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LECTURE 10: MULTI-PERIOD MODEL FUTURES & SWAPS

Overview

1. Futures
    - Forwards versus Futures Price
    - Interest Rate Forwards and Futures
    - Currency Futures
    - Commodity Futures
        - Backwardation and Contango
2. Swaps

Futures Contracts

- Exchange-traded “forward contracts”
- Typical features of futures contracts
  - Standardized, specified delivery dates, locations, procedures
  - A clearinghouse
    - Matches buy and sell orders
    - Keeps track of members’ obligations and payments
    - After matching the trades, becomes counterparty
- Differences from forward contracts
  - Settled daily through mark-to-market process → low credit risk
  - Highly liquid → easier to offset an existing position
  - Highly standardized structure → harder to customize

Example: S&P 500 Futures (cont.)

- Notional value: $250 x Index
- Cash-settled contract
- Open interest: total number of buy/sell pairs
- Margin and mark-to-market
  - Initial margin
  - Maintenance margin (70-80% of initial margin)
  - Margin call
  - Daily mark-to-market
- Futures prices vs. forward prices
  - The difference negligible especially for short-lived contracts
  - Can be significant for long-lived contracts and/or when interest rates are correlated with the price of the underlying asset

Futures: Margin Balance

- Mark-to-market proceeds and margin balance for 8 long futures:

TABLE 5.8 Mark-to-market proceeds and margin balance over 10 weeks from long position in 8 S&P 500 futures contracts. The final row represents expiration of the contract.

Week	Multiplier ($)	Futures Price	Price Change	Margin Balance($)
0	2000.00	1100.00	—	220,000.00
1	2000.00	1027.99	−72.01	76,233.99
2	2000.00	1037.88	9.89	96,102.01
3	2000.00	1073.23	35.35	166,912.96
4	2000.00	1048.78	−24.45	118,205.66
5	2000.00	1090.32	41.54	201,422.13
6	2000.00	1106.94	16.62	234,894.67
7	2000.00	1110.98	4.04	243,245.86
8	2000.00	1024.74	−86.24	71,046.69
9	2000.00	1007.30	−17.44	36,248.72
10	2000.00	1011.65	4.35	44,990.57

Forwards versus Futures Pricing

- Price of Forward using EMM is

$$0 = E_t^Q\left[\rho_T\left(F_{0,T} - S_T\right)\right]$$
$$= E_t^Q[\rho_T]\left(F_{0,T} - E_t^*[S_T]\right) - \mathrm{cov}_t^Q[\rho_T, S_T]$$

Special case:

$$F_{0,T} = E_t^Q[S_T]$$

Homework: If I do it with M, then I get …

- Value of Futures contract is always zero.
Each period there is "dividend" stream $\phi_t - \phi_{t-1}$ and $\phi_T = S_T$

$0 = E_T^Q[\rho_{t+1}(\phi_{t+1} - \phi_t)]$ for all t

since ρ_{t+1} is known at t

$\phi_t = E_t^Q[\phi_{t+1}]$ and $\phi_T = S_T$

$$\phi_t = E_t^Q[S_T]$$

General: Futures price process is always a martingale

Delete???

Uses of Index Futures

- Why buy an index futures contract instead of synthesizing it using the stocks in the index? Lower transaction costs
- Asset allocation: switching investments among asset classes
- Example: Invested in the S&P 500 index and temporarily wish to temporarily invest in bonds instead of index. What to do?
  - Alternative #1: Sell all 500 stocks and invest in bonds
  - Alternative #2: Take a short forward position in S&P 500 index

TABLE 5.10 Effect of owning the stock and selling forward, assuming that $S_0 = \$100$ and $F_{0,1} = \$110$.

	Cash Flows		
Transaction	**Today**	**1 year, $S_1 = \$80$**	**1 year, $S_1 = \$130$**
Own Stock @ \$100	−\$100	\$80	\$130
Short Forward @ \$110	0	\$110 − \$80	\$110 − \$130
Total	−\$100	\$110	\$110

Uses of Index Futures (cont.)

- \$100 million portfolio with β of 1.4 and $r_f = 6\%$

1. Adjust for difference in \$ amount
   - 1 futures contract \$250 x 1100 = \$275,000
   - Number of contracts needed \$100mill/\$0.275mill = 363.636
2. Adjust for difference in β

   363.636 x 1.4 = 509.09 contracts

Forward Rate Agreements

- FRAs: over-the-counter contracts that guarantee a borrowing or lending rate on a given notional principal amount
- Settlement:
  - In arrears: $\left(r_{qrtly} - r_{FRA}\right) \times$ notional principal
  - At the time of borrowing: notional principal $\times \frac{\left(r_{qrtly} - r_{FRA}\right)}{1 + r_{qrtly}}$
- FRAs can be synthetically replicated with zero-coupon bonds

Eurodollar Futures

- Contract specifications

FIGURE 5.7

Specifications for the Eurodollar futures contract.

Where traded	Chicaco Mercantile Exchange
Size	3-month Eurodollar time deposit, $1 million principal
Months	Mar, Jun, Sep, Dec, out 10 years, plus 2 serial months and spot month
Trading ends	5 A.M. (11 A.M. London) on the second London bank business day immediately preceding the third Wednesday of the contract month.
Delivery	Cash settlement
Settlement	100 − British Banker's Association Futures Interest Settlement Rate for 3-Month Eurodollar Interbank Time Deposits. (This is a 3-month rate annualized by multiplying by 360/90.)

