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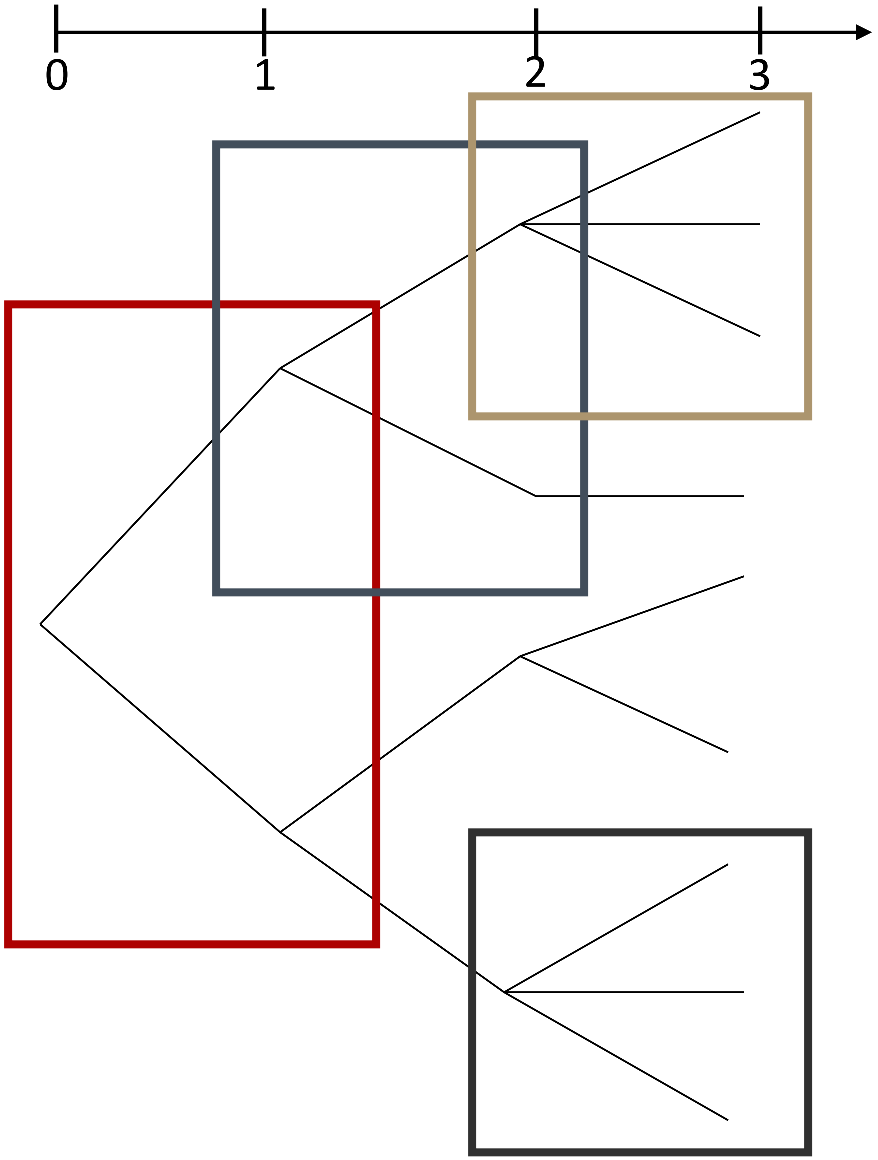
LECTURE 07: MULTI-PERIOD MODEL

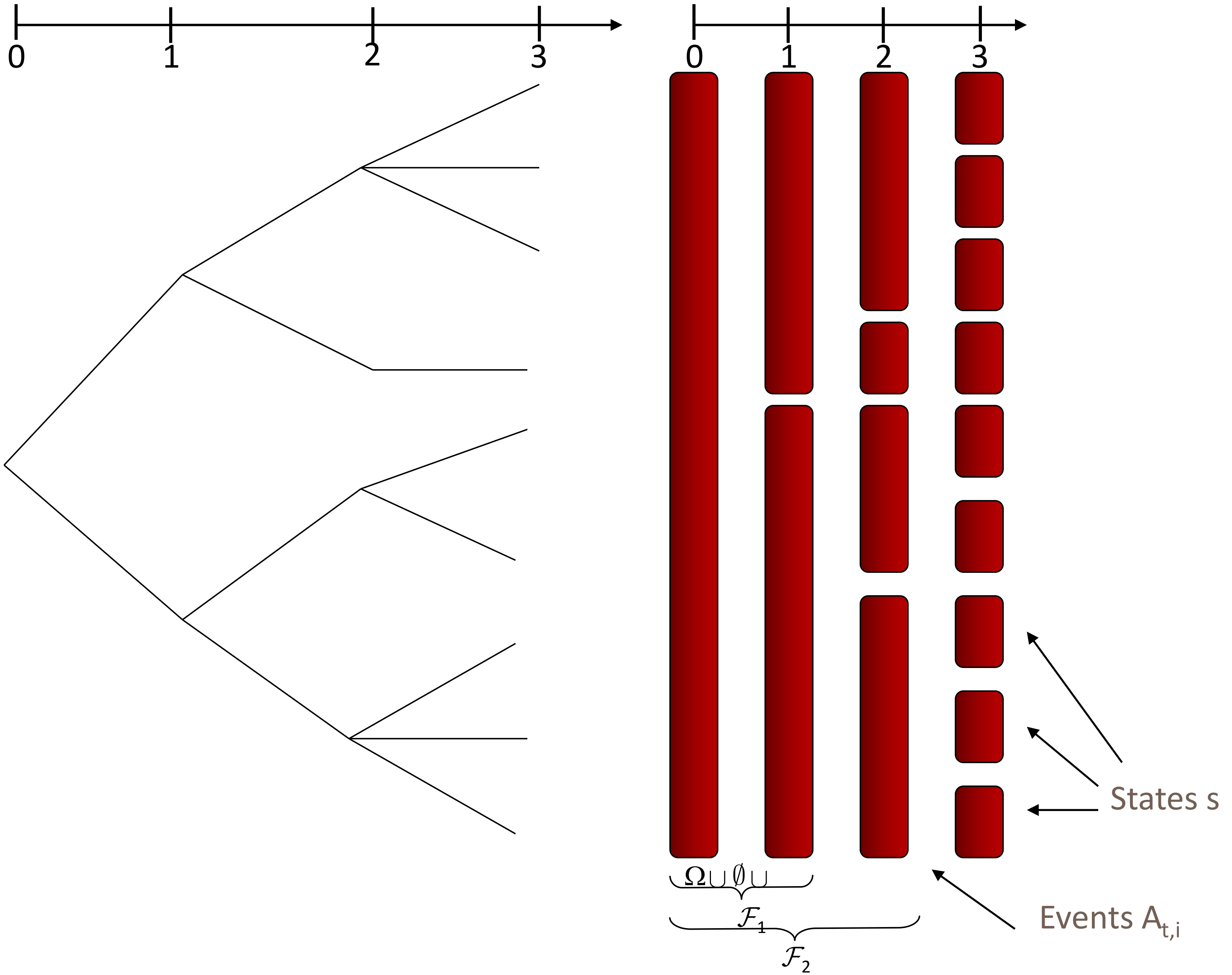
Overview

1. Generalization to a multi-period setting
    - Trees, modeling information and learning
        - Partitions, Algebra, Filtration
    - Security structure/trading strategy
        - Static vs. dynamic completeness
2. Pricing
    - Multi-period SDF and event prices
    - Martingale process – EMM
    - Forward measure
3. Ponzi scheme and Rational Bubbles



many one period models

how to model information?


0
1
2
3
$\Omega \cup \emptyset \cup$
$\mathcal{F}_1$
$\mathcal{F}_2$
States s
Events $A_{t,i}$

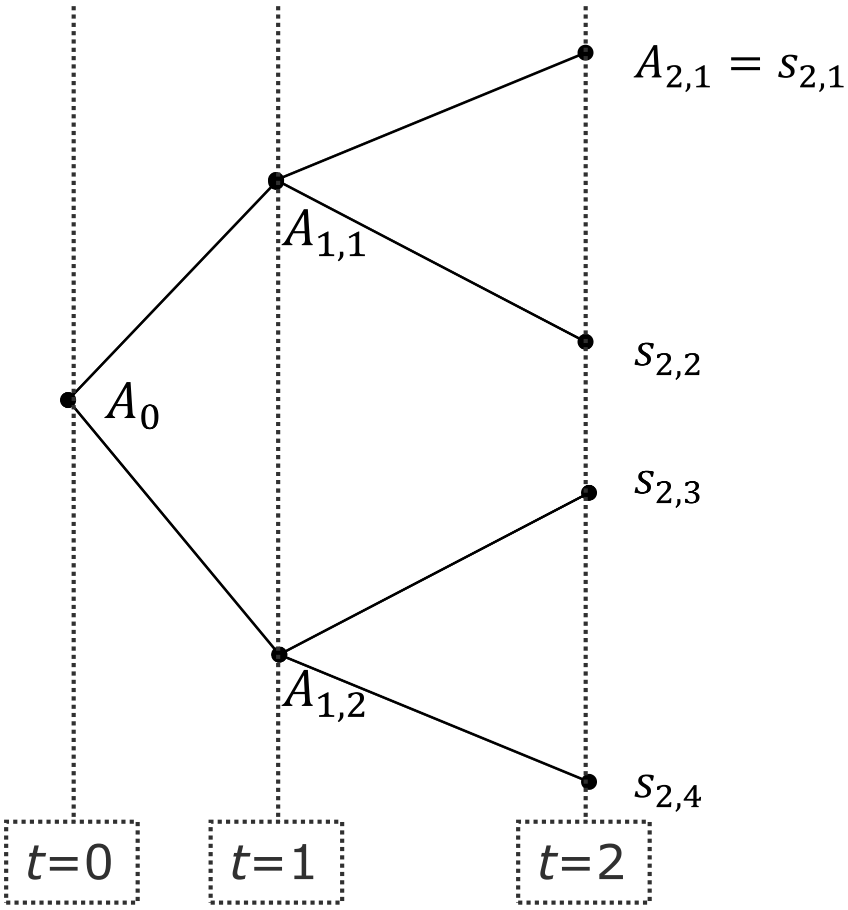
Modeling information over time

- Partition
- Field/Algebra
- Filtration

Some probability theory

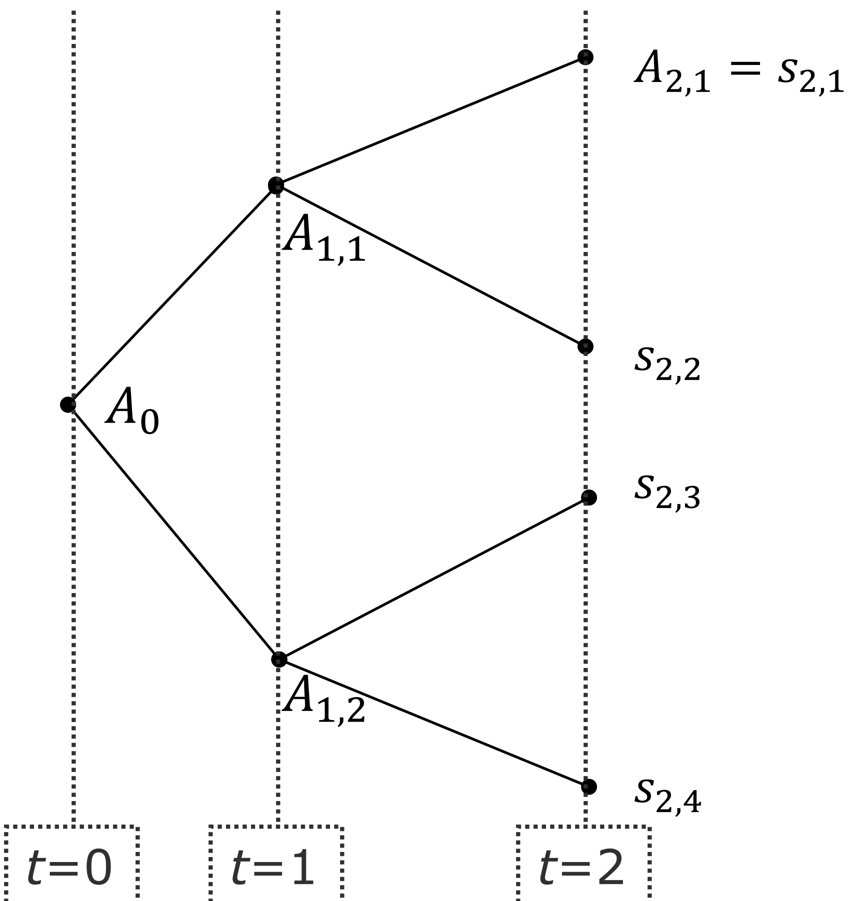
- **Measurability:** A random variable $y(s)$ is measure w.r.t. algebra $\mathcal{F}$ if
  - Pre-image of $y(s)$ are events (elements of $\mathcal{F}$)
    - for each $A \in \mathcal{F}, y(s) = y(s')$ for each $s \in A$ and $s' \in A$
    - $y(A) \coloneqq y(s), s \in A$
- **Stochastic process:** A collection of random variables $y_t(s)$ for $t = 0, \ldots, T$
- Stochastic process is **adapted to filtration $\mathcal{F}$** $= \{\mathcal{F}_u\}_{u=t}^{T}$ if each $y_t(s)$ is measurable w.r.t. $\mathcal{F}_t$
  - Cannot see in the future

