Market Liquidity and Funding Liquidity*

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Abstract

We provide a model that links an asset's market liquidity — i.e., the ease with which it is traded — and traders' funding liquidity — i.e., the ease with which they can obtain funding. Traders provide market liquidity, and their ability to do so depends on their availability of funding. Conversely, traders' funding, i.e., their capital and the margins they are charged, depend on the assets' market liquidity. We show that, under certain conditions, margins are destabilizing and market liquidity and funding liquidity are mutually reinforcing, leading to liquidity spirals. The model explains the empirically documented features that market liquidity (i) can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) is subject to "flight to quality", and (v) co-moves with the market, and it provides new testable predictions.

Keywords: Liquidity Risk Management, Liquidity, Liquidation, Systemic Risk, Leverage, Margins, Haircuts, Value-at-Risk, Credit Crunch, Counterparty Credit Risk, Fat Tails

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Trading requires capital. When a trader — e.g. a dealer, hedge fund, or investment bank — buys a security, he can use the security as collateral and borrow against it, but he cannot borrow the entire price. The difference between the security's price and collateral value, denoted as the margin, must be financed with the trader's own capital. Similarly, shortselling requires capital in the form of a margin; it does not free up capital. Therefore, the total margin on all positions cannot exceed a trader's capital at any time.

Our model shows that the funding of traders affects — and is affected by — market liquidity in a profound way. When funding liquidity is tight, traders become reluctant to take on positions, especially "capital intensive" positions in high-margin securities. This lowers market liquidity, leading to higher volatility. Further, under certain conditions, low future market liquidity increases the risk of financing a trade, thus increasing margins.

Based on the links between funding and market liquidity, we provide a unified explanation for the main empirical features of market liquidity. In particular, our model implies that market liquidity (i) can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) is subject to "flight to quality" or "flight to liquidity," and (v) co-moves with the market. The model has several new testable implications that link margins and dealer funding to market liquidity: We predict that (i) a shock to speculators' capital is a state variable affecting market liquidity and risk premia, (ii) a reduction in capital reduces market liquidity, especially if capital is already low (a non-linear effect) and for high-margin securities, (iii) margins increase in illiquidity if the fundamental value is difficult to determine, and (iv) speculators' returns are negatively skewed (even if they trade securities without skewness in the fundamentals).

Our model is similar in spirit to Grossman and Miller (1988) with the added feature that speculators face the real-world funding constraint discussed above. In our model, different customers have offsetting demand shocks, but arrive sequentially to the market. This creates a temporary order imbalance. Speculators smooth price fluctuations, thus providing market liquidity. Speculators finance their trades through collateralized borrowing from financiers

who set the margins to control their value-at-risk (VaR). We derive the competitive equilibrium of the model and explore its liquidity implications. We define market liquidity as the difference between the transaction price and the fundamental value, and funding liquidity as a speculator's scarcity (or shadow cost) of capital.