Eurodollar Futures

- Very similar in nature to an FRA with subtle differences
  - The settlement structure of Eurodollar contracts favors borrowers
  - Therefore the rate implicit in Eurodollar futures is greater than the FRA rate ⇒ Convexity bias
- The payoff at expiration: [Futures price - (100 - r_{LIBOR})] x 100 x \$25
- Example: Hedging \$100 million borrowing with Eurodollar futures:

TABLE 7.4 Results from hedging \$100m in borrowing with 98.23 short Eurodollar futures.

	Cash Flows			
	June		September	
Borrowing Rate:	1.5%	2%	1.5%	2%
Borrow \$100m	+100m	+100m	−101.5m	−102.0m
Gain on 98.23 Short Eurodollar Contracts	−0.294695m	0.196464m		
Gain Plus Interest			−0.299115m	0.200393m
Net			−101.799m	−101.799m

Treasury Bond/Note Futures

- Contract specifications

FIGURE 7.6

Specifications for the Treasury-note futures contract.

Where traded	CBOT
Underlying	6% 10-year Treasury note
Size	$100,000 Treasury note
Months	Mar, Jun, Sep, Dec, out 15 months
Trading ends	Seventh business day preceding last business day of month. Delivery until last business day of month.
Delivery	Physical T-note with at least 6.5 years to maturity and not more than 10 years to maturity. Price paid to the short for notes with other than 6% coupon is determined by multiplying futures price by a conversion factor. The conversion factor is the price of the delivered note ($1 par value) to yield 6%. Settlement until last business day of the month.

Treasury Bond/Note Futures (cont.)

- Long T-note futures position is an obligation to buy a 6% bond with maturity between 6.5 and 10 years to maturity
- The short party is able to choose from various maturities and coupons: the "cheapest-to-deliver" bond
- In exchange for the delivery the long pays the short the "invoice price."

$$\text{Invoice price} = (\text{Futures price x } \underbrace{\text{conversion factor}}_{\text{Price of the bond if it were to yield 6\%}}) + \text{accrued interest}$$

TABLE 7.5 Prices, yields, and the conversion factor for two bonds. The futures price is 97.583. The short would break even delivering the 8-year 7% bond, and lose money delivering the 7-year 5% bond. Both bonds make semiannual coupon payments.

Description	8-Year 7% Coupon, 6.4% Yield	7-Year 5%, 6.3% Yield
Market Price	103.71	92.73
Price at 6% (Conversion Factor)	106.28	94.35
Invoice Price (Futures × Conversion Factor)	103.71	92.09
Invoice − Market	0	−0.66

Currency Contracts

- Currency prepaid forward
  - Suppose you want to purchase ¥1 one year from today using $s
  - $F_{0,T}^{P} = x_0 e^{-r_y T}$ *(price of prepaid forward)*
    - where x_0 is current ($/ ¥) exchange rate, and r_y is the yen-denominated interest rate
    - Why? By deferring delivery of the currency one loses interest income from bonds denominated in that currency
- Currency forward
  - $F_{0,T} = x_0 e^{(r-r_y)T}$
    - r is the $-denominated domestic interest rate
    - $F_{0,T} > x_0$ *if* $r > r_y$ (domestic risk-free rate exceeds foreign risk-free rate)

Currency Contracts: Pricing (cont.)

- Synthetic currency forward: borrowing in one currency and lending in another creates the same cash flow as a forward contract
- Covered interest arbitrage: offset the synthetic forward position with an actual forward contract

TABLE 5.12 Synthetically creating a yen forward contract by borrowing in dollars and lending in yen. The payoff at time 1 is ¥1 − $0.009367.

	Cash Flows			
	Year 0		Year 1	
Transaction	**$**	**¥**	**$**	**¥**
Borrow $x_0 e^{-r_y}$ Dollar at 6% ($)	+0.008822	—	−0.009367	—
Convert to Yen @ 0.009 $/¥	−0.008822	+0.9802	—	—
Invest in Yen-Denominated Bill (¥)	—	−0.9802	—	1
Total	0	0	−0.009367	1

