On the Optimal Inflation Rate

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Motivation

- What should the (long-run) optimal inflation rate be?
- What role do financial frictions play?
  - Can financial frictions destroy the superneutrality of money?
- Should emerging markets, with less developed financial markets, have a higher inflation rate/target?

Inflation Target

Table 4.1. Inflation Targeters

	Inflation Targeting Adoption Date[1]	Unique Numeric Target = Inflation	Current Inflation Target (percent)	Forecast Process	Publish Forecast
Emerging market countries					
Israel	1997:Q2	Y	1–3	Y	Y
Czech Republic	1998:Q1	Y	3 (+/–1)	Y	Y
Korea	1998:Q2	Y	2.5–3.5	Y	Y
Poland	1999:Q1	Y	2.5 (+/–1)	Y	Y
Brazil	1999:Q2	Y	4.5 (+/–2.5)	Y	Y
Chile	1999:Q3	Y	2–4	Y	Y
Colombia	1999:Q3	Y	5 (+/–0.5)	Y	Y
South Africa	2000:Q1	Y	3–6	Y	Y
Thailand	2000:Q2	Y	0–3.5	Y	Y
Mexico	2001:Q1	Y	3 (+/–1)	Y	N
Hungary	2001:Q3	Y	3.5 (+/–1)	Y	Y
Peru	2002:Q1	Y	2.5 (+/–1)	Y	Y
Philippines	2002:Q1	Y	5–6	Y	Y
Industrial countries					
New Zealand	1990:Q1	Y	1–3	Y	Y
Canada	1991:Q1	Y	1–3	Y	Y
United Kingdom	1992:Q4	Y	2	Y	Y
Australia	1993:Q1	Y	2–3	Y	Y
Sweden	1993:Q1	Y	2 (+/–1)	Y	Y
Switzerland	2000:Q1	Y	<2	Y	Y
Iceland	2001:Q1	Y	2.5	Y	Y
Norway	2001:Q1	Y	2.5	Y	Y

Source: IMF, WEO, Sept. 2005

Literature

- Money as store of value = bubble

\Friction	OLG	Incomplete Markets + idiosyncratic risk	
Risk	deterministic	endowment risk borrowing constraint	investment risk
Only money	Samuelson	Bewley	
With capital	Diamond	Aiyagari	Angeletos
		Risk tied up with individual	capital

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\Friction	OLG	Incomplete Markets + idiosyncratic risk	
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With capital	Diamond	Aiyagari	Angeletos $q = 1$
	$f'(k^*) = r^*$, Dynamic inefficiency $r < r^*, K > K^*$	Inefficiency $r < r^*, K > K^*$	capital shock

depends on
price of capital q

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			Basic "I Theory"
With capital	Diamond	Aiyagari	cash flow shock
	$f'(k^*) = r^*$, Dynamic inefficiency $r < r^*, K > K^*$	Inefficiency $r < r^*, K > K^*$	Pecuniary externality Inefficiency $r > r^*, K < K^*$
			$r^m = g$

Main results

- HH portfolio choice
  - Physical capital: w/ idiosyncratic risk + dividend
  - Money: w/o idiosyncratic risk + no dividend (bubble)
    - Tilted inefficiently towards money

- Money growth ⇒ inflation ⇒ "tax on money"
- ⇒ lowers real interest rate ⇒ tilts portfolio choice
- ⇒ boosts physical investment ⇒ higher economic growth
- ⇒ raises real interest rate (partially undoes inflation tax)
- Pecuniary externality:
  - individual households do not take this GE effect into account.
  - Planner who can print money and distribute seignorage can improve growth + Pareto welfare.
- Derive optimal money growth rate/inflation rate