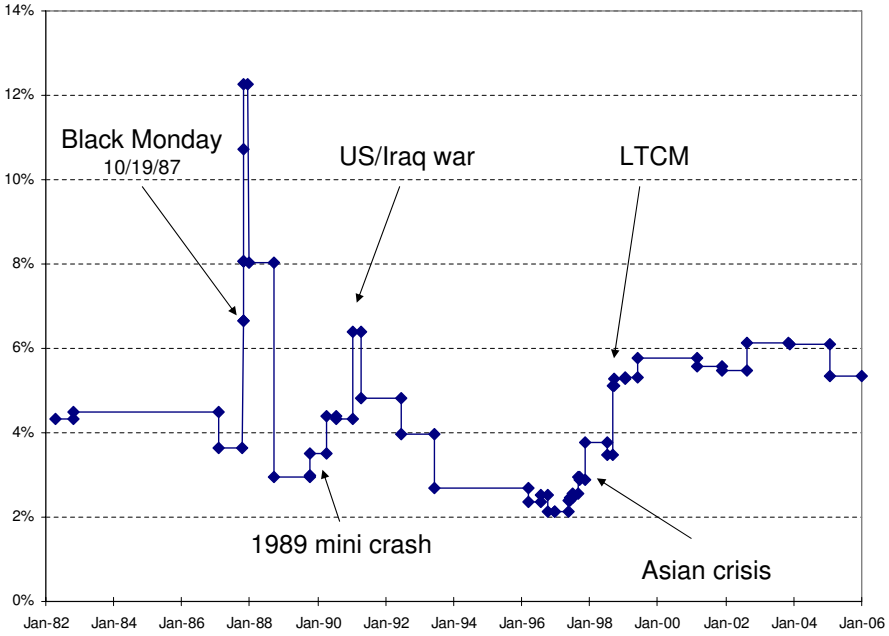

Figure 1: **Margins for S&P500 Futures.** The figure shows margin requirements on S&P500 futures for members of the Chicago Mercantile Exchange as a fraction of the value of the underlying S&P500 index multiplied by the size of the contract. (Initial or maintenance margins are the same for members.) Each dot represents a change in the dollar margin.

We first analyze the properties of margins. We show that margins can increase in illiquidity when margin-setting financiers are unsure whether price changes are due to fundamental news or to liquidity shocks and fundamentals have time-varying volatility. This happens when a liquidity shock leads to price volatility, which raises the financier's expectation about future volatility. Figure 1 shows that margins did increase empirically for S&P 500 futures during the liquidity crises of 1987, 1990, and 1998. We denote margins as "destabilizing" if they can increase in illiquidity, and note that anecdotal evidence from prime brokers suggests that margins often behave in this way.

The model also shows that margins can, in contrast, decrease in illiquidity and thus be

"stabilizing." This happens when financiers know that prices diverge due to temporary market illiquidity and know that liquidity will be improved shortly as complementary customers arrive. This is because a current price divergence from fundamentals provides a "cushion" against future adverse price moves, making the speculator's position less risky in this case. In summary, our model predicts that margins depend on market conditions and are more destabilizing in specialized markets in which financiers cannot easily distinguish fundamental shocks from liquidity shocks or predict when a trade converges.

Turning to the implications for market liquidity, we first show that, as long as speculator capital is so abundant that there is no risk of hitting the funding constraint, market liquidity is naturally at its highest level and is insensitive to marginal changes in capital and margins. However, when speculators hit their capital constraints — or risk hitting their capital constraints over the life of a trade — then they reduce their positions and market liquidity declines.

When margins are destabilizing or speculators have large existing positions, there can be multiple equilibria and liquidity can be *fragile*. In one equilibrium, markets are liquid, leading to favorable margin requirements for speculators, which in turn helps speculators make markets liquid. In another equilibrium, markets are illiquid, resulting in larger margin requirements (or speculator losses), thus restricting speculators from providing market liquidity. Importantly, any equilibrium selection has the property that small speculator losses can lead to a discontinuous drop of market liquidity. This "sudden dry-up" or fragility of market liquidity is due to the fact that with high levels of speculator capital, markets must be in a liquid equilibrium, and, if speculator capital is reduced enough, the market must eventually switch to a low-liquidity/high-margin equilibrium.[1] The events following the Russian default and LTCM collapse in 1998 are a vivid example of fragility of liquidity since a relatively small shock had a large impact. Compared to the total market capitalization of the US stock and bond markets, the losses due to the Russian default were minuscule but, as Figure 1 shows,

[1]Fragility can also be caused by asymmetric information on the amount of trading by portfolio insurance traders (Gennotte and Leland (1990)), and by losses on existing positions (Chowdhry and Nanda (1998)).

caused a shiver in world financial markets.