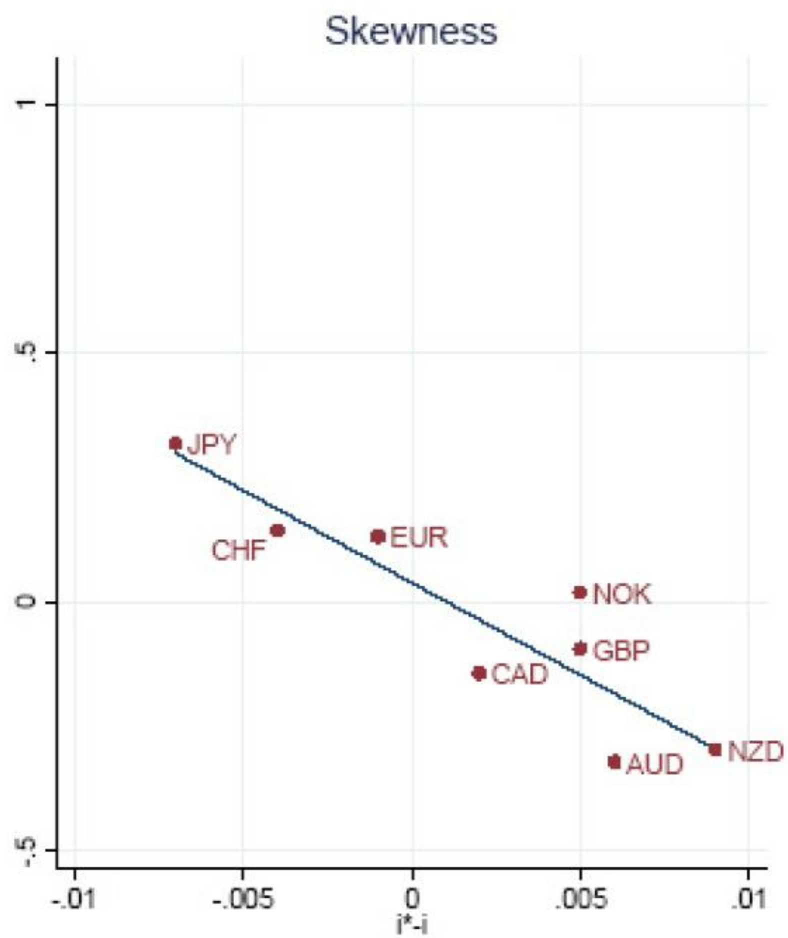
Interest Rate Parity – FX Carry Trade

- Covered Interest Rate Parity
  - Forward/futures hedged with offsetting currency portfolio
- Uncovered Interest Rate Parity
  - Carry trade:
    - buy high interest rate currency and
    - sell short interest rate currency (funding currency)
    - And hope exchange rate does move against you
  - Carry trade with forward/futures (unhedged)

Skewness of FX Carry Trade Returns

- Up the stairs and down the elevator

- Brunnermeier, Pedersen & Nagel (2012)



Commodity Forwards

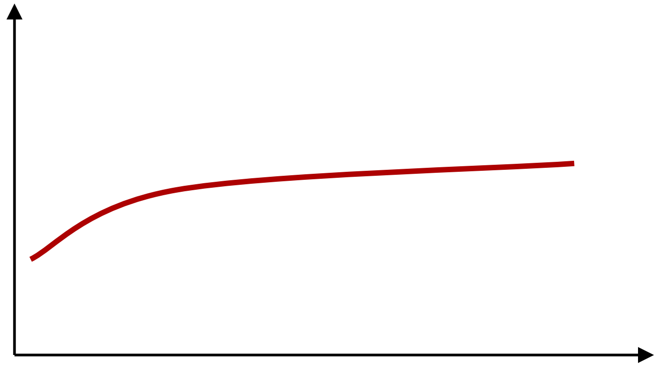
- *Commodity* forward prices can be described by the same formula as that for *financial* forward prices:

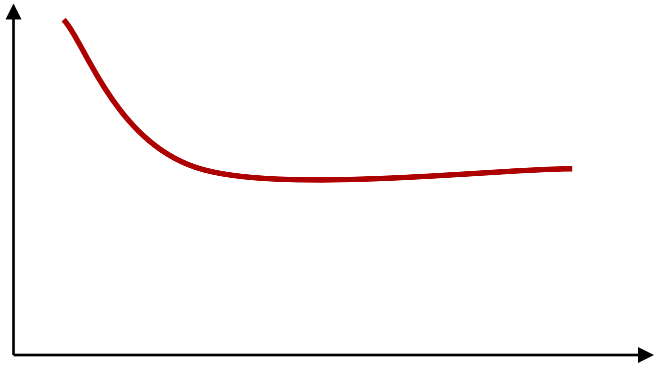
$$F_{0,T} = S_0 e^{(r-\delta)T}$$

  - For financial assets: δ is dividend yield
  - For commodities: δ is commodity lease rate
    - rate is return an investor makes from buying and lending out the commodity.
    - Can be backed out from forward prices

Commodity Forwards

- **Forward curve** (or the **forward strip**): Set of prices for different expiration dates for a given commodity
  - upward-sloping, then the market is in **contango**.
  - downward sloping, the market is in **backwardation**.
  - Note that forward curves can have portions in backwardation and portions in contango.





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2. Swaps
   - Commodity Swaps
   - Interest Rate Swaps

Introduction to Swaps

- **Swap:** contract calling for an exchange of payments, on one or more dates, determined by the difference in two prices.
- means to hedge a *stream* of risky payments.
- A single-payment swap is the same thing as a cash-settled forward contract.

Example of a Commodity Swap

- An industrial producer, IP Inc., needs to buy 100,000 barrels of oil 1 year from today and 2 years from today.
- The forward prices for deliver in 1 year and 2 years are $20 and $21/barrel.
- The 1- and 2-year zero-coupon bond yields are 6% and 6.5%.

Example of Commodity Swap

- IP wants to guarantee the cost of oil for the next 2 years
- Enter long forward contracts for 100,000 barrels in each of the next 2 years.
- The PV of this cost per barrel is

$$\frac{\$20}{1.06} + \frac{\$21}{1.065^2} = \$37.383$$

- Thus, IP could pay an oil supplier \$37.383, and the supplier would commit to delivering one barrel in each of the next two years.
- **notional amount** of the swap, e.g. 100,000 barrels determines the magnitude of the payments when the swap is settled financially
- A **prepaid swap** is a single payment today for multiple deliveries of oil in the future. (multiple prepaid forwards)

Example of Commodity Swap

- With a prepaid swap, the buyer might worry about the resulting credit risk. Therefore, a better solution is to defer payments until the oil is delivered, while still fixing the total price.

$$\frac{x}{1.06} + \frac{x}{1.065} = \$37.383$$

- Any payment stream with a PV of $37.383 is acceptable. Typically, a swap will call for equal payments in each year.
  - For example, the payment per year per barrel, x, will have to be $20.483 to satisfy the following equation:
- We then say that the 2-year swap price is $20.483.

Economics of Swaps

- Swap
  - Multiple forward contracts (futures if exchange traded)
  - Implicit borrowing/lending arrangement
- Example
  - Swap price of \$20.483/barrel. Relative to the forward curve price of \$20 in 1 year and \$21 in 2 years, we overpay by \$0.483 in $t = 1$, and we underpay by \$0.517 in $t = 2$.

