Asset Pricing under Asym. Information

Rational Expectation Equilibria

Classification of Models

Unit Demand Auctions

2^{nd}-Price

RET

Affiliated Values

Share Auctions

Constant v

Random v

Double Auction

Private Info

Uniform - Price Discrimination

Asset Pricing under Asymmetric Information Rational Expectations Equilibrium

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A Classification of Market Microstructure Models

- simultaneous submission of demand schedules
  - competitive rational expectation models
  - strategic share auctions
- sequential move models
  - screening models in which the market maker submits a supply schedule first
    - static
      - ◇ uniform price setting
      - ◇ limit order book analysis
    - dynamic sequential trade models with multiple trading rounds
  - strategic market order models where the market maker sets prices ex-post

Auctions - Overview

- Unit demand versus divisible good (share) auctions
- Signal structure:
  - common value
  - private value (liquidity, non-common priors)
  - affiliated values
- Auction Formats:
  - Open-outcry auctions: English auctions (ascending-bid, progressive), Dutch auctions (descending-bid)
  - Sealed-bid auctions: First-price auction, second-price auction
  - Share auctions: uniform-price (Dutch) auction, discriminatory price auction

Results in (Unit Demand) Auction Theory - A Refresher -

1. "Strategic equivalence" between Dutch auction and first price sealed-bid auction
(English auction is more informative than second-price auction.)
2. Second-price auction: Bidding your own private value is a (weakly) dominant strategy (Groves Mechanism)
3. Revenue Equivalence Theorem (RET)

2^{nd}-Price Auction: Private Value

- Model Setup
  - Private value: v^i
  - Highest others' bid: $B_{\max}^{-i} = \max_{j \neq i} \left\{ b^1, ..., b^j, ..., b^I \right\}$
- *Claim:* Bidding own value v^i is (weakly) dominant strategy
- Proof (note similarity to Groves Mechanism):
  - Overbid, i.e. $b^i > v^i$:
    - If $B_{\max}^{-i} \geq b^i$, he wouldn't have won anyway.
    - If $B_{\max}^{-i} \leq v^i$, he wins the object whether he bids b^i or v^i.
    - If $v^i < B_{\max}^{-i} < b^i$, he wins and gets negative utility instead of 0 utility.
  - Underbid, i.e. $b^i < v^i$:
    - If $b^i < B_{\max}^{-i} < v^i$, he loses instead of $u\left(v^i - B_{\max}^{-i}\right) > 0$.

Revenue Equivalence Theorem

- *Claim:* Any auction mechanism with risk-neutral bidders leads to the same *expected* revenue if
  1. mechanism also assigns the good to the bidder with the highest signal
  2. bidder with the lowest feasible signal receives zero surplus
  3. $v \in [\underline{V}, \overline{V}]$ from common, strictly increasing, atomless distribution
  4. private value OR
     pure common value with independent signals S^i with $v = f(S^1, ..., S^I)$.
- Proof (Sketch):
  Taken from book p. 185

Proof of RET

- Suppose the expected payoff $U^i(v^i)$ if $S^i = v^i$.
- If v^i-bidder mimics a $(v^i + \Delta v)$-bidder,
  - payoff = payoff of a $(v^i + \Delta v)$-bidder with the difference, that he values it Δv less than $(v^i + \Delta v)$-bidder, if he wins
  - prob of winning: $P(v^i + \Delta)$ if he mimics the $(v^i + \Delta v)$-bidder.
  - in any mechanism bidder should have no incentive to mimic somebody else, i.e.

$$U(v^i) \geq U(v^i + \Delta v) - \Delta v \Pr(v^i + \Delta v)$$

.

- $(v^i + \Delta v)$-bidder should not want to mimic v^i-bidder, i.e.

$$U(v^i + \Delta v) \geq U(v^i) + \Delta v \Pr(v^i)$$

.

