Comparative Valuation Dynamics in Models with Financing Frictions

Overview

Today's Lecture:
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March 18, 2019

Today's Agenda

1. Overview of the project
2. Today's outline
    1. Continuous-time recursive utility (Duffie-Epstein-Zin)
    2. Baseline model with complete markets
    3. Elasticity of intertemporal substitution
    4. Introduction to shock elasticities as a diagnostic tool

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  - Macroeconomic quantity implications
  - Asset pricing implications
  - Welfare consequences and policy ramifications

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- **Approach**: Nesting model

Recursive utility

1. Discrete-time Kreps-Porteus recursive preference specification

$$U_t = \left[[1 - \exp(-\delta\epsilon)] (C_t)^{1-\rho} + \exp(-\delta\epsilon)\mathbb{R}(U_{t+\epsilon} \mid \mathfrak{F}_t)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

$$\mathbb{R}(U_{t+\epsilon} \mid \mathfrak{F}_t) = \left(\mathbb{E}\left[(U_{t+\epsilon})^{1-\gamma} \mid \mathfrak{F}_t \right] \right)^{\frac{1}{1-\gamma}}$$

2. $\mathbb{R}(U_{t+\epsilon} \mid \mathfrak{F}_t)$: certainty equivalent with parameter γ
3. Recursion governed by 3 key parameters
    - δ – subjective discount rate
    - $1/\rho$ – IES
    - γ – relative risk aversion
4. Special case: $\rho = \gamma$

$$U_t = \left(\mathbb{E}\left[\delta \int_0^\infty \exp(-\delta s) (C_{t+s})^{1-\gamma} \, ds \mid \mathfrak{F}_t \right] \right)^{\frac{1}{1-\gamma}}$$

Recursive utility risk adjustment

$$\mathsf{R}(U_{t+\epsilon} \mid \mathfrak{F}_t) = \left(\mathbb{E}\left[(U_{t+\epsilon})^{1-\gamma} \mid \mathfrak{F}_t\right]\right)^{\frac{1}{1-\gamma}}$$

1. Construct logarithmic counterpart

$$\mathsf{r}(\log U_{t+\epsilon}) = \frac{1}{1-\gamma} \log \mathbb{E}\left(\exp\left[(1-\gamma)\log U_{t+\epsilon}\right] \mid \mathfrak{F}_t\right)$$

2. Posit $dU_t = U_t \mu_{u,t} dt + U_t \sigma_{u,t} \cdot dB_t$
3. Ito's Lemma: $d \log U_t = \mu_{u,t} dt - \frac{1}{2}|\sigma_{u,t}|^2 dt + \sigma_{u,t} \cdot dB_t$.
4. Derivative: $\frac{d}{d\epsilon}\mathsf{r}(\log U_{t+\epsilon})|_{\epsilon=0} = \mu_{u,t} - \frac{\gamma}{2}|\sigma_{u,t}|^2$

Includes an adjustment for the local variance.

Continuous-time recursion (Duffie, Epstein, Lions)

Recall

$$U_t = \left[\left[1 - \exp(-\delta\epsilon)\right] (C_t)^{1-\rho} + \exp(-\delta\epsilon) \mathbb{R}(U_{t+\epsilon} \mid \mathfrak{F}_t)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

1. Logarithmic counterpart

$$\begin{aligned} &\log U_t \\ &= \frac{1}{1-\rho} \log \left[\left[1 - \exp(-\delta\epsilon)\right] (C_t)^{1-\rho} + \exp(-\delta\epsilon) \exp\left[(1-\rho) \mathrm{r}(\log U_{t+\epsilon} \mid \mathfrak{F}_t)\right] \right] \end{aligned}$$

2. Subtract $\log U_t$ from both sides and differentiate with respect to ϵ:

$$0 = \frac{\delta}{1-\rho} \left[\left(\frac{C_t}{U_t} \right)^{1-\rho} - 1 \right] + \mu_{u,t} - \frac{\gamma}{2} |\sigma_{u,t}|^2$$

3. $\rho = 1$ limit

$$0 = \delta \left(\log C_t - \log U_t \right) + \mu_{u,t} - \frac{\gamma}{2} |\sigma_{u,t}|^2$$

Stochastic discount factor over an interval

1. Use homogeneity of the utility recursion and compute three marginal utilities: associated with the two CES recursions:

$$
\begin{aligned}
MC_t &= [1 - \exp(-\delta\epsilon)](C_t)^{-\rho}(U_t)^{\rho} \\
MR_t &= \exp(-\delta\epsilon)(R_t)^{-\rho}(U_t)^{\rho} \\
MU_{t,t+\epsilon} &= (U_{t+\epsilon})^{-\gamma}(R_t)^{\gamma}
\end{aligned}
$$

   where R_t is the date t risk adjusted continuation value.

2. Form the stochastic discount ratio as

$$
\begin{aligned}
\frac{S_{t+\epsilon}}{S_t} =& \frac{MR_t MU_{t,t+\epsilon} MC_{t+\epsilon}}{MC_t} \\
=& \exp(-\epsilon\delta)\left(\frac{C_{t+\epsilon}}{C_t}\right)^{-\rho}\left[\frac{U_{t+\epsilon}}{\mathbb{R}\left(U_{t+\epsilon} \mid \mathfrak{F}_t\right)}\right]^{\rho-\gamma}
\end{aligned}
$$

Stochastic discount factor

1. Discrete-time SDF over an interval ϵ:

$$\frac{S_{t+\epsilon}}{S_t} = \exp(-\epsilon\delta)\left(\frac{C_{t+\epsilon}}{C_t}\right)^{-\rho}\left[\frac{U_{t+\epsilon}}{\mathbb{R}\left(U_{t+\epsilon} \mid \mathfrak{F}_t\right)}\right]^{\rho-\gamma}$$

2. Depict continuous-time evolution of SDF as:

$$dS_t = S_t\mu_{s,t}dt + S_t\sigma_{s,t} \cdot dB_t$$

3. Depict a valuation or cumulative return process:

$$dA_t = A_t\mu_{a,t}dt + A_t\sigma_{a,t} \cdot dB_t.$$

   where AS is a positive martingale. Martingale restriction:

$$\mu_{a,t} + \mu_{s,t} + \sigma_{a,t} \cdot \sigma_{s,t} = 0.$$

4. Risk free rate $r_t = -\mu_{s,t}$; risk price vector $\pi_t = -\sigma_{s,t}$.

Stochastic discount factor characterization

Recall: $\frac{S_{t+\epsilon}}{S_t} = \exp(-\epsilon\delta) \left(\frac{C_{t+\epsilon}}{C_t}\right)^{-\rho} \left[\frac{U_{t+\epsilon}}{\mathbb{R}(U_{t+\epsilon}|\mathfrak{F}_t)}\right]^{\rho-\gamma}$

1. Consumption

$$d \log C_t = \mu_{c,t} dt - \frac{1}{2}|\sigma_{c,t}|^2 dt + \sigma_{c,t} \cdot dB_t$$

2. Continuation value

$$d \log U_t = \mu_{u,t} dt - \frac{1}{2}|\sigma_{u,t}|^2 dt + \sigma_{u,t} \cdot dB_t$$

3. Local coefficients (prices):

$$\sigma_{s,t} = -\rho\sigma_{c,t} + (\rho - \gamma)\sigma_{u,t}$$

$$\mu_{s,t} - \frac{1}{2}|\sigma_{s,t}|^2 = -\delta - \rho\mu_{c,t} + \frac{\rho}{2}|\sigma_{c,t}|^2 dt + \frac{(\rho-\gamma)(\gamma-1)}{2}|\sigma_{u,t}|^2$$