Model setup

- In each period j
  - HH enters with physical capital k_t & nominal money m_t
  - Produce output $Ak_t\Delta t$
  - Real cash flow shock $z_j = \sigma\varepsilon_j k_j\sqrt{\Delta t}$
  - Transfer from government τw (proportional to wealth)
  - Decide
    - Investment rate ι $k'_{j+1} = [(1 + \Phi(\iota) - \delta)\Delta t]k_j$
    - Adjustment cost function $\Phi(\iota) = \frac{1}{\kappa}\log(1+\kappa\iota)$
  - Portfolio & consumption choice
    - Purchase/sell physical capital x_j^k = portfolio share
    - Consume c_j

$$\max_{\{c_j, k_{j+1}, m_{j+1}, \iota_j\}_{j=0}^{\infty}} E\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+\rho\Delta t}\right)^j \log c_j \cdot \Delta t\right]$$

Time-line within period

Model setup

- Consumption good is numeraire
- q price of physical capital
  real value of all physical capital qK_j
- p real value of all nominal wealth pK_j

- M_j aggregate nominal money supply
  - grows at a rate μ
  - Seignorage income is $\frac{\mu\Delta t}{1+\mu\Delta t} pK_j$
- $\wp_j := \frac{M_j}{pK_j}$ is the price level

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Model setup

- HH's budget constraint

$$(c_j + \iota_j k_j)\Delta t + q k_{j+1} + \frac{m_{j+1}}{\mathcal{P}_j} =$$
$$A k_j \Delta t + z_j + q\left(1 + (\Phi(\iota_j) - \delta)\Delta t\right) k_j + R^m_{j-1}\frac{m_j}{\mathcal{P}_{j-1}} + \tau w_j$$

- Government's budget constraint
  - Seignorage income

$$S_j := \frac{M_j - M_{j-1}}{M_j} p K_j = \left(1 - \frac{1}{1+\mu\Delta t}\right) p K_j = \frac{\mu \Delta t}{1+\mu\Delta t} p K_j.$$

  - Distribution through transfers τ

$$\frac{w_j}{(p+q)K_j} S_j = \underbrace{\frac{p}{p+q}\frac{\mu\Delta t}{1+\mu\Delta t}}_{=:\tau} w_j$$

Optimality conditions

- Optimal investment rate ι^*
  - $q = \frac{1}{\Phi'(\iota^*)} = 1 + \kappa\iota^*$ Tobin's q
- Optimal consumption
  - $c^* = \frac{\rho}{1+\rho\Delta t} w'$ due to log utility
    - Where $w' = R^k qk + R^m \frac{m}{\wp} + \tau w$ wealth just prior to consumption
    - $R^k = 1 + \left(\frac{A-\iota^*}{q} + \underbrace{\Phi(\iota^*) - \delta}_{g:=}\right)\Delta t + \frac{\sigma}{q}\varepsilon\sqrt{\Delta t}$ "capital return"
    - $R^m = \frac{1+g\Delta t}{1+\mu\Delta t} = 1 + \frac{g-\mu}{1+\mu\Delta t}\Delta t$ "money return"
    - $R^p(x^k) := x^k R^k + (1-x^k)R^m + \tau$ "portfolio return"
- Optimal Portfolio

$$\max_{x^k} \frac{1}{1+\rho\Delta t} \alpha_1 E[\log R^p(x^k)]$$

$$E[\log R^p(x^k)] = E\left[\left(R^p(x^k) - 1\right) - \frac{1}{2}(R^p(x^k) - 1)^2\right] + o(\Delta t) =$$

$$\approx \left(\Phi(\iota^*) - \delta - \frac{q}{p+q}\mu + x^k\left(\frac{A-\iota^*}{q} + \mu\right) - \frac{1}{2}(x^k)^2\frac{\sigma^2}{q^2}\right)\Delta t$$