  - Lending the counterparty from $t = 1$ to $t = 2$o arranged at $t = 0$. The interest rate on this loan is

    0.517 / 0.483 – 1 = 7%.
  - Has to be consistent with implied forward rate in bond prices: 1- and 2-year zero-coupon bond yields of 6% and 6.5%

Swap Counterparty

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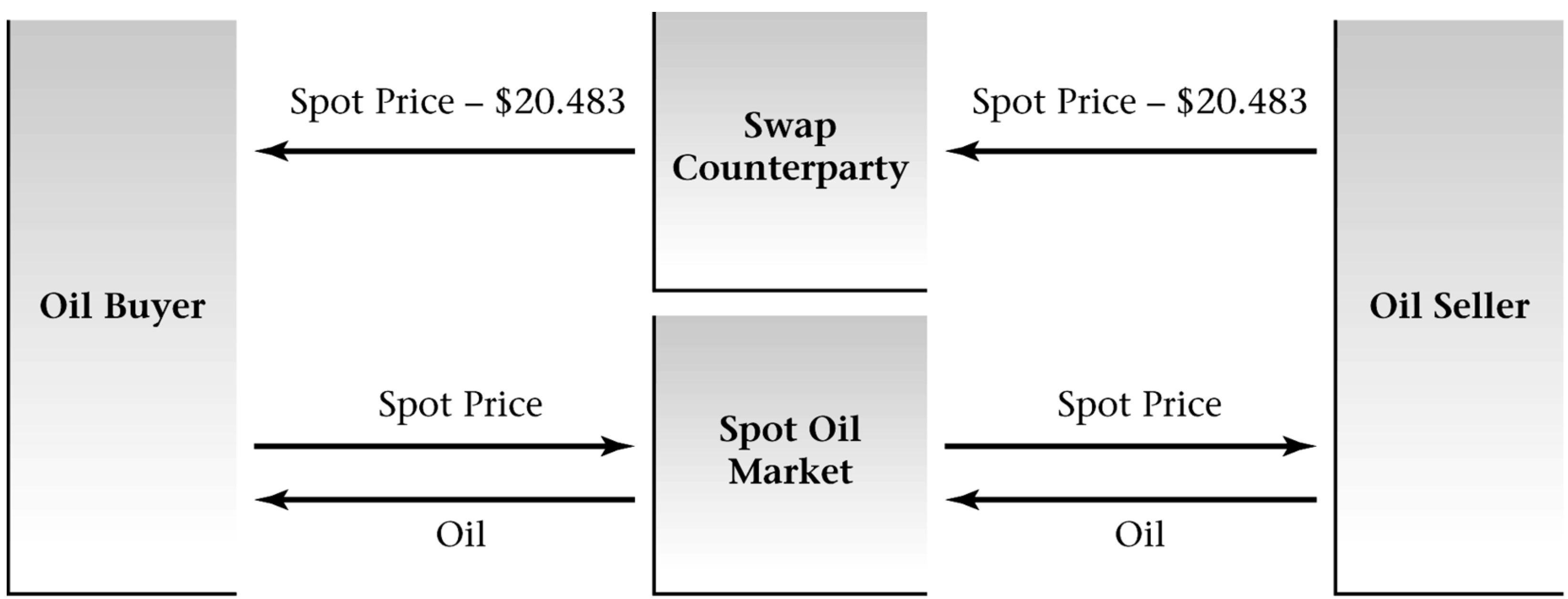
- The situation where the dealer matches the buyer and seller is called a **back-to-back transaction** or "matched book" transaction.



Market Value of a Swap

- The market value of a swap is **zero at inception**.
- Once the swap is struck, its market value will generally no longer be zero because:
  - **forward prices** for oil and interest rates **will change** over time;
  - even if prices do not change, the market value of swaps will change over time due to the **implicit borrowing and lending**.
- Exit swap buy entering into an offsetting swap with the original counterparty or not
- The market value of the swap is the difference in the PV of payments between the original and new swap rates.

Interest Rate Swaps

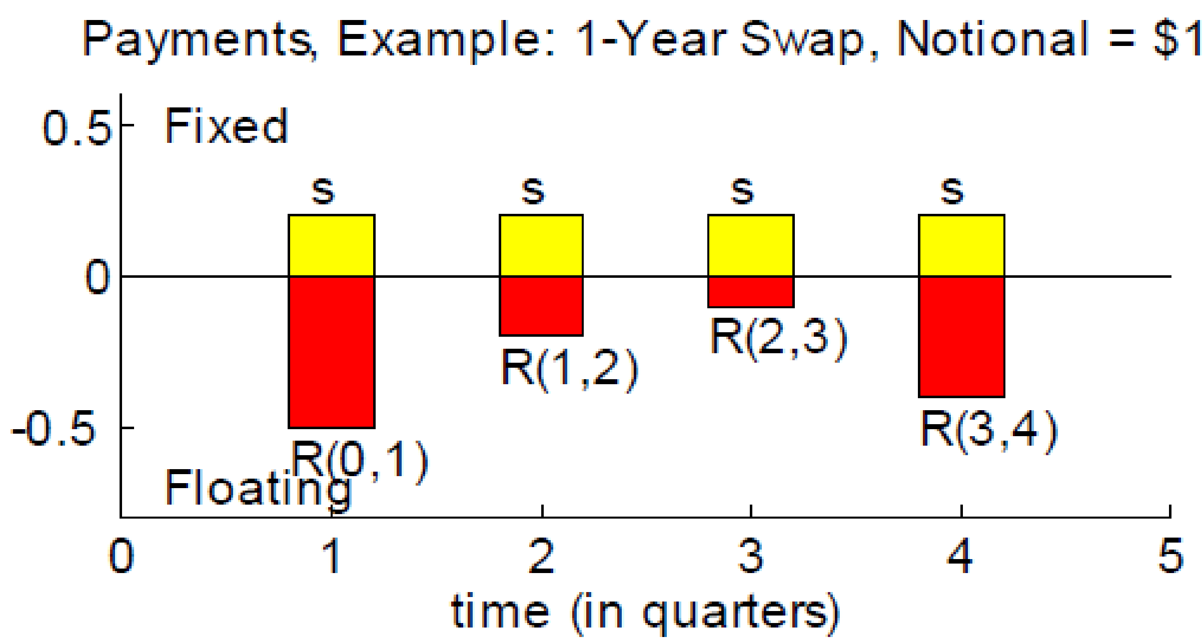
- Two arms: Fixed and floating arm
- Notional principle of the swap is the amount on which the interest payments are based.
- Swap term or swap tenor: life of swap
- Settlement:
  - If swap payments are made at the end of the period (when interest is due), the swap is said to be settled in arrears.