- Combining both inequalities leads to

$$\Pr(v^i) \leq \frac{U^i(v^i + \Delta v) - U^i(v^i)}{\Delta v} \leq \Pr(v^i + \Delta v)$$

Affiliated Values - Milgrom & Weber (1982)

- Affiliated Values - MLRP
- Model Setup
  Bidder i's signal: S^i
  Highest of other bidders' signals: $S_{\max}^{-i} := \max_{j \neq i} \left\{ S^j \right\}_{j \neq i}$
  Define two-variable function:
  $V^i(x, y) = E\left[v^i | S^i = x, S_{\max}^{-i} = y \right]$
- Optimal bidding strategy:
  - Second-price auction

$$b^i(x) = V^i(x, x)$$

  - First-price auction: Solution to ODE

$$\frac{\partial b^i(x)}{\partial x} = \left[V^i(x, x) - b^i(x) \right] \frac{f_{S_{\max}^{-i}}(x|x)}{F_{S_{\max}^{-i}}(x|x)}$$

  where f and F are the pdf and cdf of the conditional distribution of $S_{\max}^{-i}$, respectively.

Affiliated Values - Milgrom & Weber (1982)

- Revenue ranking with risk-neutral bidders:
  - English auction > second-price auction > first price auction
  - (Latter ranking might change with risk aversion Maskin & Riley 1984, Matthew 1983)
- END OF REFRESHER!

Share Auctions - Overview

1. Value v is commonly known
   illustrate multiplicity problem, role of random supply
2. Random value v, but symmetric information
   a) general demand function (no individual stock endowments)
   b) linear equilibria (with individual endowments)
3. Random value v and asymmetric information
   (CARA Gaussian setup)

Commonly Known Value v
— Illustration of Multiplicity Problem —

- Wilson (1979)
- Model Setup
  - I bidders/traders submit demand schedules
  - everybody knows value $\bar{v}$
  - non-random supply $X^{\text{sup}} = 1$ (normalization)
- *Benchmark:* unit demand auction $p^* = \bar{v}$
- *Share auctions:* Each bidder is a monopsonist who faces the *residual* supply curve.
- *Claim:* $p^* = \frac{\bar{v}}{2}$ is also an equilibrium if agents submit demand schedules $x(p) = \frac{1-2p/(I\bar{v})}{I-1}$.

Commonly Known Value v
— Illustration of Multiplicity Problem —

Proof:

- Market clearing: $Ix(p^*) = 1 \Rightarrow p^* = \frac{\bar{v}}{2}$.
- Trader i's residual supply curve: $X^{\text{sup}} - [(I-1)\,x(p)] = 1 - [1 - 2p/(I\bar{v})] = \frac{2p}{I\bar{v}}$.
- Residual demand = residual supply: $x^i(p^*) = \frac{2p^*}{I\bar{v}}$.
- Trader i's profit is $(\bar{v} - p)\,x^i(p) = (\bar{v} - p)\,\frac{2p}{I\bar{v}}$.
- By choosing $x^i(p)$, trader i effectively chooses the price p.
- Take FOC of $(\bar{v} - p)\,\frac{2p}{I\bar{v}}$ w.r.t. p: $(\bar{v})\,\frac{2}{I\bar{v}} - \frac{4p}{I\bar{v}} = 0$
- $\Rightarrow p^* = \frac{\bar{v}}{2}$ and $x^i = \frac{2(\bar{v}/2)}{I\bar{v}} = 1/I$.

Commonly Known Value v — Illustration of Multiplicity Problem —

- *Generalizations:* Any price $p^* \in [0, \bar{v})$ can be sustained in equilibrium if bidders simultaneously submit the following demand schedules:

$$x^i(p) = \frac{1}{I}\left[1 + \beta_p\left(p^* - p\right)\right], \text{ where } \beta_p = \frac{1}{(I-1)(\bar{v} - p^*)}$$

- Proof: Homework!

