



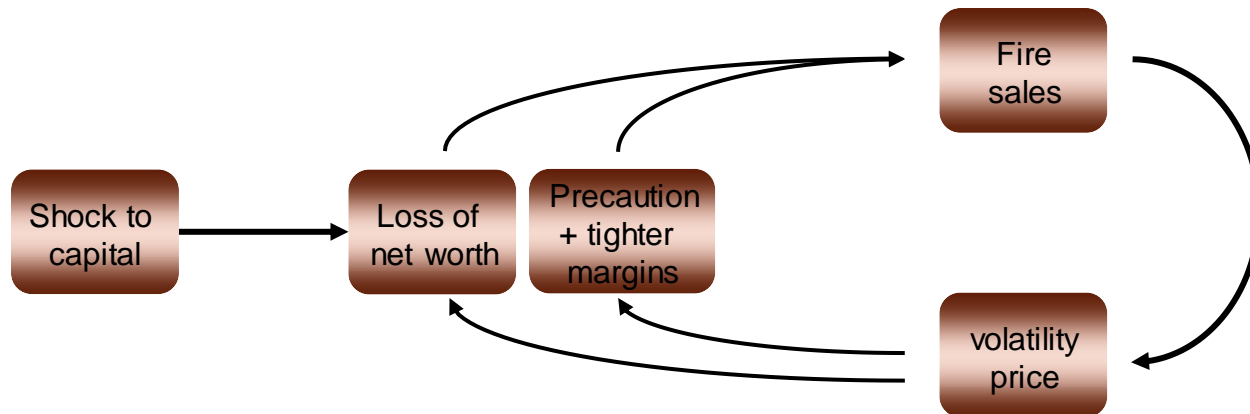
# PRINCETON INITIATIVE: BUBBLES

## MARKUS BRUNNERMEIER

Princeton University

# Systemic risk – a broad definition

- Systemic **risk build-up** during (credit) bubble ... and materializes in a crisis
  - “Volatility Paradox” → contemp. measures inappropriate
- Spillovers/contagion – **externalities**
  - Direct contractual: domino effect (interconnectedness)
  - Indirect: price effect (fire-sale externalities)  
credit crunch, liquidity spirals



- *Adverse GE response* → **amplification, persistence**



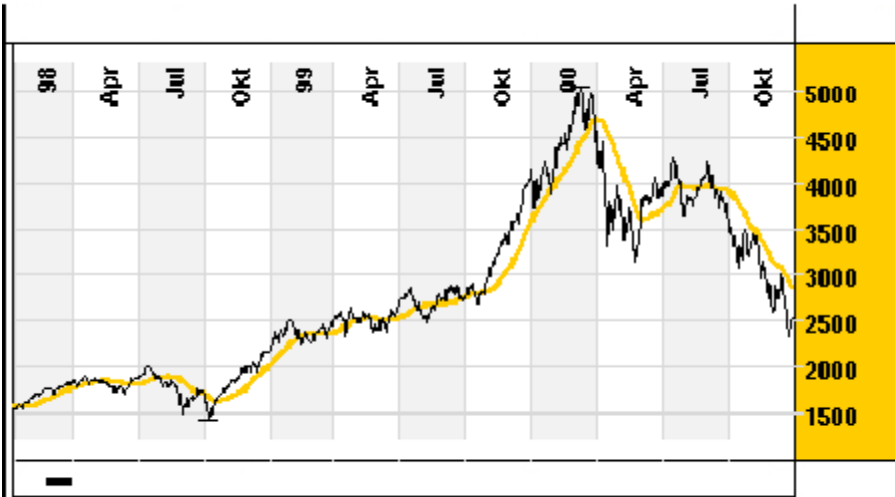
# BUBBLES AND CRASHES

DILIP ABREU AND MARKUS BRUNNERMEIER

Princeton University

# Internet bubble

- 1990's

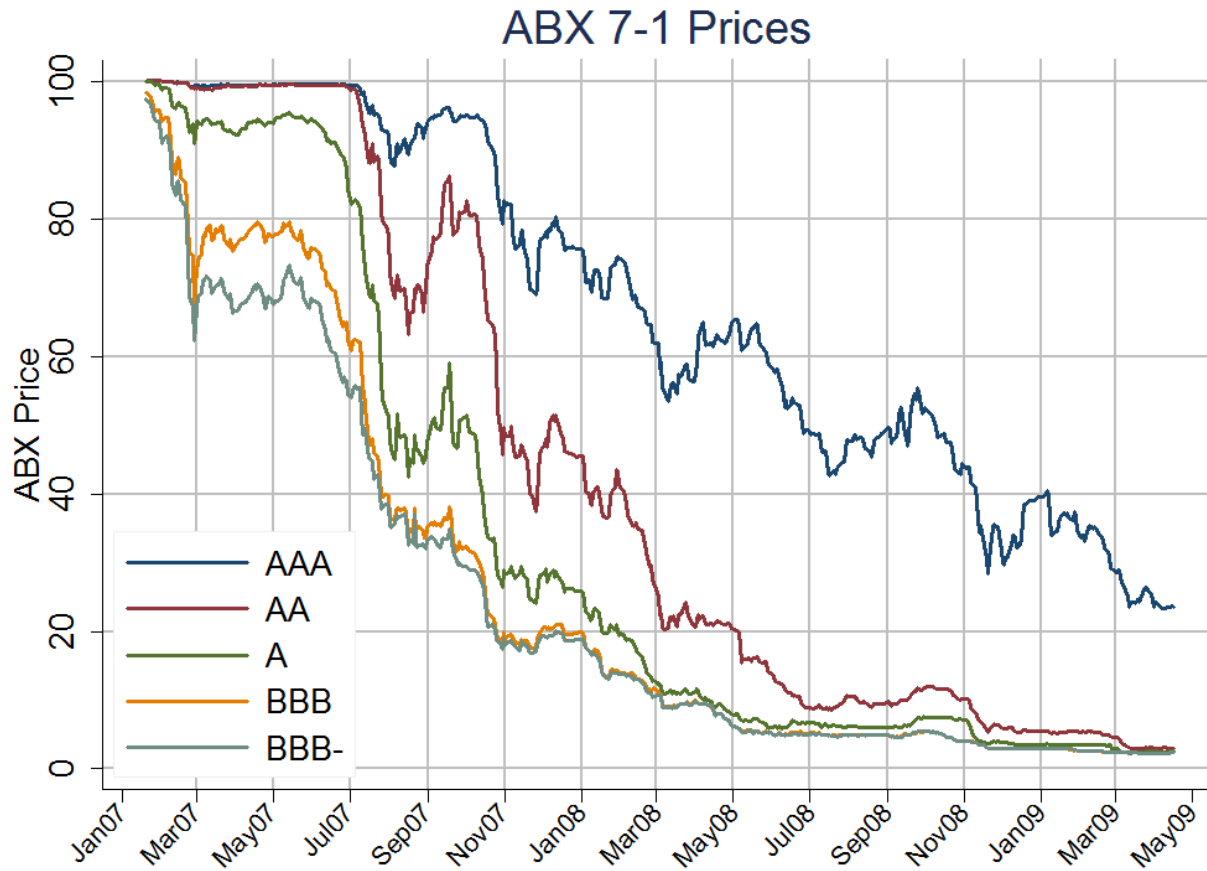


Loss of ca. **60 %**  
from high of \$ 5,132

Loss of ca. **85 %**  
from high of Euro 8,583

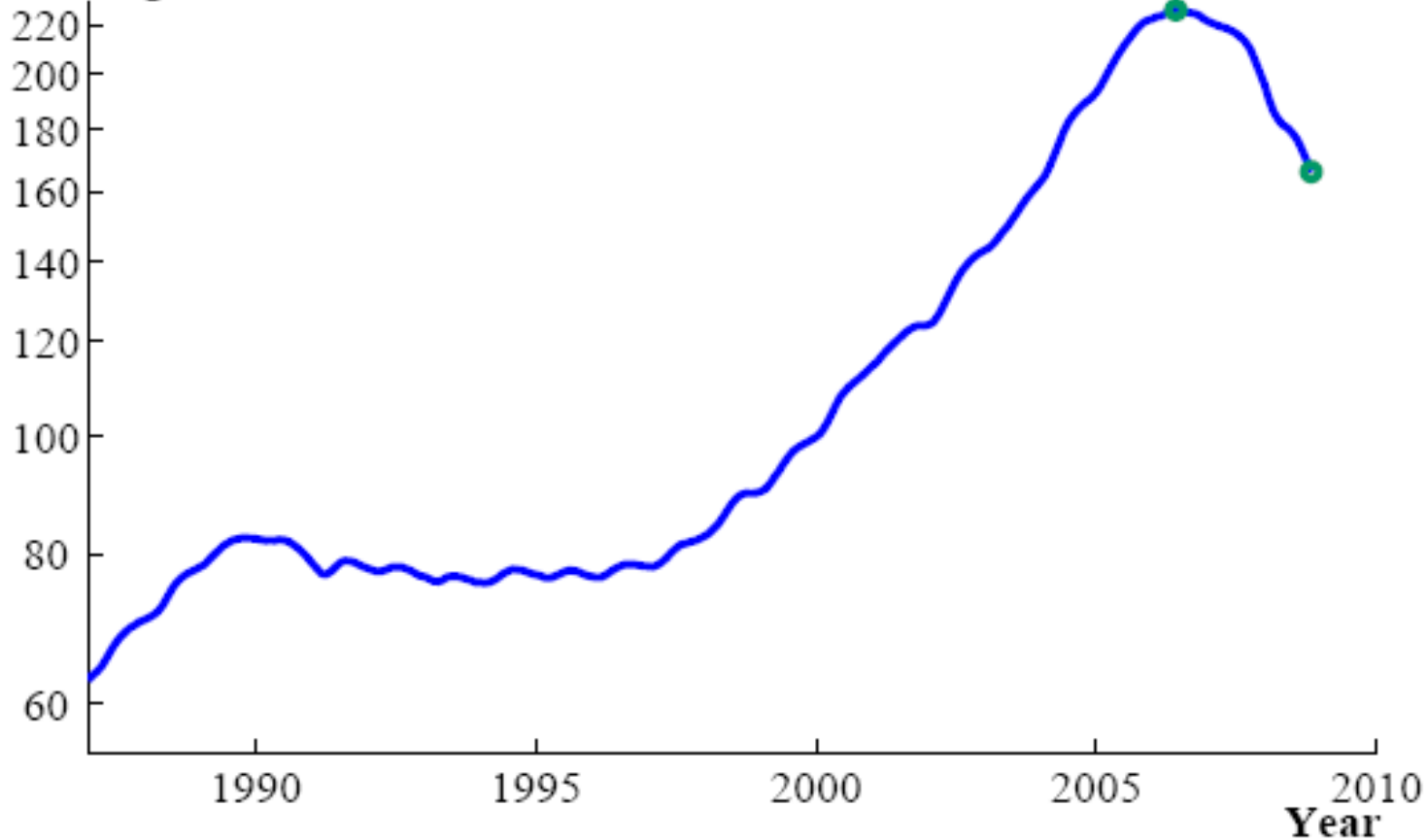
- Why do bubbles persist?
- Do professional traders ride the bubble or attack the bubble (go short)?
- What happened in March 2000?