An example of an interest rate swap

- XYZ has \$200M of floating-rate debt at LIBOR
- prefers fixed-rate debt with 3 years to maturity.
- XYZ could enter a swap, in which they
  - receive a floating rate and
  - pay the fixed rate, which is $R = 6.9548\%$.

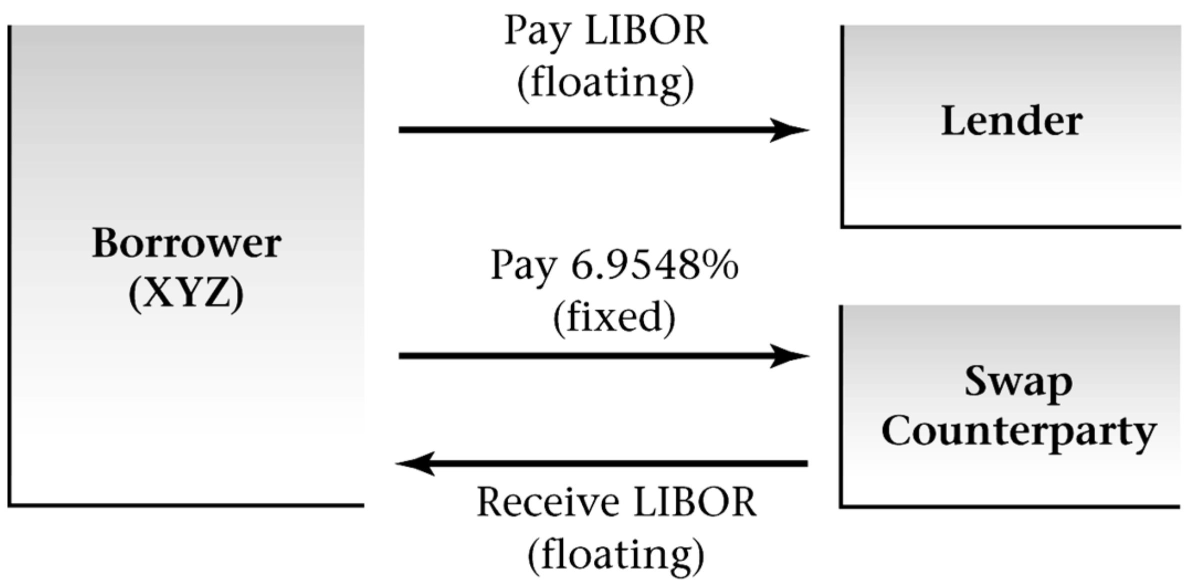
Swap Payoffs

- Counterparties swap fixed for floating $\propto$ notational N



- Fixed leg = fixed payment + N at maturity, value falls with i
- Floating leg = floating payment + N at maturity, value $= N$