Commonly Known Value v

- Graphical Illustration for $I = 2$

- Each bidders is indifferent between any demand schedule as long as it goes through the optimal point.
- $\Rightarrow$ multiple equilibria
- *Way out:* Introduce random supply $X^{\text{sup}} = u$

Value v is Random - No Private Info

- Model setup:
  - Value v is random - no private info
  - all traders have same utility function $U(\cdot)$
  - X^{sup}:
    1. deterministic/non-random X^{sup}
       $\Rightarrow$ apply previous section and use certainty equivalence (Wilson)
    2. random supply $X^{\text{sup}} = u$

Value v is Random - No Private Info

- *Necessary Condition:* Any I bidder, symmetric strategy Nash equilibrium in continuously differentiable (downward sloping) demand functions with random supply $X^{\text{sup}} = u$ is characterized by

$$0 = E_v \left[U'\left(\left(v - p\right) x\left(p\right)\right) \left[v - p + \frac{x\left(p\right)}{\left(I - 1\right) \partial x\left(p\right) / \partial p} \right] \right],$$

provided a equal tie breaking rule applies.

Proof:

- Since $x^*(p)$ is invertible, all bidders can infer the random supply u from the equilibrium price p. In other words, each equilibrium price p' corresponds to a certain realization of the random supply u'. Bidders trade conditional to the equilibrium price by submitting demand schedules. Thus they implicitly condition their bid on the random supply u.

Value v is Random - No Private Info

- Every bidder i prefers his equilibrium strategy $x^{i,*}(p)$ to any other demand schedule $x^i(p) = x^{i,*}(p) + h^i(p)$. Let us focus on pointwise deviations at a single price p', that is, for a certain realization u' of u. For a given aggregate supply u', bidder i's utility, is $E_v[U((v - p(x^i))x^i)]$.
- Deviating from $x^{i,*}_{p'}$ alters the equilibrium price p'. The marginal change in price for a given u' is given by totally differentiating the market clearing condition $x^i_{p'} + \sum_{-i \in \mathbb{I} \setminus i} x^{-i,*}(p) = u'$. That is, it is given by

$$\frac{dp}{dx^i} = -\frac{1}{\sum_{-i \in \mathbb{I} \setminus i} \partial x^{-i,*} / \partial p}.$$

Value v is Random - No Private Info

- The optimal quantity $x^{i,*}_{p'}$ for trader i satisfies the first-order condition

$$E_v[U'(\cdot)(v - p + x^{i,*}_{p'} \frac{1}{\sum_{-i \in \mathbb{I} \setminus i} \partial x^{-i,*} / \partial p})] = 0$$

for a given u'. This first-order condition has to hold for any realization u' of u, that is for any possible equilibrium price p'. For distributions of u that are continuous without bound, this differential equation has to be satisfied for all $p \in \mathbb{R}$. Therefore, the necessary condition is

$$E_v\left[U'(\cdot)\left(v - p + \frac{x^{i,*}(p)}{\sum_{-i \in \mathbb{I} \setminus i} \partial x^{-i,*} / \partial p}\right)\right] = 0.$$

- For a specific utility function $U(\cdot)$, explicit demand functions can be derived from this necessary condition.

Value v is random - No private info II
Special Cases I: Risk Neutrality

- For *risk neutral* bidders $U'(\cdot)$ is a constant.
- $p = E[v] + \underbrace{[\sum_{-i \in \mathbb{I} \setminus i} \frac{\partial x^{-i,*}}{\partial p}]^{-1} x^{i,*}(p)}_{\text{bid shading}}.$
- Imposing symmetry, $x(p) = (E[v] - p)^{\frac{1}{I-1}} k_0$, where $k_0 = p(0)$.
- inverse of it is $p(x) = E[v] - \underbrace{(1/k_0)^{(I-1)} (x)^{(I-1)}}_{\text{bid shading}}.$
- Note that equilibrium demand schedules are only linear for the two-bidder case.

Value v is random - No private info II
Special Cases II: CARA utility

- $U(W) = -e^{-\rho W}$
- FOC: $\frac{\int e^{-\rho x^{i,*} v} v f(v) dv}{\int e^{-\rho x^{i,*} v} f(v) dv} - p + \underbrace{[\sum_{-i \in \mathbb{I} \setminus i} \frac{\partial x^{-i,*}}{\partial p}]^{-1}}_{\text{bid shading}} x^{i,*} = 0,$
- where $f(v)$ is the density function of v.
- Homework: Check above FOC!
- Note: The integral is the derivative of the log of the moment generating function, $(ln\Phi)'(-\rho x(p))$.

