Asset Pricing under Asymmetric Information
Bubbles & Limits to Arbitrage

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Overview

- All agents are rational
  - Bubbles under symmetric information
  - Bubbles under asymmetric information
- Interaction between rational arbitrageurs and behavioral traders - Limits to Arbitrage
  - Fundamental risk
  - Noise trader risk + Endogenous short horizons of arbs
  - Synchronization risk

Historical Bubbles

- 1634-1637 Dutch Tulip Mania (Netherlands)
- 1719-1720 Mississippi Bubble (France)
- 1720 South Sea Bubble (England)
- 1990 Japan Bubble
- 1999 Internet/Technology Bubble

Event	Tulipmania	Crisis of 1763	Crisis of 1772
Time	1634-1637	1763	1772-1773
Place	Netherlands	Amsterdam, Hamburg, Berlin	England, Scotland
Bubble asset	Tulips	Grains, sugar	Stocks and futures of East India Company, turnpikes, canals, enclosures, building construction

Type of bubble asset	Commodity	Commodity	Securities, real estate
Holder of asset	Small-town dealers, tavern-keepers, horticulturalists	Merchant bankers	London speculators, business men
Financier of asset	Sellers of bulbs	Amsterdam investors	Ayr Bank, country banks
Expansive monetary policy	No	No	Yes

Lending boom	No	Yes	Yes
Foreign capital inflows	No	No	No
Financial deregulation	No	No	Yes
Severe recession	No	No	Yes
Banking crisis	No	Yes	Yes
Spillover to other countries	No	Yes	Yes
Leaning	No	No	Yes
Pricking	No	No	No
Use of quantity instruments	No	No	No

Only cleaning	No	Yes	No
Sources	Garber (1989, 1990), Kindleberger (2005)	Kindleberger (2005), Schnabel & Shin (2004)	Hamilton (1956), Hoppit (1986), Kindleberger (2005), Sheridan (1960)

Event	Latin America Mania	Railway mania	Panic of 1857
Time	1824-1825	1840s	1856-1857
Place	England	England	United States
Bubble asset	Securities of Real and imaginary South American governments and mines, joint stock companies	Railway related stocks and property classes, corn	Railroad stocks and bonds, land
Type of bubble asset	Securities, commodity	Securities, commodity	Securities, real estate

Holder of asset	Widely held	Widely held	
Financier of asset	Country banks, Bank of England	Bank of England, government	Banks, foreign investors
Expansive monetary policy	Yes	Yes	Yes
Lending boom	Yes	Yes	Yes
Foreign capital inflows	No	Yes	Yes
Financial deregulation	No	No	No
Severe recession	Yes	Yes	Yes

Banking crisis	Yes	Yes	Yes
Spillover to other countries	Yes	No	Yes
Leaning	Yes	No	No
Pricking	Possibly	No	No
Use of quantity instruments	No	No	No
Only cleaning	Yes	Yes	Yes
Sources	Bordo (1998), Conant (1915), Kindleberger (2005), Neal (1995)	Evans (1849), Kindleberger (2005), Ward-Perkins (1950), WEO (2003)	Calomiris & Schweikhart (1991), Kindleberger (2005)

Event	Gründerkrise	Chicago real estate boom	Crisis of 1882
Time	1872-1873	1881-1883	1881-1882
Place	Germany, Austria	Chicago	France
Bubble asset	Stocks, railroads, houses, land property	New-built apartments, houses from foreclosure proceedings, land	Stocks of new banks

Type of bubble asset	Securities, real estate	Real estate	Securities
Holder of asset		Widely held	Widely held
Financier of asset	Banks	Households	Banks, caisses de reports, individuals
Expansive monetary policy	Yes	Yes	No
Lending boom	Yes	No	Yes
Foreign capital inflows	Yes	No	Yes

Financial deregulation	Yes	No	No
Severe recession	Yes	No	Yes
Banking crisis	Yes	No	Yes
Spillover to other countries	Yes	No	No
Leaning	Yes	No	No
Pricking	No	No	No
Use of quantity instruments	No	No	No
Only cleaning	No	No	Yes
Sources	Burhop (2009), Conant (1915), McCartney (1935)	Hoyt (1933)	Conant (1915), Kindleberger (2005), White (2007)