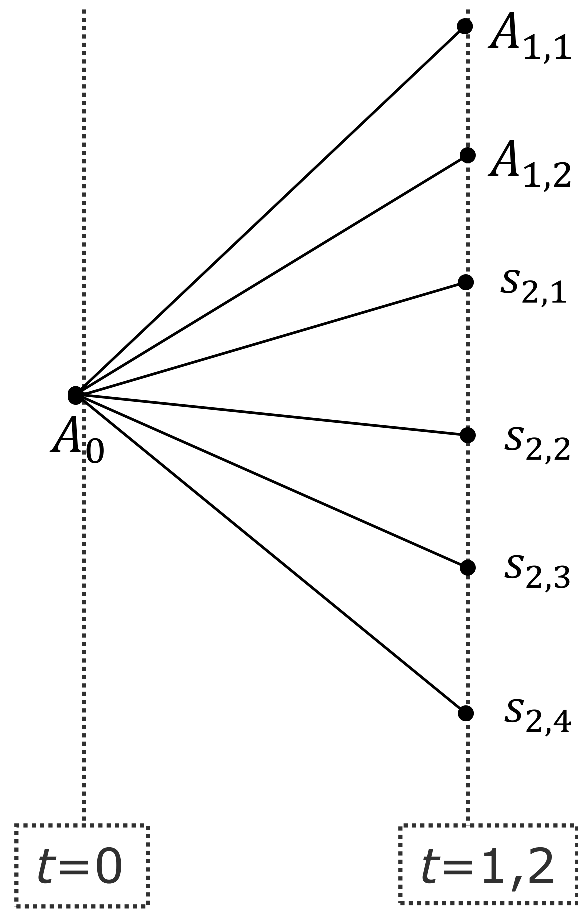
Multiple period Event Tree



- Last period events have prob., $\pi_{2,1}, \ldots, \pi_{2,4}$.
- To be consistent, the probability of an event is equal to the sum of the probabilities of its successor events.
  - E.g. $\pi_{1,1} = \pi_{2,1} + \pi_{2,2}$.

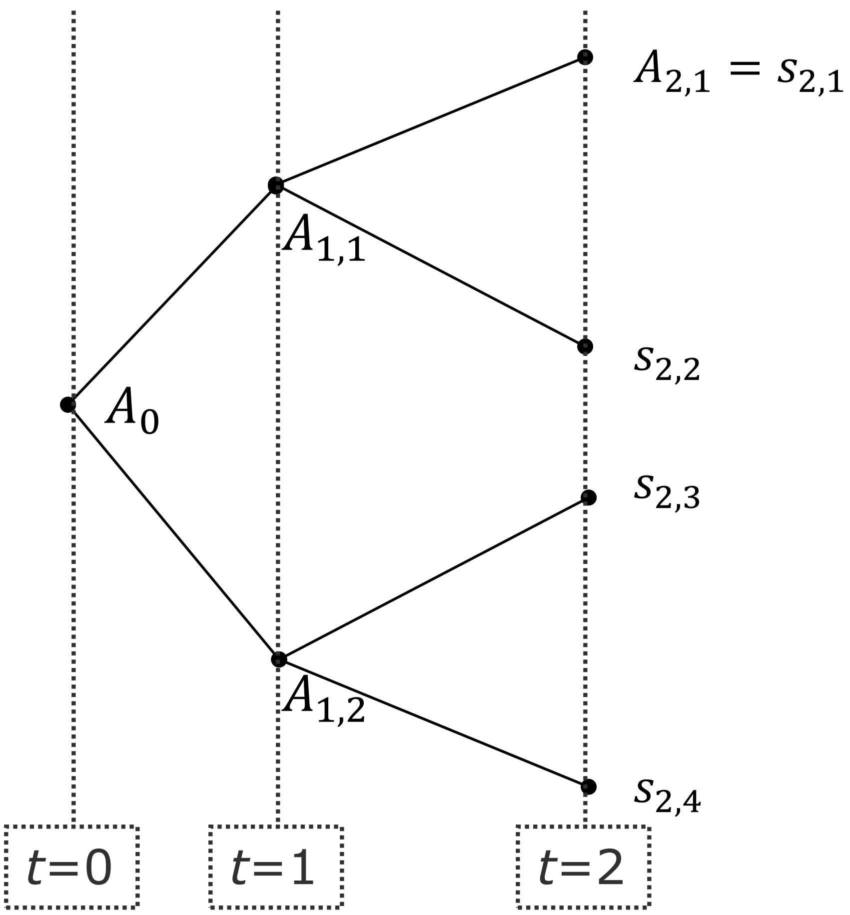
2 Ways to reduce to One Period Model

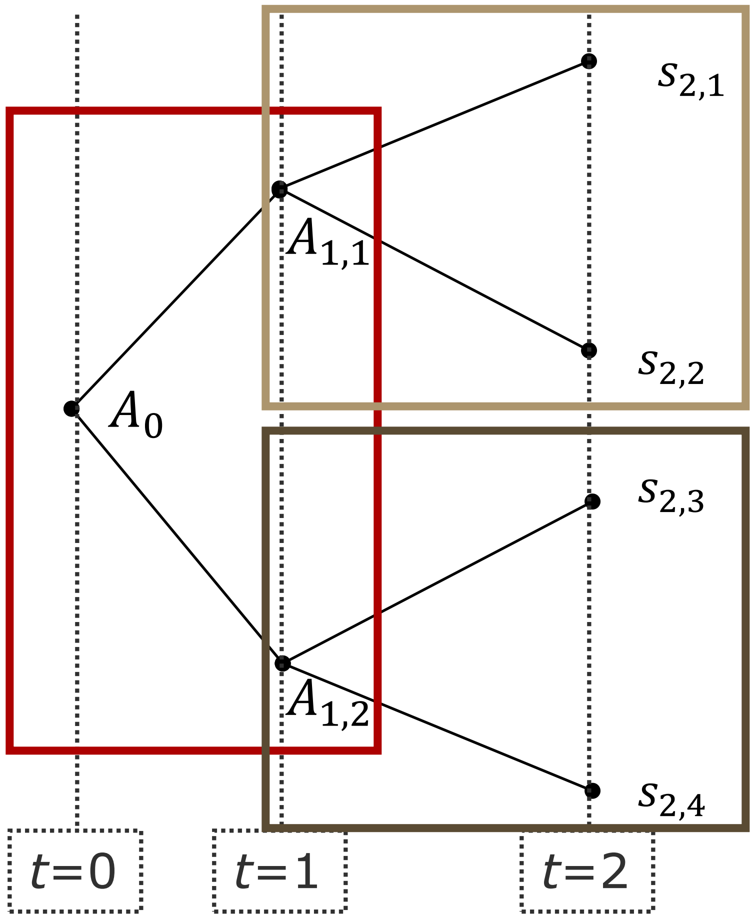




Debreu

2 Ways to reduce to One Period Model





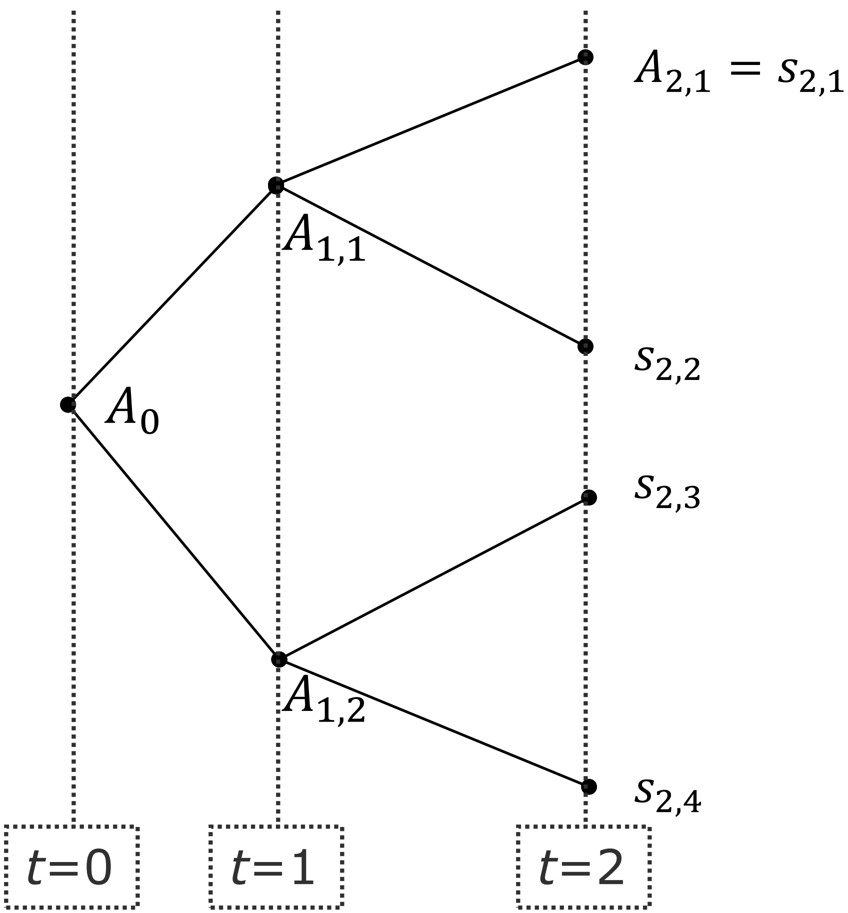
Overview: from static to dynamic…

Asset holdings	Dynamic strategy (adapted process)
Asset payoff x Payoff of portfolio holding	Next period's payoff $x_{t+1} + p_{t+1}$ Payoff of a strategy
span of assets	Marketed subspace of strategies
Market completeness	a) Static completeness (Debreu)
	b) Dynamic completeness (Arrow)
No arbitrage w.r.t. holdings	No arbitrage w.r.t strategies
States $s = 1, \ldots, S$	Events $A_{t,i}$, states $s_{t,j}$

Overview: …from static to dynamic

State prices q_s	Event prices $q_{t,i}$
Risk free rate R^f	Risk free rate R_t^f varies over time
DiscFactor: $\rho = 1/R^f$	Discount factor from t to 0: ρ_t
Risk neutral prob. $\pi_s^Q = q_s R^f$	Risk neutral prob. $\pi^Q(A_{t,i}) = \frac{q_{t,i}}{\rho_t}$
Pricing kernel $p^j = E[m^* x^j]$ $1 = E[m^*]R^f$	Pricing kernel $M_t p_t^j = E_t[M_{t+1}(p_{t+1}^j + x_{t+1}^j)]$ $M_t = R_t^f E_t[M_{t+1}]$