# Credit bubble 2004-2006



# US House price index – Case-Shiller

Housing Price Index (Jan 2000=100, ratio scale)



# Do (rational) professionals ride the bubble?

- South Sea Bubble (1710-1720)
  - Issac Newton
    - 04/20/1720 sold shares at £ 7,000 profiting £3,500
    - Re-entered the market later – ending up losing £20,000
    - “I can calculate the motions of the heavenly bodies, but not the madness of people”
- Internet Bubble (1992 – 2000)
  - Druckenmiller of Soros’ Quantum Fund didn’t think that the party would end so quickly.
    - “We thought it was the eighth inning, and it was the ninth.”
  - Julian Robertson of Tiger Fund refused internet stocks.
- Housing bubble (2007)
  - Chuck Prince “Dance as long as the music is playing”

# Stylized facts

- Initial innovation justifies some price increase
- Momentum leads to price overshooting
  - Extrapolative expectations
- Many market participants seem to be aware that the “price is too high” but keep on holding the asset
  - “Play as long as the music is playing”
- Resell-option is crucial for speculative bubbles
- Minsky moment – triggered by “trivial news”
- Credit bubbles lead to extra amplification effects in downturn (since they can impair financial sector)
  - subprime borrowing was only 4% of US mortgage market



# || Minsky moment – Wile E. Coyote Effect



# Overview of Bubble Literature

- Rational bubbles

- Difference equation 
$$b_t = E_t^Q \left[ \frac{1}{1+r} b_{t+1} \right]$$
- No zero-sum argument

- OLG and incompleteness frictions (morning lecture)

- Samuelson, Triole, ... Bewley, ... Noise trader risk (DSSW)

- Informational frictions

- Synchronization Risk (Abreu & Brunnermeier 2003)

- Delegated investment friction

- Allen & Gorton 1993, Allen & Gale 2000, Shleifer & Vishny 1997

- Heterogeneous beliefs bubbles

- Harrison & Kreps 1978, Scheinkman & Xiong, Hong & Stein <sup>15</sup>

# On Market Efficiency

- Keynes (1936)  $\Rightarrow$  bubble can emerge
  - "It might have been supposed that *competition between expert professionals*, possessing judgment and knowledge beyond that of the average private investor, would correct the vagaries of the ignorant individual left to himself."
- Friedman (1953), Fama (1965)

Efficient Market Hypothesis  $\Rightarrow$  no bubble emerges

- "If there are many sophisticated traders in the market, they may cause these "bubbles" to burst before they really get under way."

# || Limits to Arbitrage

- Fundamental risk (Campell & Kyle 1993)
  - Risk that fundamental overturns mispricing
- Noise trader risk (DSSW)
  - Risk that irrational traders drive price even further from fundamentals
- Synchronization risk
  - One trader alone cannot correct mispricing (can sustain a trade only for a limited time)
  - Risk that other rational traders do not act against mispricing (in sufficiently close time)
  - Relatively unimportant news can serve as synchronization device and trigger a large price correction

# Timing Game - Synchronization

- (When) will behavioral traders be overwhelmed by rational arbitrageurs?
- *Collective* selling pressure of arbitrageurs *more than suffices* to burst the bubble.
- Rational arbitrageurs understand that an *eventual* collapse is inevitable.  
But when?
- Delicate, difficult, dangerous ***TIMING GAME !***

# Elements of the Timing Game

- *Coordination* at least  $\kappa > 0$  arbs have to be 'out of the market'
- *Competition* only *first*  $\kappa < 1$  arbs receive pre-crash price.
- *Profitable ride* ride bubble as long as possible.
- *Sequential Awareness*

## *A Synchronization Problem arises!*

- Absent of sequential awareness  
competitive element dominates  $\Rightarrow$  and bubble burst immediately.
- With sequential awareness  
incentive to TIME THE MARKET  $\Rightarrow$  "delayed arbitrage"  
 $\Rightarrow$  persistence of bubble

# Overview

- Introduction
- Model setup
- Preliminary analysis
- Persistence of bubbles
- Public events
- Price cascades and rebounds
- Empirical evidence & Hedge funds
  - Brunnermeier & Nagel (2004)



- Common action of  $\kappa$  arbs
- Sequential awareness

Random  $t_0$  with  $F(t_0) = 1 - e^{-\lambda t_0}$

$$p_t = e^{gt}$$

$$\bar{\beta} p_t$$

$$(1 - \beta(\cdot)) p_t$$

1

0

**paradigm shift**

- internet 90's
- railways
- etc.

$t_0$   
random  
starting  
point

$t_0 + \eta\kappa$   
 $\kappa$  traders  
are aware of  
the bubble

$t_0 + \eta$   
all traders  
are aware of  
the bubble

$t_0 + \bar{\tau}$   
bubble bursts  
for exogenous  
reasons

$1/\eta$

maximum life-span of the bubble  $\bar{\tau}$



# Payoff structure

- Focus: “when does bubble burst”
- $t_0$  is only random variables, all other variables are CK
- Cash payoff (difference)
  - Sell one share at  $t - \Delta$  instead of at  $t$

$$p_{t-\Delta}e^{r\Delta} - p_t$$

$$\text{where } p_t = \begin{cases} e^{gt} & \text{prior to crash} \\ (1 - \beta(t - t_0))e^{gt} & \text{after the crash} \end{cases}$$

- Price at the time of bursting (tie breaking rule)
  - Pre crash price for first random orders up to  $\kappa$

# Payoff structure, Trading

- Small transaction costs  $ce^{rt}$
- Risk-neutrality but max/min stock position
  - Max long position
  - Max short position
  - Due to capital constraints, margin requirements etc.
- Definition 1: trading equilibrium
  - Perfect Bayesian Nash Equilibrium
  - Belief restriction: trader who attacks at time  $t$  believes that all traders who became aware of the bubble prior to her also attack at  $t$ .

# || Sell out condition for $\Delta \rightarrow 0$ periods

- Sell out at  $t$   
if

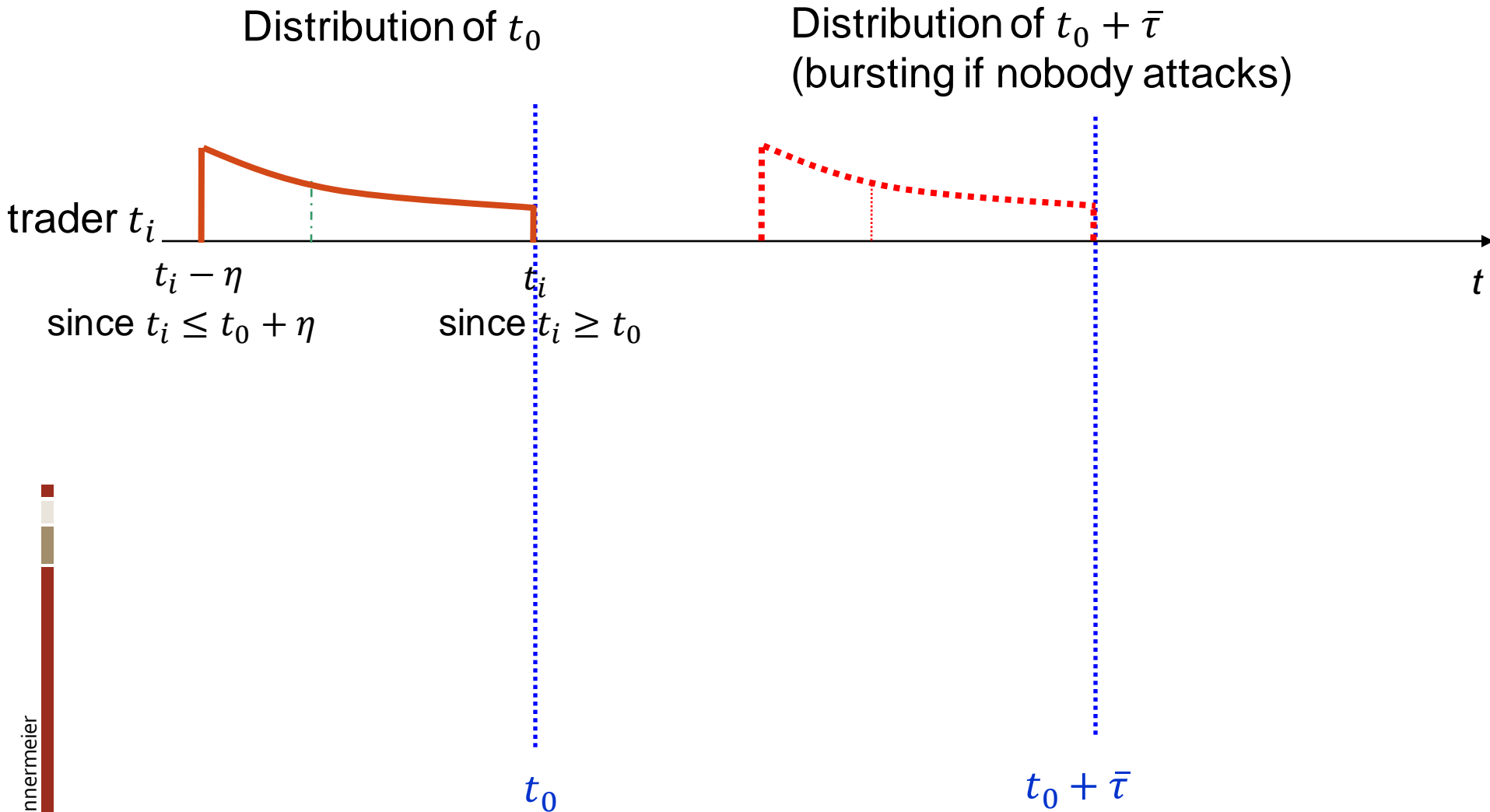
$$\underbrace{\Delta h(t|t_i) E_t[\beta p_t | \cdot]}_{\text{benefit of attacking}} \geq (1 - \Delta h(t|t_i)) \underbrace{(g - r) p_t \Delta}_{\text{cost of attacking}}$$