Further, when markets are illiquid, market liquidity is highly sensitive to further changes in funding conditions. This is due to two liquidity spirals: first, a "*margin spiral*" emerges if margins are increasing in market illiquidity because a reduction in speculator wealth lowers market liquidity, leading to higher margins, tightening speculators' funding constraint further, and so on. For instance, Figure 1 shows how margins gradually escalated within a few days after Black Monday in 1987. Second, a "*loss spiral*" arises if speculators hold a large initial position that is negatively correlated with customers' demand shock. In this case, a funding shock increases market illiquidity, leading to speculator losses on their initial position, forcing speculators to sell more, causing a further price drop, and so on.[2] These liquidity spirals reinforce each other, implying a larger total effect than the sum of their separate effects. Paradoxically, liquidity spirals imply that a larger shock to the customers' demand for immediacy leads to a reduction in the provision of immediacy during such times of stress. Consistent with our predictions, Mitchell, Pulvino, and Pedersen (2007) find significant liquidity-driven divergence of prices from fundamentals in the convertible bond markets after capital shocks to the main liquidity providers, namely convertible arbitrage hedge funds.

In the cross-section we show that the ratio of illiquidity to margin is the same across all assets for which speculators provide market liquidity. This is the case since speculators optimally invest in securities that have the greatest expected profit (i.e. illiquidity) per capital use (determined by the asset's dollar margin). This common ratio is determined in equilibrium by the speculators' funding liquidity i.e. capital scarcity.

Our model thus provides a natural explanation for the *commonality* of liquidity across assets since shocks to speculators' funding constraint affect all securities. This may help explain why market liquidity is correlated across stocks (Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001) and Huberman and Halka (2001)), and across stocks and bonds

[2]The loss spiral is related to the multipliers that arise in Grossman (1988), Kiyotaki and Moore (1997), Shleifer and Vishny (1997), Chowdhry and Nanda (1998), Xiong (2001), Kyle and Xiong (2001), Gromb and Vayanos (2002), Morris and Shin (2004), Plantin, Sapra, and Shin (2005) and others. To our knowledge, our paper is the first to model the margin spiral and the interaction between the two multipliers.

(Chordia, Sarkar, and Subrahmanyam (2005)). In support of the idea that commonality is driven at least in part by our funding-liquidity mechanism, Chordia, Sarkar, and Subrahmanyam (2005) find that "money flows ... account for part of the commonality in stock and bond market liquidity." Moreover, their finding that "during crisis periods, monetary expansions are associated with increased liquidity" is consistent with our model's prediction that the effects are largest when traders are near their constraint. Coughenour and Saad (2004) provide further evidence of the funding-liquidity mechanism by showing that the co-movement in liquidity among stocks handled by the same NYSE specialist firm is higher than for other stocks, commonality is higher for specialists with less capital, and decreases after a merger of specialists.

Next, our model predicts that market liquidity declines as fundamental volatility increases, which is consistent with the empirical findings of Benston and Hagerman (1974) and Amihud and Mendelson (1989).[3] Further, the model can shed new light on "flight to quality," referring to episodes in which risky securities become especially illiquid. In our model, this happens when speculator capital deteriorates. Indeed, a reduction in speculators' capital induces them to provide liquidity mostly in securities that do not use much capital (low volatility stocks with lower margins), implying that the liquidity differential between high-volatility and low-volatility securities increases. This capital effect means that illiquid securities are predicted to have more liquidity risk.[4] Recently, Hendershott, Moulton, and Seasholes (2006) test these predictions using inventory positions of NYSE specialists as a proxy for funding liquidity. Their findings support our hypotheses that market liquidity of high volatility stocks is more sensitive to changes in inventory shocks and that this is more pronounced at times of low funding liquidity. Moreover, Pastor and Stambaugh (2003) and Acharya and Pedersen

[3]The link between volatility and liquidity is shared by the models of Stoll (1978), Grossman and Miller (1988), and others. What sets our theory apart is that this link is connected with margin constraints. This leads to testable differences since, according to our model, the link is stronger when speculators are poorly financed, and high-volatility securities are more affected by speculator wealth shocks — our explanation of flight to quality.

[4]In Vayanos (2004) liquidity premia increase in volatile times. Fund managers become effectively more risk averse because higher fundamental volatility increases the likelihood that their performance falls short of a threshold, leading to costly performance-based withdrawal of funds.

(2005) document empirical evidence consistent with flight to liquidity and the pricing of this liquidity risk.