Event	Panic of 1893	Norwegian crisis	Real estate bubble in the US
Time	1890-1893	1895-1905	1920-1926
Place	Australia	Norway	United States
Bubble asset	Mining shares, land property	Land property, new homes, real estate shares	Residential housing, securitization
Type of bubble asset	Securities, real estate	Real estate	Real estate
Holder of asset	Borrowers, banks, foreign investors	Construction sector, manufacturers, brokers, stock market investors	Households, banks
Financier of	Pastoral	Commercial	Non-

Expansive monetary policy	Yes	Yes	Yes
Lending boom	Yes	Yes	Yes
Foreign capital inflows	Yes	Yes	No
Financial deregulation	No	No	Yes
Severe recession	Yes	No	No
Banking crisis	Yes	Yes	Yes
Spillover to other countries	Yes	No	No

Leaning	No	No	Yes
Pricking	No	No	No
Use of quantity instruments	No	No	Yes
Only cleaning	Yes	Yes	No
Sources	Conant (1915), Lauck (1907), Merrett (1997)	Gerdrup (2003)	White (2009)

Event	German stock price bubble	US stock price bubble	Lost decade
Time	1927	1928-1929	1985-2003
Place	Germany	United States	Japan
Bubble asset	Stocks	Stocks, real estate	Stocks, real estate
Type of bubble asset	Securities	Securities, Real estate	Securities, Real estate
Holder of asset	Wealthy individuals, institutional investors, banks	Widely held	Widely held
Financier of asset	Banks, foreign investors	Domestic banks, later private investors, corporation	Trusts, banks

Expansive monetary policy	No	No	Yes
Lending boom	No	Yes	Yes
Foreign capital inflows	Yes	Yes	No
Financial deregulation	No	No	No
Severe recession	Yes	Yes	Yes
Banking crisis	Yes	Yes	Yes
Spillover to other countries	No	Yes	No

Leaning	Yes	Yes	Yes
Pricking	Yes	Yes	Yes
Use of quantity instruments	Yes	No	Yes
Only cleaning	No	No	No
Sources	Voth (2003)	Kindleberger (2005), White (1990)	Kaufman et al. (2003), Kindleberger (2005), Patrick (1998), Posen (2003)

Event	Scandinavian crisis: Norway	Scandinavian crisis: Sweden	Scandinavian crisis: Finland
Time	1988-1992	1990-1992	1991-1992
Place	Norway	Sweden	Finland
Bubble asset	Commercial real estate, housing	Commercial real estate	Land, dwellings
Type of bubble asset	Real estate	Real estate	Securities, Real estate

Holder of asset	Firms, households		Households, business
Financier of asset	Banks	Banks, finance companies	Banks
Expansive monetary policy	No	No	No
Lending boom	Yes	Yes	Yes
Foreign capital inflows	Yes	No	Yes
Financial deregulation	Yes	Yes	Yes
Severe recession	Yes	Yes	Yes

Banking crisis	Yes	Yes	Yes
Spillover to other countries	No	No	No
Leaning	No	Yes	Yes
Pricking	No	No	No
Use of quantity instruments	No	Yes	Yes
Only cleaning	Yes	No	No
Sources	Gerdrup (2003), Vale (2004)	Englund (1999), Herring, Wachter (1998)	Nyberg (1994)

Event	Asian crisis	Dotcom bubble	Real estate bubble in Australia
Time	1997-1998	1995-2000	2002-2004
Place	Thailand	United States	Australia
Bubble asset	Stocks, housing, commercial real estate	New technology company stocks	Dwelling
Type of bubble asset	Securities, real estate	Securities	Real estate
Holder of asset		Households, retail investors	Households

Financier of asset	Finance and securities companies, banks	Venture capitalists	Banks, mortgage originators
Expansive monetary policy	Yes	Yes	No
Lending boom	Yes	No	Yes
Foreign capital inflows	Yes	Yes	No
Financial deregulation	Yes	Yes	Yes
Severe recession	Yes	No	No