Long-run risk model with complete markets

1. The long-run risk processes (Z, V):

$$dZ_t = -\lambda_z Z_t dt + \sqrt{V_t}\sigma_z \cdot dB_t$$
$$dV_t = -\lambda_v (V_t - 1) dt + \sqrt{V_t}\sigma_v \cdot dB_t$$

2. The A-K production technology with adjustment costs

$$\frac{dK_t}{K_t} = \left[\Phi\left(\frac{I_t}{K_t}\right) + Z_t - \alpha_k\right] dt + \sqrt{V_t}\sigma_k \cdot dB_t$$

3. Φ is the concave and increasing installation cost function
4. The economy's resource constraint:

$$C_t + I_t = aK_t$$

Planner problem

1. State vector $X \doteq (Z, V)$
2. Capital evolution in logarithms:

$$d\log K_t = \left[\Phi\left(\frac{I_t}{K_t}\right) + Z_t - \alpha_k - \frac{V_t|\sigma_k|^2}{2}\right] dt + \sqrt{V_t}\sigma_k \cdot dB_t$$

2. Homogeneity properties of the model lead to: $\log U_t = \log K_t + \xi(X_t)$.
3. HJB equation for planner problem:

$$0 = \max_{c+i=a}\left\{\frac{\delta}{1-\rho}\left(c^{1-\rho}\exp\left((\rho-1)\xi\right) - 1\right) + \Phi(i) + z - \alpha_k - \frac{1}{2}v|\sigma_k|^2 \right.$$
$$\left. + \mu_x \cdot \partial_x \xi + \frac{1}{2}\mathrm{tr}\left(\sigma_x' \partial_{xx'}\xi \sigma_x\right) + \frac{1-\gamma}{2}|\sqrt{v}\sigma_k + \sigma_x'\partial_x\xi|^2\right\}$$

where c (consumption-to-capital ratio) and i (investment-to-capital ratio).

Aggregate Wealth

1. Marginal utility of consumption satisfies

$$MC_t = \delta C_t^{-\rho} U_t^{\rho}$$

2. Use (i) Euler's theorem and (ii) total wealth = value of capital stock to obtain

$$Q_t K_t = \frac{U_t}{MC_t} = \frac{1}{\delta} \left(\frac{C_t}{K_t} \right)^{\rho} \left(\frac{U_t}{K_t} \right)^{1-\rho} K_t$$

   where Q_t is the price of capital.

3. Return on wealth is exposed to direct shocks to the capital stock and also to shocks to its value Q_t.

Unitary elasticity

HJB equation for planner problem when $\rho = 1$

$$0 = \max_{c+i=a} \left\{ \delta \left(\log c - \xi \right) + \Phi(i) + z - \alpha_k - \frac{1}{2} v |\sigma_k|^2 \right.$$
$$\left. + \mu_x \cdot \partial_x \xi + \frac{1}{2} \mathrm{tr} \left(\sigma_x' \partial_{xx'} \xi \sigma_x \right) + \frac{1-\gamma}{2} |\sqrt{v} \sigma_k + \sigma_x' \partial_x \xi|^2 \right\}$$

where c (consumption-to-capital ratio) and i (investment-to-capital ratio).

1. *i* and *c* are constant independent of the Markov state.
2. Affine value function: $\xi(x) = \beta_0 + \beta_1 \cdot x$
    - 2.1 The dependence on growth state variable $\beta_{1z} z$ satisfies:

$$\beta_{1z} z = \left(\frac{1}{\delta + \lambda_z} \right) z = \mathbb{E} \left[\int_0^\infty \exp(-\delta \tau) Z_{t+\tau} d\tau \mid Z_t = z \right]$$

    - 2.2 Coefficient on the volatility state variable β_{1v} satisfies a quadratic equation

Sensitivity to changes ρ

Construct an expansion around $\rho = 1$.

1. The sign of ρ changes the quantity dynamics
2. Responses when $\rho < 1$
    a. c^* is decreasing in growth z
    b. c^* is increasing in volatility v

    and conversely when $\rho > 1$.

Models of Asset Valuation

Two channels:

1. Stochastic growth modeled as a process $G = \{G_t\}$ where G_t captures growth between dates zero and t.
2. Stochastic discounting modeled as a process $S = \{S_t\}$ where S_t assigns risk-adjusted prices to cash flows at date t.

Date zero prices of a payoff G_t are

$$\text{Price} = \mathbb{E}\left(S_t G_t | \mathfrak{F}_0\right)$$

where $\mathcal{F}_0$ captures current period information.

Stochastic discounting reflects investor preferences through the intertemporal marginal rate of substitution for marginal investors.

Impulse Problem

Ragnar Frisch (1933):

> *There are several alternative ways in which one may approach the impulse problem .... One way which I believe is particularly fruitful and promising is to study what would become of the solution of a determinate dynamic system if it were exposed to a stream of erratic shocks that constantly upsets the continuous evolution, and by so doing introduces into the system the energy necessary to maintain the swings.*

Irving Fisher (1930):

> *The manner in which risk operates upon time preference will differ, among other things, according to the particular periods in the future to which the risk applies.*

Diagnostic Tool

Transition dynamics and valuation through altering cash flow exposure to shocks.

1. Study implication on the price today of changing the exposure tomorrow on a cash flow at some future date.
2. Represent shock price elasticities by normalizing the exposure and studying the impact on the logarithms of the expected returns.
3. Construct pricing counterpart to impulse response functions.

Unpack the Term Structure of Risk Premia

1. Proportional risk premium over horizon t:

$$\log \mathbb{E}\left(\frac{G_t}{G_0}\right) - \log \mathbb{E}\left(\frac{S_t G_t}{S_0 G_0} \mid \mathfrak{F}_0\right) + \log \mathbb{E}\left(\frac{S_t}{S_0} \mid \mathfrak{F}_0\right)$$

   where the first term is the expected cash flow growth, the second is the value, and the third term is the negative of the expected risk-less return all in logarithms.

2. Counterparts to impulse response functions pertinent to valuation:
   2.1 shock-exposure elasticities
   2.2 shock-price elasticities

   These are the ingredients to risk premia, and they have a term structure induced by the changes in the investment horizons.

Hansen-Scheinkman (*Finance and Stochastics*), Borovička and Hansen ((*Journal of Econometrics*), Borovička-Hansen-Scheinkman (*Mathematical and Financial Economics*)

Construct Elasticities

1. Construct shock elasticities as counterparts to impulse response functions

2. Use (exponential) martingale $D^{(\tau)}$, perturbing an underlying positive (multiplicative) process M over the time horizon $[0, \tau)$, where:

$$\log M_t = \int_0^t \mu_m(X_s) ds + \int_0^t \sigma_m(X_s) \cdot dB_s$$
$$\log D_t^{(\tau)} = -\int_0^{t \wedge \tau} \frac{|\sigma_d(X_s)|^2}{2} ds + \int_0^{t \wedge \tau} \sigma_d(X_s) \cdot dB_s$$

where $\mathbb{E}\left(|\sigma_d(X_t)|^2\right) = 1$.

3. Shock elasticity (for example, if $\sigma_d = (1, 0)$, then $D^{(\tau)}$ perturbs in the direction of the first shock):

$$\epsilon_m(x, t) \doteq \lim_{\tau \downarrow 0} \frac{1}{\tau} \log \mathbb{E}\left[\frac{M_t}{M_0} D_t^{(\tau)} | X_0 = x\right]$$

Shock elasticity

$$\epsilon_m(x,t) \doteq \lim_{\tau \downarrow 0} \frac{1}{\tau} \log \mathbb{E}\left[\frac{M_t}{M_0} D_t^{(\tau)} | X_0 = x\right]$$

Apply to a cash-flow G, stochastic discount factor S and product SG

1. shock exposure elasticity $\epsilon_g(x,t)$;
2. shock cost elasticity $\epsilon_{sg}(x,t)$;
3. shock price elasticity $\epsilon_g(x,t) - \epsilon_{sg}(x,t)$

Interpret Elasticities

Recall

$$\epsilon_m(x,t) = \lim_{\tau \downarrow 0} \frac{1}{\tau} \log \mathbb{E}\left[\frac{M_t}{M_0} D_t^{(\tau)} | X_0 = x\right]$$

where $D^{(\tau)}$ is an exponential martingale perturbation over the time interval $[0,\tau)$.