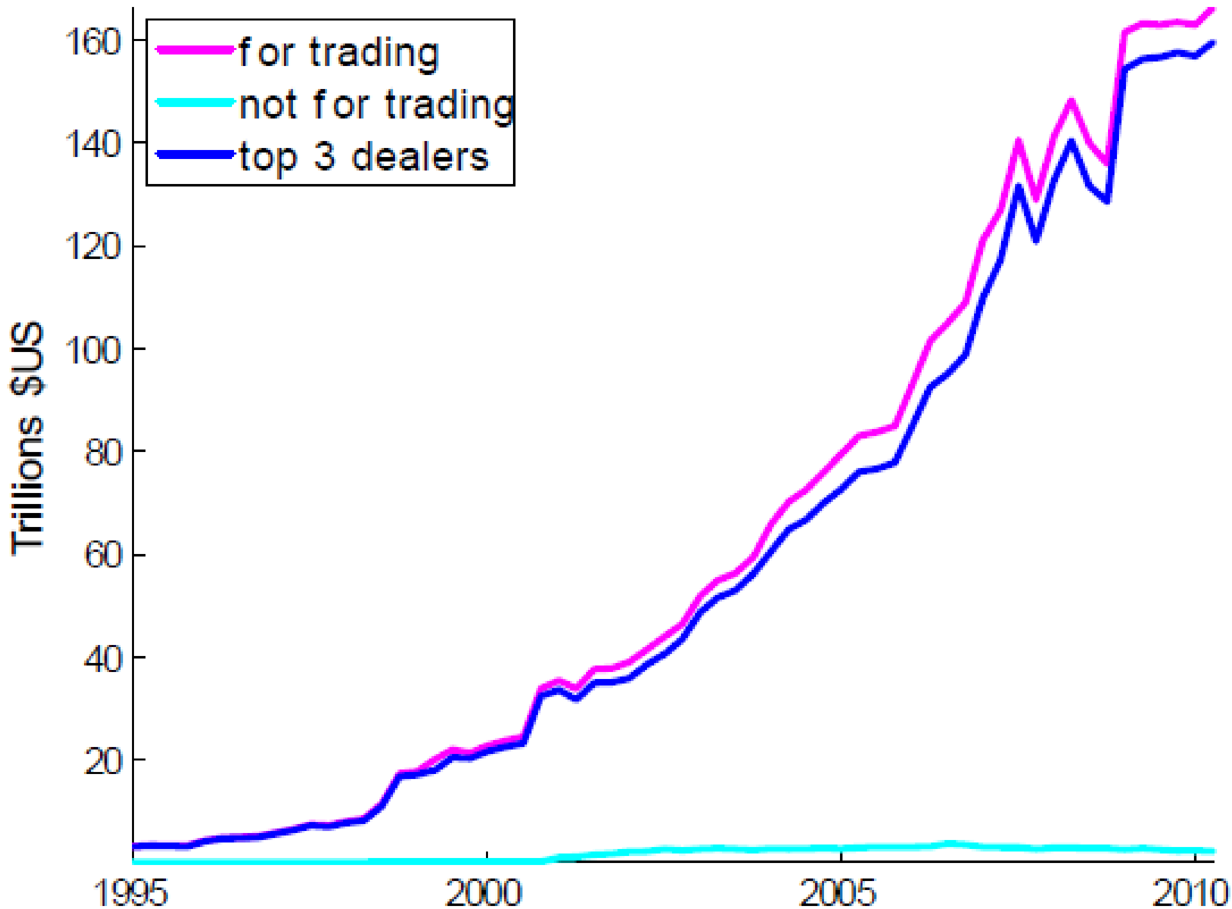
Example of Interest Rate Swap



On net, XYZ pays 6.9548%:

$$\text{XYZ net payment} = \underbrace{-\text{LIBOR}}_{\text{Floating Payment}} + \underbrace{\text{LIBOR} - 6.9548\%}_{\text{Swap Payment}} = -6.9548\%$$

Concentrated Holding of Interest Rate Swaps



Bergenau et al. 2013

Swap ate

- Relative asset pricing of swap rate R
  - relative to forward rates implied by bond prices
  - $r_0(t_{i-1}, t_i)$: implied forward interest rate from date t_{i-1} to date t_i, known at date 0, is.
  - $q(0, t_i)$: Price of zero-coupon bond maturing on t_i
  - R The fixed swap rate is.
  - n swap settlements, occurring on dates $t_i, i = 1, \ldots, n$

Swap Rate

- The requirement that the hedged swap have zero net PV is

$$\sum_{i=1}^{n} Z(0,t_i)[R - r_0(t_{i-1},t_i)] = 0$$

- Hence,

$$R = \frac{\sum_{i=1}^{n} Z(0,t_i) r_0(t_{i-1},t_i)}{\sum_{i=1}^{n} Z(0,t_i)}$$

  - where $\sum_{i=1}^{n} Z(0,t_i) r_0(t_{i-1},t_i)$ is PV of interest payments implied by the strip of forward rates, and
  - $\sum_{i=1}^{n} Z(0,t_i)$ is the PV of a \$1 annuity when interest rates vary over time.

Swap Rate

- Rewrite – to easy interpretation:

$$R = \sum_{i=1}^{n} \left[\frac{Z(0, t_i)}{\sum_{j=1}^{n} Z(0, t_j)} \right] r_0(t_{i-1}, t_i)$$

- weighted average of the implied forward
  - zero-coupon bond prices determines weights

Swap Rate

- Third way of writing

$$R = \frac{1 - Z(0, t_n)}{\sum_{i=1}^{n} Z(0, t_i)}$$

- using $r_0(t_1, t_2) = \frac{Z(0,t_1)}{Z(0,t_2)} - 1$

This equation is equivalent to the formula for the coupon on a par coupon bond.

Thus, the swap rate is the coupon rate on a par coupon bond.
(firm that swaps floating for fixed ends up with economic equivalent of a fixed-rate bond)

Swap Curve

- *swap curve:* Set of swap rates at different maturities
- consistent with the interest rate curve implied by the Eurodollar futures
    - Recall
      Eurodollar futures contract provides a set of 3-month forward LIBOR rates. In turn, zero-coupon bond prices can be constructed from implied forward rates. Therefore, we can use this information to compute swap rates.