Banking crisis	Yes	No	No
Spillover to other countries	Yes	No	No
Leaning	No	Yes	Yes
Pricking	No	Yes	No
Use of quantity instruments	No	No	Yes
Only cleaning	Yes	No	No
Sources	Collyns, Senhadji (2002), Corsetti et al. (1999)	BIS 70th/71st Annual Report, Greenspan (2002)	BIS 76th Annual Report, Bloxham, Kent (2010)

Event	Subprime housing bubble	Spanish housing bubble
Time	2007–	2008–
Place	United States	Spain
Bubble asset	Subprime mortgages	
Type of bubble asset	Real estate	Real estate
Holder of asset	Widely held	Widely held

Financier of asset	Banks	Banks, saving banks
Expansive monetary policy	Yes	Yes
Lending boom	Yes	Yes
Foreign capital inflows	Yes	Yes
Financial deregulation	Yes	No
Severe recession	Yes	Yes
Banking crisis	Yes	Yes
Spillover to other countries	Yes	No

Leaning	No	Yes
Pricking	No	No
Use of quantity instruments	No	Yes
Only cleaning	Yes	No
Sources	Brunnermeier (2009), Gorton and Metrick (2012)	BoS (2012), Carballo-Cruz (2011)

A Technology Company

- Company X introduced a revolutionary wireless communication technology.
- It not only provided support for such a technology but also provided the informational content itself.
- It's IPO price was $1.50 per share. Six years later it was traded at $ 85.50 and in the seventh year it hit $ 114.00.
- The P/E ratio got as high as 73.
- The company never paid dividends.

The Story of RCA in 1920's

Company:	Radio Corporate of America (RCA)
Technology:	Radio
Years:	1920s

Figure : **RCA's Stock Price from Dec 25 to Dec 50.**

- RCA peaked at $ 397 in Feb. 1929, down to $ 2.62 in May 1932
- RCA's stock price was below $ 25 at least until 1950

NASDAQ and "Neuer Markt"

Figure : **NASDAQ and Neuer Markt during "Technology Bubble".**

Bubbles under Symmetric Information

- Keynes' distinction between speculation and long-run investment:
  - Speculation: Buy "overvalued" in the hope to sell it to someone else at an even higher price
  - Investing: Buy and hold strategy
- Fundamental value: Was ist das?
  "highest WTP" if one forces agents to buy & hold the asset

no uncertainty:	discounted value of dividends
uncertainty w/ risk-neutral agent:	*expected* discounted value
uncertainty w/ risk-averse agents:	take expectations *w.r.t. EMM*

Bubbles under Symmetric Information

- Problem of Keynes' buy and hold definition of fundamental value:
  - Retrade does also occur to dynamically complete the market (not only for speculation).
  - With retrade a different allocation can be achieved and hence the EMM is different.
  - Allow for retrade and take EMM which leads to highest fundamental value.

Bubbles under Symmetric Information

- with stochastic discount factor m_t (or pricing kernel m_t^*) the price of an asset is given by

$$m_t p_t = E_t \left[m_{t+1} \left(p_{t+1} + d_{t+1} \right) \right]$$

  where m_{t+1} is related to MRS (divided by prob. of state)
- Alternatively one can also write pricing equation in terms of the equivalent martingale measure

$$p_t = E_t^{\hat{Q}} \left[\frac{1}{1 + r_t^f} \left(p_{t+1} + d_{t+1} \right) \right]$$

- Securities with Finite Maturity
  - Reiterate pricing equation
  - Backwards induction rules out bubbles