appreciation rate

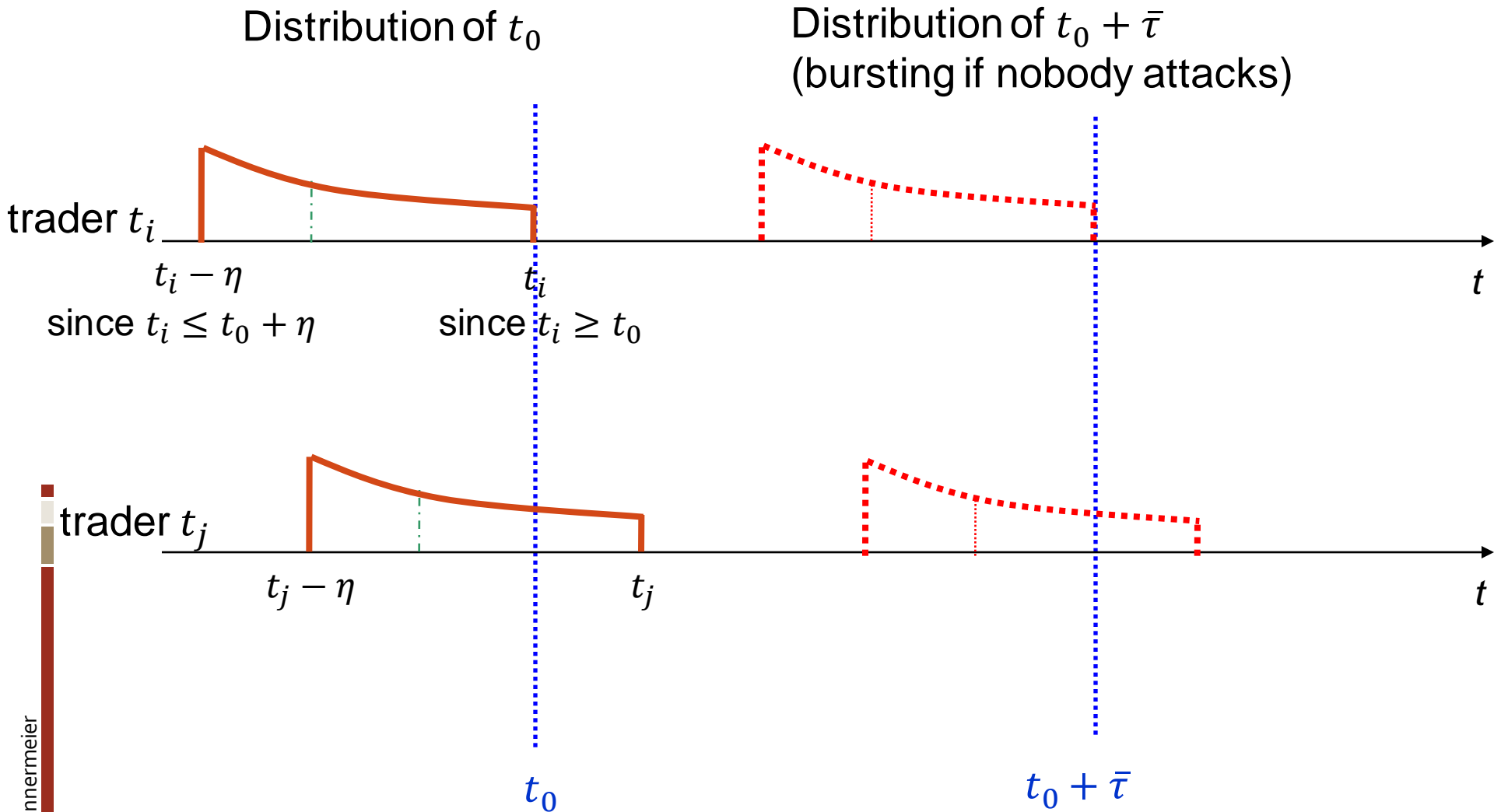
$$h(t|t_i) \geq \frac{g - r}{\beta^*}$$

- RHS  $\rightarrow (g - r)$  as  $t \rightarrow \infty$ 
  - Bursting date:  $T^*(t_0) = \min\{T(t_0 + \eta\kappa), t_0 + \bar{\tau}\}$

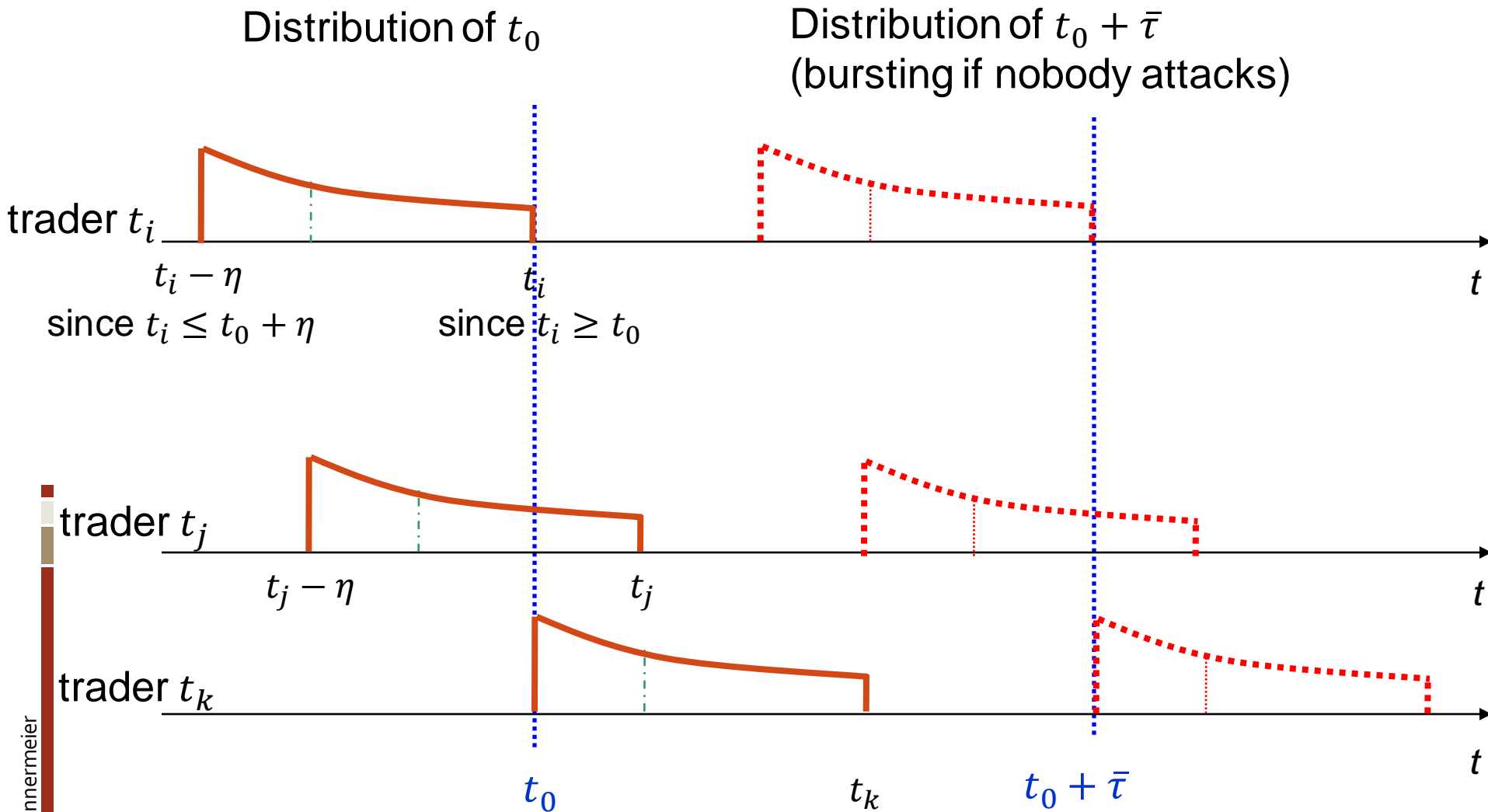
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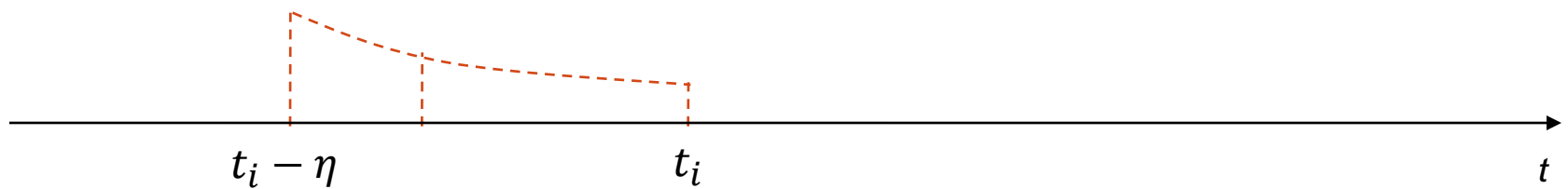


# Sequential awareness



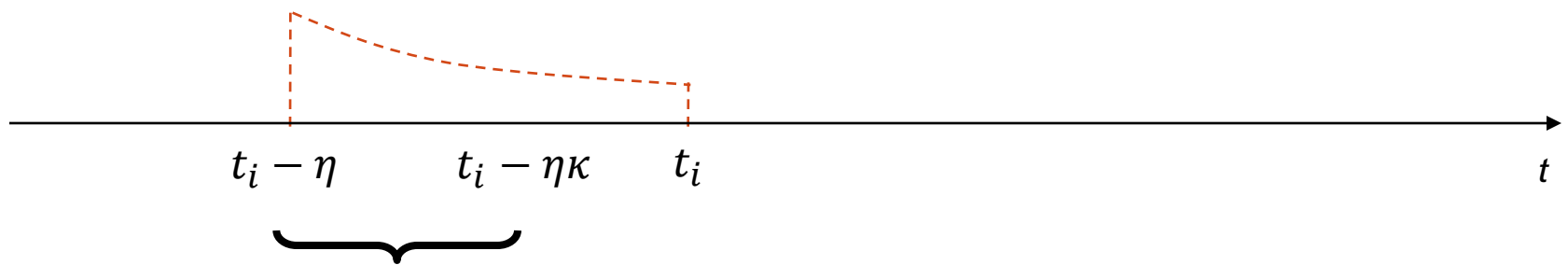
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⇒ **Bubble bursts at  $t_0 + \eta\kappa$**   
when  $\kappa$  traders are aware of the bubble



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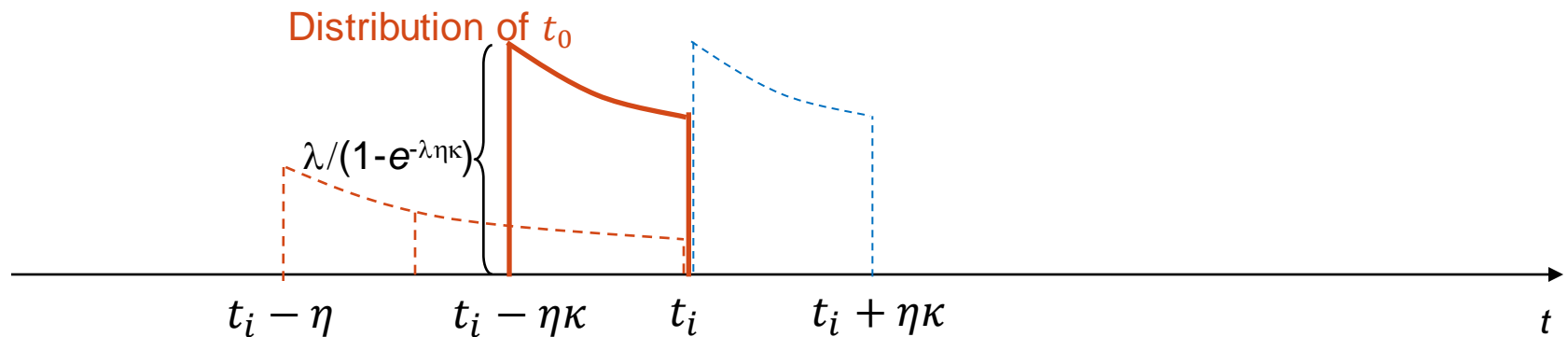


If  $t_0 < t_i - \eta\kappa$ , the bubble would have burst already.