Multiple period Event Tree



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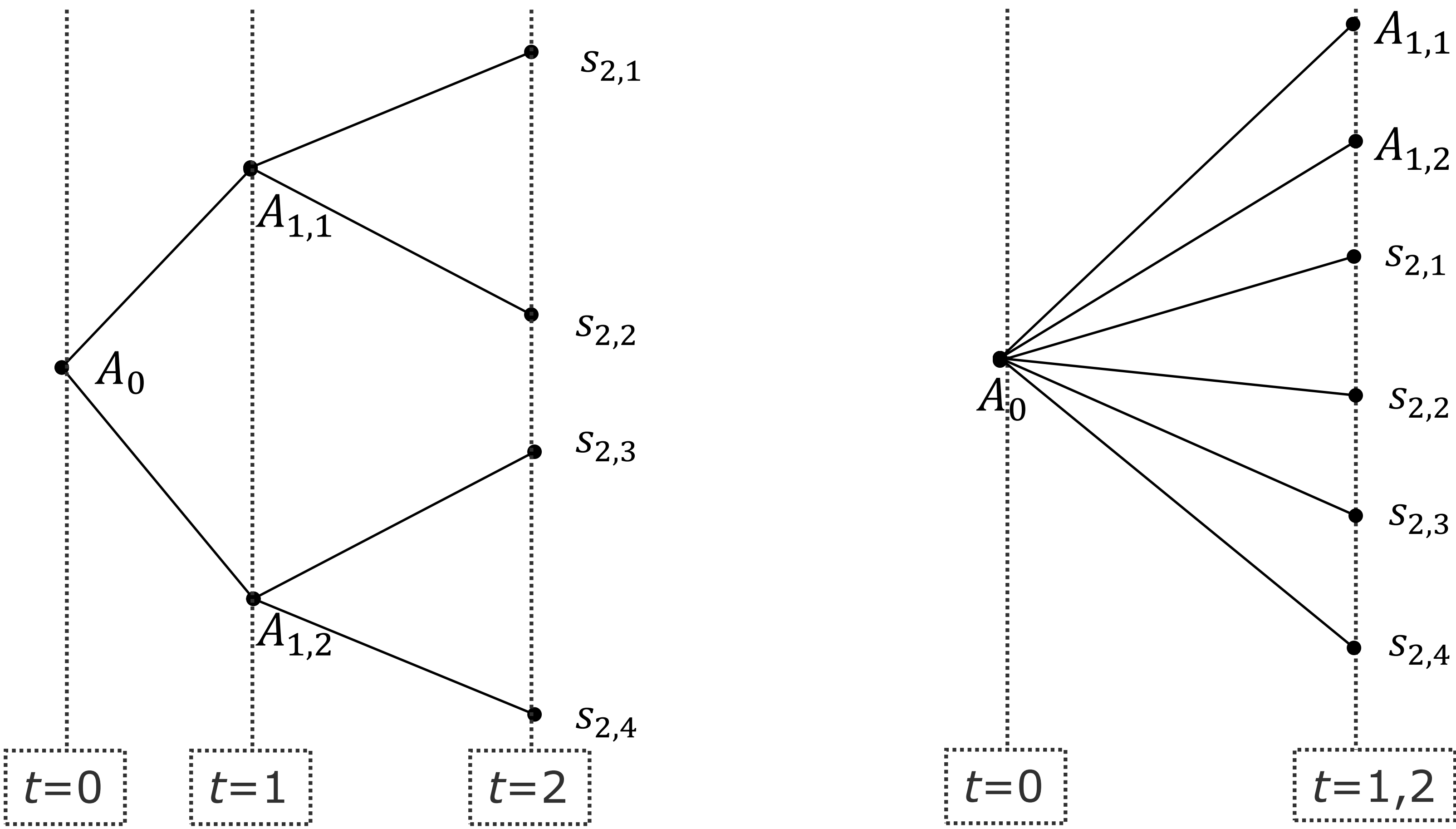
Static Complete Markets

Debreu

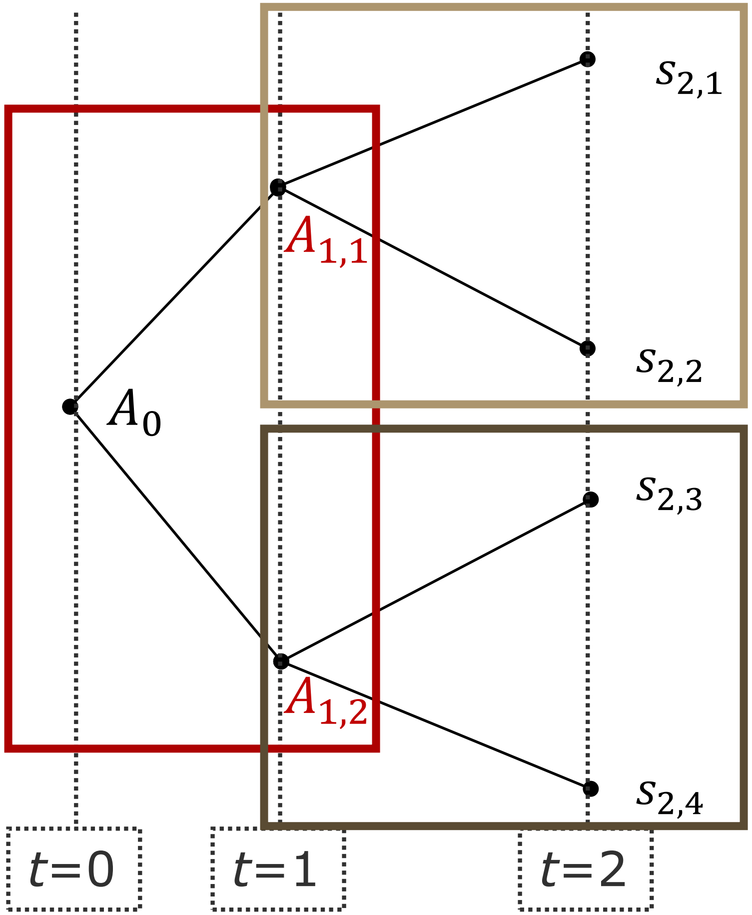


6 independently traded assets needed

All trading occurs at $t = 0$

Dynamic Completion

Arrow (1953)



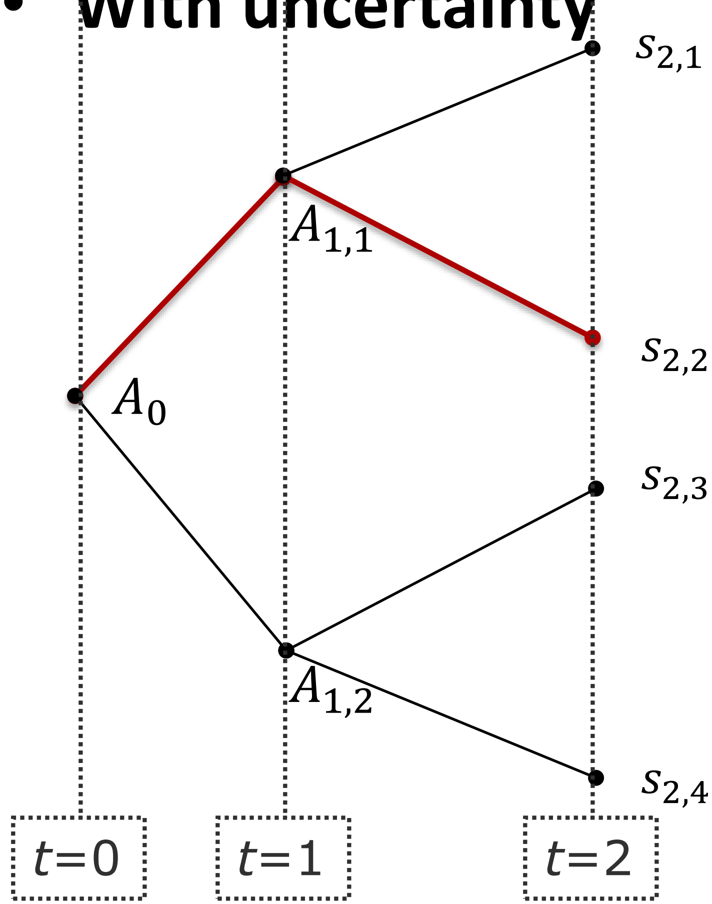
- Assets can be retraded
  - Conditional on event $A_{1,1}$ or $A_{1,2}$
- Completion with
  - Short-lived assets
    - Pays only next period
  - Long-lived assets
    - Payoff over many periods
- Trading strategy $h(A_{t,i})$

Completion with Short-lived Assets

- **Without uncertainty**:
  - No uncertainty and *T* periods (*T* can be infinite)
  - *T* one period assets,
    from period 0 to period 1, from period 1 to 2, etc.
  - Let p_t be the price of the short-term bond that begins in period t and matures in period $t+1$.
- Completeness requires
  Transfer of wealth between any two periods t and t', not just between consecutive periods.
  - Roll over short-term bonds
  - Cost of strategy: $p_t \cdot p_{t+1} \cdot\cdot\, p_{t'-1}$

Completion with Short-lived Assets

- **With uncertainty**



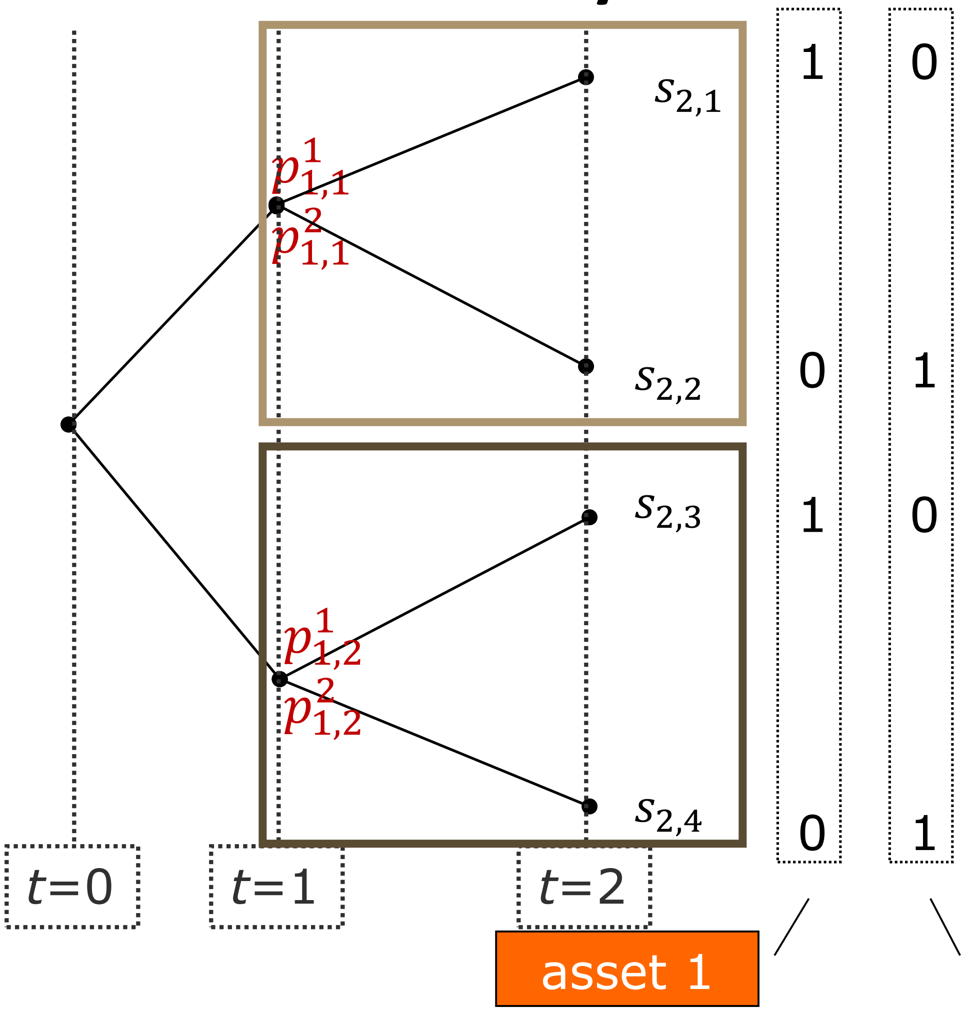
- $p^{A_{t,i}}$= price of an Arrow-Debreu asset that pays one unit in event $A_{t,i}$. want to transfer wealth from event A_0 to event-state $s_{2,2}$.
- Go backwards:
  - in event $A_{1,1}$, buy one event-state $s_{2,2}$ asset for a price $p^{A_{2,2}}$.
  - In event A_0, buy $p^{A_{2,2}}$ shares of event $A_{1,1}$ assets.
- Today's cost $p^{A_{2,2}}p^{A_{1,1}}$. The payoff is one unit in event $s_{2,2}$ and nothing otherwise.

Completion with Long-lived Assets

- Without uncertainty:
  - T-period model ($T < \infty$).
  - Single asset
    - Discount bond maturing in T.
    - Tradable in each period for p_t.
  - T prices (not simultaneously, but sequentially)
  - Payoff can be transferred from period t to period $t' > t$ by purchasing the bond in period t and selling it in period t'.