Bubbles under Symmetric Information

- Securities with Infinite Maturity
  - Backwards induction argument fails since there is no well defined final period
  - "Lack of market clearing at $t = \infty$"
  - Split the price in a fundamental component p_t^f and a bubble component b_t.
  - By pricing equation, we get the following expectational difference equation

$$b_t = E_t^{\hat{Q}} \left[\frac{1}{1 + r_t^f} b_{t+1} \right]$$

  - Example 1: deterministic bubble
    $\Rightarrow$ has to grow at the risk-free rate
  - Example 2 (Blanchard & Watson 1982): (risk-neutral investors)
    - bubble bursts in each period with prob. $(1 - \pi)$, persists with prob. π
    - $\Rightarrow$ bubble has to grow by a factor $\frac{1+r_t^f}{\pi}$ (if it doesn't burst)

Bubbles under Symmetric Information

- How can we rule out bubbles?
  - Negative bubbles (Blanchard & Watson 1982, Diba & Grossman 1988)
    - For $b_t < 0$ difference equation implies that p_t will become negative.
    - Free disposal rules out negative prices.
  - Positive bubbles on assets with positive net supply if $g < r$ (Brock, Scheinkman, Tirole 85, Santos & Woodford 97)
    - Argument: *(bubbles would outgrow the economy if $r > g$)*
    - At any point in time $t + \tau$, the aggregate wealth of the economy contains bubble component b_τ.
    - NPV_t of aggregate wealth $W_{t+\tau}$ does not converge to zero as $\tau \to \infty$
    - If aggregate consumption$_{t+\tau}$ is bounded or grows at a rate $g < r$, $\text{NPV}_{t+\tau}(C_{t+\tau}) \to 0$ as $\tau \to \infty$.
    - Household wealth exceeds PV of C for all $t + \tau$ sufficiently far in the future.
    - This is inconsistent with optimization since household would consume part of wealth.

Bubbles under Symmetric Information

- 5 Counter Examples (Santos and Woodford (1997)):
  - Example 1: fiat money (=bubble) in OLG models
    - allows (better) intergeneral transfers
    - without bubble households want to save more and hence MRS "implicit r"$< g$
      (can lead to overaccumulation of private capital and hence, dynamic inefficiency (see also Abel et al. (1989)))
    - Geerolf (2014) overturns result with OECD data: sufficient conditions for dynamic efficiency are not satisfied (e.g. Japan is unambiguously inefficient)
  - Example 2: ...
  - *Common theme:*
    Pure existence of a bubble enlarges the trading space. leads to different allocation and EMM.

Overlapping Generations

- Samuelson (1958) considers an infinite-horizon economy with two-period lived overlapping agents
  - Population growth rate $= n$
- Preferences given by $u(c_t^t, c_{t+1}^t)$
  - Pareto optimal allocation satisfies $\frac{u_1}{u_2} = 1 + n$
- OLG economies have multiple equilibria that can be Pareto ranked

OLG: Multiple Equilibria

- Assume:

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \beta \log c_{t+1}^t$$

Endowment: $y_t^t = e$, $y_{t+1}^t = 1 - e$

- Assume complete markets and interest rate r
- Agents FOC implies:

$$\frac{c_{t+1}^t}{\beta c_t^t} = 1 + r$$

  - For $r = n$, this corresponds to the *Pareto Solution*
  - For $r = \frac{1-e}{\beta e} - 1$, agents will consume their endowment
- Autarky solution is clearly *Pareto inferior*

OLG: Completion with Durable Asset

- Autarky solution is the **unique** equilibrium implemented in a sequential exchange economy
  - Young agents cannot transfer wealth to the next period
  - . . . relates to Chris Sims's lecture
- A durable asset provides a *store of value*
  - Effective store of value reflects *market liquidity*
  - Pareto solution can be attained as a competitive equilibrium in which the price level grows at same rate as the population, i.e. $b_{t+1} = (1 + n)b_t$
  - Old agents trade durable asset for young agents' consumption goods

OLG: Production

- Diamond (1965) introduces a capital good and production
  - Constant-returns-to-scale (CRS) production:

$$Y_t = F(K_t, L_t)$$

- Optimal level of capital is given by the *golden rule*, i.e.

$$f'(k^\star) = n$$

  Here, lowercase letters signify **per capita** values
- Individual (and firm) optimization implies that:

$$\frac{u_1}{u_2} = 1 + r = 1 + f'(k)$$

  It is possible that $r < n \Rightarrow k > k^\star \Rightarrow$ Pareto inefficient