Swap Curve

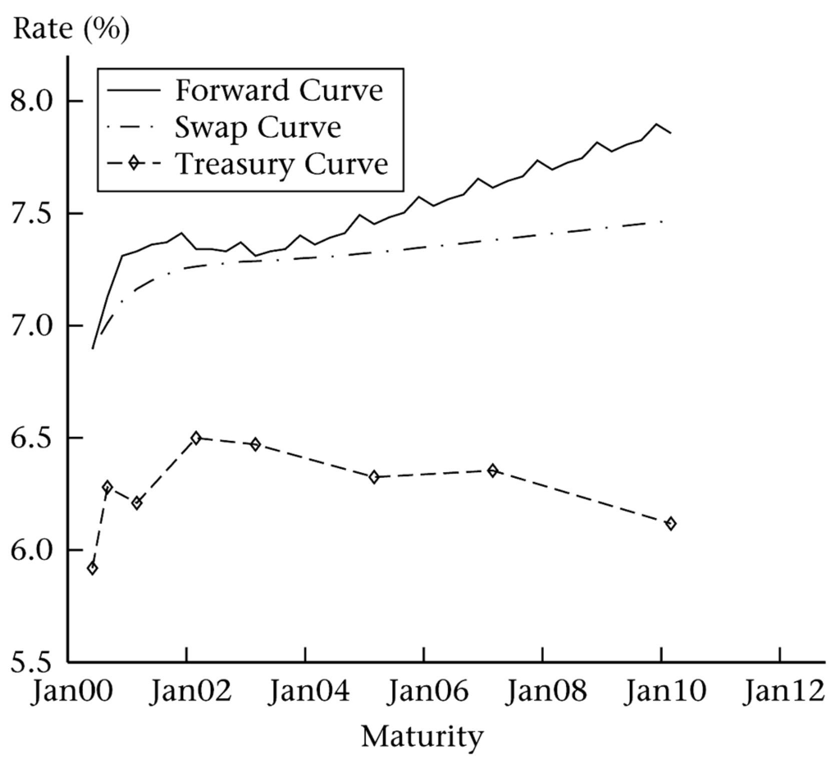
TABLE 8.4 3-month LIBOR forward rates implied by Eurodollar futures prices with maturity dates given in the first column. Prices are from June 7, 2000.

Maturity Date, t_i	Eurodollar Futures Price	Implied Quarterly Rate, $r(t_i, t_{i+1})$	Implied June 2000 Price of \$1 Paid on Maturity Date, t_i, $P(0, t_i)$	Swap Rate
Jun-00	93.18	0.01724	—	6.895%
Sep-00	92.95	0.01782	0.9830	7.011%
Dec-00	92.77	0.01827	0.9658	7.109%
Mar-01	92.75	0.01832	0.9485	7.162%
Jun-01	92.72	0.01840	0.9314	7.201%
Sep-01	92.70	0.01842	0.9146	7.228%
Dec-01	92.67	0.01852	0.8980	7.252%
Mar-02	92.74	0.01835	0.8817	7.263%

For example, the December swap rate can be computed using equation (8.3): (1 – 0.9485)/ (0.9830 + 0.9658 + 0.9485) = 1.778%. Multiplying 1.778% by 4 to annualize the rate gives the December swap rate of 7.109%.

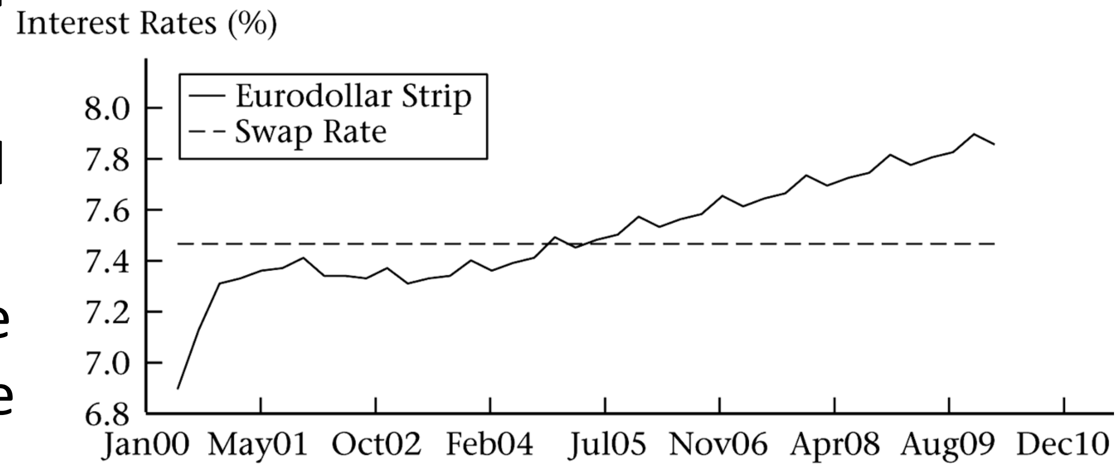
Swap Curve

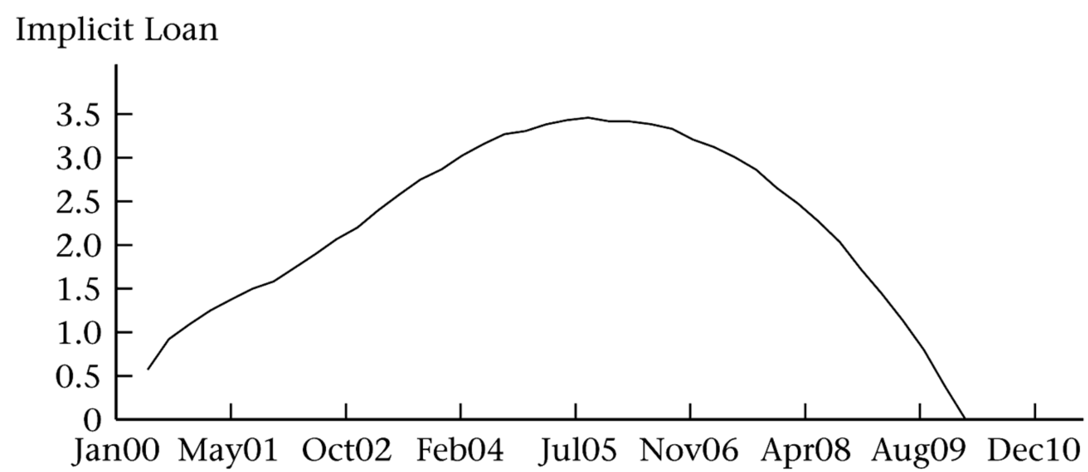
- **Swap spread** is the difference between swap rates and Treasury-bond yields for comparable maturities.