Completion with Long-lived Assets

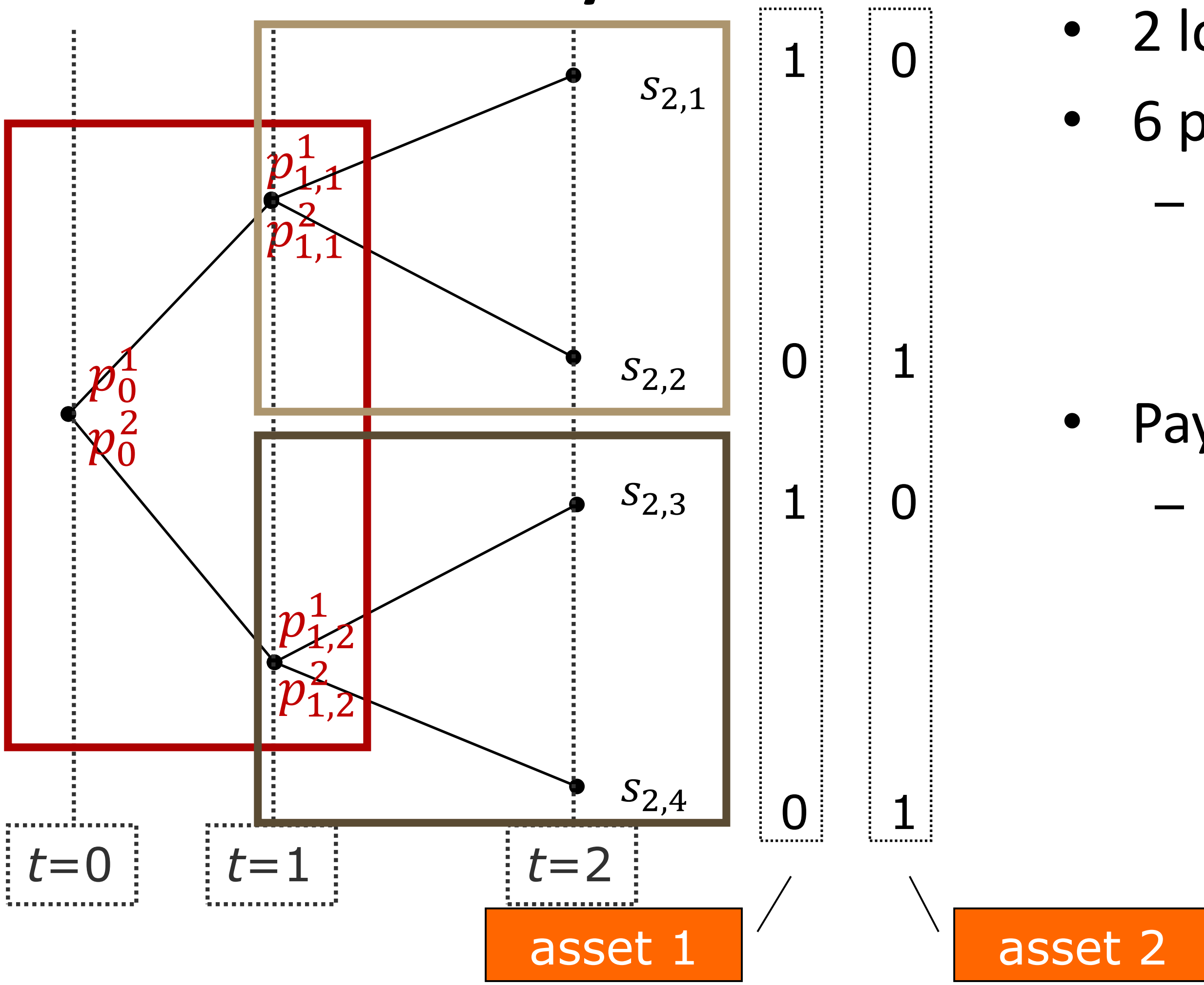
- **With uncertainty**



- At $t = 1$ it is as if one has 2 Arrow-Debreu securities (in each event $A_{1,i}$).
  - From perspective of $t = 0$ it is as if one has 4 Arrow-Debreu assets at $t = 1$.

Completion with Long-lived Assets

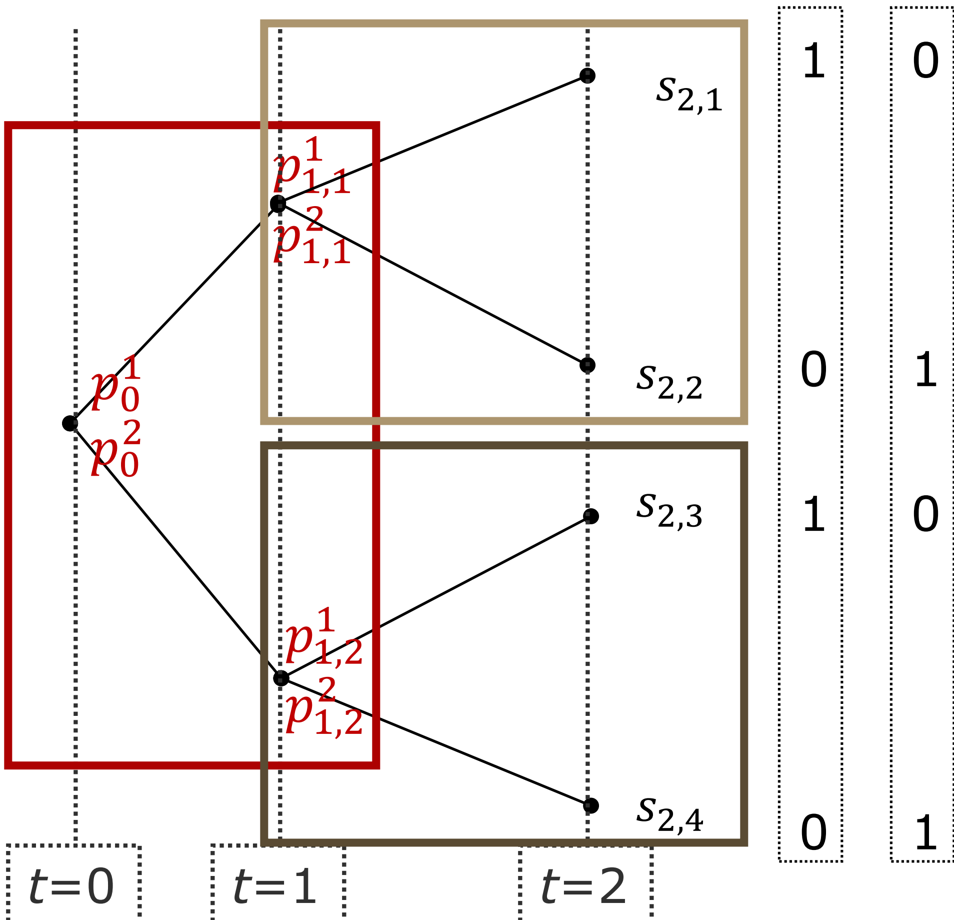
- **With uncertainty**



- 2 long-lived assets
- 6 prices
  - Each asset is traded in 3 events
- Payoff
  - In $t = 1$ is endogenous price p_1

One-period holding

- Call "trading strategy $[j, A_{t,i}]$" the cash flow of asset j that is purchased in event $A_{t,i}$ and is sold one period later.



6 trading strategies:
$[1, A_0]$,$[1, A_{1,1}]$,$[1, A_{1,2}]$,
$[2, A_0]$,$[2, A_{1,1}]$, $[2, A_{1,1}]$

(Note that this is potentially sufficient to span the complete space.)

Extended Payoff Matrix

6x6 payoff matrix.

Asset	$[1, A_0]$	$[2, A_0]$	$[1, A_{1,1}]$	$[2, A_{1,1}]$	$[1, A_{1,2}]$	$[2, A_{1,2}]$
event A_0	$-p_0^1$	$-p_0^2$	0	0	0	0
event $A_{1,1}$	$p_{1,1}^1$	$p_{1,1}^2$	$-p_{1,1}^1$	$-p_{1,1}^2$	0	0
event $A_{1,2}$	$p_{1,2}^1$	$p_{1,2}^2$	0	0	$-p_{1,2}^1$	$-p_{1,2}^2$
state $s_{2,1}$	0	0	1	0	0	0
state $s_{2,2}$	0	0	0	1	0	0
state $s_{2,3}$	0	0	0	0	1	0
state $s_{2,4}$	0	0	0	0	0	1

This matrix is full rank/regular (and hence the market complete) if the red framed submatrix is regular (of rank 2).

When Dynamically Complete?

- Is the red-framed submatrix of rank 2?
- Payoffs are endogenous future prices
- There are cases in which $(p^1_{1,1}, p^2_{1,1})$ and $(p^1_{1,2}, p^2_{1,2})$ are collinear in equilibrium.
    - Example: If per capita endowment is the same in event $A_{1,1}$ and $A_{1,2}$, in state $s_{2,1}$ and $s_{2,3}$, and in state $s_{2,2}$ and $s_{2,4}$, respectively, and if the probability of reaching state $s_{1,1}$ after event $A_{1,1}$ is the same as the probability of reaching state $s_{2,3}$ after event $A_{1,2}$
      → submatrix is singular (only of rank 1).
    - then events $A_{1,1}$ and $A_{1,2}$ are effectively identical, and we may collapse them into a single event.

Accidental Incompleteness

- A *random* square matrix is of full rank (regular). So outside of special cases, the red-framed submatrix is of full ("almost surely").
- The 2x2 submatrix may still be singular *by accident*.
- In that case it can be made regular again by applying a small perturbation of the returns of the long-lived assets, by perturbing aggregate endowment, the probabilities, or the utility function.
- *Generically*, the market is dynamically complete.

Dynamic Completeness in General

- *branching number* = The maximum number of branches fanning out from any event.
- = number of assets necessary for dynamic completion.
- Generalization by Duffie and Huang (1985): continuous time → continuity of events → but a small number of assets is sufficient.
- The large power of the event space is matched by continuously trading few assets, thereby generating a continuity of trading strategies and of prices.

Example: Black-Scholes Formula

- Cox, Ross, Rubinstein binominal tree model of B-S
- Stock price goes up or down (follows binominal tree) interest rate is constant
- Market is dynamically complete with 2 assets
  - Stock
  - Risk-free asset (bond)
- Replicate payoff of a call option with (dynamic Δ-hedging)
- (later more)

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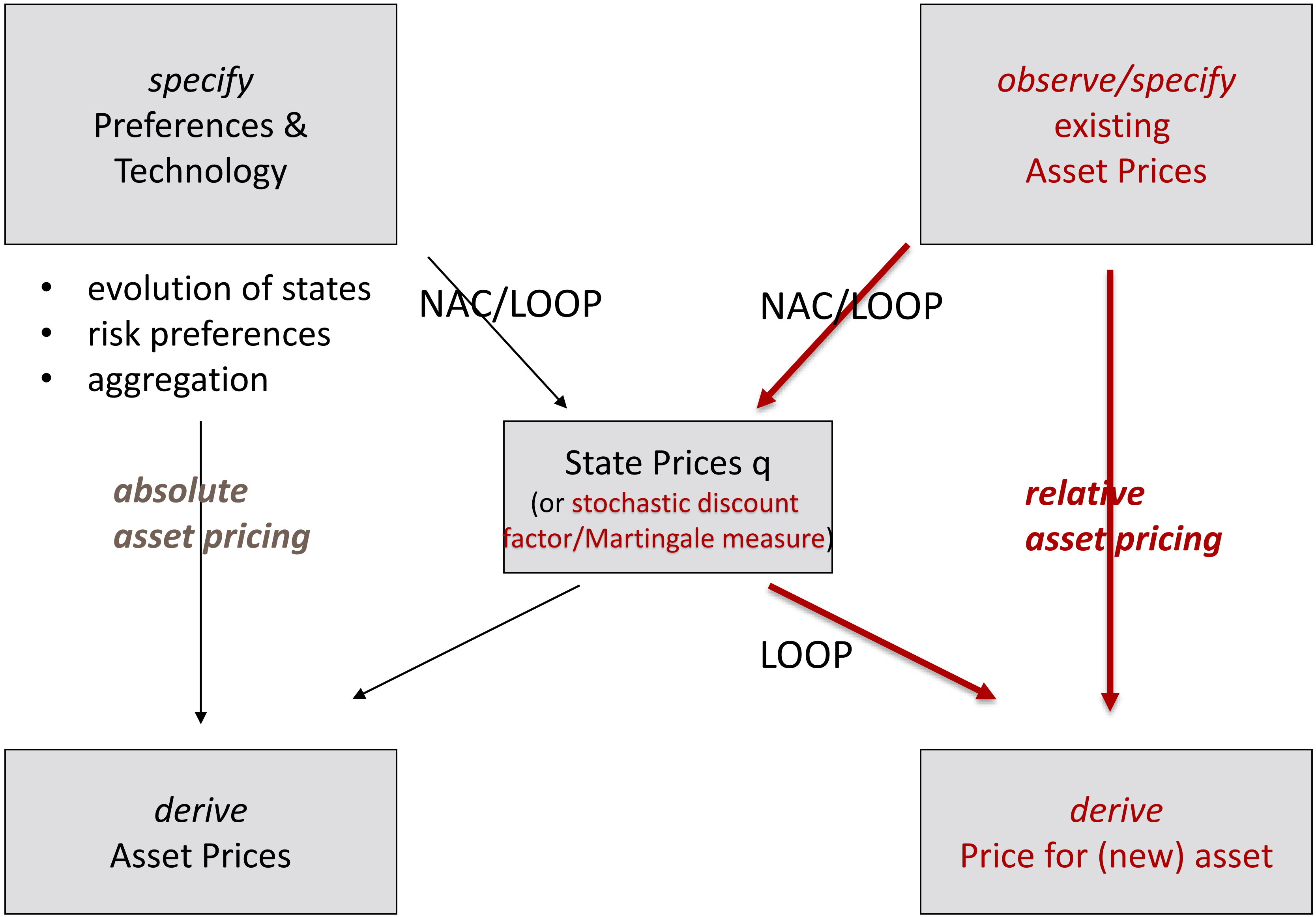
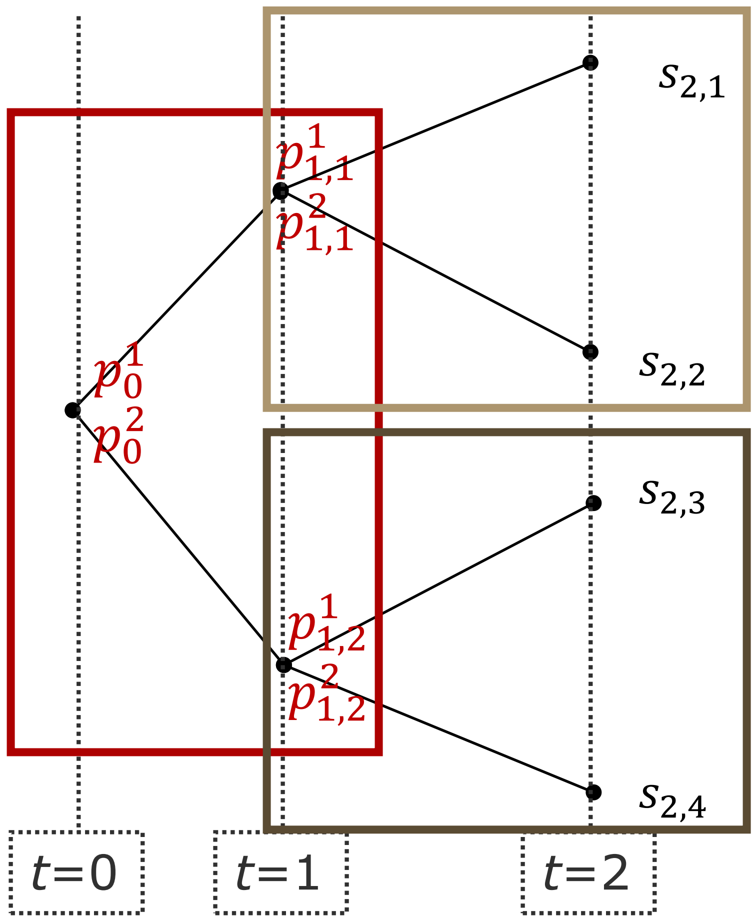