Two interpretations:

1. Change in probability measure - local impulse response
2. Change in cash flow exposure - local risk return

Depend on current state and horizon.

What do These Elasticities Contribute?

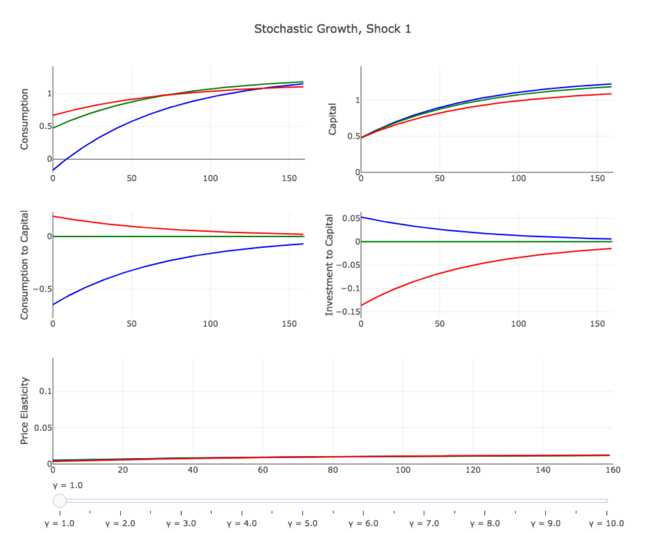
1. What shocks investors do care about as measured by expected return compensation?
2. How do these compensations vary across states and over horizons?
3. How do the shadow compensation differ across agent type?

Inputs for the computations

1. σ_k only depends on the first shock while σ_z depends on both shocks. The numbers were chosen to better track the observed consumption dynamics.
2. The computations used a first-order small noise, large risk aversion parameterization. In this approximation γ only alters the implied deterministic steady states and not the impulse responses to shocks.

Shock Elasticity: Shock 1

Blue: $\rho = 0.5$ Green: $\rho = 1$ Red: $\rho = 2$



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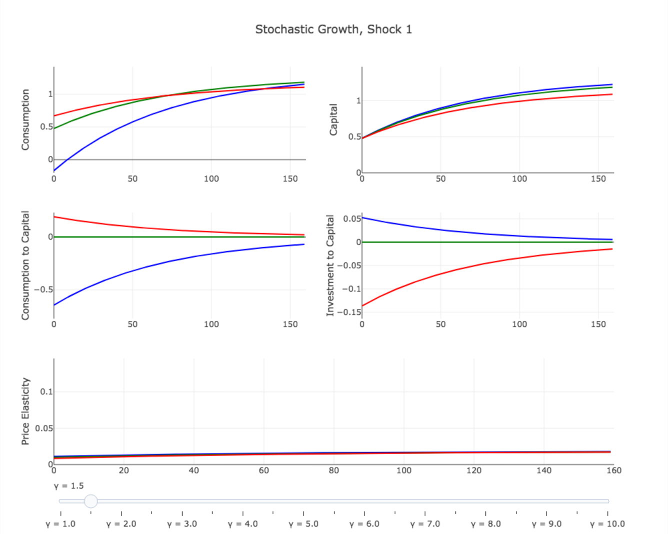

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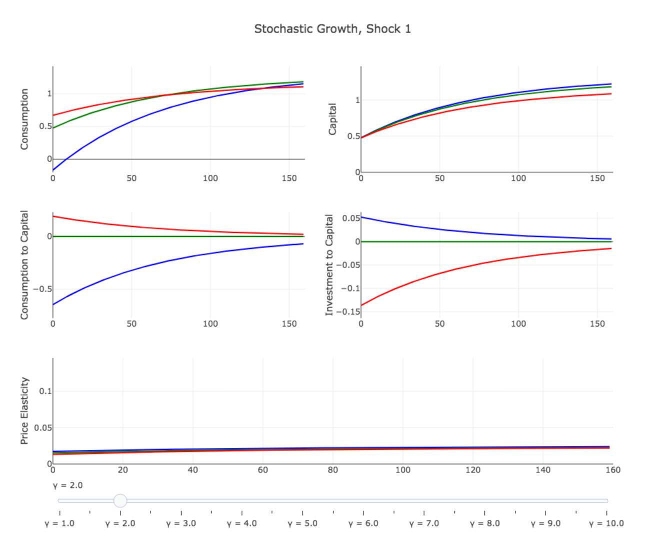

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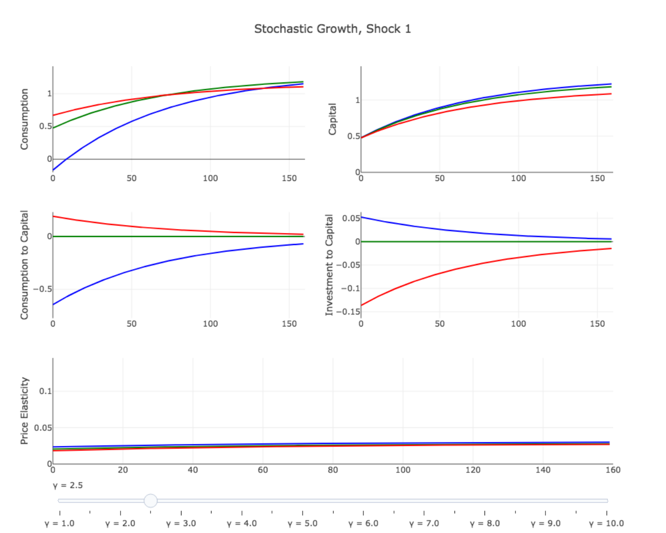

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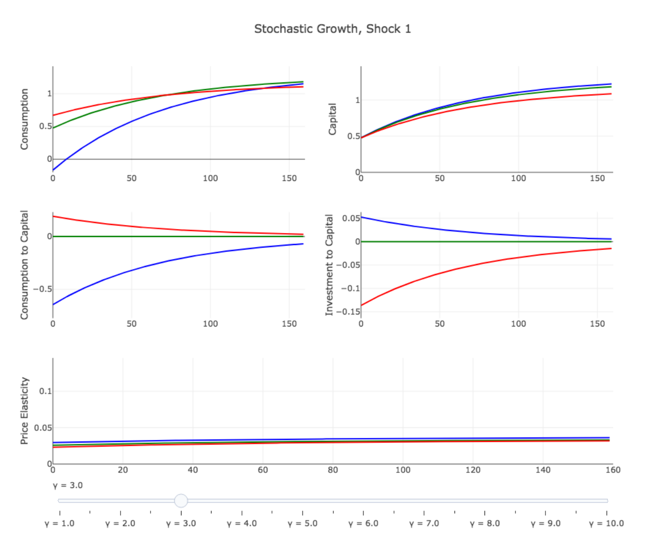