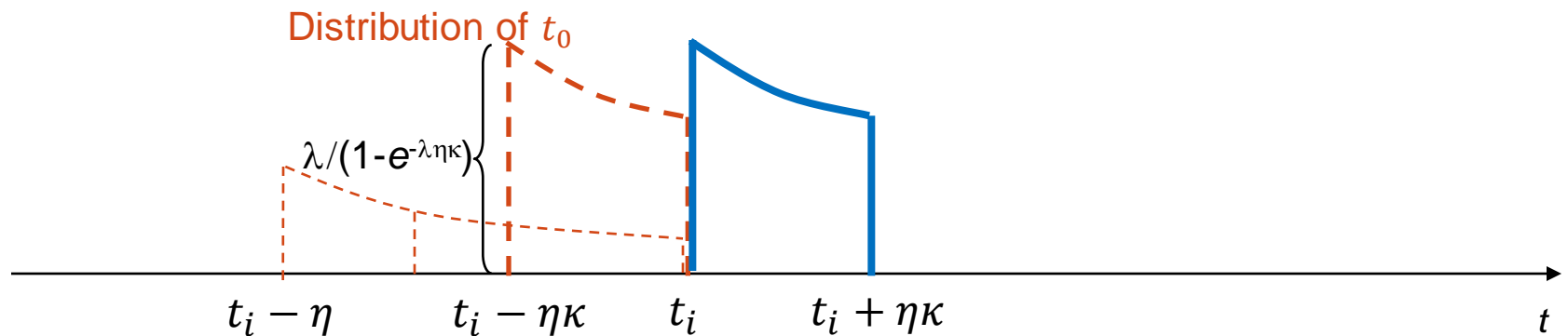
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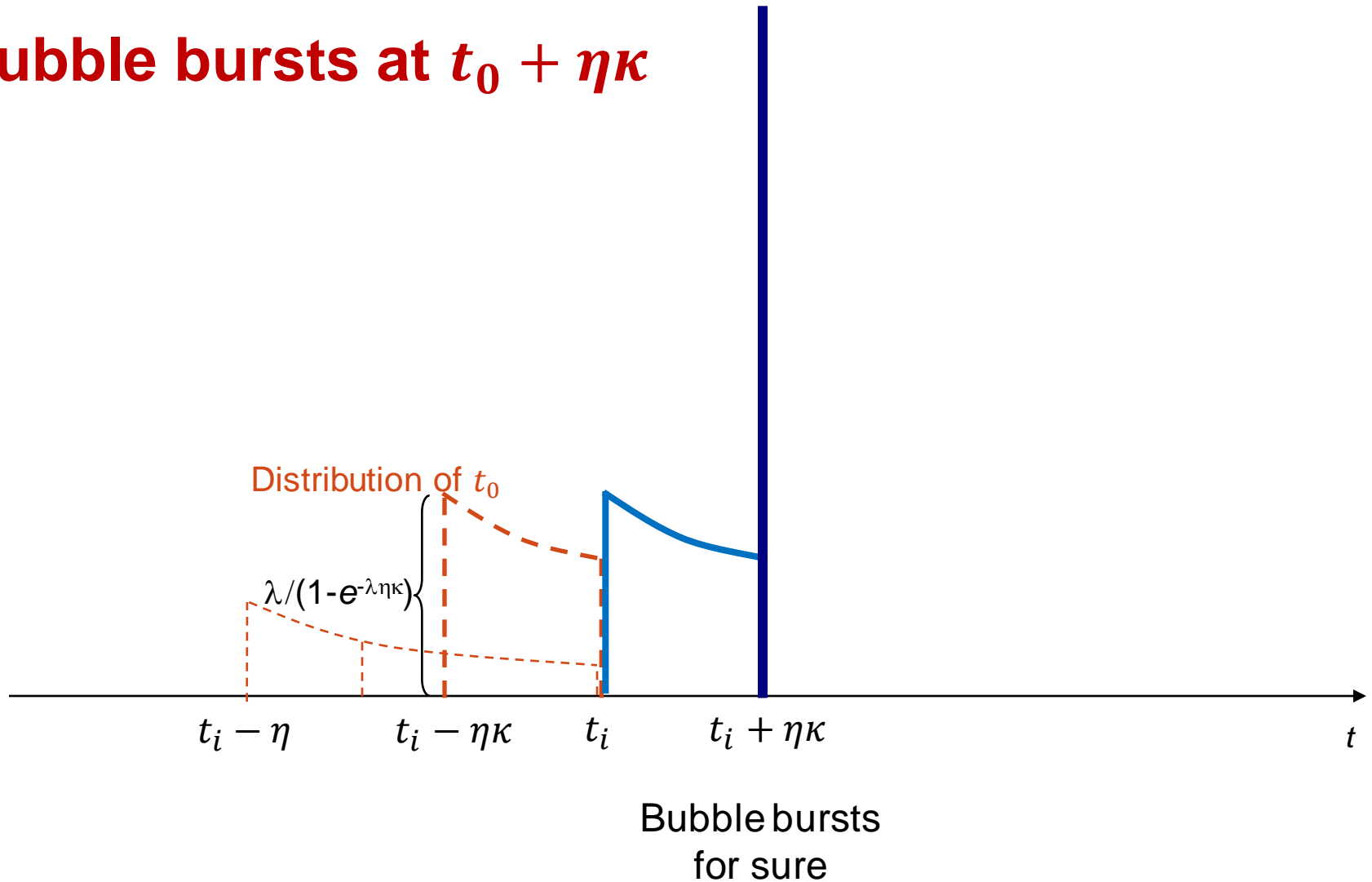
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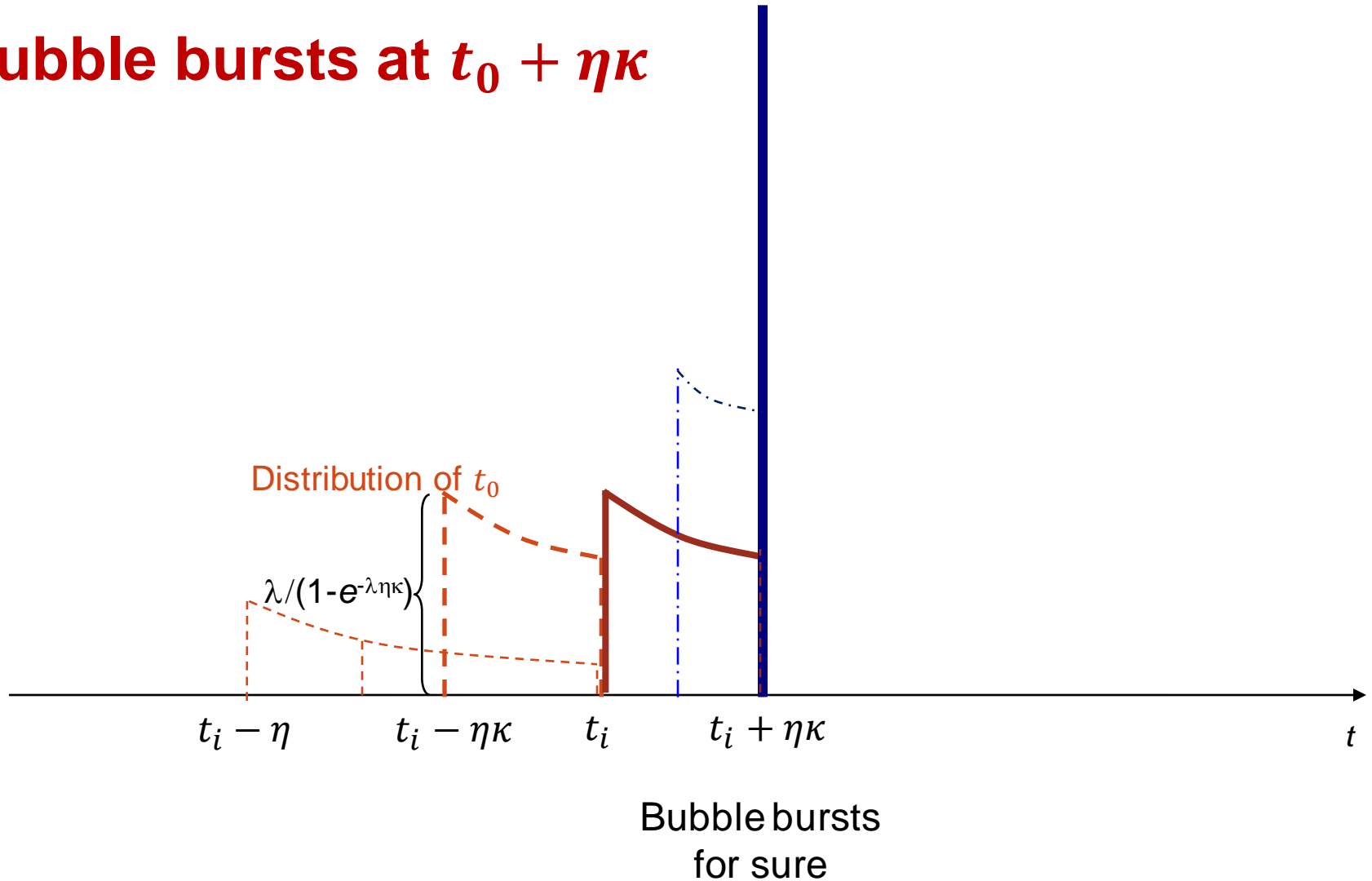