Market-making firms are often net long in the market. For instance, Ibbotson (1999) reports that security brokers and speculators have median market betas in excess of one. Therefore, capital constraints are more likely to be hit during market downturns, and this, together with the mechanism outlined in our model, helps to explain why sudden liquidity dry-ups occur more often when markets decline. Further, capital constraints affect the liquidity of all securities, leading to co-movement as explained above. The fact that this effect is stronger in down markets could explain that co-movement in liquidity is higher during large negative market moves as documented empirically by Hameed, Kang, and Viswanathan (2005).

Finally, the very risk that the funding constraint becomes binding limits speculators' provision of market liquidity. Our analysis shows that speculators' optimal (funding) risk management policy is to maintain a "safety buffer." This affects initial prices, which increase in the covariance of future prices with future shadow costs of capital (i.e., with future funding illiquidity).

Our paper is related to several literatures.[5] Most directly related are the models with margin-constrained traders: Grossman and Vila (1992) and Liu and Longstaff (2004) derive optimal strategies in a partial equilibrium with a single security; Chowdhry and Nanda (1998) focus on fragility due to dealer losses; and Gromb and Vayanos (2002) derive a general equilibrium with one security and study welfare and liquidity provision. Our paper contributes to the literature by considering the simultaneous effect of margin constraints on multiple securities and by examining the nature of those margin constraints. Stated simply, the existing theoretical literature uses a fixed or decreasing margin constraint — say \$5,000 per contract

[5]Market liquidity is the focus of market microstructure (Stoll (1978), Ho and Stoll (1981, 1983), Kyle (1985), Glosten and Milgrom (1985), Grossman and Miller (1988)), and is related to the limits of arbitrage (DeLong, Shleifer, Summers, and Waldmann (1990), Shleifer and Vishny (1997), Abreu and Brunnermeier (2002)). Funding liquidity is examined in corporate finance (Shleifer and Vishny (1992), Holmström and Tirole (1998,2001)) and banking (Diamond and Dybvig (1983), Allen and Gale (1998, 2004, 2005)). Funding and collateral constraints are also studied in macroeconomics (Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Lustig (2004)), and general equilibrium with incomplete markets (Geanakoplos (1997, 2003)). Finally recent papers consider illiquidity with constrained traders (Attari, Mello, and Ruckes (2005), Bernardo and Welch (2004), Brunnermeier and Pedersen (2005), Eisfeldt (2004), Morris and Shin (2004), and Weill (2004)).

— and studies what happens when trading losses cause agents to hit this constraint, whereas we study how market conditions lead to changes in the margin requirement itself — e.g., an increase from \$5,000 to \$15,000 per futures contract as happened in October 1987 — and the resulting feedback effects between margins and market conditions.

We proceed as follows. We describe the model (Section 1) and derive our four main new results: (i) margins increase with market illiquidity when financiers cannot distinguish fundamental shocks from liquidity shocks and fundamentals have time-varying volatility (Section 2); (ii) this makes margins destabilizing, leading to sudden liquidity dry ups and margin spirals (Section 3); (iii) liquidity crises simultaneously affect many securities, mostly risky high-margin securities, resulting in commonality of liquidity and flight to quality (Section 4); and (iv) liquidity risk matters even before speculators hit their capital constraints (Section 5). Then we outline how our model's new testable predictions may be helpful for a novel line of empirical work that links measures of speculators' funding conditions to measures of market liquidity (Section 6). Finally, we describe the real-world funding constraints for the main liquidity providers, namely market makers, banks, and hedge funds (Appendix A), and provide proofs (Appendix B).

