

Optimal Monetary and Macroprudential Policy

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Problem. Consider one of the models from the lecture (model number 3). There are two types of agents: households and intermediaries. Capital carries the idiosyncratic risk of $\tilde{\sigma}$. Households have to bear the full idiosyncratic risk when they hold capital, while intermediaries can diversify some of the risk and hold only $\phi\tilde{\sigma}$, $\phi \in (0, 1)$. Aggregate risk is $\sigma = 0$. Capital produces the same output of a per unit, regardless of whether it is held by households or intermediaries. If agents invest ι_t per unit of capital, capital growth rate is $\Phi(\iota_t) - \delta = \log(\kappa\iota_t + 1)/\kappa - \delta$.

As in the lecture, assume that the policy maker can control the portfolio weight on money ϑ_t through the choice of the money growth rate $\hat{\mu}_t^M$. Assume that the money growth rate does not have any volatility component. Recall that if money growth rate is the only policy tool, then the fraction of capital that intermediaries hold ψ_t satisfies

$$\frac{\psi}{\eta}\phi^2\tilde{\sigma}^2 = \frac{1-\psi}{1-\eta}\tilde{\sigma}^2 \quad \Rightarrow \quad \psi = \frac{\eta}{\eta + \phi^2(1-\eta)}. \quad (1)$$

Now, suppose that the planner has another policy tool. The planner can impose a constraint (upper bound) on the amount of capital intermediaries or households can hold, and push ψ below the level (1) by constraining intermediaries or above that level by constraining households. Without policy intervention, condition (1) equates the idiosyncratic risk premium of households and intermediaries. If the constraint is imposed on the intermediaries, then the households determine the idiosyncratic risk premium that all agents earn on capital. If instead the constraint is imposed on households, intermediaries determine the risk premium of capital.

(a) For a policy that pushes capital allocation to level ψ_t by placing an upper bound on the amount of capital that one type of agents can hold, and which sets the world portfolio weight on money to ϑ_t , derive the law of motion of the wealth share of intermediaries η_t .

The planner faces a control problem with controls ϑ and ψ , state variable η with law of motion you found in part (a), and payoff flow of

$$\lambda \log \eta + (1 - \lambda) \log(1 - \eta) + \frac{\log(1 - \vartheta)}{\rho\kappa} - \frac{\rho\kappa + 1}{\rho\kappa} \log(\rho\kappa + 1 - \vartheta)$$

$$-\frac{(1-\vartheta)^2\tilde{\sigma}^2}{2\rho}\left(\lambda\frac{\psi^2}{\eta^2}\phi^2+(1-\lambda)\frac{(1-\psi)^2}{(1-\eta)^2}\right). \quad (2)$$

Recall that $\vartheta^*(\hat{\sigma}^2)$ is defined as a function that maximizes

$$\frac{\log(1-\vartheta)}{\rho\kappa}-\frac{\rho\kappa+1}{\rho\kappa}\log(\rho\kappa+1-\vartheta)-\frac{(1-\vartheta)^2\hat{\sigma}^2}{2\rho}.$$

(b) Write down the HJB equation for the planner's problem

(c) Given $V'(\eta)$ and the policy choice of ψ , what is the optimal choice of ϑ ? Use the function $\vartheta^*(\cdot)$ to express your answer.

Next, consider maximization with respect to ψ , which the planner can choose freely. To characterize some elements of the optimal policy, assume that $V(\eta)$ is a concave function, and denote by η^* the point at which $V(\eta)$ achieves its maximum.

(d) Characterize the optimal choice of ψ as a function of η and $V'(\eta)$.

(e) Find the level $\psi^*(\eta)$ that maximizes payoff flow (2). Show that the optimal choice of ψ at η^* is given by $\psi^*(\eta^*)$.

(f) Show that the planner will always constrain one of the two agent groups when $V'(\eta) > 0$.

(g) Show that it is never optimal for the planner to give all capital to households when $V'(\eta) \geq 0$ (i.e. $\eta \leq \eta^*$). Show that it is never optimal to give all capital to intermediaries when $V'(\eta) \leq 0$ (i.e. $\eta \geq \eta^*$).