Value v is random - No private info II
Special Cases III: CARA-Gaussian setting

- in addition: $v \sim \mathcal{N}(\mu, \sigma_v^2)$
- Integral term simplifies to $E[v] - \rho x(p) Var[v]$
- $p = \underbrace{E[v] - \rho Var[v]\ x^{i,*}(p)}_{\text{value of marginal unit}} + \underbrace{\frac{1}{\sum_{-i \in \mathbb{I} \setminus i} \frac{\partial x^{-i,*}}{\partial p}}}_{\text{bid shading}}\ x^{i,*}(p).$
- Impose symmetry, $p(x) = E[v] - \rho Var[v]\ \frac{I-1}{I-2} x - k_1 (x)^{I-1}$.
- Inverse for $k_1 = 0$, $x^i(p) = \frac{I-2}{I-1} \frac{E[v]-p}{\rho Var[v]}$
- This also illustrates that demand functions are only linear for $I \geq 3$ and for the constant $k_1 = 0$.

Double Auction View

- Model Setup
  - CARA-Gaussian setup
  - *Individual* endowment for each trader z^i
  - Aggregate random supply u. Total supply is $u + \sum_i z^i$. (only u is random)
  - Each trader's allocation is then $x^i = z^i + \Delta x^i (p^*)$
  - still symmetric information
- Focus on linear demand schedules:
- **Step 1:** Conjecture linear demand schedules $\Delta x^i = a^i - b^i p$ for all i (strategy profile)

Double Auction View

Residual supply is $u - \sum_{j \neq i} \left(a^j - b^j p\right) = \Delta x^i$

$$\Leftrightarrow p = \underbrace{\left(\sum_{j \neq i} a^j - u\right) / \left(\sum_{j \neq i} b^j\right)}_{:=\tilde{p}_0} + \underbrace{1/\sum_{j \neq i} b^j}_{:=1/\lambda^i} \Delta x^i$$

- **Step 2:** By conditioning on p, trader i can choose his demand for each realization of u (or $\tilde{p}_0$).
- **Step 3:** Best response
  Trader i's objective

$$\left(E[v] - \tilde{p}_0 - 1/\lambda^i \Delta x^i\right) \Delta x^i + E[v] z^i - \frac{1}{2}\rho^i Var[v] \left(z^i + \Delta x^i\right)^2$$

Double Auction View

Take FOC w.r.t. Δx^i

$$E[v] - \tilde{p}_0 - \frac{2}{\lambda^i}\Delta x^i - \rho^i Var[v]\left(z^i + \Delta x^i\right) = 0$$

SOC:

$$-\frac{2}{\lambda^i} - \rho^i Var[v] < 0 \Longleftrightarrow \lambda^i \notin \left[-\frac{2}{\rho^i Var[v]}, 0\right]$$

Best response is

$$\Delta x^i(\tilde{p}_0) = \frac{E[v] - \tilde{p}_0 - \rho^i Var[v]\, z^i}{2/\lambda^i + \rho^i Var[v]}$$

Double Auction View

In terms of price

$$\Delta x^i(p) = \frac{\lambda^i\left\{\eta^i\tau_v\left(E[v]-p\right)-z^i\right\}}{\eta^i\tau_v+\lambda^i} = \underbrace{\frac{\lambda^i\left\{\eta^i\tau_v E[v]-z^i\right\}}{\eta^i\tau_v+\lambda^i}}_{:=a^i} - \underbrace{\frac{\lambda^i\eta^i\tau_v}{\eta^i\tau_v+\lambda^i}}_{:=b^i} p$$