Only works as long as market completeness doesn't change

No Arbitrage



- No dynamic trading strategy
  - No cost today about some positive payoff along the tree
  - Negative cost today and no negative payoff along the tree

- No dynamic trading strategy

  = no (static) arbitrage in each subperiod

Existence of Multi-period SDF M

- No Arbitrage $\Leftrightarrow$ there exists $m_{t+1} \gg 0$ for each one period subproblem
  - such that $p_t = E_t[m_{t+1}(p_{t+1} + x_{t+1})]$

- Define multi-period SDF (discounts back to $t = 0$)

$$M_{t+1} = m_1 \cdot m_2 \cdot \ldots \cdot m_{t+1}$$

(waking along the event tree)

adapted process that is measurable w.r.t. filtration $\{F_{t+1}\}^T{}_t$

$$M_t p_t = E_t[M_{t+1}(p_{t+1} + x_{t+1})]$$

Solving it forward
And apply LIE

The Fundamental Pricing Formula

- To price an arbitrary asset x,
  portfolio of STRIPped cash flows, $x_1^j, x_2^j, \dots x_\infty^j$,
  where x_t^j denotes the cash-flows in event $A_{t,s}$
- The price of asset x^j is simply the sum of the prices of its STRIPed payoffs, so

$$p_0^j = \sum_t E_0[M_t x_t^j]$$

Pricing Kernel M_t^*

- Recall $m_{t+1}^* = proj(m_{t+1} | < X_{t+1} >)$
    - That is, there exists h_t^* s.t. $m_{t+1}^* = X_{t+1} h_t^*$ and
      $$p_t = E_t[X'_{t+1} m_{t+1}^*] = E_t[X'_{t+1} X_{t+1} h_t^*]$$
      $$= E_t[X'_{t+1} X_{t+1}] h_t^*$$
    - $h_t^* = (E_t[X'_{t+1} X_{t+1}])^{-1} p_t$
    - Hence, $m_{t+1}^* = X_{t+1} (E_t[X'_{t+1} X_{t+1}])^{-1} p_t$

- Define $M_{t+1}^* = m_1^* \cdot m_2^* \cdot \ldots \cdot m_{t+1}^*$
- Part of asset span

Aside: Alternative Formula for m^*

- $\frac{M^*_{t+1}}{M^*_t} = m^*_{t+1} = X_{t+1}(E_t[X'_{t+1}X_{t+1}])^{-1}p_t$
  - where $E_t[X'_{t+1}X_{t+1}]$ is a second moment $(J \times J)$ matrix

- Expressed in covariance-matrix, Σ_t

$$m^*_{t+1} = E[m^*_{t+1}] + \left[p_t - E[m^*_{t+1}]E[X_{t+1}]\right]' \Sigma_t^{-1}(X_{t+1} - E[X_{t+1}])$$

- In excess returns, R^e and now return $\Sigma_t \equiv Cov_t[R^e_{t+1}]$

$$m^*_{t+1} = \frac{1}{R^F_t} - \frac{1}{R^F_t} E[R^e_{t+1}]' \Sigma_t^{-1}(R^e_{t+1} - E[R^e_{t+1}])$$

- Continuous time analogous

$$\frac{dM^*}{M} = -r^F dt - \left(\mu + \frac{D}{P} - r^F\right)' \Sigma^{-1} dz$$

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Martingales

- Let X_1 be a random variable and
let x_1 be the realization of this random variable.
- Let X_2 be another random variable and assume that the distribution of X_2 depends on x_1.
- Let X_3 be a third random variable and assume that the distribution of X_3 depends on x_1, x_2.

- Such a sequence of random variables, $(X_1, X_2, X_3, \ldots)$ is called a stochastic process.

- A stochastic process is a martingale if $E[X_{t+1}|x_t, \ldots] = x_t$

History of the Word Martingale

- “martingale” originally refers to a sort of pants worn by “Martigaux” people living in Martigues located in Provence in the south of France. By analogy, it is used to refer to a strap in equestrian. This strap is tied at one end to the girth of the saddle and at the other end to the head of the horse. It has the shape of a fork and divides in two.
- In comparison to this division, martingale refers to a strategy which consists in playing twice the amount you lost at the previous round. Now, it refers to any strategy used to increase one's probability to win by respecting the rules.
- The notion of martingale appears in 1718 (The Doctrine of Chance by Abraham de Moivre) referring to a strategy that makes you sure to win in a fair game.
- See also www.math.harvard.edu/~ctm/sem/**martingales**.pdf

$M_t p_t$ is Martingale

- $M_t p_t = E_t[M_{t+1} p_{t+1}] + E_t[M_{t+1} x_{t+1}]$

- … but
  1. Dividend payments x_{t+1}

consider

fund that reinvests

Prices are Martingales…

- Samuelson (1965) has argued that prices have to be martingales in equilibrium.
- $p_t = \frac{1}{1+r^f_{t,t+1}} E^Q_t [p_{t+1} + x_{t+1}]$
- … 3 "buts" consider
  1. Dividend payments — fund that reinvests dividends
  2. Discounting — discounted process
  3. Risk aversion — risk-neutral measure π^Q_t

Equivalent Martingale Measure

- risk-neutral probabilities

$$\pi^*_{A_t} = \frac{\pi_{A_t} M_{A_t}}{\rho_{A_t}}$$

where ρ_{A_t} is the discount-factor from event A_t to 0.
(state dependent)

- Discount everything back to $t = 0$.

- Why not "upcount"/compound to $t = T$?

Risk-Forward Pricing Measure

- $P_s(t,T)$ be the time- s price of a bond purchased at time t with maturity T, with $s < t < T$.
- The fundamental pricing equation is

$$P_t(t,T) = E_t[m_{t+1} P_{t+1}(t+1,T)]$$

- Dividing the pricing equation for a generic asset j by this relation and rearranging we get

$$\frac{p_t^j}{P_t(t,T)} = E_t\left[\frac{P_{t+1}(t+1,T) \quad m_{t+1}\left(x_{t+1}^j + p_{t+1}^j\right)}{E_t[m_{t+1}P_{t+1}(t+1,T)]P_{t+1}(t+1,T)}\right] = E_t^{F_T}\left[\frac{x_{t+1}^j + p_{t+1}^j}{P_{t+1}(t+1,T)}\right]$$

Forward price   Forward price   Forward price

- Where $\pi_s^{F_T} = \frac{\pi_s P_{t+1,s}(t+1,T) m_{t+1,s}}{E_t[m_{t+1}P_{t+1}(t+1,T)]}$.
- Useful for pricing of bond options and
- coincides with the risk-neutral measure
  - for $\text{t} + 1 = T$.
  - … (connection with expectations hypothesis?)

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Ponzi Schemes: Infinite Horizon Max.-problem

- Infinite horizon allows agents to borrow an arbitrarily large amount without effectively ever repaying, by rolling over debt forever.
  - Ponzi scheme - allows infinite consumption.

- Example
  - Consider an infinite horizon model, no uncertainty, and a complete set of short-lived bonds.
  - z_t is the amount of bonds maturing in period t.

Ponzi Schemes: Rolling over Debt Forever

- The following consumption path is possible:

$$c_t = y_t + 1$$

- Note that agent consumes *more than his endowment, y_t, in each period, **forever*** financed with *ever increasing debt*

- Ponzi schemes
  - can never be part of an equilibrium.
  - destroys the existence of a utility maximum because the choice set of an agent is unbounded above.
  - additional constraint is needed.

Ponzi Schemes: Transversality

- The constraint that is typically imposed on top of the budget constraint is the *transversality condition*,

$$\lim_{t\to\infty} p_t^{bond} z_t \geq 0$$

- This constraint implies that the value of debt cannot diverge to infinity.
  - More precisely, it requires that all debt must be redeemed eventually (i.e. in the limit).

Fundamental and Bubble Component

- Our formula

$$M_t p_t = E_t[M_{t+1}(p_{t+1} + x_{t+1})]$$

or

$$M_t p_t = E_t M_{t+1} p_{t+1} + E_t M_{t+1} x_{t+1}$$

- Solve forward – (many solutions)

$$\Rightarrow p_0 = \underbrace{\sum_{t=1}^{\infty} E_0[M_t x_t]}_{\text{fund. value}} + \underbrace{\lim_{T\to\infty} E_0 M_T p_T}_{\text{bubble comp.}}$$

Money as a Bubble

$$p_0 = \underbrace{\sum_{t=1}^{\infty} M_t}_{\text{fundamental value}} + \underbrace{\lim_{T\to\infty} M_T p_T}_{\text{bubble comp}}$$

- The fundamental value = price in the static-dynamic model.
- Repeated trading gives rise to the possibility of a bubble.
- Fiat money as a store of value can be understood as an asset with no dividends.
  The fundamental value of such an asset would be zero.
  But in a world of frictions fiat money can have positive value (a bubble) (e.g. in Samuelson 195X, Bewley, 1980).
- In asset pricing theory, we often rule out bubbles simply by imposing $\lim_{T\to\infty} M_T p_T = 0$

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4. Multi-Factor Model – Empirical Strategy