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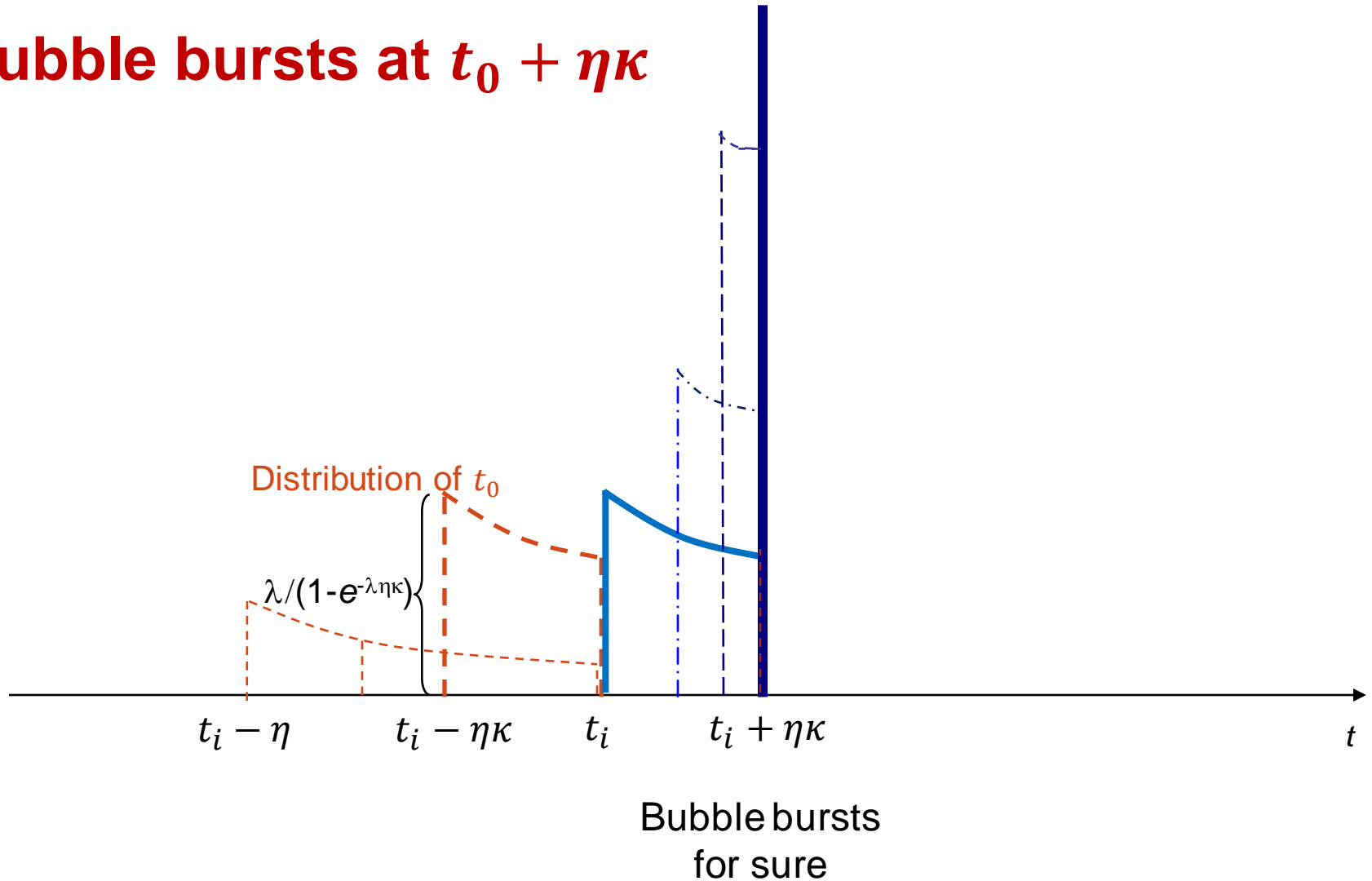
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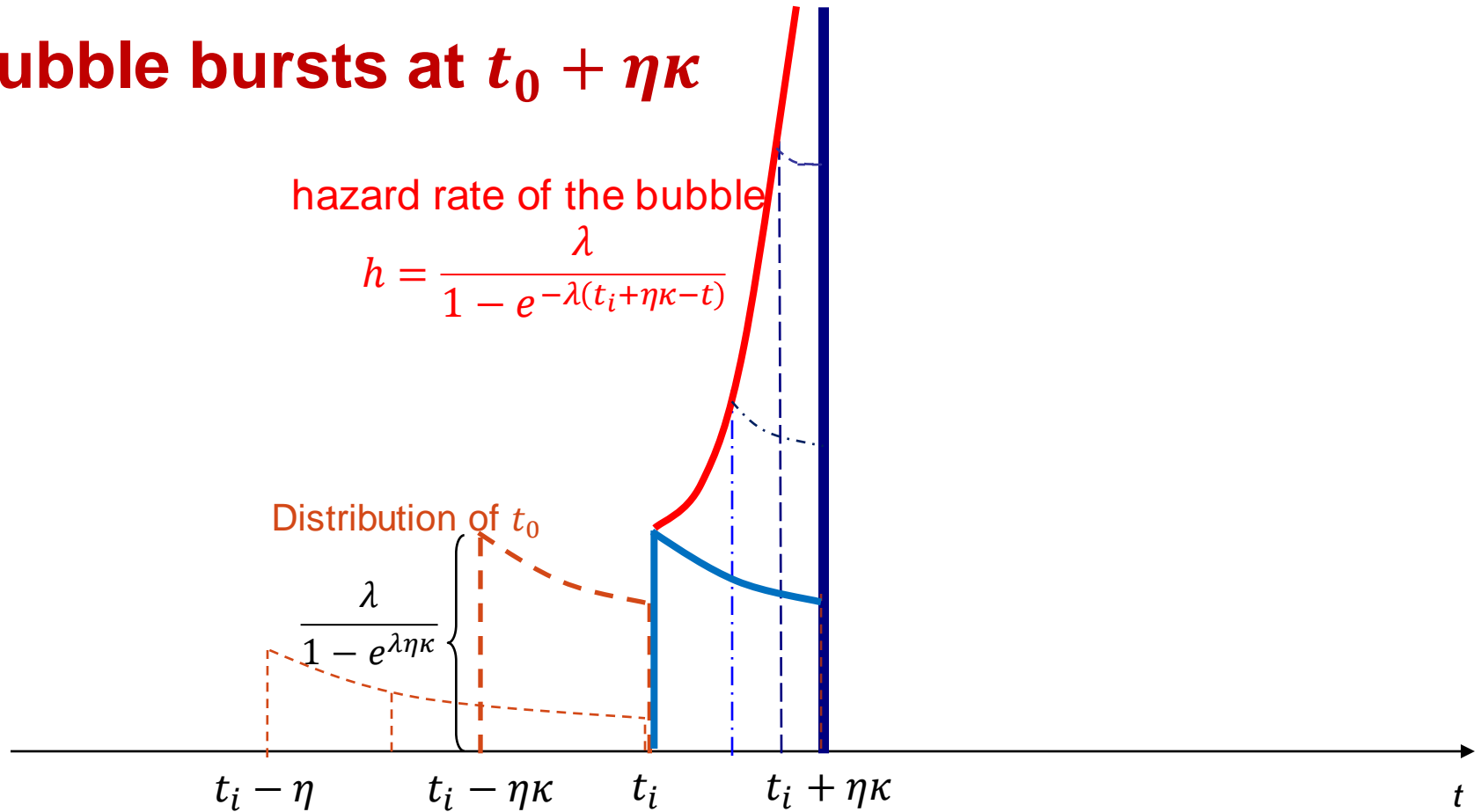
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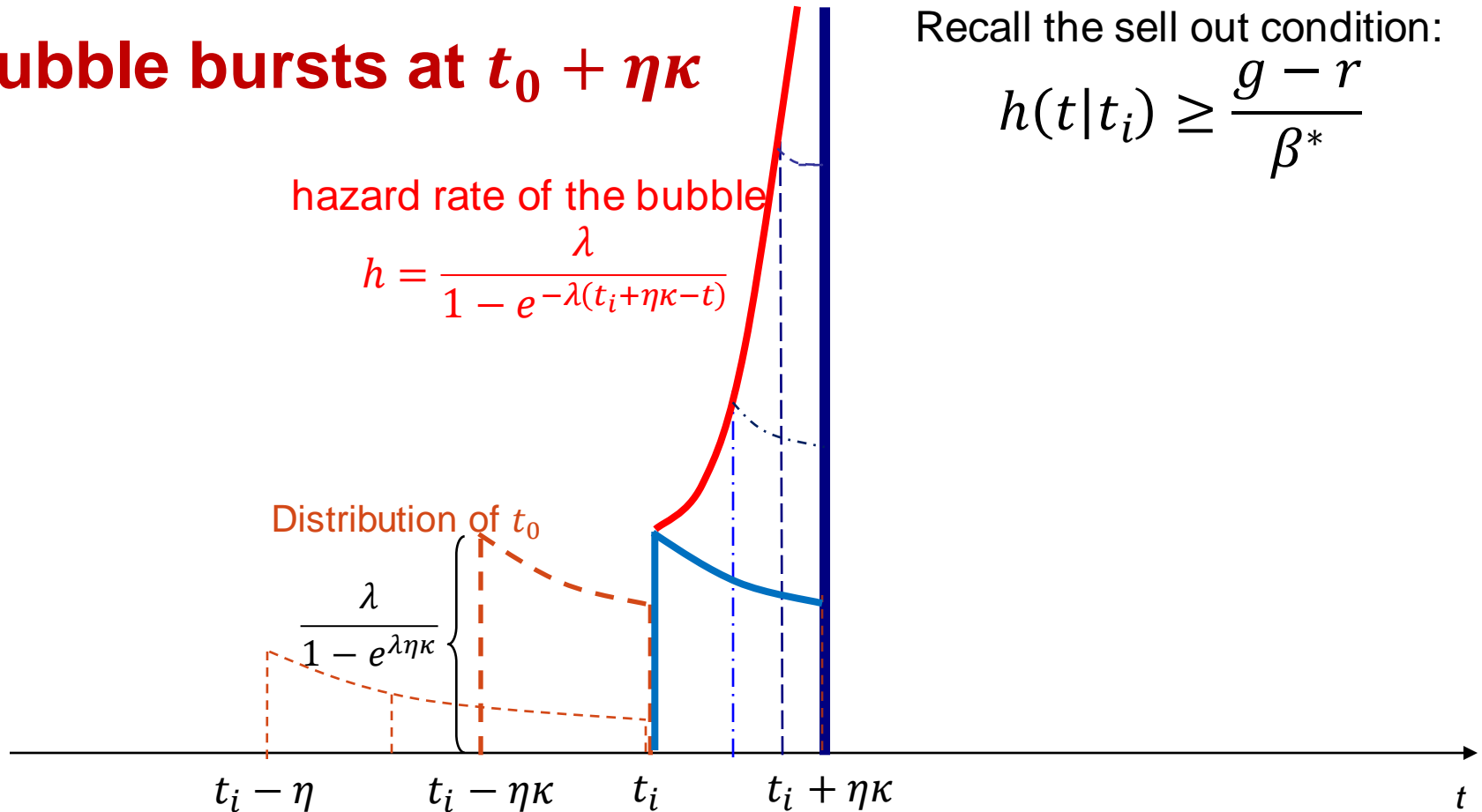