- **Step 4:** Impose Rationality
  In symmetric equilibrium $b=b^i$, $\lambda=\lambda^i\ \forall i$. Hence, $\sum_{j\neq i} b^j=\lambda^i$ becomes $(I-1)\,b=\lambda$.
- Replacing λ

$$b=\frac{(I-1)\,b\eta\tau_v}{\eta\tau_v+(I-1)\,b}=\frac{I-2}{I-1}\eta\tau_v \Rightarrow \lambda=(I-2)\,\eta\tau_v$$

Double Auction View

- Note that only for $I \geq 3$ a symmetric equilibrium exists. and
$$a^i = \frac{I-2}{I-1}\eta\tau_v E[v] - \frac{I-2}{I-1} z^i$$
- Put everything together
$$x^i(p) = z^i + \Delta x^i = \frac{I-2}{I-1}\frac{E[v]-p}{\rho Var[v]} + \frac{1}{I-1} z^i$$

Difference of Strategic Outcome to Competitive REE

1. Trading
   - traders are less aggressive
   - endowments matter for holdings
     - Why? Price "moves against you"
2. Excess "equilibrium" payoff

$$E[Q] = \rho Var[v] \left[\frac{1}{I} \sum_i z^i + \frac{I-1}{I-2} \frac{u}{I} \right]$$

   - For $u = 0$, same as in competitive case. (Check homework)
   - For $u > 0$, abnormally high - cost for liquidity (noise) traders (sell when price is low)
   - For $u < 0$, abnormally low - cost for liquidity (noise) traders (buy when price is high)
3. As $I \to \infty$, convergence to competitive REE with sym. info

Value v is Random & Private Info Kyle (1989)

- Kyle (1989)
  (similar to Hellwig 1980 setting, all traders receive signal $S^i = v + \varepsilon^i$)
- Simpler Model Setup (here):
  - CARA Gaussian setup
  - Signal structure (line in Grossman-Stiglitz 1980)
    - M uninformed traders
    - N informed traders, who observe same signal S
- Focus on linear demand functions only

Value v is Random & Private Info Kyle (1989)

- **Step 1:** Conjecture symmetric, linear demand schedules
  for uninformed: $\Delta x^{un} = a^{un} - b^{un} p$
  for informed: $\Delta x^{in} = a^{in} - b^{in} p + c^{in} \Delta S$
  Define
  price impace (slope) $\lambda = N b^{in} + M b^{un}$
  'residual slope for informed' $\lambda^{in} = (N-1)\, b^{in} + M b^{un}$
  'residual slope for uninformed' $\lambda^{un} = N b^{in} + (M-1)\, b^{un}$
  intercept $A = N a^{in} + M a^{un}$
  Equilibrium price is

$$p = \frac{1}{\lambda}\left(A - u + N c^{in} \Delta S\right)$$

Informed traders

- **Step 2:** no info inference
- **Step 3:** Best response same as before, just replace mean and variance, by conditional mean and variance

Value v is Random & Private Info Kyle (1989)

Best response (as a function of price) is

$$\begin{aligned}
\Delta x^{in}(p) &= \frac{\lambda^{in}\left\{\eta^{in}\tau_{v|S}\left(E[v|S]-p\right)-z^{in}\right\}}{\eta^{in}\tau_{v|S}+\lambda^{in}} \\
&= \frac{\lambda^{in}\left\{\eta^{in}\tau_{v|S}\left(E[v]+\frac{\tau_\varepsilon}{\tau_{v|S}}\Delta S-p\right)-z^{in}\right\}}{\eta^{in}\tau_{v|S}+\lambda^{in}} \\
&= \underbrace{\frac{\lambda^{in}\left\{\eta^{in}\tau_{v|S}E[v]-z^{in}\right\}}{\eta^{in}\tau_{v|S}+\lambda^{in}}}_{:=a^{in}} - \underbrace{\frac{\lambda^{in}\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S}+\lambda^{in}}}_{:=b^{in}}p \\
&\quad + \underbrace{\frac{\lambda^{in}\eta^{in}\tau_\varepsilon}{\eta^{in}\tau_{v|S}+\lambda^{in}}}_{:=c^{in}}\Delta S
\end{aligned}$$