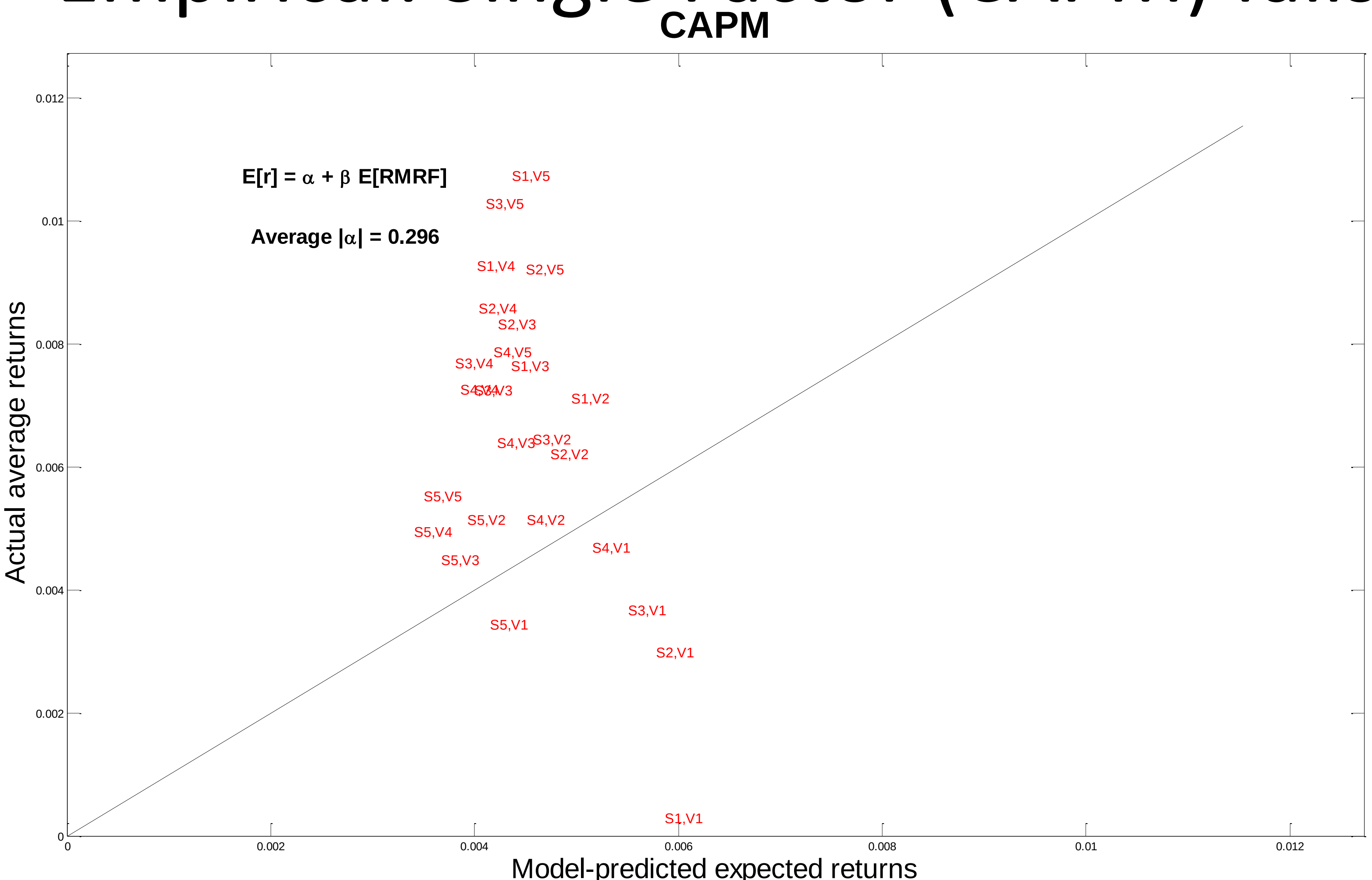
Time-varying R_t^* (SDF)

- If one-period SDF m_{t+1}^* is not time-varying (i.e. distribution of m_{t+1}^* is i.i.d., then
  - Expectations hypothesis holds
  - Investment opportunity set does not vary
  - Corresponding R_{t+1}^* of **single factor** state-price beta model can be easily estimate (because over time one more and more observations about R_{t+1}^*)
- If not, then m_t^* (or corresponding R_t^*)
  - depends on state variable
  - **multiple factor** model

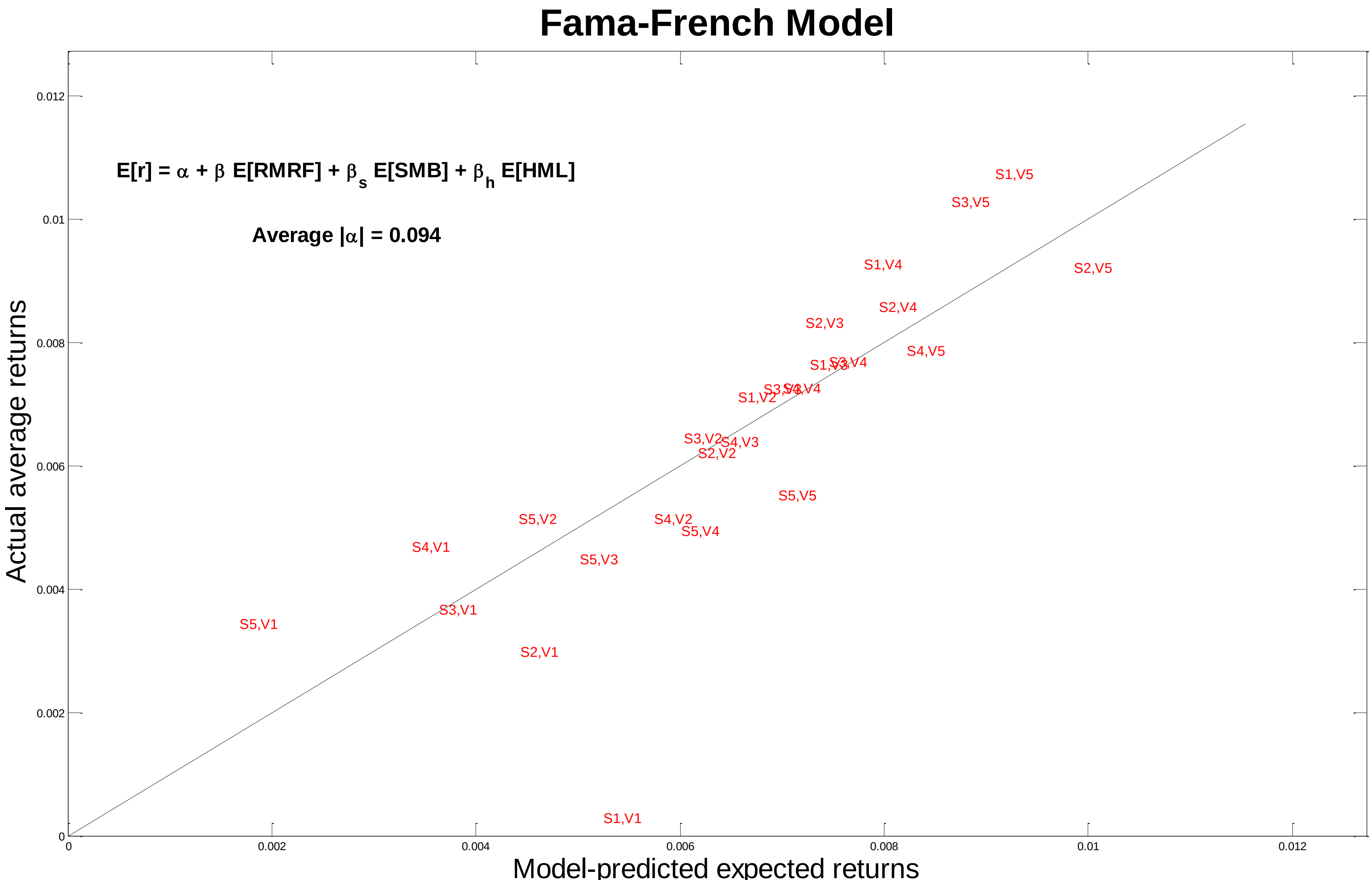
R_t^* depends on State Variable

- $R_t^* = R^*(z_t)$, with state variable z_t
- Example:
  - $z_t = 1$ or 2 with equal probability
  - Idea:
    - Take all periods with $z_t = 1$ and figure out $R^*(1)$
    - Take all periods with $z_t = 2$ and figure out $R^*(2)$
  - Can one do that?
    - No – hedge across state variables
- Potential state-variables: predict future return

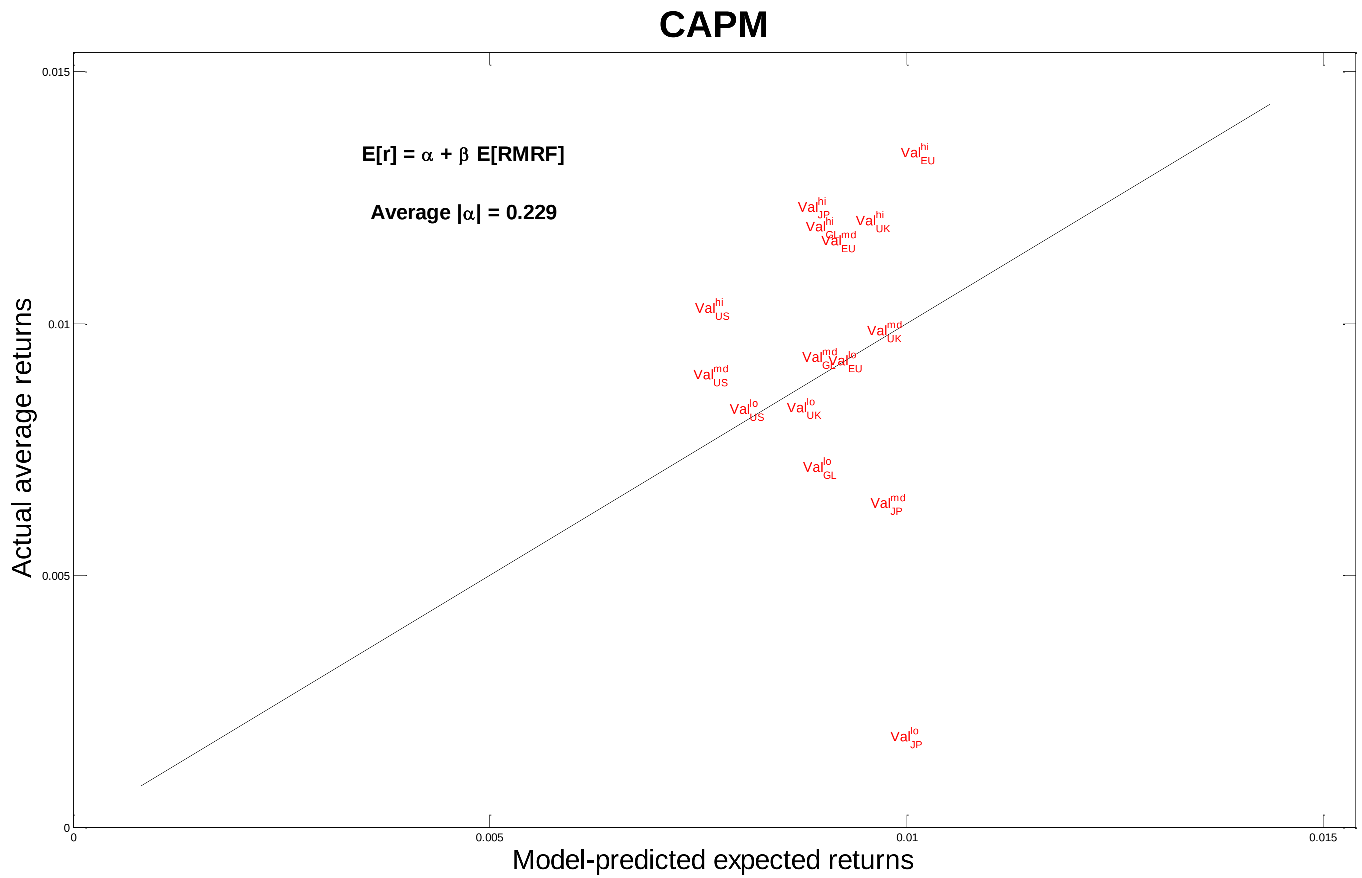
Empirical: Single Factor (CAPM) fails



Three Factor Model works



International data: Out of Sample Test



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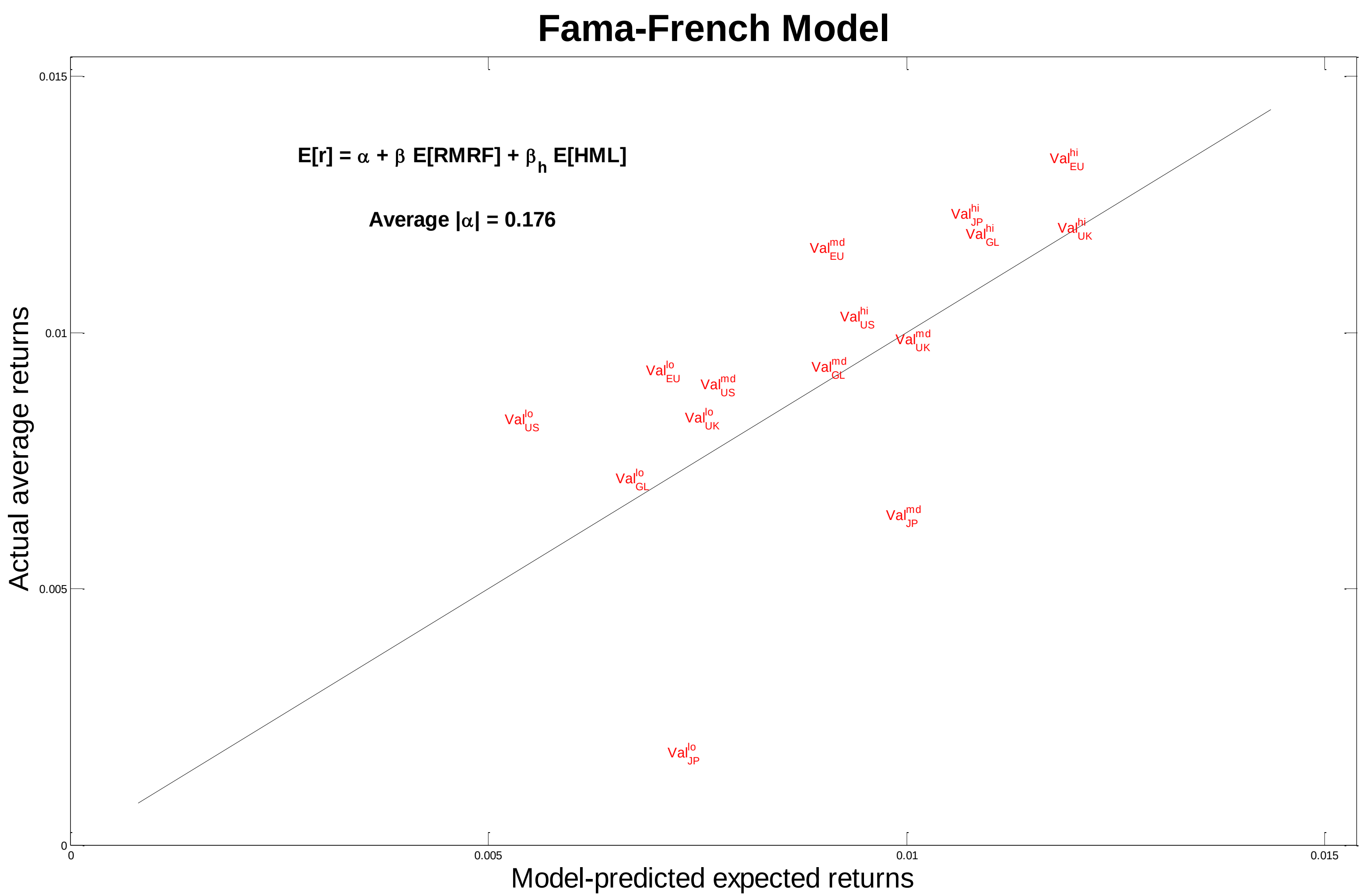