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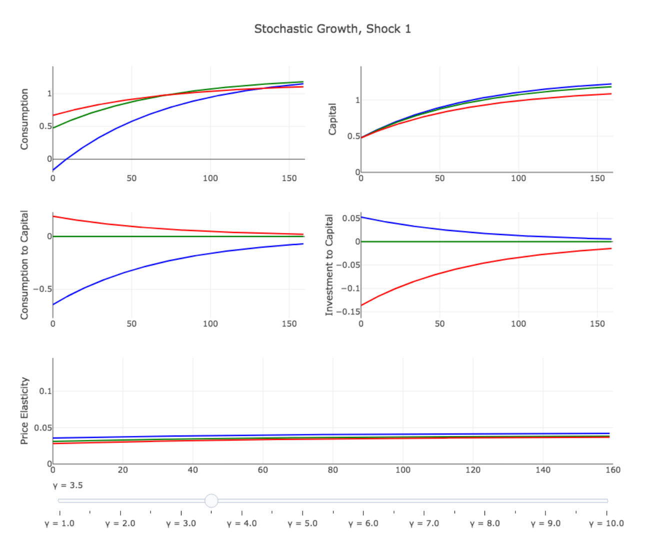

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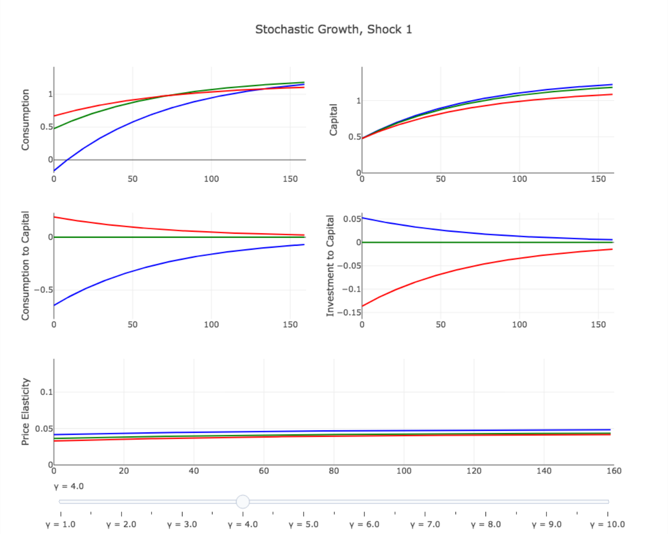

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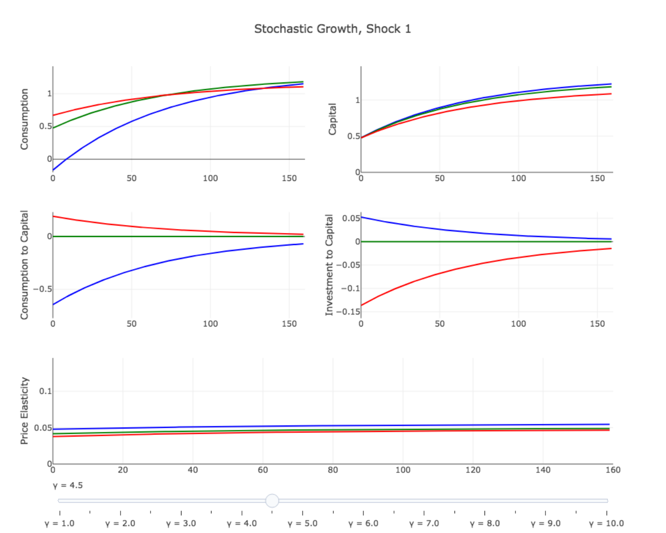

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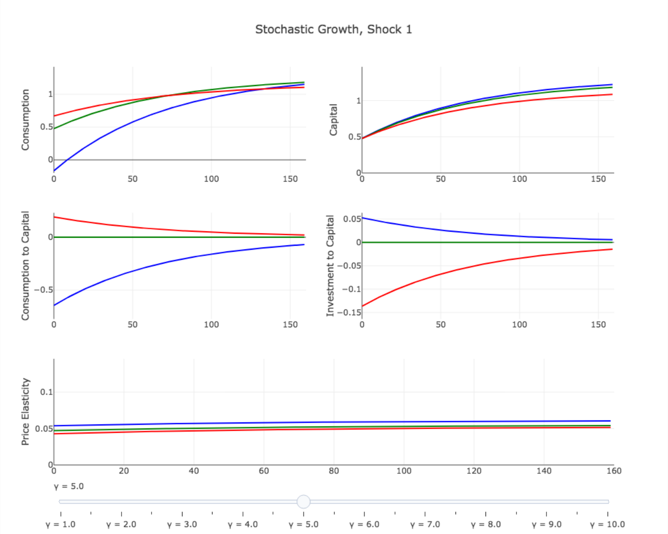

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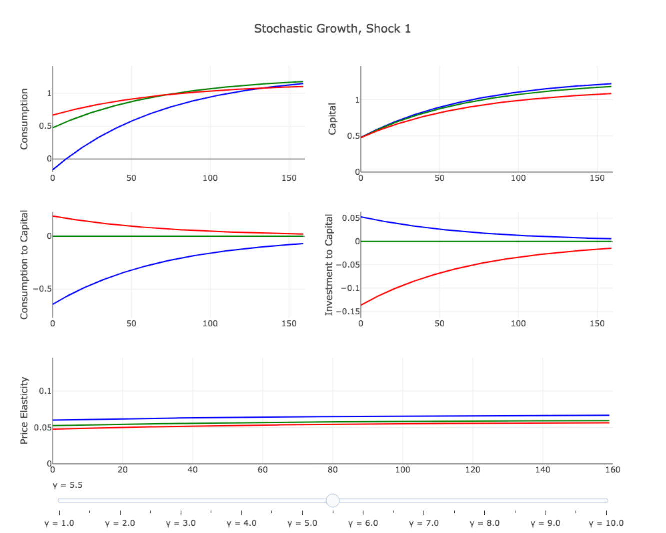

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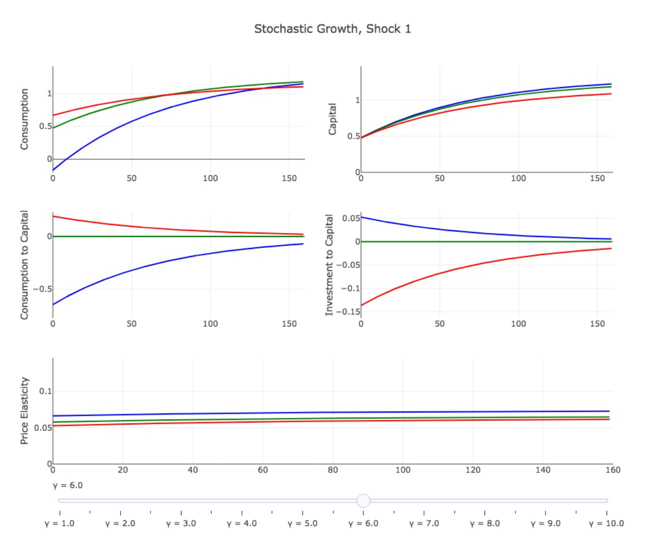

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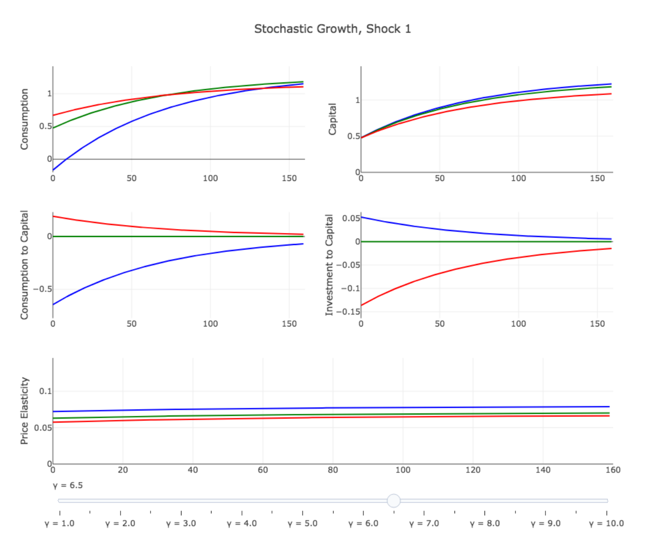