1 Model

Setup. The economy has J risky assets, traded at times $t = 0, 1, 2, 3$. At time $t = 3$, each security j pays off v^j, a random variable defined on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$. There is no aggregate risk and the risk-free interest rate is normalized to zero, so the fundamental value of each stock is its conditional expected value of the final payoff $v_t^j = E_t\left[v^j\right]$. Fundamental volatility has an autoregressive conditional heteroscedasticity (ARCH) structure. Specifically, v_t^j evolves according to

$$v_{t+1}^j = v_t^j + \Delta v_{t+1}^j = v_t^j + \sigma_{t+1}^j \varepsilon_{t+1}^j \,, \tag{1}$$

where all ε_t^j are i.i.d. across time and assets with a standard normal cumulative distribution function Φ with zero mean and unit variance, and the volatility σ_t^j has dynamics

$$\sigma_{t+1}^j = \underline{\sigma}^j + \theta^j |\Delta v_t^j|, \tag{2}$$

where $\underline{\sigma}^j, \theta^j \geq 0$. A positive θ^j implies that shocks to fundamentals increase future volatility.

There are three groups of market participants: "customers" and "speculators" trade assets while "financiers" finance speculators' positions. The group of customers consists of three risk-averse agents. At time 0, customer $k = 0, 1, 2$ has a cash holding of W_0^k bonds and zero shares, but finds out that he will experience an endowment shock of $\mathbf{z}^k = \{z^{1,k}, ..., z^{J,k}\}$ shares at time $t = 3$, where $\mathbf{z}$ are jointly normally distributed random variables such that the aggregate endowment shock is zero, $\sum_{k=0}^{2} z^{j,k} = 0$.

With probability $(1 - a)$, all customers arrive at the market at time 0 and can trade securities in each time period 0, 1, 2. Since their aggregate shock is zero, they can share risks and have no need for intermediation.

The basic liquidity problem arises because customers arrive sequentially with probability a, which gives rise to order imbalance. Specifically, in this case customer 0 arrives at time 0, customer 1 arrives at time 1, and customer 2 arrives at time 2. Hence, at time 2 all customers are present, at time 1 only customers 0 and 1 can trade, and at time 0 only customer 0 is in the market.

Before a customer arrives in the marketplace, his demand is $\mathbf{y}_t^k = 0$, and after he arrives he chooses his security position each period to maximize his exponential utility function $U(W_3^k) = -\exp\{-\gamma W_3^k\}$ over final wealth. Wealth W_t^k, including the value of the anticipated endowment shock of $\mathbf{z}^k$ shares, evolves according to

$$W_{t+1}^k = W_t^k + \left(\mathbf{p}_{t+1} - \mathbf{p}_t\right)' \left(\mathbf{y}_t^k + \mathbf{z}^k\right). \tag{3}$$

The total demand shock of customers who have arrived in the market at time t is denoted

by $Z_t := \sum_{k=0}^{t} \mathbf{z}^k$.

The early customers' trading need is accommodated by speculators who provide liquidity/immediacy. Speculators are risk-neutral and maximize expected final wealth W_3. Speculators face the constraint that the total margin on their position $\mathbf{x}_t$ cannot exceed their capital W_t:

$$\sum_j \left(x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-} \right) \leq W_t \tag{4}$$

where $x_t^{j+} \geq 0$ and $x_t^{j-} \geq 0$ are the positive and negative parts of $x_t^j = x_t^{j+} - x_t^{j-}$, respectively, and $m_t^{j+} \geq 0$ and $m_t^{j-} \geq 0$ are the dollar margin on long and short positions, respectively. The institutional features related to this key constraint for different types of speculators like hedge funds, banks, and market makers are discussed in detail in Appendix A.

Speculators start out with a cash position of W_0 and zero shares, and their wealth evolves according to

$$W_t = W_{t-1} + \left(\mathbf{p}_t - \mathbf{p}_{t-1} \right)' \mathbf{x}_{t-1} + \eta_t \,, \tag{5}$$

where η_t is an independent wealth shock arising from other activities, e.g., a speculator's investment banking arm. If a speculator loses all his capital, $W_t \leq 0$, he can no longer invest because of the margin constraint (4), i.e. he must choose $x_t = 0$. We let his utility in this case be $\varphi_t W_t$, where $\varphi_t \geq 0$. Limited liability corresponds to $\varphi_t = 0$, and a proportional bankruptcy cost (e.g., monetary, reputational, or opportunity costs) corresponds to $\varphi_t > 0$. We focus on the case in which $\varphi_2 = 1$, that is, negative consumption equal to time-2 the dollar loss, and we discuss φ_1 in Section 5. Our results would be qualitatively the same with other bankruptcy assumptions.[6]

The financier sets the margins to limit his counterparty credit risk. Specifically, the financier ensures that the margin is large enough to cover the position's π-value-at-risk (where

[6]We could allow the speculators to raise new capital as long as this takes time. Indeed, the model would be the same if the speculators could raise capital only at time 2 (and in this case we need not assume that the customers' endowment shocks z^j aggregate to zero).