Fama-MacBeth 2 Stage Method

- Stage 1: Use *time series* data to obtain estimates for each individual stock's β^j

$$R_t^j - R^f = \alpha + \beta^j (R_t^m - R_t^f) + \epsilon_t^j$$

(e.g. use monthly data for last 5 years)

Note: $\widehat{\beta}^j$ is just an estimate [around true β^j]

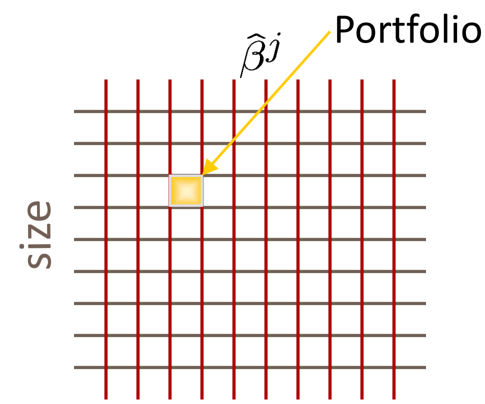
- Stage 2: Use *cross sectional* data and estimated β^js to estimate SML

$$R^j_{\text{next month}} = a + b\widehat{\beta}^j + e^j$$

b=market risk premium

CAPM β–Testing Fama French (1992)

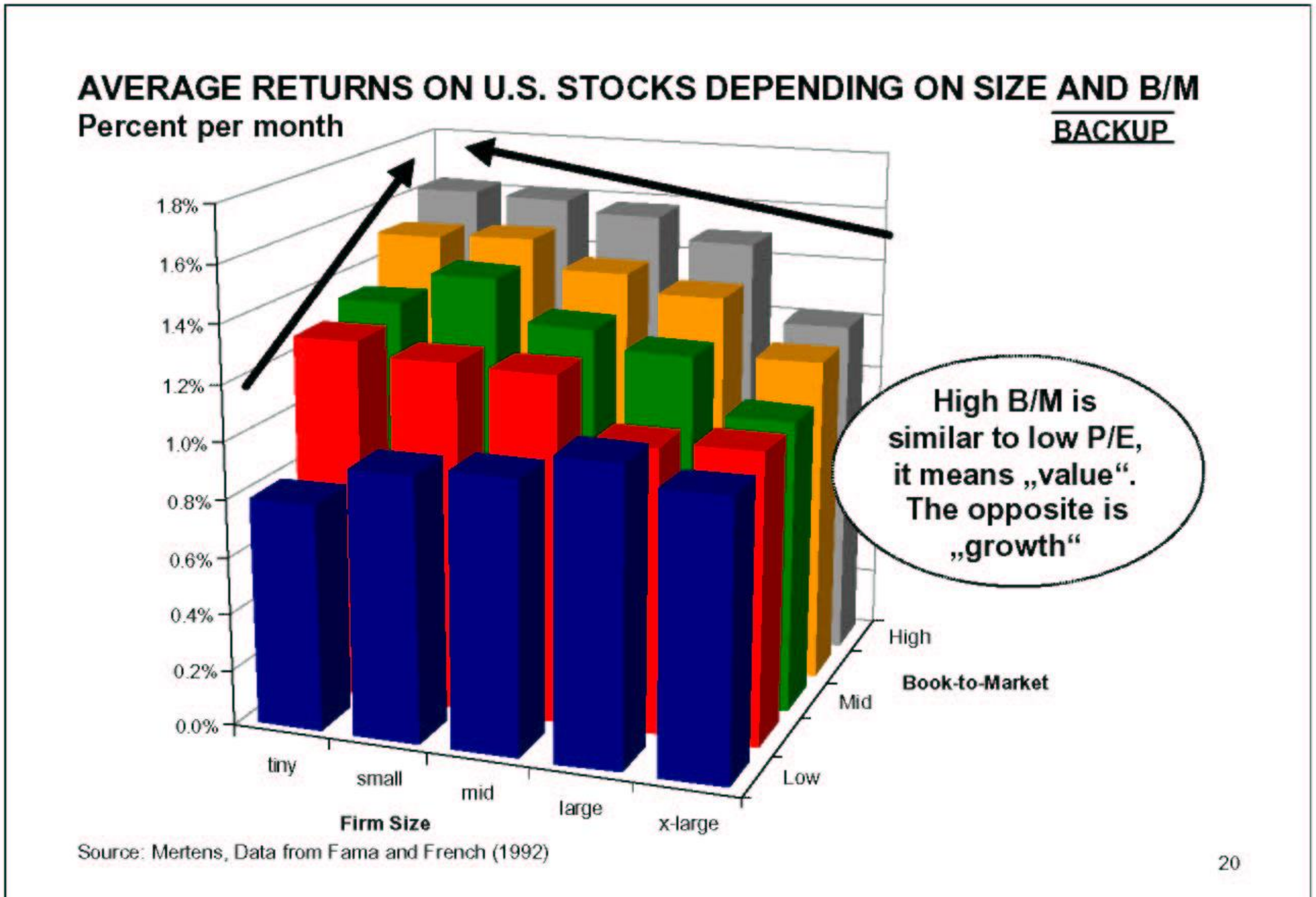
- Using newer data slope of SML b is not significant (adding size and B/M)
- Dealing with econometrics problem:
  - $\hat{\beta}^j$ s are only noisy estimates, hence estimate of b is biased
  - Solution:
    - Standard Answer: Find instrumental variable
    - Answer in Finance: Derive $\hat{\beta}$ estimates for portfolios
      - Group stocks in 10 x 10 groups sorted to size and estimated $\hat{\beta}^j$
      - Conduct Stage 1 of Fama-MacBeth for portfolios
      - Assign all stocks in same portfolio same β
      - Problem: Does not resolve insignificance



- *CAPM predictions:* b is significant, all other variables insignificant
- *Regressions:* size and B/M are significant, b becomes insignificant
  - Rejects CAPM

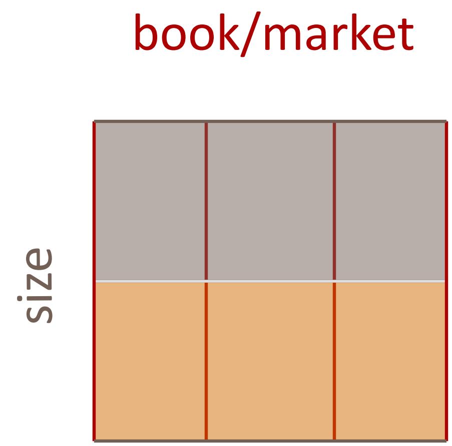
Book to Market and Size

Small „value“ companies have higher returns



Fama French Three Factor Model

- Form 2x3 portfolios
  - Size factor (SMB)
    - Return of small minus big
  - Book/Market factor (HML)
    - Return of high minus low



- For $R_t^j - R_t^f = \alpha^p + \beta^p(R_t^m - R_t^f)$

  αs are big and βs do not vary much

- For .$R_t^p - R_t^f = \alpha^p + \beta^p(R_t^m - R_t^f) + \gamma^p \text{SMB}_t + \delta^p \text{HML}_t$

  (for each portfolio p using time series data)

  αs are zero, coefficients significant, high R^2.

Fama French Three Factor Model

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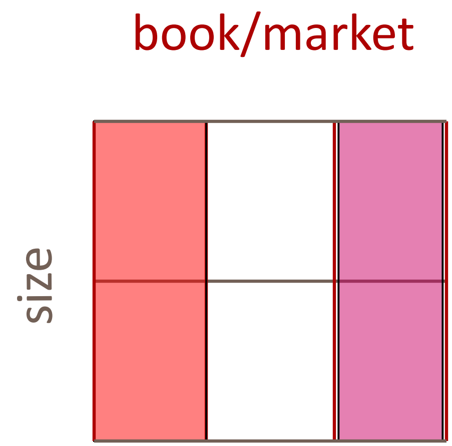