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⇒ **Bubble bursts at  $t_0 + \eta\kappa$**

Recall the sell out condition:

$$h(t|t_i) \geq \frac{g - r}{\beta^*}$$

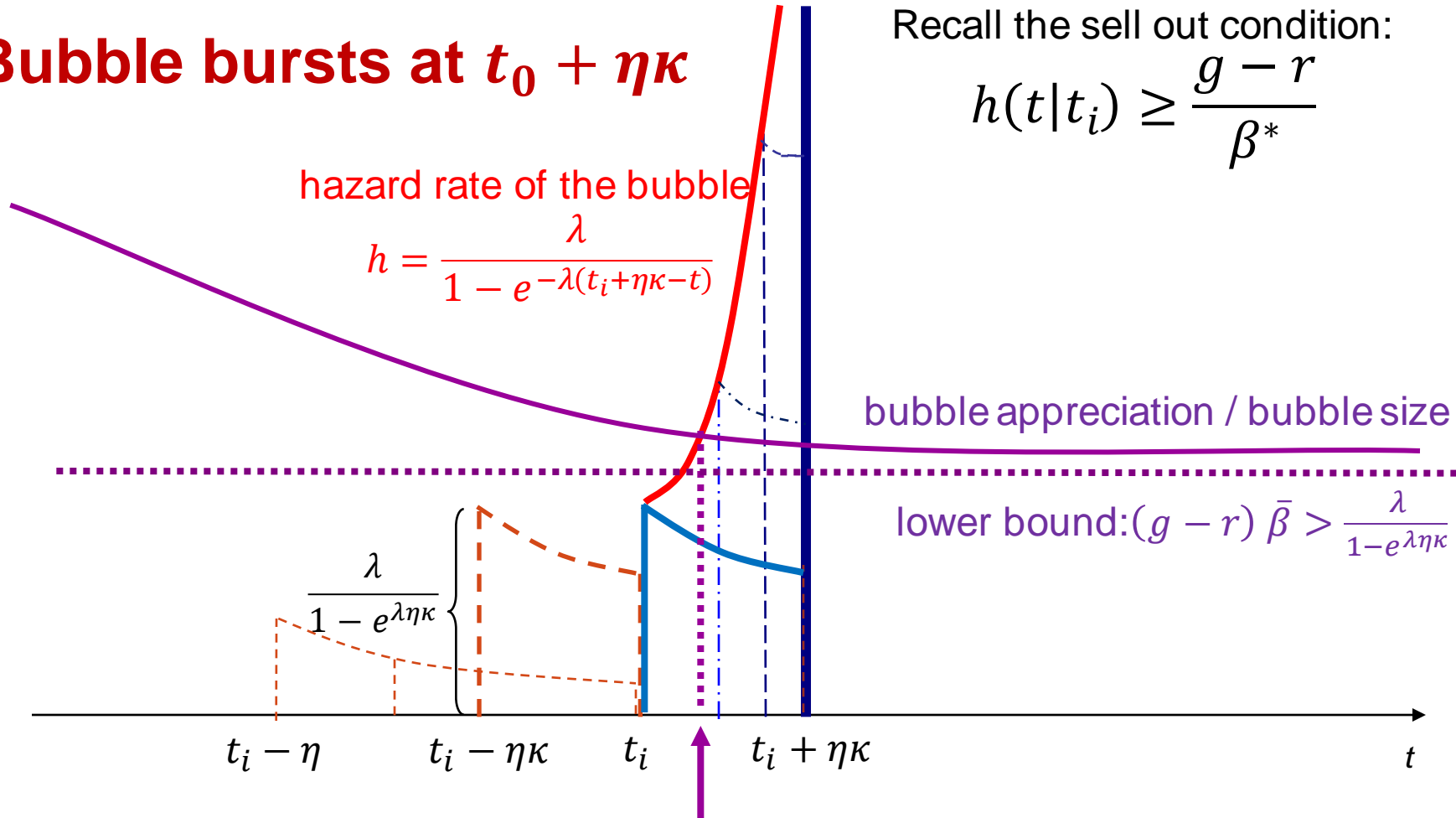


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Recall the sell out condition:

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optimal time to attack  $t_i + \tau_i$  ⇒ **“delayed attack is optimal”**



# Endogenous Crash for large enough $\bar{\tau}$ (i.e. $\bar{\beta}$ )

- Proposition 3: Suppose  $\frac{\lambda}{1-e^{-\lambda\eta\kappa}} > \frac{g-r}{\bar{\beta}}$ 
  - Unique trading equilibrium
  - Traders begin attacking after a delay of  $\tau^*$  periods
  - Bubble bursts due to endogenous selling pressure at a size of  $p_t$  times