SOC $\lambda^{in} \notin \left[-2\eta\tau_{v|S}, 0\right] \Rightarrow b^{in} > 0.$

Value v is Random & Private Info Kyle (1989)

- **Step 4:** Impose Rationality
  (For $M = I$ is sym. info case.)
  Rewrite $b^{in} = \frac{\lambda^{in}\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S}+\lambda^{in}}$ as $b^{in}\frac{\eta^{in}\tau_{v|S}+\lambda^{in}}{\eta^{in}\tau_{v|S}} = \lambda^{in}$ and notice
  $\lambda = \lambda^{in} + b^{in}$

$$\lambda = b^{in}\frac{\eta^{in}\tau_{v|S}+\lambda^{in}}{\eta^{in}\tau_{v|S}} + b^{in} \text{ and}$$

$$\text{def for } \lambda^{un} : \quad Mb^{un} = b^{in}\frac{\eta^{in}\tau_{v|S}+\lambda^{in}}{\eta^{in}\tau_{v|S}} - (N-1)\,b^{in}$$

Value v is Random & Private Info
Kyle (1989)

- **Uninformed traders:**
  - **Step 2:** Information Inference from $p = \frac{1}{\lambda}\left(A - u + Nc^{in}\Delta S\right)$

$$E\left[v|p\right] = E\left[v\right] + \frac{\lambda}{Nc^{in}}\left(\frac{\phi\tau_\varepsilon}{\tau_{v|p}}\right)\left[p - \frac{A}{\lambda}\right] \text{ and } \tau_{v|p} = \tau_v +$$

$$\text{where } \phi = \frac{N^2\left(c^{in}\right)^2\tau_u}{N^2\left(c^{in}\right)^2\tau_u + \tau_\varepsilon}$$

$$= \frac{N^2\left(b^{in}\right)^2\tau_u\tau_\varepsilon}{N^2\left(b^{in}\right)^2\tau_u\tau_\varepsilon + \left(\tau_{v|S}\right)^2} \text{ since } c^{in} = \frac{\tau_\varepsilon}{\tau_{v|S}}b^{in}$$

Value v is Random & Private Info Kyle (1989)

- **Step 3:** Best response $\Delta x^{un}(p) =$

$$= \frac{\lambda^{un}\left\{\eta^{un}\tau_{v|p}\left(E[v|p]-p\right)-z^{un}\right\}}{\eta^{un}\tau_{v|p}+\lambda^{un}}$$

$$= \frac{\lambda^{un}\left\{\eta^{un}\tau_{v|p}\left(E[v]-\frac{1}{Nc^{in}}\frac{\phi\tau_{\varepsilon}}{\tau_{v|p}}(A-\lambda p)-p\right)-z^{un}\right\}}{\eta^{un}\tau_{v|p}+\lambda^{un}}$$

$$= \underbrace{\frac{\lambda^{un}\left\{\eta^{un}\tau_{v|p}\left(E[v]-\frac{1}{Nc^{in}}\frac{\phi\tau_{\varepsilon}}{\tau_{v|p}}A\right)-z^{un}\right\}}{\eta^{un}\tau_{v|p}+\lambda^{un}}}_{:=a^{un}}$$

$$-\underbrace{\frac{\lambda^{un}\eta^{un}\tau_{v|p}\left(1-\frac{\lambda}{Nc^{in}}\frac{\phi\tau_{\varepsilon}}{\tau_{v|p}}\right)}{\eta^{un}\tau_{v|p}+\lambda^{un}}}_{:=b^{un}}p$$

- **Step 4:** Equate coeff. (fcns of b^{in}). Solve for polynomial.