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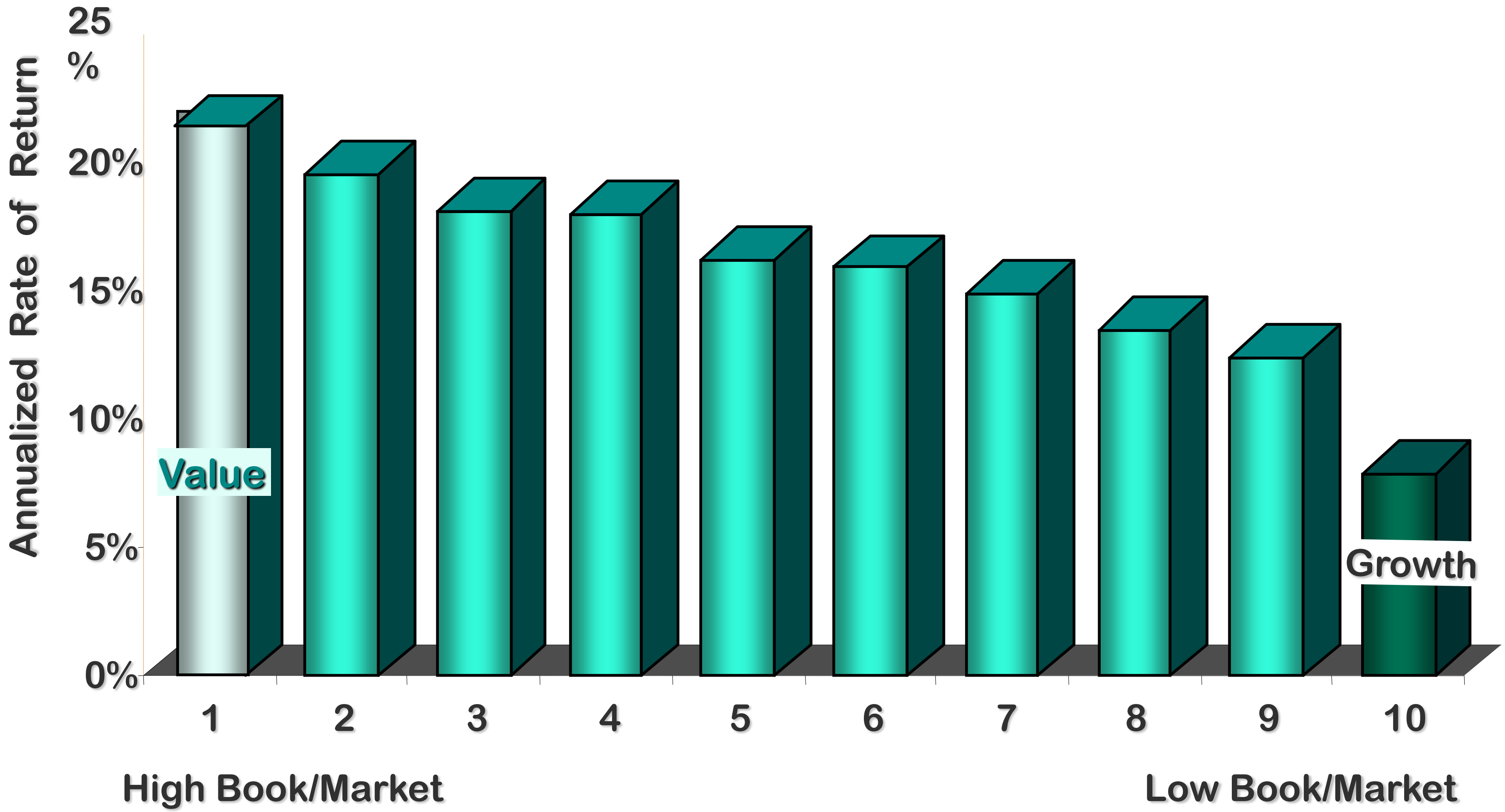
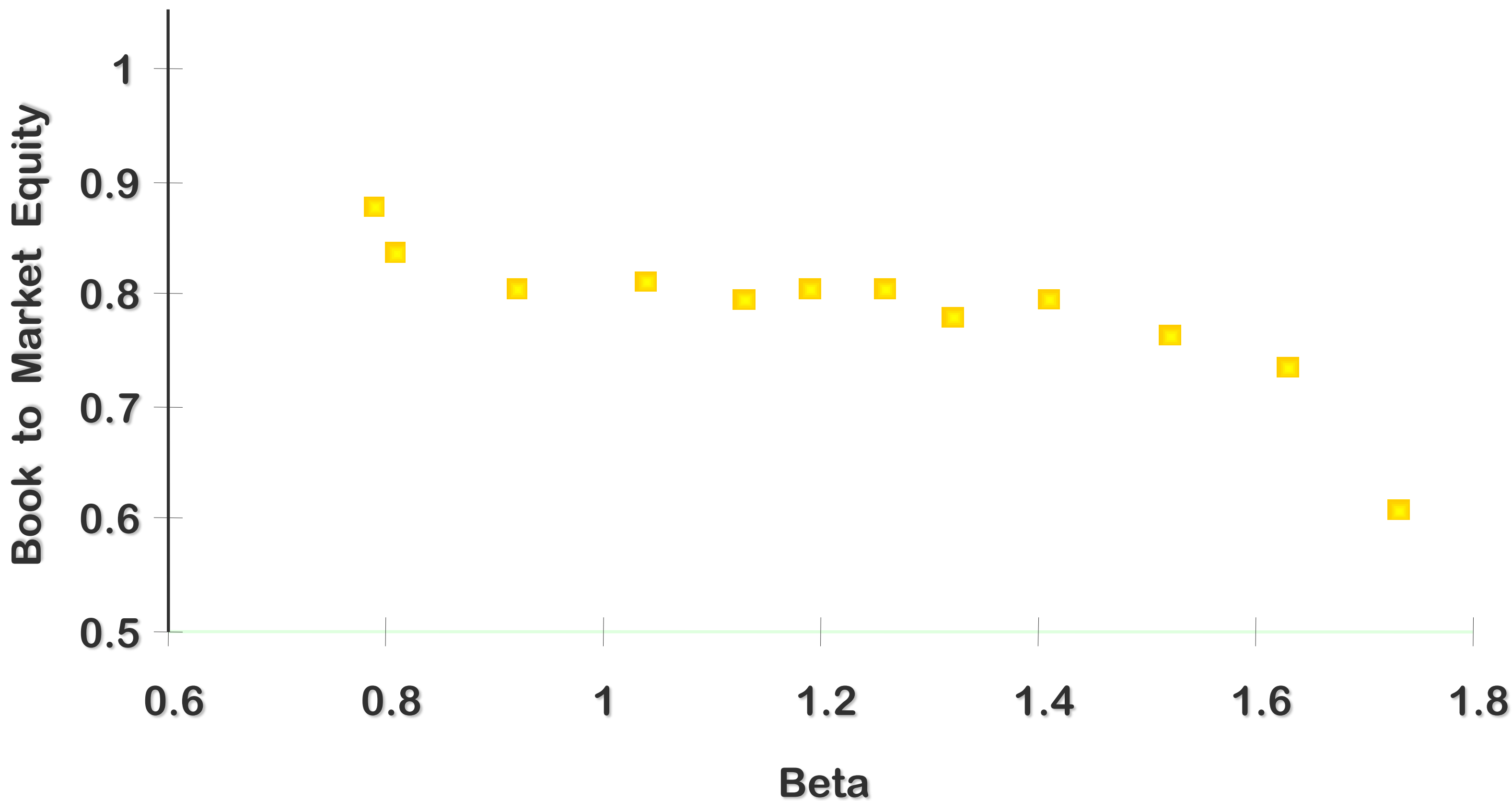
  (for each portfolio p using time series data)

  α^ps are zero, coefficients significant, high R^2.

Book to Market as a Predictor of Return

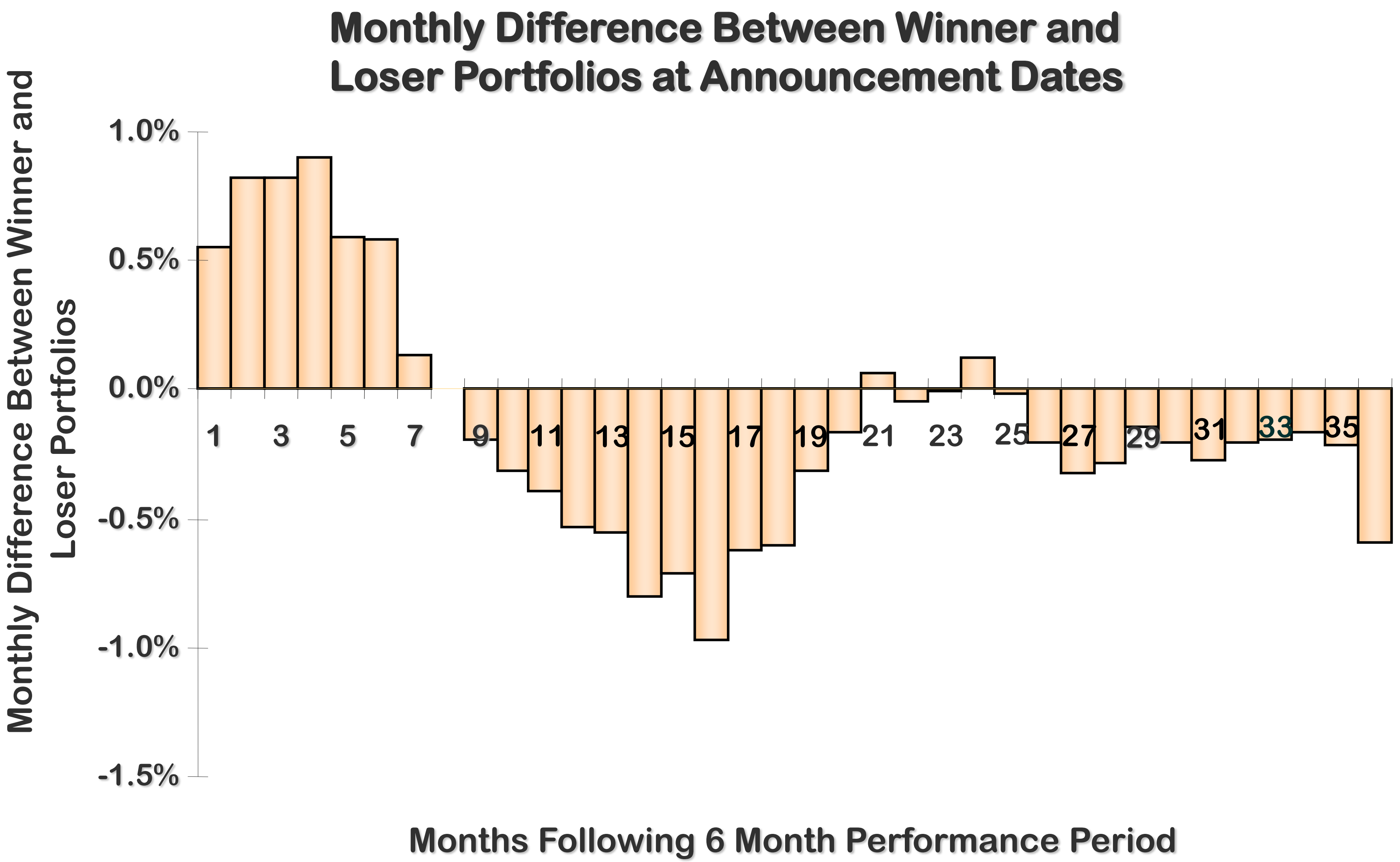


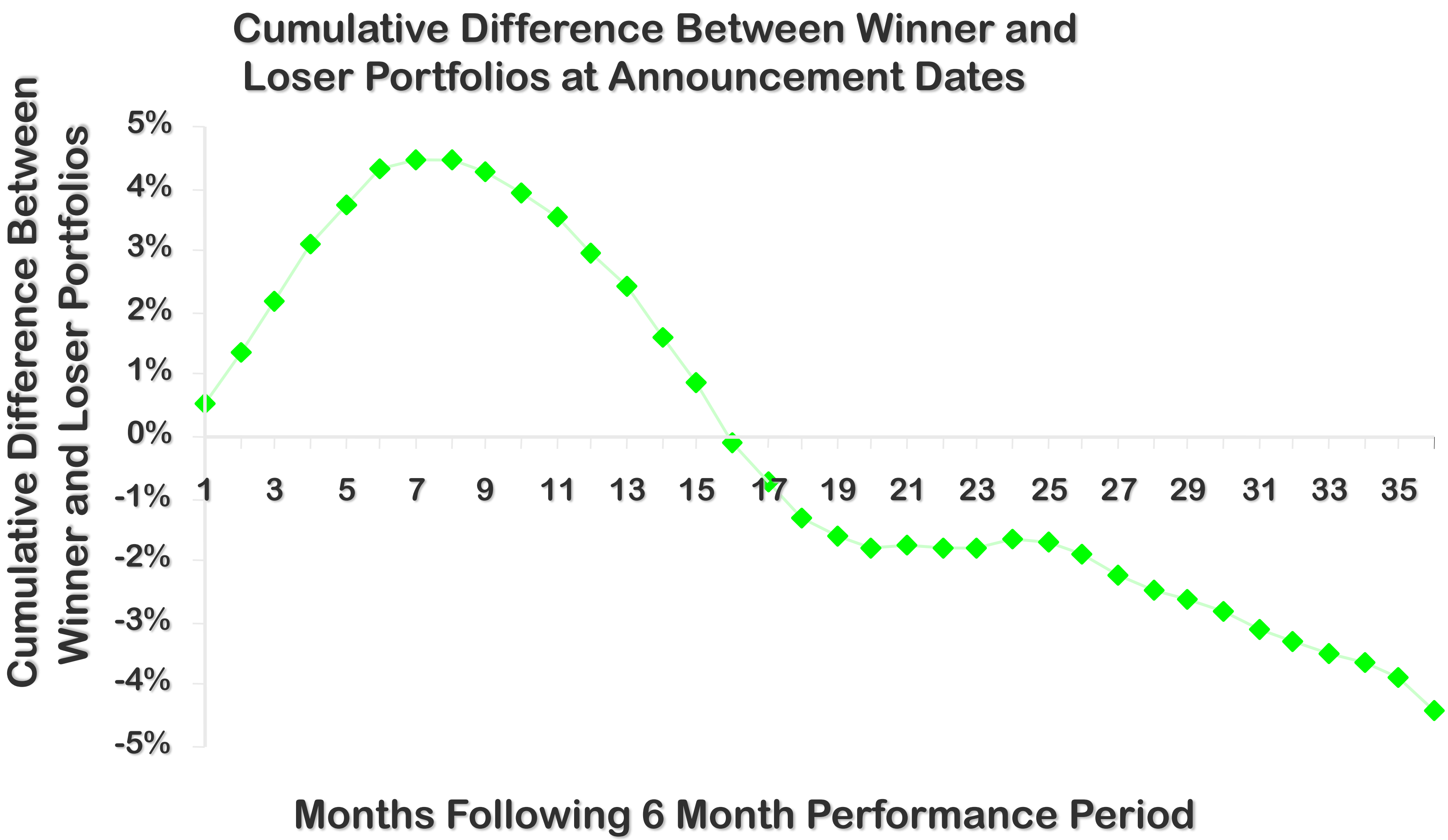
Book to Market Equity of Portfolios Ranked by Beta



Adding Momentum Factor

- 5x5x5 portfolios
- Jegadeesh & Titman 1993 JF rank stocks according to performance to past 6 months
  - Momentum Factor
    Top Winner minus Bottom Losers Portfolios


Monthly Difference Between Winner and Loser Portfolios at Announcement Dates
Monthly Difference Between Winner and Loser Portfolios
1.0%
0.5%
0.0%
-0.5%
-1.0%
-1.5%
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
Months Following 6 Month Performance Period


Cumulative Difference Between Winner and Loser Portfolios at Announcement Dates
Cumulative Difference Between Winner and Loser Portfolios
5%
4%
3%
2%
1%
0%
-1%
-2%
-3%
-4%
-5%
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
Months Following 6 Month Performance Period