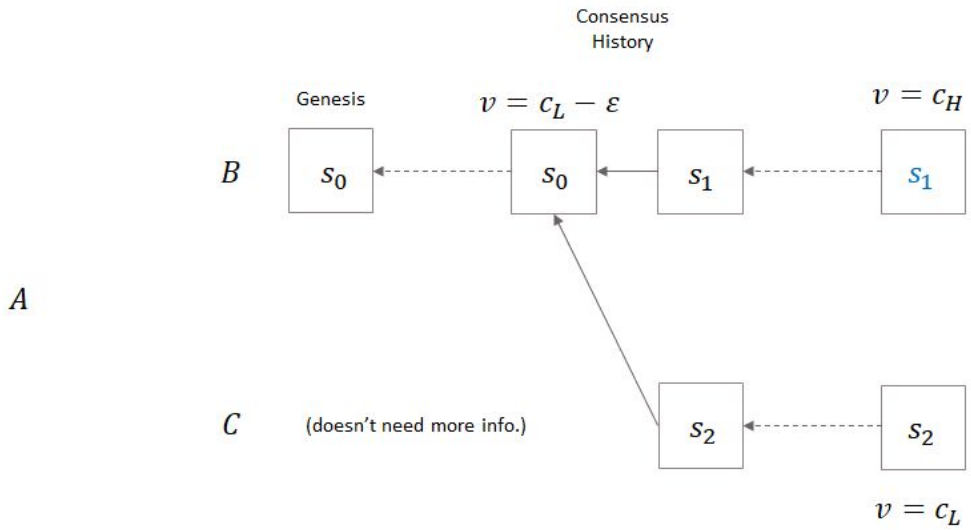
Long-range attack: *A* benefits from attack, loses trust with *C*

$$U_A = V(s_1) + V_A$$

$$\Rightarrow \text{IC: } V_A \leq u_{AC}$$

# Example: PoW blockchain

- PoW consensus algorithm: Longest chain rule (**objective**)
  - Any user (old or new) can determine current state

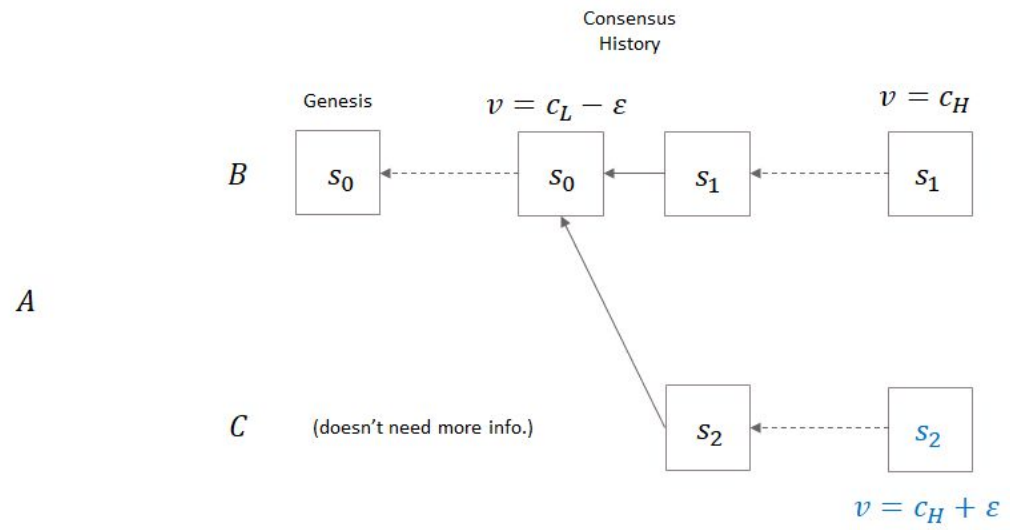


Honest mining: Consensus is  $s_1$

$$U_A = V(s_1)$$

# Example: PoW blockchain

- PoW consensus algorithm: Longest chain rule (**objective**)
  - Any user (old or new) can determine current state



Double-spend: Consensus is  $s_2$ , pay physical resource cost

$$U_A = V(s_2) - (c_H - c_L)$$

$$\Rightarrow \text{IC: } V_A = V(s_2) - V(s_1) \leq c_H - c_L$$



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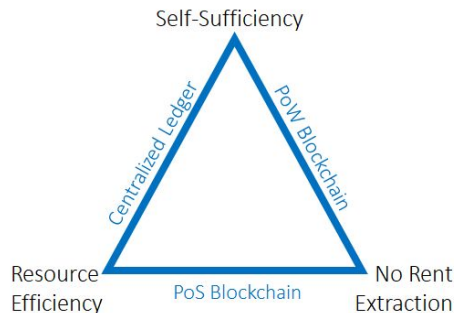
# Statement of Blockchain Trilemma

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- In order to achieve consensus in equilibrium, it must be that for any attacking coalition,

$$V_A \leq \underbrace{r}_{\text{rents}} + \underbrace{u}_{\text{ext. trust}} + \underbrace{c}_{\text{resource cost}}$$

- Impossible to have all three properties:



# Statement of Blockchain Trilemma

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- In order to achieve consensus in equilibrium, it must be that for any attacking coalition,

$$V_A \leq \underbrace{r}_{\text{rents}} + \underbrace{u}_{\text{ext. trust}} + \underbrace{c}_{\text{resource cost}}$$

- Depends on features of **mechanism**, **external environment**, and **consensus algorithm**
  - Rents/value of attack: Features of mechanism
  - External trust: Feature of environment
  - Resource cost: Feature of consensus algorithm





# Proof sketch: Mimicking Lemma

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- **Always possible to present new user with a cryptographically valid alternate history**
  - Centralized system: Give new user entirely different ledger
  - PoS blockchain: Long-range attack
  - PoW blockchain: Standard double-spend
- **Extends to arbitrary hybrid consensus algorithms**
  - Social messages + digital signatures + PoW are sufficient to create valid ledger
  - Who can attack?
    - Depends on writing permissions/possibilities for collusion
  - How much does it cost to attack?
    - Digital signature: Ex-post loss of **rents**
    - Social message: Ex-post loss of **external trust**
    - PoW: Ex-ante **resource cost**

# Possession vs. Ownership: Enforcement

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- Blockchain as a ledger for all kinds of assets– not just cryptocurrencies
- Who will enforce the ledger?



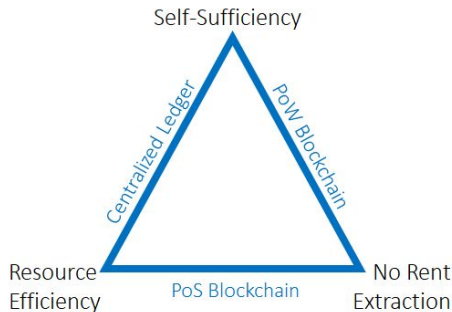
*You see, in this world,  
there are two types of  
people, my friend– those  
with loaded guns, and  
those who dig. You dig.*

- So far: Ignored distinction between **ownership** and **possession**
  - Ownership is traded in a market
  - Possession is conferred by previous possessor and must be **enforced**
    - E.g. Owning a house with squatters inside
- Cryptocurrency is special: No need to enforce any agreements

# Conclusion

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- Blockchain Trilemma:



- Guiding framework to answer questions about how records should be kept
  - What security assumptions underlie different models of record-keeping?
  - Local external trust: Globally scalable with blockchain
- Ownership vs. possession: Record-keeping is useful only if there's enforcement