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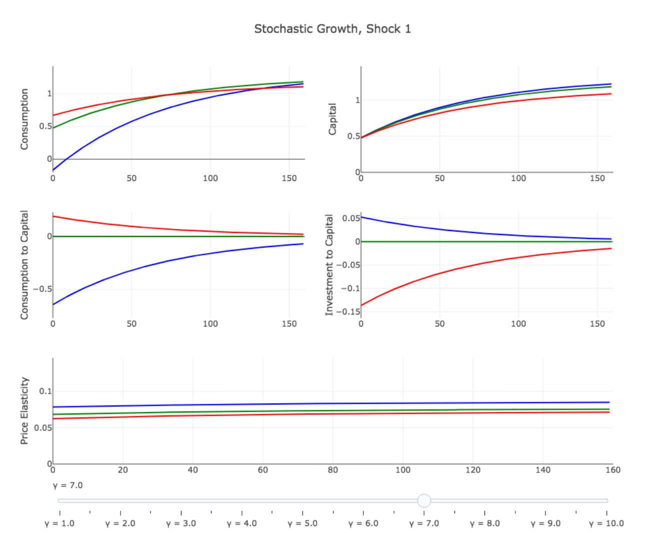

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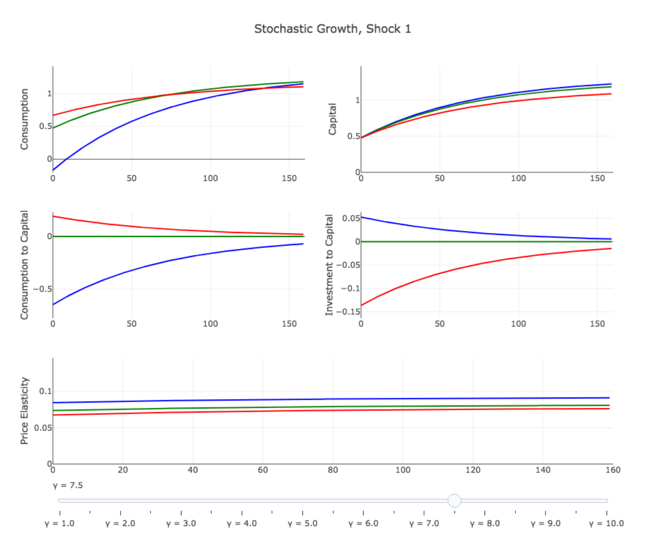

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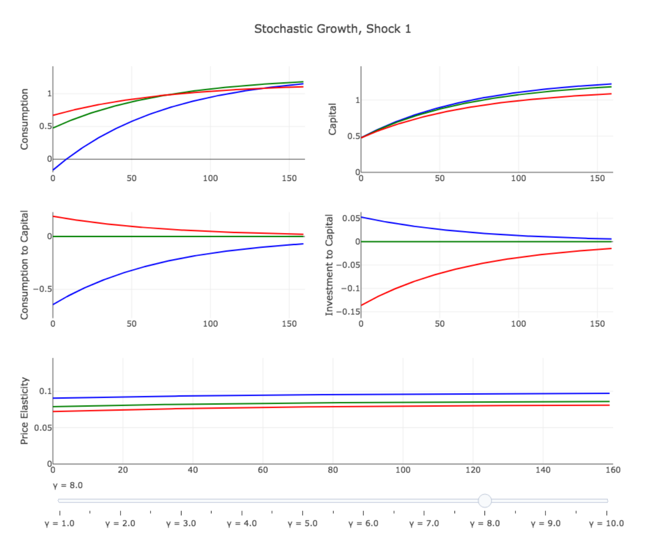

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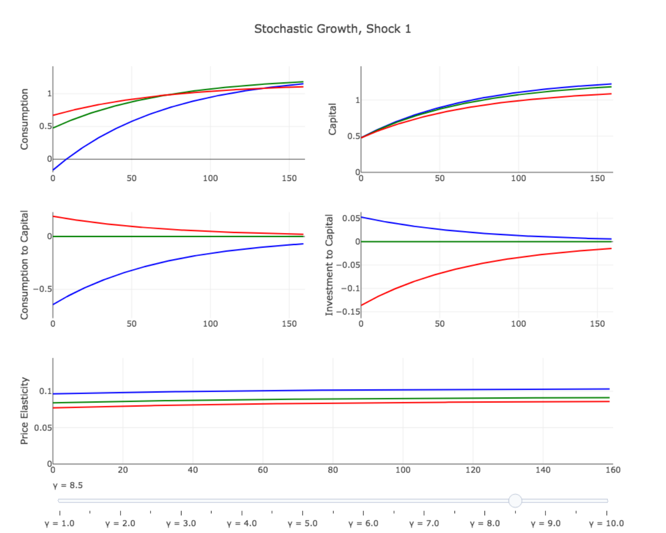

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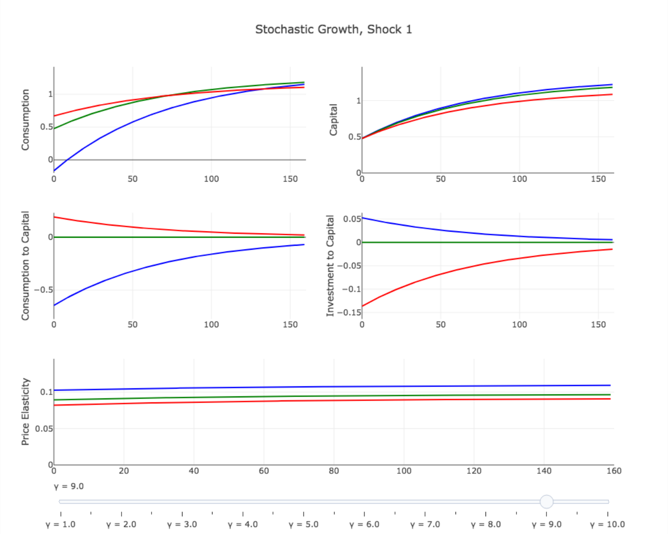

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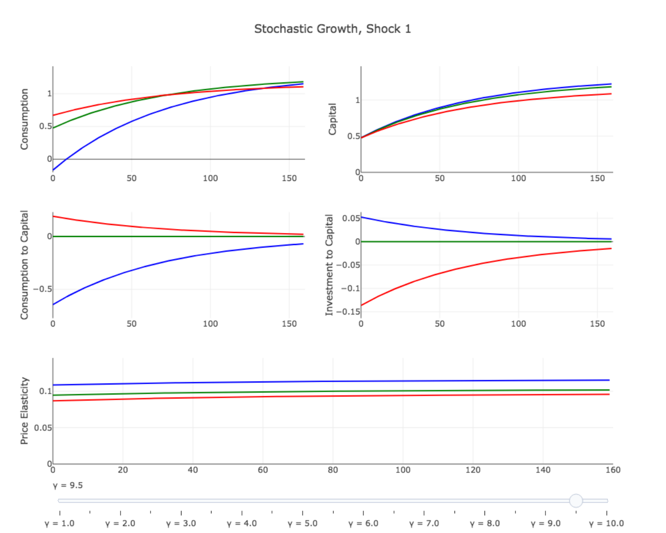