OLG: Production & Efficiency

- Diamond recommends issuing government debt at interest rate r
- Tirole (1985) introduces a rational bubble asset trading at price b_t:

$$b_{t+1} = \frac{1 + r_{t+1}}{1 + n} b_t$$

- Both solutions *crowd out* investment and increase r
  - If baseline economy is inefficient, then an appropriately chosen debt issuance or bubble size can achieve Pareto optimum with $r = n$

OLG: Crowding-out vs. Crowding-in

- Depending on the framework, government debt and presence of bubbles can have two opposite effects:
  1. **Crowding-out** refers to the decreased investment to increase in the supply of capital
  2. **Crowding-in** refers to increased investment due to improved risk transfer
- Woodford (1990) explores both of these effects

Bubbles and Credit Frictions

- Samuelson-Tirole model implications are hard to reconcile with the following stylized facts:
  1. bubbles seem to pop up & burst (not deterministic) in reality
  2. bubbles are associated with consumption booms, *as well as* rapid expansions in capital stock and output
- Martin and Ventura (2012) address these shortcomings in an OLG framework by introducing:
  - investor sentiment shocks
  - capital market imperfections
- Takeaway: bubbles are not only reduce inefficient investments, but also increase efficient ones

Bubbles and Credit Frictions: Model

- Risk-neutral individuals; utility: $U_{it} = E_t[c_{it+1}]$
  - each generation contains a measure 1 of individuals
  - live for two periods & supply 1 unit of labour when young
- Technology: $F(l_t, k_t) = l_t^{1-\alpha} k_t^{\alpha}$
  - fraction $\epsilon \in [0, 1]$ of productive individuals produce 1 unit of capital with one unit of output; unproductive produce $\delta < 1$ units of capital with 1 unit of output
- Financial Friction: no borrowing allowed $\Rightarrow$ unproductive investors have to make own investments

Bubbles and Credit Frictions: Model

- Dynamics of capital stock in the presence of bubbles:

$$k_{t+1} = \begin{cases} Ask_t^\alpha + (1-\delta)b_t^P - \delta b_t & \text{if } \dfrac{b_t + b_t^P}{(1-\epsilon)sk_t^\alpha} < 1 \\ sk_t^\alpha - b_t & \text{if } \dfrac{b_t + b_t^P}{(1-\epsilon)sk_t^\alpha} \geq 1 \end{cases}$$

- **Crowding-out**: when old sell bubble to young, consumption grows and investment falls; bubble crowds out unproductive investments first, then productive investments. Average investment efficiency rises and crowding-out effect minimized.
- **Crowding-in**: when productive young sell bubble to unproductive young, productive investments replace unproductive ones. This further raises average investment efficiency.

Bubbles under Asymmetric Information

- "Dynamic Knowledge Operator"

$$\mathcal{K}_t^i(E) = \left\{ \omega \in \Omega^{dynamic} : \mathcal{P}_i^t(\omega) \subseteq E \right\}$$

- Expected Bubbles versus Strong Bubbles
  - **expected bubble:**
    $p_t >$ every agents marginal valuation at a date state (t, ω)
  - **strong bubble:** (arbitrage?)
    $p_t >$ all agents know that no possible dividend realization can justify this price.

Necessary Conditions for Bubbles

- Model setup in Allen, Morris & Postlewaite 1993: risky asset pays dividend $d_T(\omega) : \Omega \longmapsto \mathbb{R}_+$ at $t = T$
- Necessary Conditions for Expected Bubbles
  1. Initial allocation (interim) Pareto inefficient (Tirole 1982)
     - rational traders is not willing to buy "bubble asset" since some traders have realized their gains leaving a negative sum game for the buyers
  2. Short-sale constraint strictly binds at some future time in some contingency for all i
     - only don't sell to the position limit now, since shorting might be more profitable in the future