$$\beta^* = \frac{1 - e^{-\lambda\eta\kappa}}{\lambda} (g - r)$$

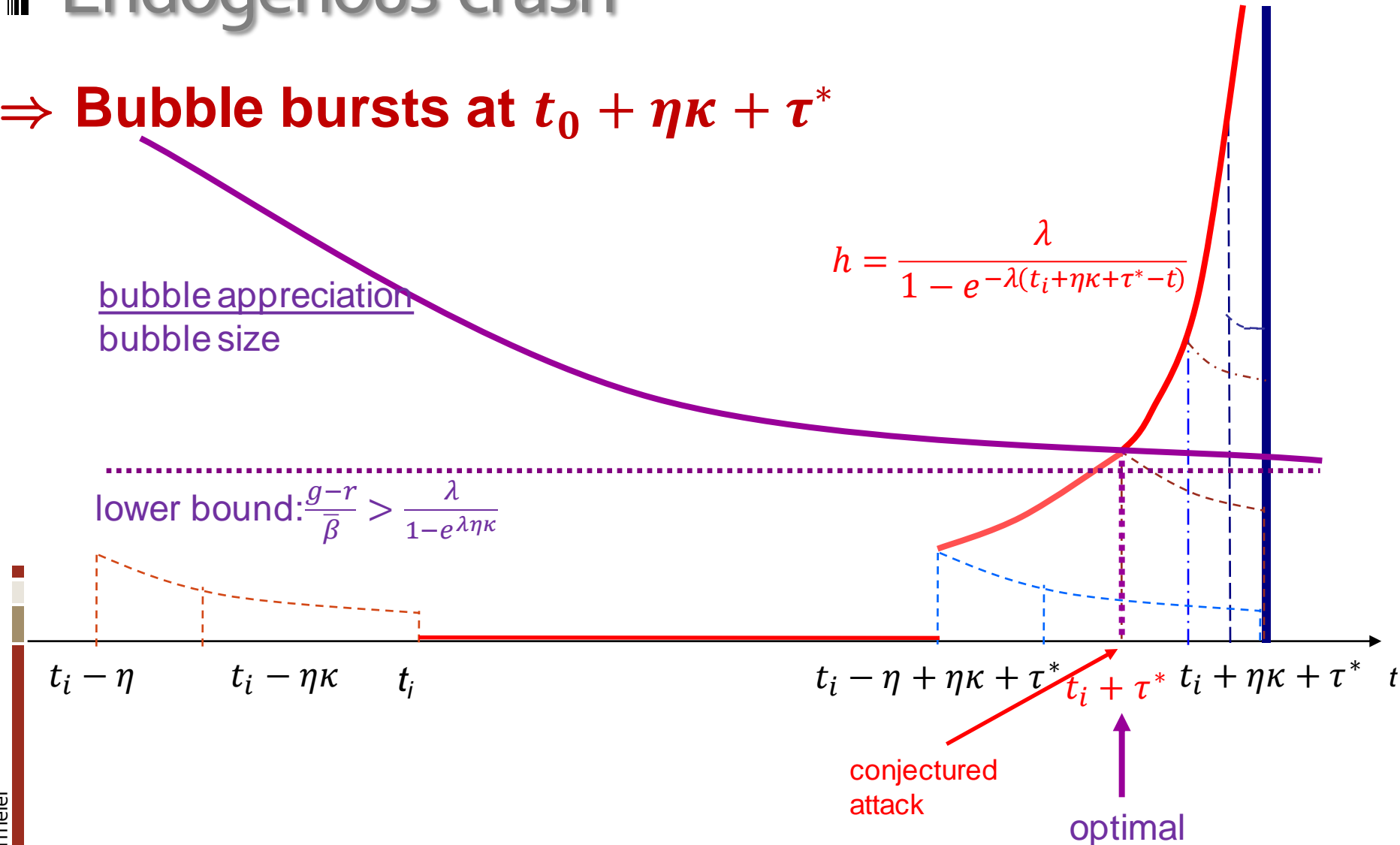
# Endogenous crash

⇒ **Bubble bursts at  $t_0 + \eta\kappa + \tau^*$**

bubble appreciation  
bubble size

$$h = \frac{\lambda}{1 - e^{-\lambda(t_i + \eta\kappa + \tau^* - t)}}$$

lower bound:  $\frac{g-r}{\beta} > \frac{\lambda}{1 - e^{-\lambda\eta\kappa}}$

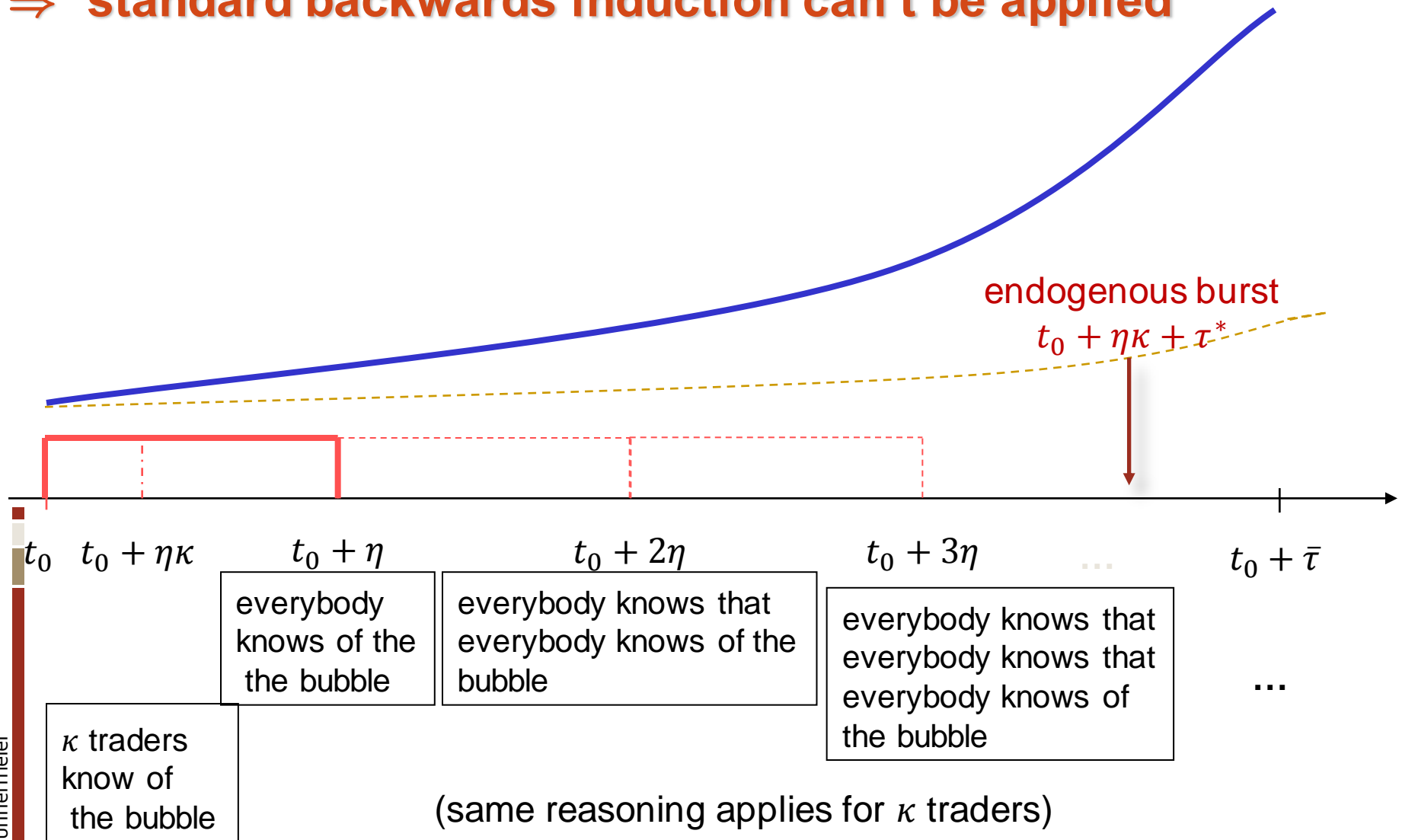


# Exogenous crash for low $\bar{\tau}$ (i.e. low $\bar{\beta}$ )

- Proposition 2: Suppose  $\frac{\lambda}{1-e^{-\lambda\eta\kappa}} \leq \frac{g-r}{\bar{\beta}}$ .
  - Unique trading equilibrium
  - Traders begin attacking after a delay of  $\tau^1 < \bar{\tau}$  periods.
  - Bubble does not burst due to endogenous selling pressure prior to  $t_0 + \bar{\tau}$ .

# Lack of common knowledge

⇒ **standard backwards induction can't be applied**



# Role of synchronizing events

- News may have an impact disproportionate to any intrinsic informational (fundamental) content
  - News can serve as a synchronization device
- Fads & fashion in information
  - Which news should traders coordinate on?
- When “synchronized attach” fails, then the bubble is temporarily strengthened

# Setting with synchronizing events

- Focus on news with no info content (sunspots)
- Synchronizing events occur with Poisson arrival rate
  - Note that pre-emption argument does not apply since event occurs with zero probability
- Arbitrageurs who are aware of the bubble become increasingly worried about it over time.
  - Only traders who became aware of the bubble more than  $\tau_e$  periods ago observe (look out for) this synchronizing event.

# || Synchronizing events – market rebounds

- Proposition 5: In 'responsive equilibrium'  
Sell out a) always at the time of the public event  $t_e$ ,  
b) after  $t_i + \tau^{**}$  (where  $\tau^{**} < \tau^*$ )  
except after a failed attack at  $t$ , re-enter the market  
for  $t \in (t_e, t_e - \tau_e + \tau^{**})$ .
- Intuition for re-entering the market
  - For  $t_e < t_0 + \eta\kappa + \tau_e$  attack fails, agents learn  $t_0 > t_e - \tau_e - \eta\kappa$
  - Without public event, they would have learnt this only at  $t_e + \tau_e - \tau^{**}$
  - Density that bubble burst for endogenous reasons is zero

# Conclusion of Bubbles and Crashes

- Bubbles
  - Dispersion of opinion among arbs causes a synchronization problem which makes coordinated price correction difficult.
  - Arbitrageurs time the market and ride the bubble  
⇒ Bubbles persist
- Crashes
  - Can be triggered by unanticipated news without any fundamental content, since
  - It might serve as synchronization device.
- Rebound
  - Can occur after a failed attack which temporarily strengthens the bubble



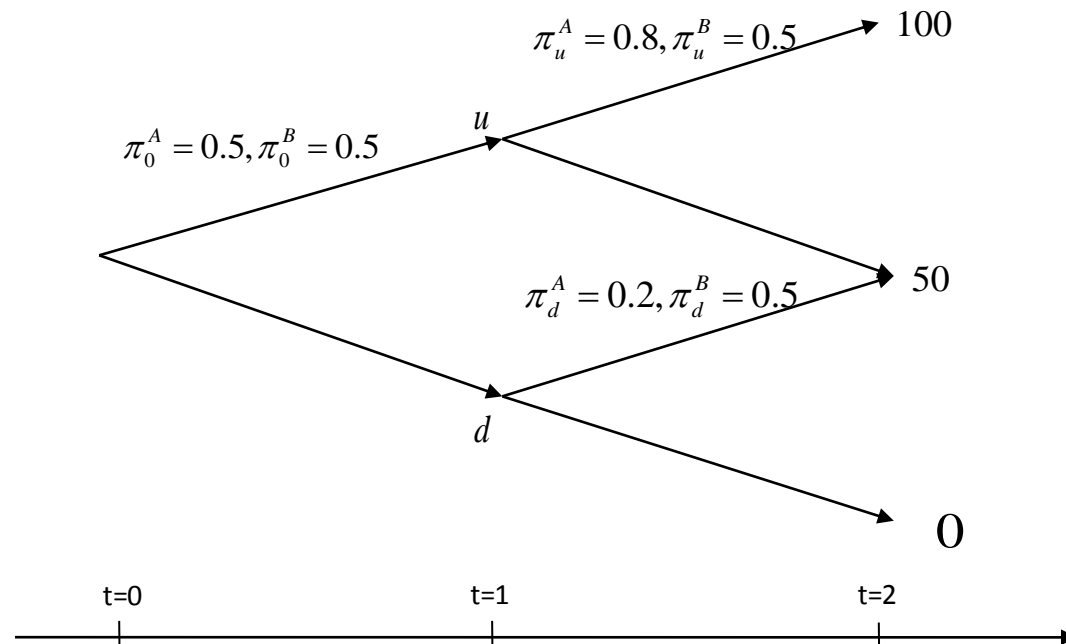


# HETEROGENEOUS BELIEFS BUBBLES

HARRISON AND KREPS, SCHEINKMAN AND XIONG

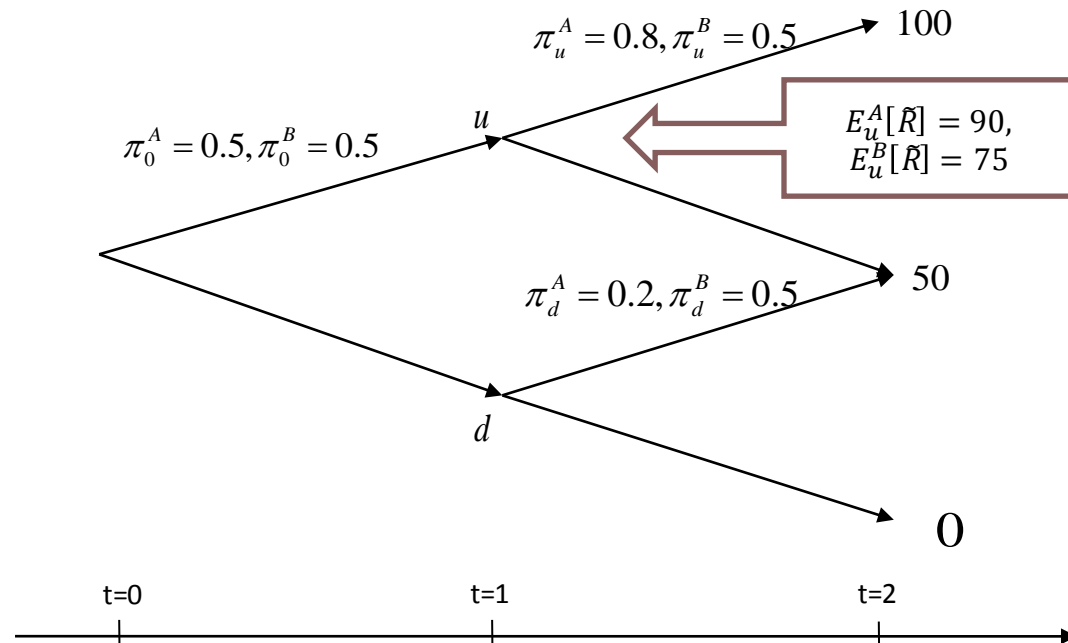
# Bubbles with Trading Cost – simplified example

- Two risk-neutral agents: A and B.
  - An asset with fixed supply, 1 unit equally divided bw A & B.
  - Heterogeneous beliefs; short-sales prohibited.
  - Harrison and Kreps (1978), Morris (1996), Scheinkman and Xiong (2003).