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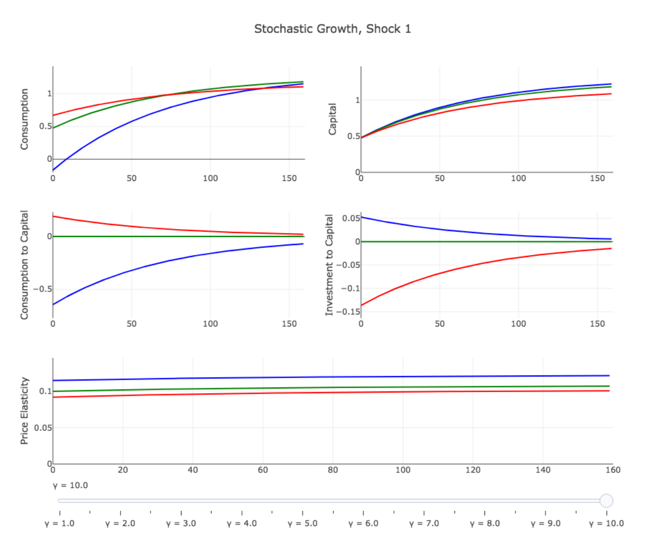

Shock Elasticity: Shock 2

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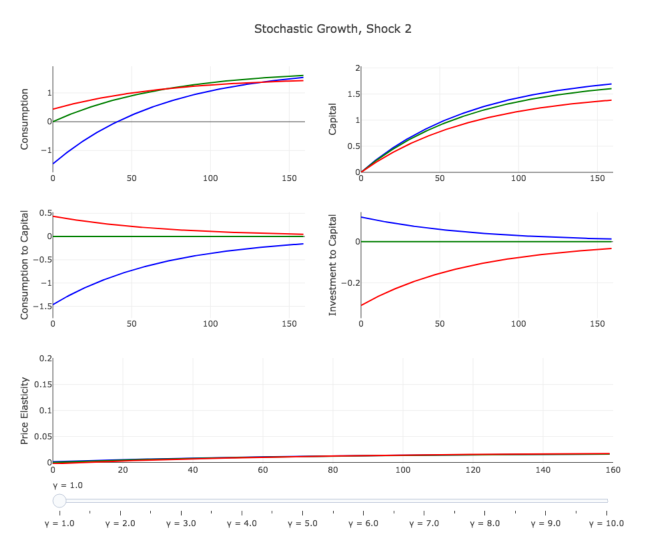

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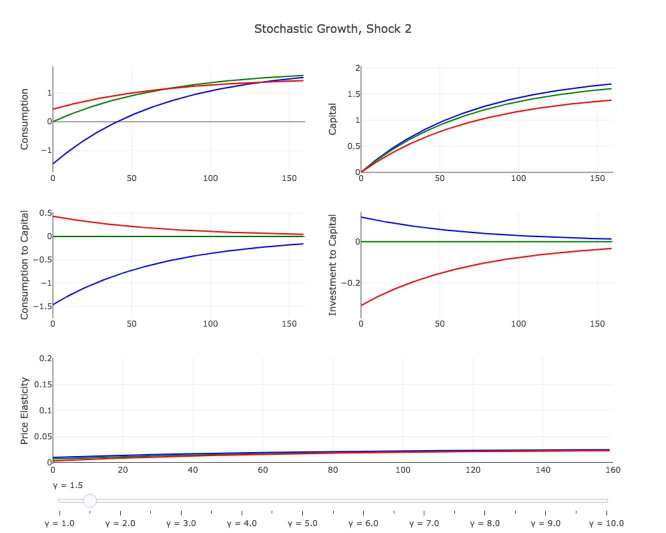

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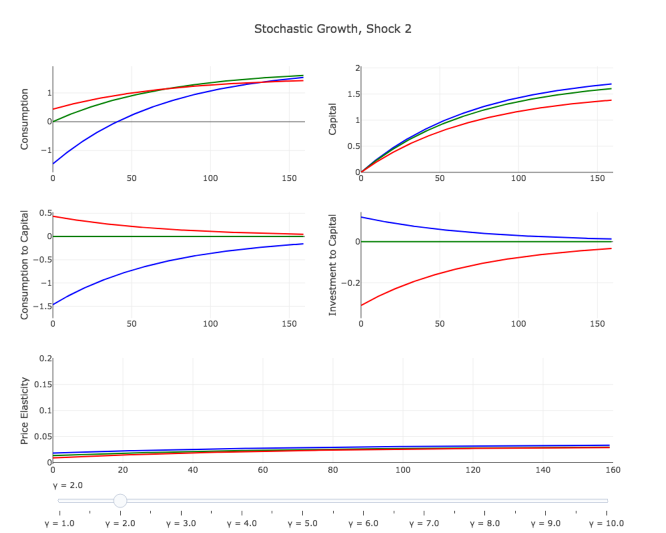

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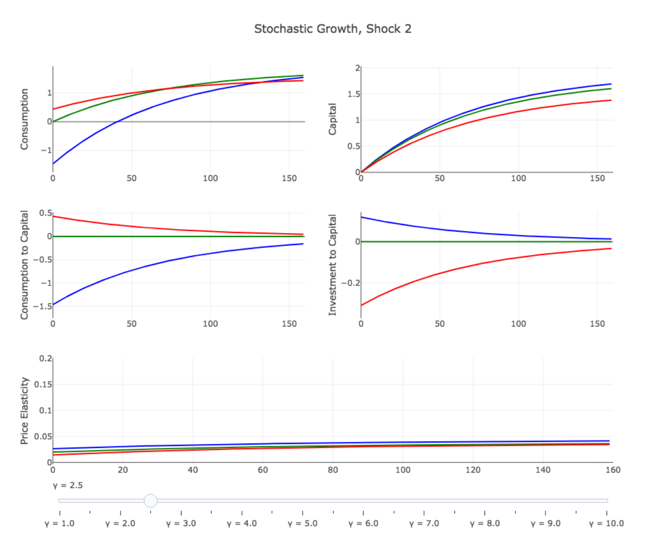

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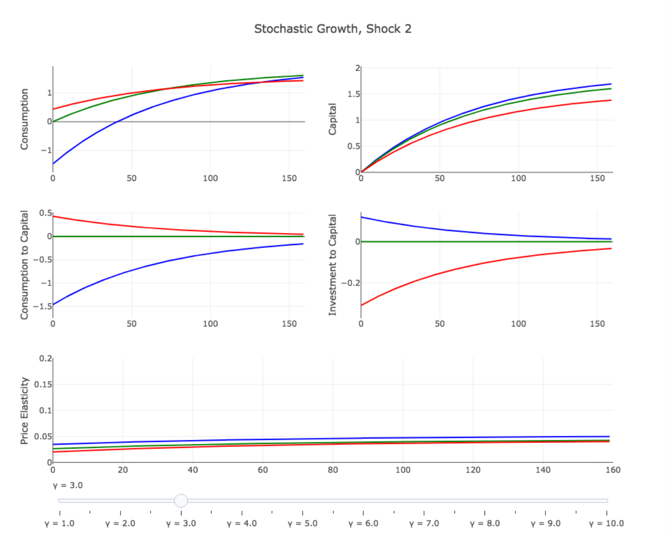
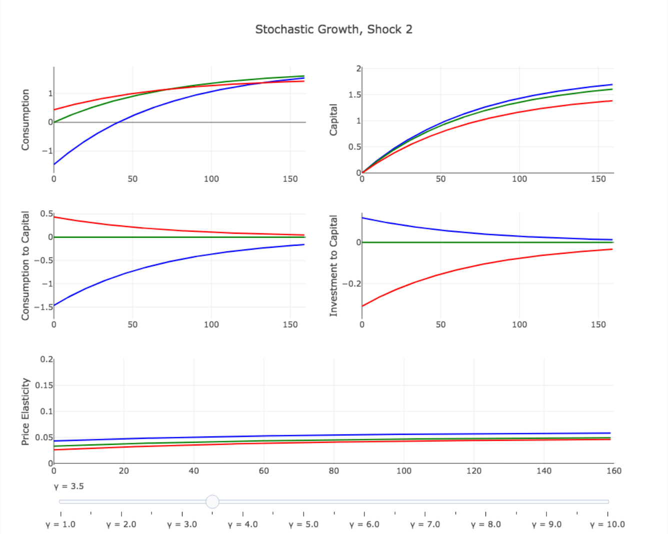