Necessary Conditions for Bubbles

- Additional Necessary Conditions for Strong Bubbles
  1. asymmetric information is necessary since traders must believe that the other traders do not know this fact.
  2. Net trades of all traders cannot be CK (since CK of actions negates asymmetric info about events) $\Rightarrow$ no bubbles in economies with only two types of traders.
- Morris, Postelwaite & Shin (1995)-Model setup
  - now, all agents are risk-neutral
  - $p_T = d_T$ and $p_t = \max_i E_t^i \left[p_{t+1} | \mathcal{P}_t^i \right]$ for all $\omega \in \Omega$ and $t = 1, ..., T$.
  - Let's focus on ω, where $d_T = 0$, $E_T^{d_T=0} : \{\omega \in \Omega | d_T(\omega) = 0\}$

Necessary Conditions for Bubbles

- **Main Result:** Strong bubble can be ruled out at time t if $\mathcal{K}_t^G \mathcal{K}_{t+1}^G \cdots \mathcal{K}_{T-1}^G \left(E_T^{d_T=0} \right) = \{\omega \in \Omega | p_t(\omega) = 0\}$
  - (That is, it is mutual knowledge in t that in period $t+1$ it will be mutual knowsledge that ... in $(T-1)$ it will be mutual knowledge that $d_T = 0$.)
  - Sketch argument:
    if it is mutual knowledge at $T-1$ that $d_T = 0$, then $p_{T-1} = 0$.
  - if it is mutual knowledge at $T-2$ that $p_{T-1} = 0$, then $p_{T-2} = 0$.
  - ...
  - Since knowledge can only improve over time. If it is at t already $(T-t)$-mutual knowledge that $d_T = 0$, $p_t = 0$.

Limits to Arbitrage - Overview

- Efficient Market Hypothesis - 3 levels of justifications
  - All traders are rational, since behavioral will not survive in the long-run (their wealth declines)
  - Behavioral trades cancel each other on average
  - Rational arbitrageurs correct all mispricing induced by behavioral traders.
- Fama/Friedman contra Keynes
  - "If there are many sophisticated traders in the market, they may cause these "bubbles" to burst before they really get under way." (Fama 1965)
  - "It might have been supposed that competition between expert professionals, possessing judgment and knowledge beyond that of the average private investor, would correct the vagaries of the ignorant individual left to himself." (Keynes 1936)

Limits to Arbitrage - Overview

- Reasons for limits to arbitrage
  - Fundamental risk
  - Noise trader risk (DSSW 1990a, Shleifer & Vishny 1997)
  - Synchronization risk (Abreu & Brunnermeier 2002, 2003)
- Special case of market frictions (incl. liquidity)

Noise Trader Risk

- *Idea:* Arbitrageurs do not fully correct the mispricing caused by noise traders due to
  - arbitrageurs short horizons
  - arbitrageurs risk aversion (face noise trader risk)
- Noise traders survive in the long-run
  (they are not driven out of the market.)

Noise Trader Risk - DSSW 1990a

- Model Setup of DSSW 1990a
  - OLG model
    - agents live for 2 periods
    - make portfolio decisions when they are young
  - 2 assets
safe asset s	pays fixed real dividend r
	perfect elastic supply
	numeraire, i.e. $p_s = 1$
unsafe asset u	pays fixed real dividend r
	no elastic supply of $X^{sup} = 1$
	price at $t = p_t$
    - Fundamental value of s = Fundamental value of u (perfect substitutes)
  - agents
    - mass of $(1 - \mu)$ of arbitrageurs
    - mass of μ of noise traders, who misperceive next period's price by $\rho_t \sim \mathcal{N}\left(\rho_t^*, \sigma_\rho^2\right)$, ($\rho^*$ measures bullishness)
    - CARA utility function $U(W) = -\exp\{-2\gamma W\}$ with certainty equivalent $E[W] - \gamma Var[W]$.