π is a non-negative number close to zero, e.g., 1%):

$$\pi = Pr(-\Delta p^j_{t+1} > m^{j+}_t \,|\, \mathcal{F}_t) \tag{6}$$

$$\pi = Pr(\ \Delta p^j_{t+1} > m^{j-}_t \,|\, \mathcal{F}_t)\,. \tag{7}$$

Equation (6) means that the margin on a long position m^+ is set such that price drops larger than the margin only happen with a small probability π. Similarly, (7) means that price increases larger than the margin on a short position only happen with small probability. Clearly, the margin is larger for more volatile assets. The margin depends on financiers' information set $\mathcal{F}_t$. We consider two important benchmarks: "informed financiers" who know the fundamental value and the liquidity shocks z, $\mathcal{F}_t = \sigma\{\mathbf{z}, \mathbf{v}_0, \ldots, \mathbf{v}_t, \mathbf{p}_0, \ldots, \mathbf{p}_t, \eta_1, \ldots, \eta_t\}$, and "uninformed financiers" who only observe prices, $\mathcal{F}_t = \sigma\{\mathbf{p}_0, \ldots, \mathbf{p}_t\}$. This margin specification is motivated by the real-world institutional features described in Appendix A. Theoretically, Stiglitz and Weiss (1981) show how credit rationing can be due to adverse selection and moral hazard in the lending market, and Geanakoplos (2003) considers endogenous contracts in a general-equilibrium framework of imperfect commitment.

We let Λ^j_t be the (signed) deviation of the price from fundamental value

$$\Lambda^j_t = p^j_t - v^j_t\,, \tag{8}$$

and we define our measure of market illiquidity as the absolute amount of this deviation, $|\Lambda^j_t|$. We consider competitive equilibria of the economy:

Definition 1 *An* equilibrium *is a price process $\boldsymbol{p}_t$ such that (i) $\boldsymbol{x}_t$ maximizes the speculators' expected final profit subject to the margin constraint (4), (ii) each $\boldsymbol{y}^k_t$ maximizes k-customers expected utility after their arrival at the marketplace and is zero beforehand, (iii) margins are set according to the VaR specification (6), and (iv) markets clear, $\boldsymbol{x}_t + \sum_{k=0}^{2} \boldsymbol{y}^k_t = 0$.*

Outline of Equilibrium. We derive the optimal strategies for customers and speculators using dynamic programming, starting from time 2, and working backwards. A customer's value function is denoted Γ and a speculator's value function is denoted J. At time 2, customer k's problem is

$$\begin{aligned} \Gamma_2(W_2^k, p_2, v_2) &= \max_{y_2^k} -E_2[e^{-\gamma W_3^k}] && (9) \\ &= \max_{y_2^k} -e^{-\gamma(E_2[W_3^k] - \frac{\gamma}{2} Var_2[W_3^k])} && (10) \end{aligned}$$

which has the solution

$$y_2^{j,k} = \frac{v_2^j - p_2^j}{\gamma(\sigma_3^j)^2} - z^{j,k} \tag{11}$$

Clearly, since all customers are present in the market at time 2, the unique equilibrium is $p_2 = v_2$. Indeed, when the prices are equal to fundamentals, the aggregate customer demand is zero, $\sum_k y_2^{j,k} = 0$, and the speculator also has a zero demand. We get the customer's value function $\Gamma_2(W_2^k, p_2 = v_2, v_2) = -e^{-\gamma W_2^k}$, and the speculator's value function $J_2(W_2, p_2 = v_2, v_2) = W_2$.