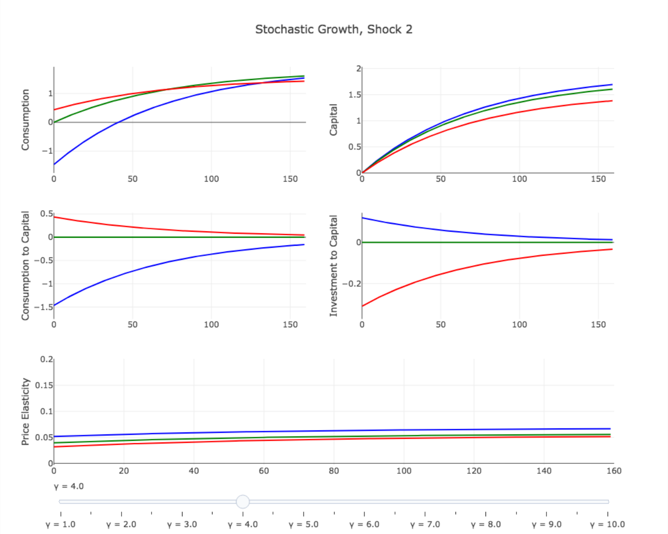
Shock Elasticity: Shock 2

Blue: $\rho = 0.5$ Green: $\rho = 1$ Red: $\rho = 2$



Shock Elasticity: Shock 2

Blue: $\rho = 0.5$ Green: $\rho = 1$ Red: $\rho = 2$



Shock Elasticity: Shock 2

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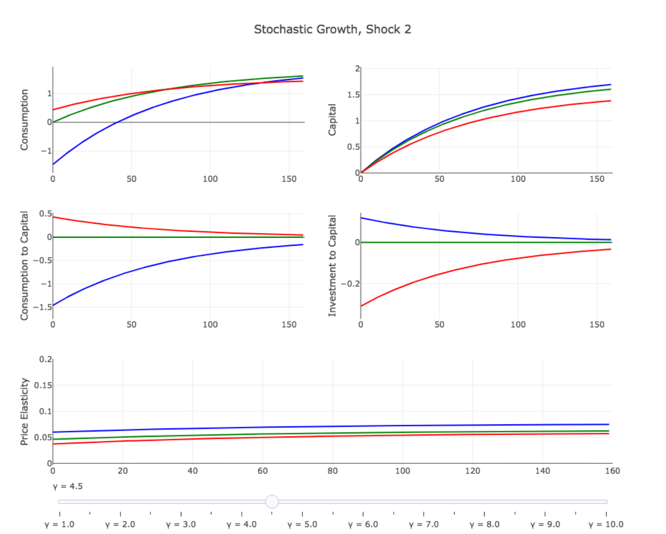

SHOCK ELASTICITY: SHOCK 2

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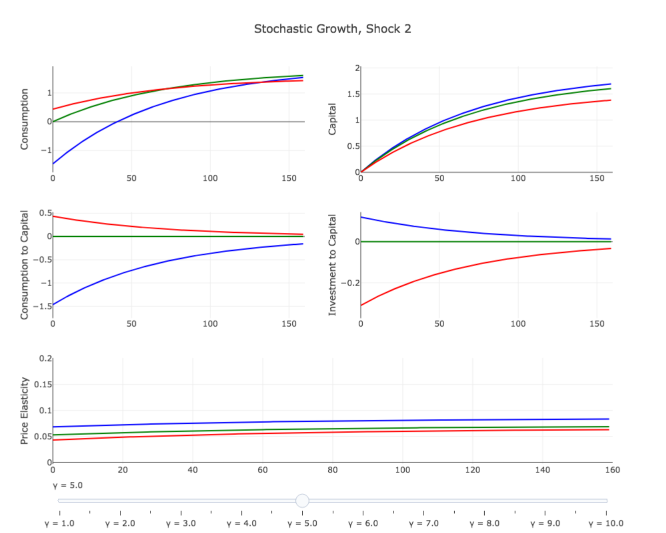

Shock Elasticity: Shock 2

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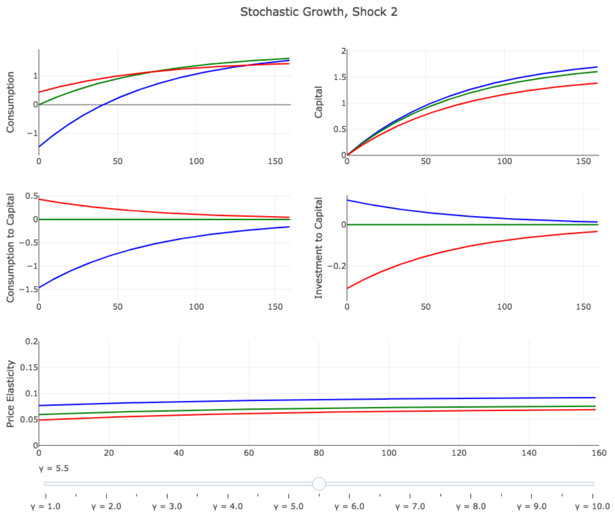

Shock Elasticity: Shock 2

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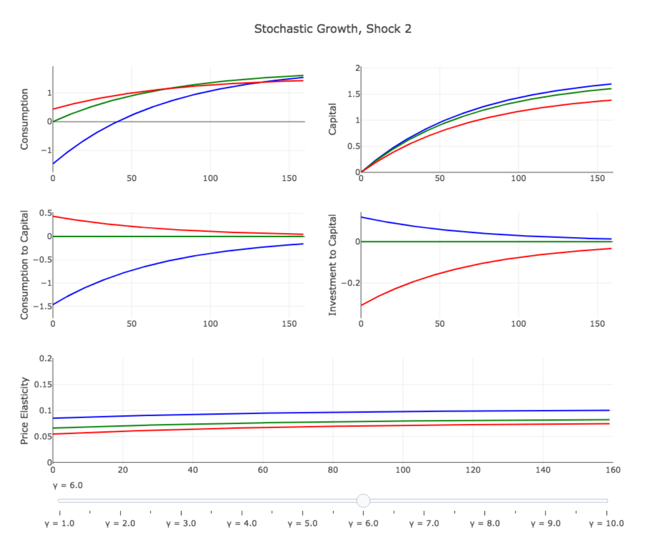

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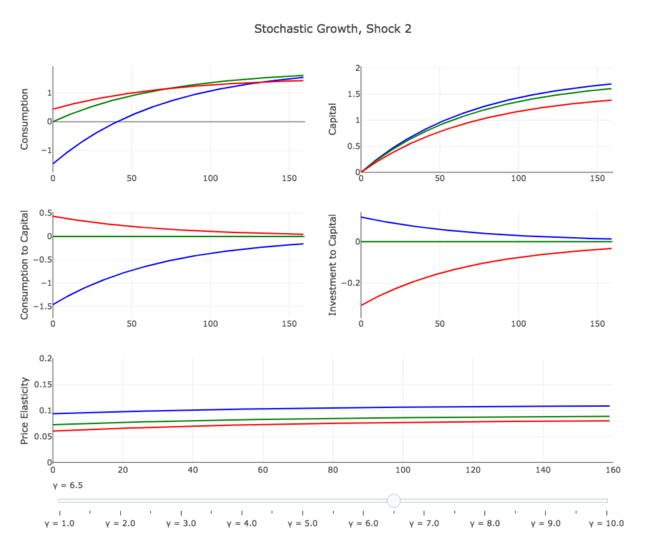

