Macro, Money and (International) Finance – Problem Set 0

January 29, 2019

Problem set prepared by Sebastian Merkel (smerkel@princeton.edu). Please let me know, if any tasks are unclear or you find mistakes in the problem descriptions. Questions about how to approach the problems are best directed to your local course TA.

1 Stochastic Calculus Review

1. Read pages 1–12 (up to "The Martingale Representation Theorem") and pages 15 and 16 (section "Kolmogorov Forward Equations") of the "Overview of Stochastic Calculus" by Yuliy Sannikov.

   Optional: read the sections on the martingale representation theorem, backward stochastic differential equations and the stochastic maximum principle.

2. Answer questions 1–3,5–7,9,17.

   Optional: answer questions 13–14,18.

2 Additional Questions

1. Derive the HJB equation for an abstract dynamic decision problem as in equations (9) and (10) in Yuliy's notes, but now suppose there is a two-dimensional state process: g depends on the arguments (X_t, Y_t, A_t) and X and Y evolve according to

$$dX_t = \mu_X(X_t, Y_t, A_t)dt + \sigma_X(X_t, Y_t, A_t)dZ_t,$$
$$dY_t = \mu_Y(X_t, Y_t, A_t)dt + \sigma_Y(X_t, Y_t, A_t)dZ_t.$$

2. Write down the HJB equation and derive the first-order conditions for consumption c and the risky asset portfolio share x for the optimal consumption and portfolio choice problem on pages 11–12, if logarithmic utility in the objective is replaced by CRRA utility with risk aversion coefficient $\gamma \neq 1$.