Simplification I: Information Monopolist

- Since $Nc^{in} = \frac{\tau_\varepsilon}{\tau_{v|S}} b^{in}$,
  $b^{un}\left(\eta^{un}\tau_{v|p} + \lambda^{un}\right) = \lambda^{un}\eta^{un}\left(\tau_{v|p} - \frac{\lambda}{b^{in}}\phi\tau_{v|S}\right)$.
- Using $\lambda = b^{in}\frac{\eta^{in}\tau_{v|S}+\lambda^{in}}{\eta^{in}\tau_{v|S}} + b^{in}$,
  RHS becomes $\lambda^{un}\eta^{un}\left(\tau_{v|p} - \left(\frac{\eta^{in}\tau_{v|S}+\lambda^{in}}{\eta^{in}\tau_{v|S}} + 1\right)\phi\tau_{v|S}\right)$ or
  $\lambda^{un}\eta^{un}\left[(1-\phi)\tau_v - \phi\eta^{in}\frac{\left(\tau_{v|S}\right)^2}{\eta^{in}\tau_{v|S}-b^{in}}\right]$.
  Since we can write $\phi = \frac{N^2\left(b^{in}\right)^2\tau_u\tau_\varepsilon}{N^2(b^{in})^2\tau_u\tau_\varepsilon+\left(\tau_{v|S}\right)^2}$, RHS is
  $$\lambda^{un}\eta^{un}\left[\frac{\tau_v-\eta^{in}\frac{\left(b^{in}\right)^2\tau_u\tau_\varepsilon}{\eta^{in}\tau_{v|S}-b^{in}}}{N^2(b^{in})^2\tau_u\tau_\varepsilon+\left(\tau_{v|S}\right)^2}\right]\left(\tau_{v|S}\right)^2 =: \zeta\left(b^{in}\right).$$

Simplification I: Information Monopolist

- Using $Mb^{un} = b^{in}\frac{\eta^{in}\tau_{v|S}+\lambda^{in}}{\eta^{in}\tau_{v|S}} - (N-1)\, b^{in}$, one can eliminate b^{un} and λ^{un}.

  ...

- Finally,

$$\frac{1}{M}\frac{\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S}-b^{in}}\left[\eta^{un}\tau_{v|p}+\left(\frac{M-1}{M}\frac{\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S}-b^{in}}+1\right)\right.$$

$$= \quad \eta^{un}\left[\frac{M-1}{M}\frac{\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S}-b^{in}}+1\right]\zeta\left(b^{in}\right)$$

Simplification II: Information Monopolist & Competitive Outsiders

- Taking the right limit:
  - As $M \to \infty$, $M\eta^{un} \to \bar{\eta}^{un}$, i.e. each individual uninformed trader becomes infinitely risk averse.
  - Why not just $M \to \infty$? uninformed trader would dominate and informed traders demand becomes relatively tiny.
- Above equation simplifies to (multiply by M and notice that $\eta^{un} \to 0$)

$$\frac{\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S} - b^{in}} \left[\frac{\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S} - b^{in}} + 1 \right] b^{in} =$$

$$= M\eta^{un} \left[\frac{\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S} - b^{in}} + 1 \right] \zeta\left(b^{in}\right)$$

Simplification I: Information Monopolist

$$\frac{\eta^{in}\tau_{v|S}}{\eta^{in}\tau_{v|S} - b^{in}} b^{in} = \bar{\eta}^{un}\zeta\left(b^{in}\right)$$

- Sub in $\zeta\left(b^{in}\right)$ [check at home!]

$$\eta^{in} b^{in} \tau_{v|S} + \eta^{in}\left(b^{in}\right)^2 \tau_u \tau_\varepsilon \left[b^{in} + \bar{\eta}^{un}\tau_{v|S}\right] =$$
$$= \bar{\eta}^{un}\tau_{v|S}\tau_v\left[\eta^{in}\tau_{v|S} - b^{in}\right]$$

- Plot both sides and one can see that the unique real root to this cubic equation is in the acceptable (recall SOC) interval $\left(0, \eta^{in}\tau_{v|S}\right)$.
- Let $\vartheta \in (0,1)$, and $b^{in*} = \vartheta\eta^{in}\tau_{v|S}$.
- Using $Mb^{un} = b^{in}\frac{\eta^{in}\tau_{v|S}+\lambda^{in}}{\eta^{in}\tau_{v|S}} \to \frac{\vartheta}{1-\vartheta}\eta^{in}\tau_{v|S}$.