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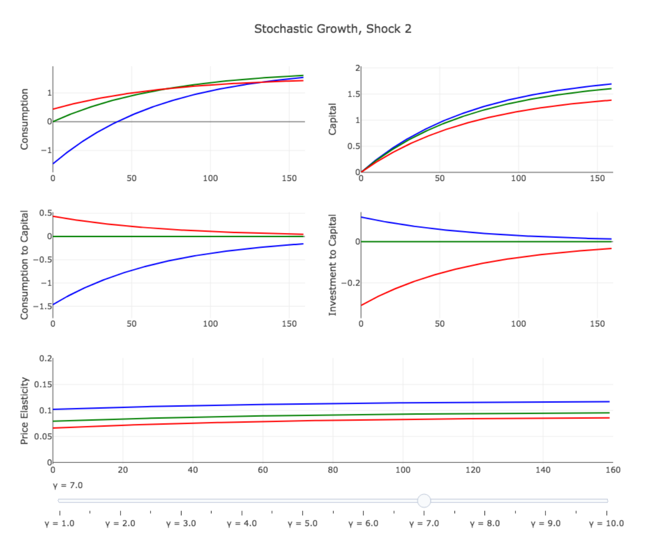

Shock Elasticity: Shock 2

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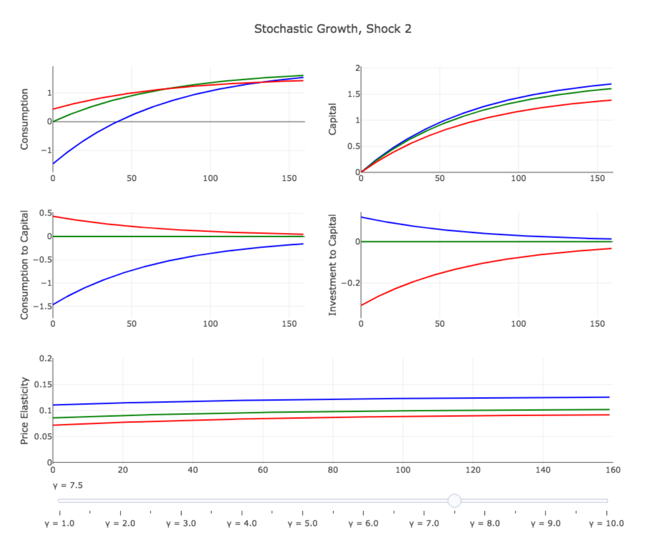

Shock Elasticity: Shock 2

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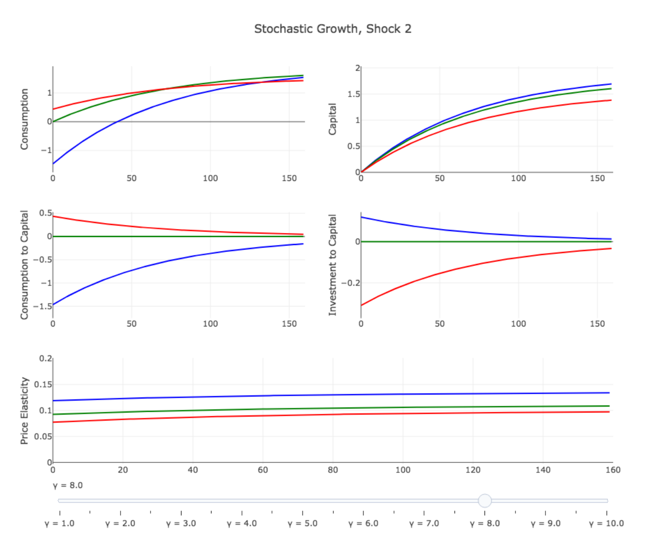

Shock Elasticity: Shock 2

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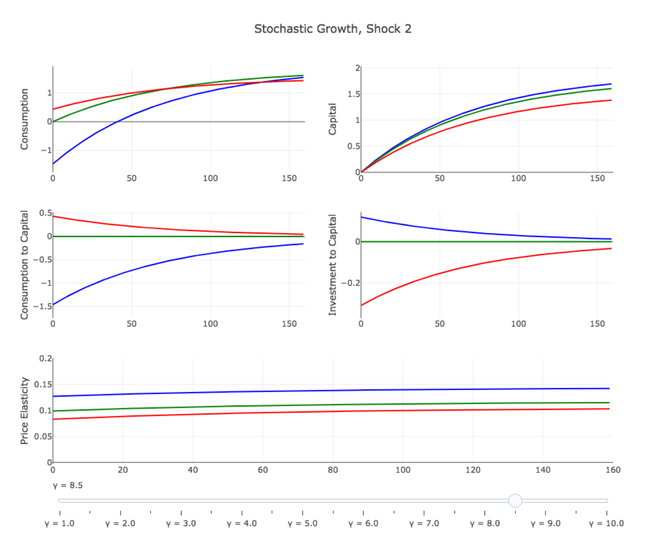

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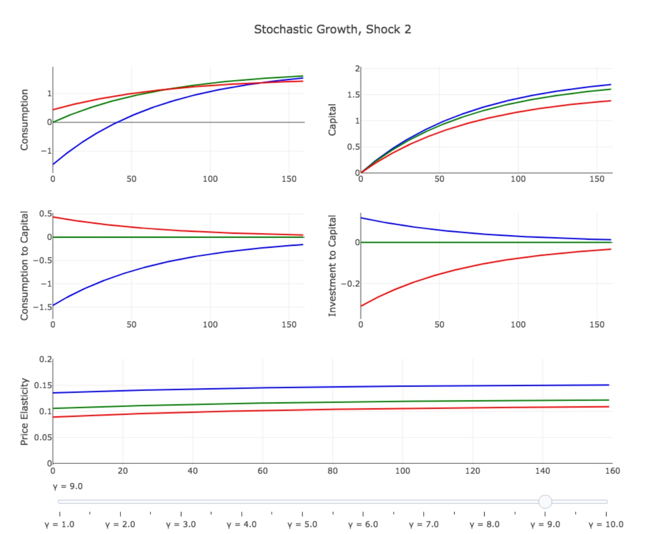

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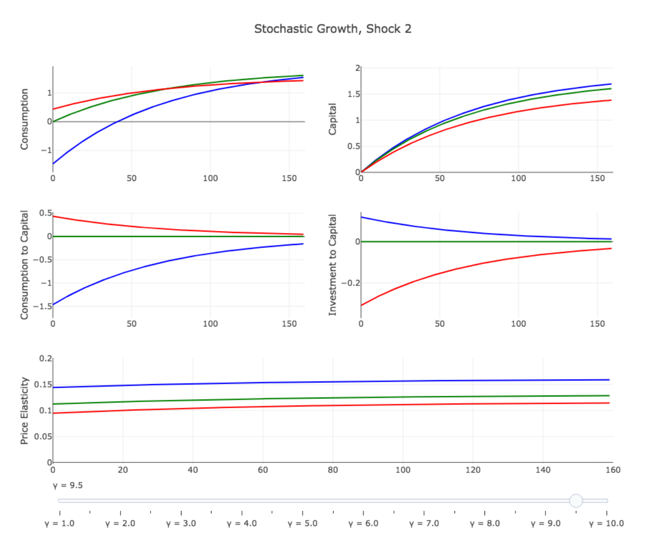

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