Noise Trader Risk - DSSW 1990a

- Individual demand
  - arbitrageur's
    $E[W] - \gamma Var[W] =$
    $c_0 + x_t^a [r + E_t[p_{t+1}] - p_t(1+r)] - \gamma (x_t^a)^2 Var_t[p_{t+1}]$
  - noise traders
    $E[W] - \gamma Var[W] =$
    $c_0 + x_t^n [r + E_t[p_{t+1}] + \rho_t - p_t(1+r)] - \gamma (x_t^a)^2 Var_t[p_{t+1}]$
  - Taking FOC
    arbitrageurs: $x_t^a = \frac{r+E_t[p_{t+1}]-(1+r)p_t}{2\gamma Var_t[p_{t+1}]}$
    noise traders: $x_t^n = \frac{r+E_t[p_{t+1}]-(1+r)p_t}{2\gamma Var_t[p_{t+1}]} + \frac{\rho_t}{2\gamma Var_t[p_{t+1}]}$
- Market Clearing
  $(1-\mu) x_t^a + \mu x_t^n = 1$

$$p_t = \frac{1}{1+r}[r + E_t[p_{t+1}] - 2\gamma Var_t[p_{t+1}] + \mu\rho_t]$$

Noise Trader Risk - DSSW 1990a

Solve recursively,

$$p_{t+1} = \frac{1}{1+r}\left[r + E_{t+1}\left[p_{t+2}\right] - 2\gamma Var_{t+1}\left[p_{t+2}\right] + \mu\rho_{t+1}\right]$$

$$E_t\left[p_{t+1}\right] = \frac{1}{1+r}\left[r + E_t\left[p_{t+2}\right] - 2\gamma Var_t\left[p_{t+2}\right] + \mu\rho^*\right]$$

we will see later that $Var_t\left[p_{t+\tau}\right]$ is a constant for all τ.
Solve first order difference equation

$$p_t = 1 + \frac{\mu\left(\rho_t - \rho^*\right)}{1+r} + \frac{\mu\rho^*}{r} - \frac{2\gamma}{r}Var_t\left[p_{t+1}\right]$$

Note that ρ_t is the only random variable. Hence,
$Var_t\left[p_{t+1}\right] = Var\left[p_{t+1}\right] = \frac{\mu^2\sigma_\rho^2}{(1+r)^2}$

$$p_t = 1 + \frac{\mu\left(\rho_t - \rho^*\right)}{1+r} + \frac{\mu\rho^*}{r} - \frac{(2\gamma)\,\mu^2\sigma_\rho^2}{r\,(1+r)}$$

Noise Trader Risk - DSSW 1990a

$$p_t = 1 + \frac{\mu\left(\rho_t - \rho^*\right)}{1+r} + \frac{\mu\rho^*}{r} - \frac{(2\gamma)\,\mu^2\sigma_\rho^2}{r\,(1+r)}$$

where

- $1 =$ fundamental value
- $\frac{\mu(\rho_t-\rho^*)}{1+r} =$ deviation due to current misperception of noise traders
- $\frac{\mu\rho^*}{r} =$ average misperception of noise traders
- $-\frac{(2\gamma)\mu^2\sigma_\rho^2}{r(1+r)} =$ arbitrageurs' risk-premium
- Homework:
  1. Check limiting cases
     1. $\gamma \to 0$
     2. $\sigma_\rho^2 \to 0$
  2. Check whether there is also a fundamental equilibrium, where $p_t = 1$ for all t
     (no risk $\Rightarrow$ arbitrageurs buy everything)

Do Noise Traders Die Out over Time (Evolutionary Argument)

- Relative Expected Returns
- Difference in returns
- $\Delta R_{n-a} = (x_t^n - x_t^a)\left[r + p_{t+1} - p_t(1+r)\right]$
- Aside 1: $(x_t^n - x_t^a) = \frac{\rho_t}{(2\gamma)Var_t[p_{t+1}]} = \frac{(1+r)^2\rho_t}{(2\gamma)\mu^2\sigma_\rho^2}$
(Note for $\mu \to 0$, $(x_t^n - x_t^a) \to \infty$)
- Aside 2: By market clearing $E\left[r + p_{t+1} - p_t(1+r)\right] = (2\gamma)\,Var_t\left[p_{t+1}\right] - \mu\rho_t = \frac{(2\gamma)\mu^2\sigma_\rho^2}{(1+r)^2} - \mu\rho_t$

$$\Rightarrow E_t\left[\Delta R_{n-a}\right] = \rho_t - \frac{(1+r)^2(\rho_t)^2}{(2\gamma)\,\mu^2\sigma_\rho^2}$$

