Macro, Money and (International) Finance – Problem Set 4

March 5, 2019

Problem set prepared by Sebastian Merkel (smerkel@princeton.edu). Please let me know, if any tasks are unclear or you find mistakes in the problem descriptions. Questions about how to approach the problems are best directed to your local course TA.

Please submit to your local TA/coordinator by Tueday, March 12, before the lecture. Do not submit your solutions to me.

1 A Simple I Theory of Money

Consider an economy with two types of agents, households and bankers. Households are exactly as in the model of lecture 5 ("I theory without I"), i.e. they can produce output goods using physical capital, which is subject to aggregate (quality) shocks σdZ as well as agent-specific idiosyncratic shocks $\tilde{\sigma} d\tilde{Z}$. In addition to capital, households can hold outside money or nominal deposits at banks.

Bankers cannot operate physical capital directly, but instead are able to invest outside equity into the capital operations of multiple households and are thereby able to diversify a part of the households' idiosyncratic risk. Specifically, a unit of capital operated by household $\tilde{i}$ earns a return of

$$dr^{k,\tilde{i}} = \left(\frac{a-\iota}{q} + \Phi(\iota) - \delta + \mu^q + \sigma\sigma^q\right) dt + (\sigma + \sigma^q)\, dZ + \tilde{\sigma} d\tilde{Z}^{\tilde{i}}.$$

This return is split into a return $dr^{k,h,\tilde{i}}$ earned by the operating household and a return $dr^{k,b,\tilde{i}}$ earned by bankers providing financing according to $dr^{k,\tilde{i}} = \alpha dr^{k,b,\tilde{i}} + (1-\alpha)\, dr^{k,h,\tilde{i}}$, where $\alpha \leq \bar{\alpha}$ is the fraction of risk that is sold off to bankers ($\bar{\alpha} \in [0,1]$ is a model parameter as in lecture 3). $dr^{k,b,\tilde{i}}$ and $dr^{k,h,\tilde{i}}$ have the same Brownian loadings as $dr^{k,\tilde{i}}$, but are permitted to have different drifts $\mu^{r,k,b}$ and $\mu^{r,k,h}$.[1] An individual banker $\tilde{j}$ cannot invest in the equity of only one household, but in a portfolio of a set of households $I(\tilde{j})$. Assume that after optimal diversification across the equities in the set $I(\tilde{j})$, the return on the portfolio of all outside equity claims of banker $\tilde{j}$ evolves according to

$$dr^{oe,\tilde{j}} = \mu^{r,k,b} dt + (\sigma + \sigma^q)\, dZ + \phi\tilde{\sigma} d\tilde{Z}^{\tilde{j}}$$

[1] The structure of the economy is such that these drifts do not depend on the specific individual $\tilde{i}$, but only on the aggregate state.

where $\phi \in [0,1]$ is a model parameter that captures the ability of bankers to diversify risk.[2] Bankers rely on two sources of funding to finance households, their own net worth and nominal deposits issued to households.

Assume that both agent types have logarithmic preferences with identical time-preference rates ρ that is not too large relative to $\tilde{\sigma}$ (and $\bar{\alpha}$ and ϕ) such that money can have positive value in equilibrium. Focus on the money equilibrium and disregard other equilibria.

1. Let η be the wealth share of bankers. Determine the drift and the volatility of η. Show that there is an equilibrium with perfect aggregate risk sharing.[3]

2. How does the system behave in the long run? Is there a steady state and if so, is it at an interior η or does one group completely dominate the economy? The answer may depend on the parameters assumed.

3. For all parameter combinations that imply the existence of a long-run steady state, determine (analytically) the steady-state values of η, ϑ, α, p, q and ι.[4]

4. Choose some parameters and solve (numerically) for the Markov equilibrium in the state variable η, plot $\eta\mu^{\eta}$, ϑ, α, p and q as a function of η.

5. Compute (numerically) the expected utility of an infinitesimally small household that enters the economy with one unit of capital and no money as a function of η.

6. How would the model change, if agents could also trade arbitrary state-contingent claims contingent on all aggregate variables (but *not* on individual-specific outcomes/idiosyncratic risk)?

[2] The precise details of the contracting space are kept deliberately vague, because they do not really matter here. Obviously, if $I(\tilde{j})$ is not a finite set, but a continuum, then the family of Brownian motions $\left(\tilde{Z}^{\tilde{i}}\right)_{\tilde{i} \in I(\tilde{j})}$ cannot be independent, if we want to generate imperfect diversification. There are different ways to generate such a pattern (the easiest is probably to place all agents on "islands", impose the restriction that banks can only invest into equity on their own island and expose household capital both to individual-specific and island-specific shocks, where the former is fully diversifiable by banks and the latter is not at all). When modelling these details, some care must be taken that all asset markets for individual claims clear.

[3] You do not need to show that this perfect risk-sharing equilibrium is the unique equilibrium in which money always has value with probability 1, though my conjecture is that it is.

[4] If you cannot find a closed-form solution, find a characterization by algebraic equations that is as tight as possible: meaning a maximum number of explicitly solved variables as a function of a minimum number of remaining variables that are only given implicitly by a set of equations.