Swap's Implicit Loan Balance

- Implicit borrowing and lending in a swap can be illustrated using the following graph, where the 10-year swap rate is 7.4667%:





Swap's Implicit Loan Balance

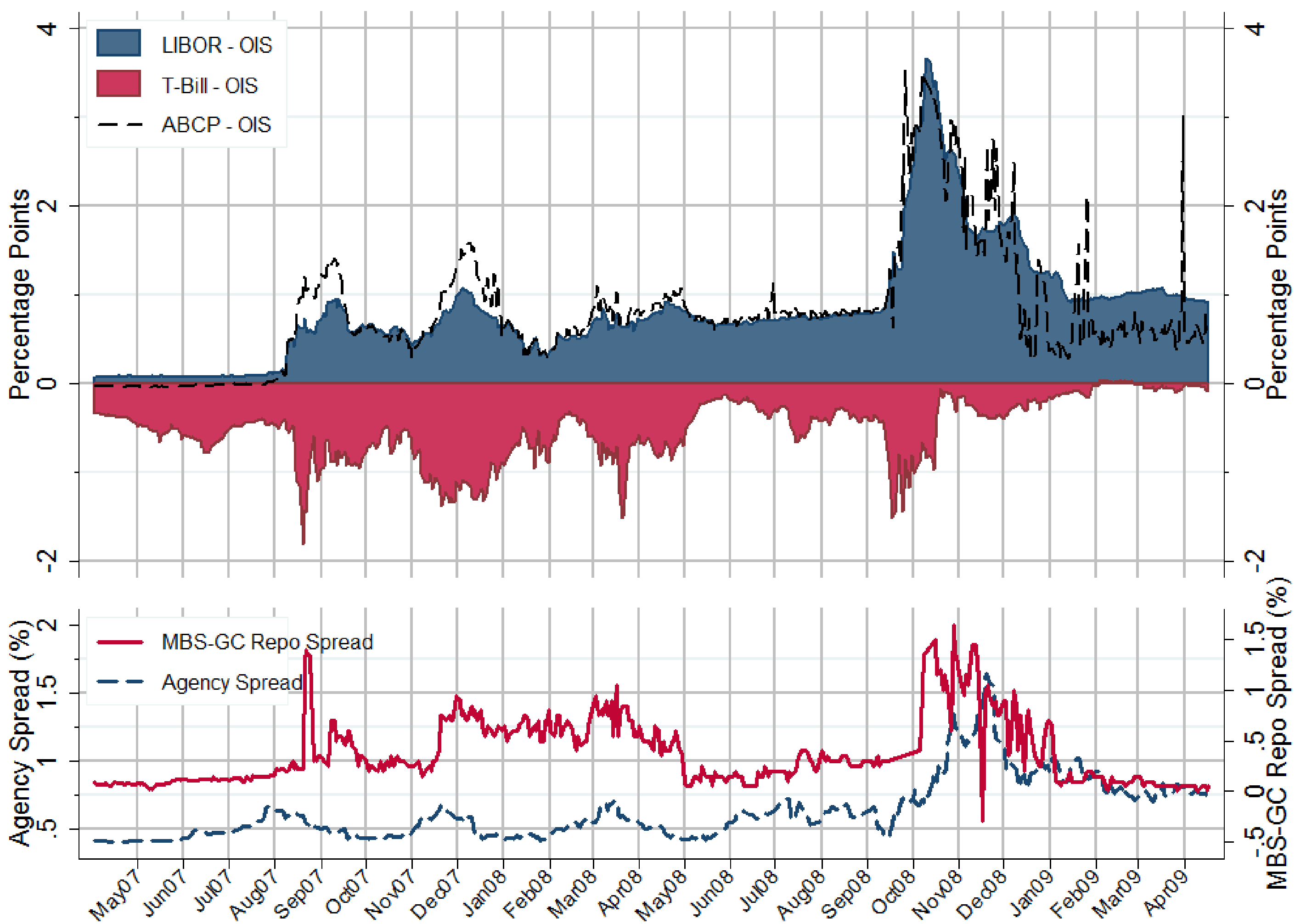
- In the above graph,
  - investor who pays fixed and receives floating.
  - in the early years:
    he pays a high rate of the swap, and hence lends money.
  - In the later years:
    Eurodollar forward rate exceeds the swap rate and the loan balance declines,
    falling to zero by the end of the swap.

  - Therefore, the credit risk in this swap is borne, at least initially, by the fixed-rate payer, who is lending to the fixed-rate recipient.

Why Swap Interest Rates?

- Interest rate swaps permit firms to separate credit risk and interest rate risk.
  - By swapping its interest rate exposure, a firm can pay the short-term interest rate it desires, while the long-term bondholders will continue to bear the credit risk.

LIBOR OIS Spread



Deferred Swap

- A deferred swap is a swap that begins at some date in the future, but its swap rate is agreed upon today.

- The fixed rate on a deferred swap beginning in *k* periods is computed as

$$R = \frac{\sum_{i=k}^{T} Z(0, t_i) r_0(t_{i-1}, t_i)}{\sum_{i=k}^{T} Z(0, t_i)}$$

Amortizing and Accreting Swaps

- An amortizing swap is a swap where the notional value is *declining* over time (e.g., floating rate mortgage).
- An accreting swap is a swap where the notional value is *growing* over time.

- The fixed swap rate is still a weighted average of implied forward rates, but now the weights also involve changing notional principle, Q_t:

$$R = \frac{\sum_{i=1}^{n} Q_{t_i} Z(0, t_i) r(t_{i-1}, t_i)}{\sum_{i=1}^{n} Q_{t_i} Z(0, t_i)} \tag{8.7}$$

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