- Taking unconditional expectations

$$E\left[\Delta R_{n-a}\right] = \rho^* - \frac{(1+r)^2\rho^* + (1+r)^2\sigma_\rho^2}{(2\gamma)\,\mu^2\sigma_\rho^2} > 0 \text{ only if } \rho^* > 0$$

Do Noise Traders Survive over Time (Evolutionary Argument)

- Taking unconditional expectations

$$E\left[\Delta R_{n-a}\right] = \rho^* - \frac{(1+r)^2 \rho^* + (1+r)^2 \sigma_\rho^2}{(2\gamma)\, \mu^2 \sigma_\rho^2} > 0 \text{ only if } \rho^* > 0$$

- "Overoptimistic/bullish" traders hold riskier positions and have higher expected returns.
- In evolutionary process, they will have more off-springs and hence they won't die out.

Myopia due to Liquidation Risk

- Why are professional arbitrageurs myopic?
- Model setup of Shleifer & Vishny (1997) [slightly modified]
  - Two assets
    - risk free bond and
    - risky stock with final value v
  - Two types of fund managers:
    - Good fund managers know fundamental value v
    - Bad fund managers have no additional information (just gamble with "other people's money").
  - Two trading rounds $t = 1$ and 2 (in $t = 3$ v is paid out)
  - Individual investors
    - entrust their money F_1 to a fund manager without knowing the fund managers' skill level - "separation of brain and money"
    - can withdraw their funds in $t = 2$
  - Noise traders submit random demand

Myopia due to Liquidation Risk

- Price setting:
  - $P_3 = v$
  - P_2 is determined by aggregate demand of fund managers and liquidity/noise traders
- Focus on case where
  1. $P_1 < v$ (asset is undervalued)
  2. $P_2 < P_1$asset price goes even further down in t_2 due to
     - sell order by noise traders
     - sell order by other informed traders
- Performanced-based fund flows (see Chevalier & Ellison 1997)
  - If price drops, the probability increases that the fund manager is "bad".
  - Individual investors withdraw their money at $t = 2$.
  - Shleifer & Vishny assume $F_2 = F_1 - aD_1 \left(1 - \frac{P_2}{P_1}\right)$, where D_1 is the amount the fund manager invested in the stock.

Myopia due to Liquidation Risk

- "Good" manager's problem who has invested in risky asset
  - He has to liquidate his position at $P_2 < P_1$ (exactly when mispricing is the largest!)
    That is, he makes losses, even though the asset was initially undervalued.
  - Due to this "early liquidation risk", at $t = 1$ a rational fund manager is reluctant to fully exploit arbitrage opportunities at $t = 1$.
  - Focus on short-run price movements ⇒ myopia of professional arbitrageurs

Synchronization Risk

- "Bubbles and Crashes" (Abreu & Brunnermeier 2003) (for bubbles)
- "Synchronization Risk and Delayed Arbitrage" (Abreu & Brunnermeier 2002)
  (for any form of mispricing)
- see power point slides (file "08 slides Eco525.ppt")

Clean vs. Lean

- Long-standing debate about the role of monetary policy with regard to asset price bubbles:
    - **"Cleaning"** view (Greenspan (1999, 2002)): mitigate the consequences of bursting bubbles rather than trying to detect and prevent asset price bubbles when they emerge
    - **"Leaning"** view (BIS): try to prevent the build-up of bubbles by reacting early on to upward-trending asset prices
- Issues related to "leaning-against the wind":
    1. Bubbles are hard to identify (as fundamental asset values)
    2. Monetary policy instruments are too blunt when aimed at containing bubbles (may overly suppress output/inflation)
    3. Bubbles could instead be tackled with financial regulation
- Recent crisis experience tilted views towards more intervention, closer to the BIS view (Stein (2013, 2014))