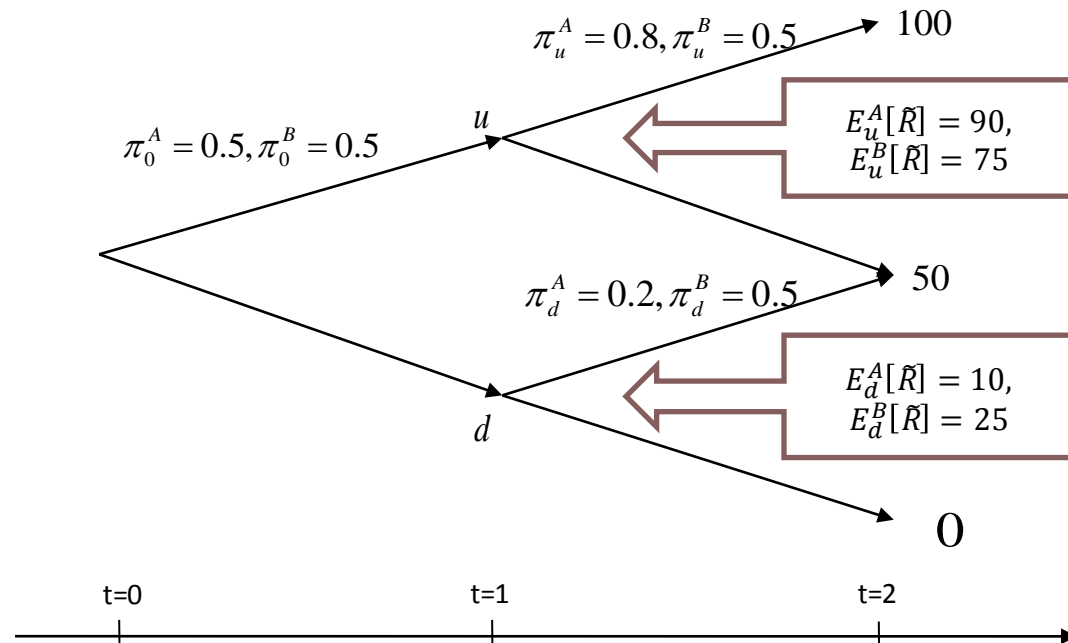
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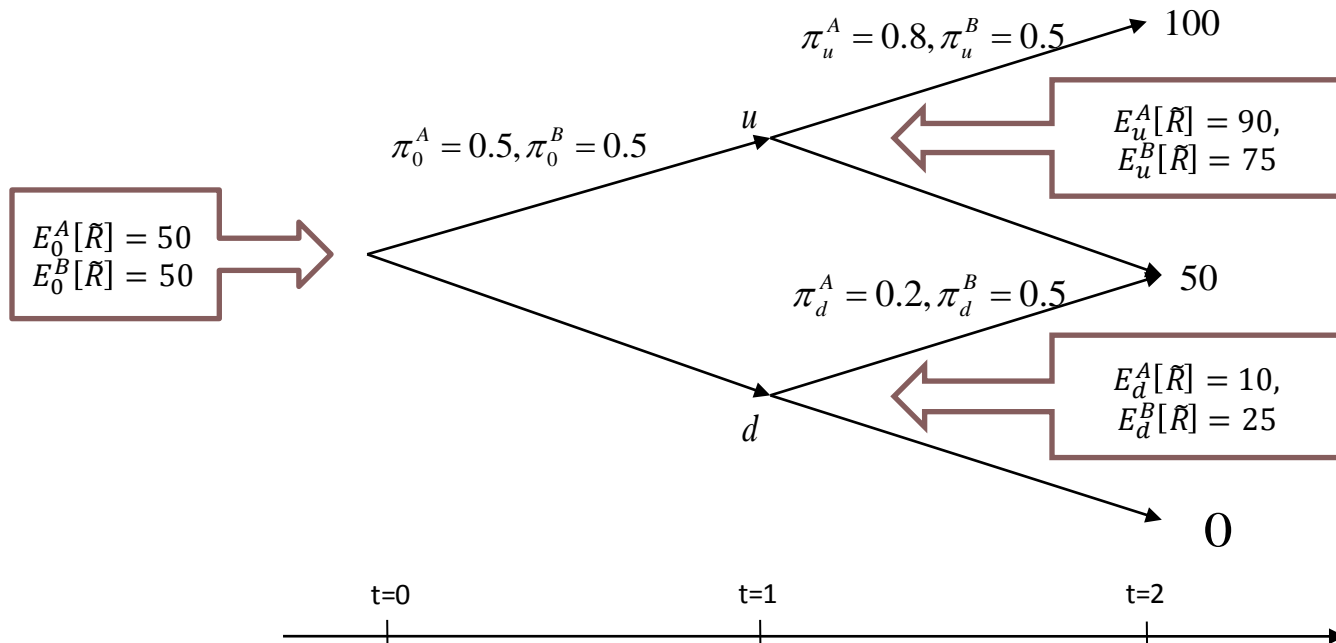
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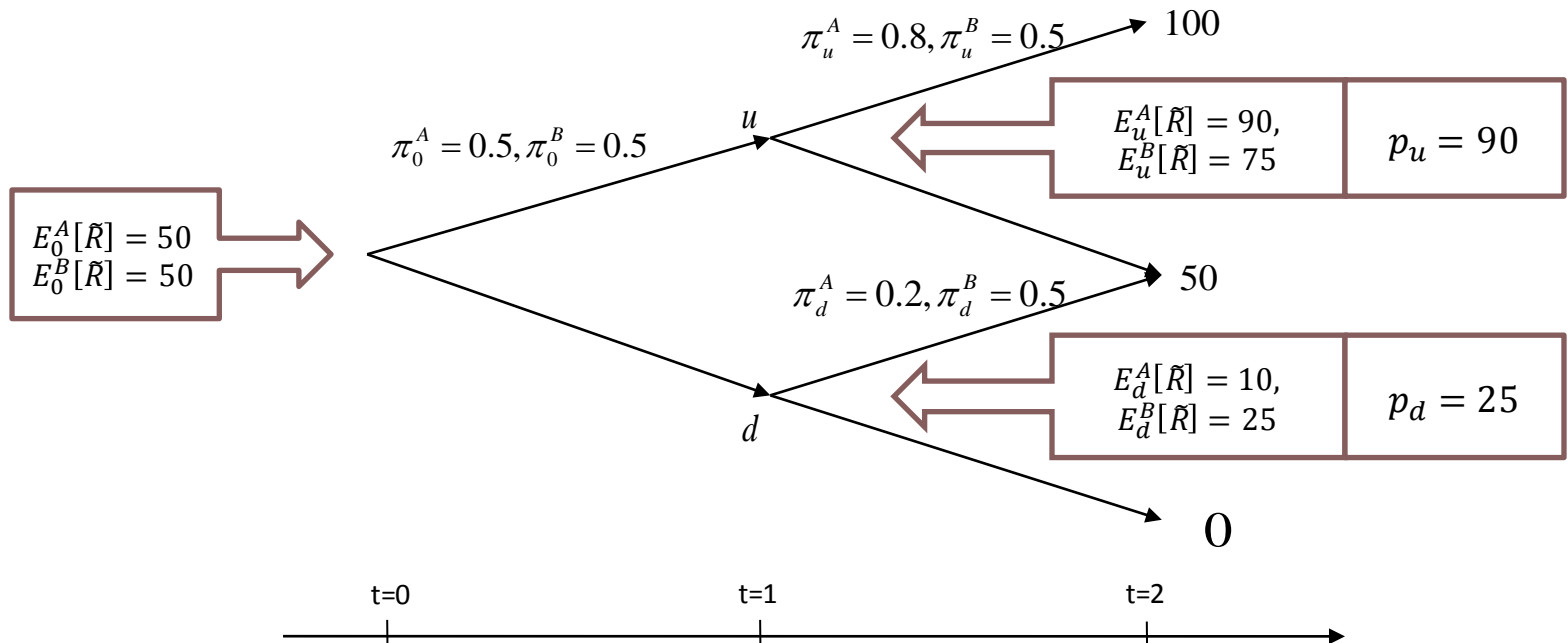
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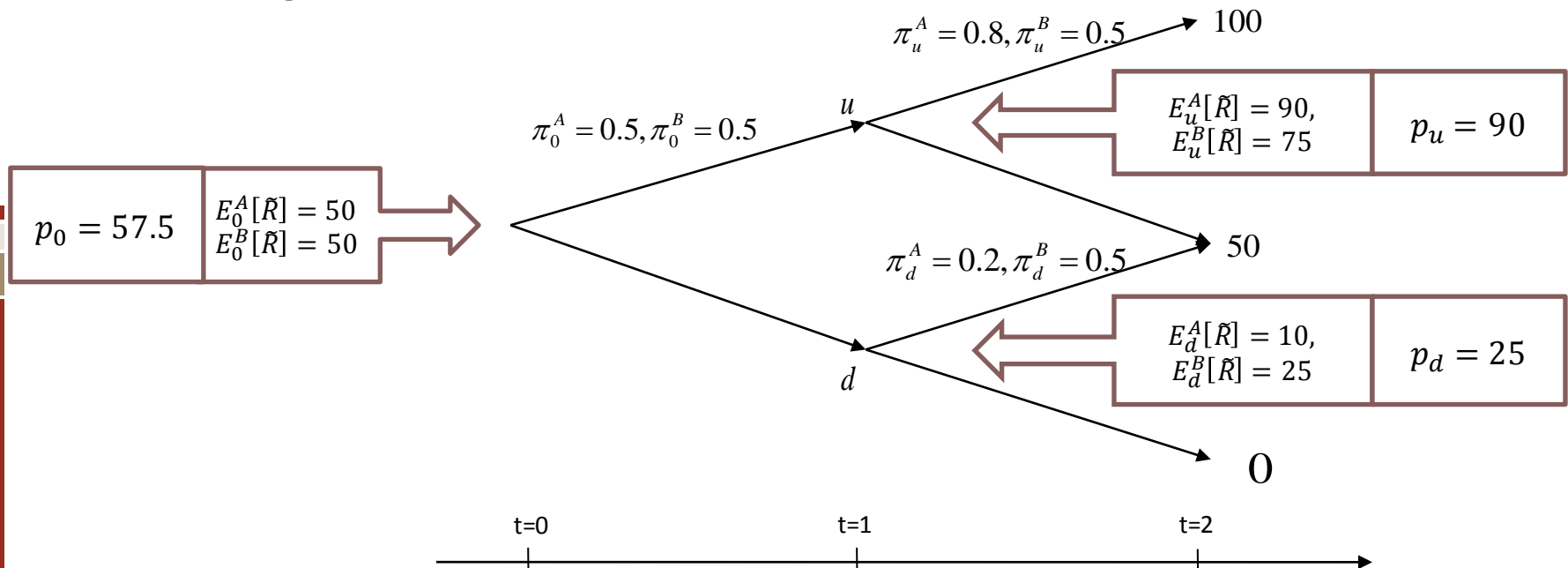
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# Welfare criteria heterogeneous beliefs

Brunnermeier & Xiong 2011

- Given a social welfare function  $W$ , allocation  $x \succcurlyeq_W x'$  if
  - $E^A [W(u^A(x), u^B(x))] \geq E^A [W(u^A(x'), u^B(x'))]$  AND
  - $E^B [W(u^A(x), u^B(x))] \geq E^B [W(u^A(x'), u^B(x'))]$

- Back to Bubble example

- Assume linear and symmetric social welfare function:

$$W(u_A, u_B) = u(c_A) + u(c_B) = c_A + c_B.$$

- At the status quo:

$$E_0^j [W(u_A, u_B)] = E_0^j [\tilde{R}] = 50, \forall j \in \{A, B\}.$$

- Suppose that trading costs  $k$  per share.
  - $k < 15$  so that trading occurs.

- In the equilibrium:

$$E_0^j [W(u_A, u_B)] = E_0^j [\tilde{R}] - \frac{k}{2} = 50 - \frac{k}{2}, \forall j \in \{A, B\}.$$



# Overview of Bubble Literature - again

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