  - $x^{k*} = \frac{q(A-\iota^*)}{\sigma^2} + \frac{q^2\mu}{\sigma^2}$

Market clearing conditions

- Goods market
  - $AK_j\Delta t = \iota^* K_j \Delta t + \frac{\rho}{1+\rho\Delta t} W_j' \Delta t$
  - $(A-\iota^*)\Delta t = \rho[\Delta t + (\Phi(\iota^*)-\delta)(\Delta t)^2](p+q)$
  - $A-\iota^* = \rho(p+q)$ for $\Delta t \to 0$
- Capital market
  - $\frac{x^k W_j}{q} = K_j \Rightarrow q\frac{K_j}{W_j} = x^k = \frac{q(A-\iota^*)}{\sigma^2} + \frac{q^2\mu}{\sigma^2}$
  - $\frac{1}{p+q} = \frac{A-\iota^*}{\sigma^2} + \frac{q\mu}{\sigma^2}$
- Money market
  - clears by Walras law

Equilibrium

- Collecting the three equations:

$$q = 1 + \kappa\iota^*$$
$$\rho(p+q) = A - \iota^*$$
$$\frac{\sigma^2}{q+p} = A - \iota^* + q\mu$$

- Equilibrium solved in terms of $\hat{\mu} \coloneqq x^k \mu$ (monotone transformation)

$$p = \frac{\sigma(1+\kappa\rho)}{\sqrt{\rho+\hat{\mu}}} - (1+\kappa A)$$
$$q = 1 + \kappa A - \frac{\kappa\rho\sigma}{\sqrt{\rho+\hat{\mu}}}$$
$$\iota^* = A - \rho\frac{\sigma}{\sqrt{\rho+\hat{\mu}}}$$

Closed form!

Welfare

- Plug in FOC in value function
- Plug in equilibrium
- All households start symmetrically

- Expected Utility of an individual household

$$V = V_0 + \frac{\frac{1}{\kappa}\log\left(1 + \kappa A - \frac{\kappa\rho\sigma}{\sqrt{\rho+\hat{\mu}}}\right) - \delta + \rho - \frac{1}{2}(\rho + \hat{\mu})}{\rho^2} + \frac{\log\left(\frac{\sigma}{\sqrt{\rho+\hat{\mu}}}\right)}{\rho}.$$

Closed form!

Optimal inflation rate

- Money growth μ affects (steady state) inflation in two ways

$$\pi = \mu - \underbrace{(\Phi(\iota^*(\mu)) - \delta)}_{g}$$

- Proposition:
  - If $\frac{\sigma}{\sqrt{\rho}} > \frac{2(A\kappa+1)}{1+2\kappa\rho}$, welfare maximizing money growth rate $\mu^* > 0$.
    - Market outcome is not even constrained Pareto efficient
    - Economic growth rate, $g > r^m$, is also higher
  - Growth maximizing $\mu^{g*} \geq \mu^*$, s.t. $p^{g*} = 0$, Tobin (1965)

$$\iota^* = A - \rho \frac{\sigma}{\sqrt{\rho + \hat{\mu}}}$$ increasing in $\hat{\mu}$

- Corollary: No super-neutrality of money
  - Nominal money growth rate affects real economy
    - No price/wage rigidity, no monopolistic competition

Optimal Inflation rate: Emerging markets

- Proposition: (comparative static)
  - μ^* does not depend on depreciation rate δ, but inflation does
  - μ^* is strictly increasing in idiosyncratic risk σ
    "Emerging markets should have higher inflation target"

Conclusion: our 3 initial questions

- What should the (long-run) optimal inflation rate be?
  - Competitive market outcome is constrained Pareto inefficient.
  - Inflation is Pigouvian & internalizes pecuniary externality!
    - HH take real interest rate as given, but
    - Portfolio choice affects economic growth and real interest rate

- What role do financial frictions play?
  - incomplete markets ⇒ no superneutrality of money
    - No price/wage rigidity needed

- Emerging markets, with less developed financial markets, should have higher inflation rate/target
  - Higher idiosyncratic risk ⇒ higher pecuniary externality