Remarks I

- 3 effects for informed monopolist
  - For given λ^{in}, price moves against informed trader $\Rightarrow$ lower b^{in}.
  - informational effect
    - For given $\tau_{v|p}$: Uninformed trader make inferences from prices $\Rightarrow$ their demand will react less strongly to increases in p. $\Rightarrow$ residual supply curve is steeper $\Rightarrow$ lower b^{in}
    - Increase in b^{in} $\Rightarrow$ $\tau_{v|p}$ increases $\Rightarrow$ makes uninformed more aggressive $\Rightarrow$ lowers λ^{in} $\Rightarrow$ higher b^{in}.
- Comparative statics

  ...

Comparative Statics

1. $Var[u] \nearrow \infty$, $\phi \searrow 0$ (price carries no info), $\bar{b}^{un} \to \bar{\eta}^{un}\tau_v$, $b^{in} \to \frac{\bar{\eta}^{un}\tau_v}{\bar{\eta}^{un}\tau_v + \eta^{in}\tau_{v|S}}\eta^{in}\tau_{v|S}$
   same as in monopoly solution with competitive "Walrasian" outsiders (homework: check this!)
2. As $Var[u] \searrow 0, (\tau_u \nearrow \infty)$, $b^{in} \to 0$, from cubic equation.
   - Actually, $\left(b^{in}\right)^2 \tau_u \to \frac{\tau_{v|S}\tau_v}{\tau_\varepsilon}$. Hence, $\phi \to \frac{\tau_v}{2\tau_v + \tau_\varepsilon} < \frac{1}{2}$.
   - Furthermore, $\bar{b}^{un} \to 0$, $\lambda^{in} \to 0$, $a^{in} \to 0$, $a^{un} \to 0$.
   - Hence, NO TRADE EQUILIBRIUM, that is $\Delta x^i(p) \to 0$, even though initial endowments $\left\{z^i\right\}_{i\in I}$ are not well diversified.
   - One needs noise to lubricate financial markets.
   - NOTE: this result hinges on the unbounded support of normal distribution.

Does Asymmetric Information Without Noise Trading Lead to Market Break Down?

1. Limit $Var[u] = 0$ in above simplified Kyle (1989) setting
   $\Rightarrow$ non-existence of an equlibrium
2. Bhattacharya & Spiegel (1991) setup:
   as before, but (i) z^{in} is random and (ii) $Var[u] = 0$
   $\Rightarrow$ non-existence of an equilibrium
   [due to unbounded support of $X^{\text{sup}}S$ (Noeldeke 1993, Hellwig1993)]
3. Finite number of signals and CARA (Noeldeke 1992)
   $\Rightarrow$ if initial allocation is inefficient a fully revealing trade equilibrium always exists.
   (with bounded support, allows construction starting from worst possible type)
4. Finite number of signals and HARA & NIARA (nonincreasing)
   $\Rightarrow$ market may break down (in very specific circumstances)

Price Discrimination vs. Uniform Pricing

- total payment:
  - uniform prices: total payment is $px^i(p)$.
  - discriminatory: total payment is $\int_0^{x^i(p)} p(q)dq$ (area below demand schedule $p(q)$).
- Discriminatory pricing eliminates equilibria with $p < \bar{v}$ (commonly known)
- Demand curves in mean variance setting (Viswanathan & Wang)
  - uniform pricing:
    $p = E[v] - \rho Var[v]\ x^{i,*}(p) + \frac{1}{\sum_{-i \in \mathbb{I} \setminus i} \frac{\partial x^{-i,*}}{\partial p}}\ x^{i,*}(p)$.
  - discriminatory pricing: (intercept & slope change)
    $p = E[v] - \rho Var[v]\ x^{i,*}(p) + \frac{1}{\sum_{-i \in \mathbb{I} \setminus i} \frac{\partial x^{-i,*}}{\partial p}}\ \frac{1}{H(u)}$.

    where $H(u) = \frac{g(u)}{1-G(u)}$ (hazard rate of random u.)