

ECO 525: Problem Set 1

November 11, 2012

To be turned in Ji Huang's mailbox in 001 Fisher Hall.

Problem 1

[Based on 4.3 of Vives(2008)] Consider a market with a single risky asset, with random fundamental value $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$, and a riskless asset (with unitary return). There are 3 types of traders: informed traders indexed in the interval $[0, 1]$, noise traders, and risk-neutral market makers. Informed traders have CARA utility function with risk aversion coefficient ρ . Each informed traders i receives a private signal $s_i = \theta + \varepsilon_i$ about θ , where θ and ε_i are uncorrelated, errors are uncorrelated across agents and normally distributed with zero mean and variance σ_ε^2 . $u \sim N(0, \sigma_u^2)$ is noisy traders' total demand for the risk asset.

Informed traders and noisy traders move first. A proportion v of informed traders submit demand schedules $X(s_i, p) = a(s_i - \bar{\theta}) + \zeta(p)$ and the rest of informed traders place market orders $Y(s_i) = c(s_i - \bar{\theta})$, where a , c , and $\zeta(p)$ are determined endogenously. Their orders are accumulated in a limit-order book $L(\cdot)$. Based on this limit-order book, competitive risk-neutral market makers set price informational efficiently:

$$p = E[\theta | L(\cdot)]$$

- (a) Derive $L(\cdot)$ and argue that $p = E[\theta | p]$;
- (b) Derive $\text{var}[p]$ and show $\text{var}[p - \bar{\theta}] + \text{var}[\theta - p] = \text{var}[\theta]$. Provide some comment.
- (c) Express a explicitly and derive c as a root of a cubic equation.
- (d) Set $v = 1$, derive the expected volume traded by informed agents, $E\left[\left|\int_0^1 X(s_i, p) di\right|\right]$
- (e) (optional) set $v = 0$, perform a comparative statics analysis of the market parameters ρ , σ_ε^2 , σ_θ^2 , and σ_u^2 .

Problem 2

[Based on 7.1 of Veldkamp(2011)] There is a continuum of ex ante identical traders, indexed by i , with CARA utility function and risk averse coefficient ρ . There are two assets. One offers a riskless return r . The other pays a risky amount $f = \theta + \varepsilon$, where $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, and θ and ε are uncorrelated. Traders can observe θ at a cost of c . The supply of the risky asset is $x \sim N(\bar{x}, \sigma_x^2)$. Solve the equilibrium asset price p and the proportion of traders who acquire information about f .