The equilibrium before time 2 depends on whether the customers arrive sequentially or all at time 0. If all customers arrive at time 0, then the simple arguments above show that $p_t = v_t$ at any time $t = 0, 1, 2$.

We are interested in the case with sequential arrival of the customers such that the speculators' liquidity provision is needed. At time 1, customers 0 and 1 are present in the market, but customer 2 has not arrived yet. As above, customer $k = 0, 1$ has a demand and value function of

$$\begin{aligned} y_1^{j,k} &= \frac{v_1^j - p_1^j}{\gamma(\sigma_2^j)^2} - z^{j,k} && (12) \\ \Gamma_1(W_1^k, p_1, v_1) &= -\exp\left\{-\gamma\left[W_1^k + \sum_j \frac{(v_1^j - p_1^j)^2}{2\gamma(\sigma_2^j)^2}\right]\right\} && (13) \end{aligned}$$

At time 0, customer $k = 0$ arrives in the market and maximizes $E_0[\Gamma_1(W_1^k, p_1, v_1)]$.

At time $t = 1$, if the market is perfectly liquid so that $p_1^j = v_1^j$ for all j, then the speculator is indifferent among all possible positions x_1. If some securities have $p_1 \neq v_1$, then the risk-neutral speculator invests all his capital such that his margin constraint binds. The speculator optimally trades only in securities with the highest expected profit per dollar used. The profit per dollar used is $(v_1^j - p_1^j)/m_1^{j+}$ on a long position and $-(v_1^j - p_1^j)/m_1^{j-}$ on a short position. A speculator's shadow cost of capital, denoted ϕ_1, is 1 plus the maximum profit per dollar used as long as he is not bankrupt:

$$\phi_1 \quad = \quad 1 + \max_j \left(\max\{ \frac{v_1^j - p_1^j}{m_1^{j+}} \, , \, \frac{-(v_1^j - p_1^j)}{m_1^{j-}} \} \right) , \tag{14}$$

where the margins for long and short positions are set by the financier as described in the next section. If the speculator is bankrupt $W_1 < 0$ then $\phi_1 = \varphi_1$. Each speculator's value function is therefore

$$J_1(W_1, p_1, v_1, p_0, v_0) = W_1 \phi_1 \, . \tag{15}$$

At time $t = 0$, the speculator maximizes $E_0[W_1 \phi_1]$ subject to his capital constraint (4).

The equilibrium prices at times 1 and 0 do not have simple expressions. However, after having derived the margin conditions, we characterize several important properties of these prices, which illuminates the connection between market liquidity, $|\Lambda|$, and speculators' funding situation.

2 Margin Setting and Liquidity (Time 1)

A key determinant of speculators' funding liquidity is their margin requirement for collateralized financing. Hence, it is important to determine the margin function, m_1, set by, respectively, informed and uninformed financiers. The margin at time 1 is set to cover a position's value-at-risk, knowing that prices equal the fundamental values in the next period 2, $p_2 = v_2$.

We consider first informed financiers who know the fundamental values v_1 and, hence, price divergence from fundamentals Λ_1. Since $\Lambda_2 = 0$, they set margins on long positions at $t = 1$ according to

$$\begin{aligned} \pi &= Pr(-\Delta p_2^j > m_1^{j+} \,|\, \mathcal{F}_1) \\ &= Pr(-\Delta v_2^j + \Lambda_1^j > m_1^{j+} \,|\, \mathcal{F}_1) \\ &= 1 - \Phi\left(\frac{m_1^{j+} - \Lambda_1^j}{\sigma_2^j}\right), \end{aligned} \tag{16}$$

which implies that

$$\begin{aligned} m_1^{j+} &= \Phi^{-1}(1-\pi)\,\sigma_2^j + \Lambda_1^j \\ &= \bar{\sigma}^j + \bar{\theta}|\Delta v_1^j| + \Lambda_1^j \end{aligned} \tag{17}$$

where we define

$$\bar{\sigma}^j = \underline{\sigma}^j \Phi^{-1}(1-\pi) \tag{18}$$

$$\bar{\theta} = \theta\ \Phi^{-1}(1-\pi)\ . \tag{19}$$

The margin on a short position can be derived similarly and we arrive at the following surprising result:

Proposition 1 (Stabilizing Margins and the Cushioning Effect) *When the financier is informed about the fundamental value and knows that prices will equal fundamentals in the next period $t = 2$, then the margins on long and short positions are, respectively,*

$$m_1^{j+} = \max\{\bar{\sigma}^j + \bar{\theta}|\Delta v_1^j| + \Lambda_1^j, 0\} \tag{20}$$

$$m_1^{j-} = \max\{\bar{\sigma}^j + \bar{\theta}|\Delta v_1^j| - \Lambda_1^j, 0\} \tag{21}$$

The more prices are below fundamentals $\Lambda_1^j < 0$, the lower is the margin on a long position

m_1^{j+}, and the more prices are above fundamentals $\Lambda_1^j > 0$, the lower is the margin on a short position m_1^{j-}. Hence, in this case illiquidity reduces margins for speculators who buy low and sell high.

The margins are reduced by illiquidity because the speculator is expected to profit when prices return to fundamentals at time 2, and this profit "cushions" the speculators from losses due to fundamental volatility. Thus, we denote the margins set by informed financiers at $t = 1$ as *stabilizing margins.*

Stabilizing margins are an interesting benchmark, and they are hard to escape in a theoretical model. However, real-world liquidity crises are often associated with increases in margins, not decreases. To capture this, we turn to the case of a financier who is uninformed about the current fundamental so that he must set his margin based on the observed prices p_0 and p_1. This is in general a complicated problem since the financier needs to filter out the probability that customers arrive sequentially, and the values of z_0 and z_1. The expression becomes simple however, if the financier's prior probability of a liquidity shock is small so that he finds it likely that $p_t^j = v_t^j$, implying a common margin $m_1^j = m_1^{j+} = m_1^{j-}$ for long and short positions in the limit:

Proposition 2 (Destabilizing Margins) *When the financier is uninformed about the fundamental value, then, as $a \to 0$, the margins on long and short positions approach*

$$m_1^j = \bar{\sigma}^j + \bar{\theta}|\Delta p_1^j| = \bar{\sigma}^j + \bar{\theta}|\Delta v_1^j + \Delta\Lambda_1^j|\,. \tag{22}$$

Margins are increasing in price volatility and market illiquidity can increase margins.

Intuitively, since liquidity risk tends to increase price volatility, and since an uninformed financier may interpret price volatility as fundamental volatility, this increases margins.[7] Equation (22) corresponds closely to real-world margin setting, which is primarily based on volatility estimates from past price movements. Equation (22) shows that illiquidity increases

[7] In the analysis of time 0, we shall see that margins can also be destabilizing when price volatility signals future liquidity risk (not necessarily fundamental risk).

margins when the liquidity shock $\Delta\Lambda_1^j$ has the same sign as the fundamental shock Δv_1^j or is greater in magnitude, but margins are reduced if the liquidity shock counterbalances a fundamental move. We denote the phenomenon that margins can increase as illiquidity rises by *destabilizing margins*. As we will see next, the information available to the financier — i.e., whether margins are stabilizing or destabilizing — has important implications for the equilibrium.

3 Fragility and Liquidity Spirals (Time 1)

We next show how speculators' funding problems can lead to liquidity spirals and fragility — the property that a small change in fundamentals can lead to a large jump in illiquidity. We show that funding problems are especially escalating with uninformed financiers, i.e. destabilizing margins. For simplicity we illustrate this with a single security $J = 1$.

Fragility. To set the stage for the main fragility proposition below, we make a few brief definitions. Liquidity is said to be *fragile* if the equilibrium price $p_t(\eta_t, v_t)$ cannot be chosen to be continuous in the exogenous shocks, namely η_t and Δv_t. Fragility arises when the excess demand for shares $x_t + \sum_{k=0}^{1} \mathbf{y}_1$ can be non-monotonic in the price. While under "normal" circumstances, a high price leads to a low total demand (i.e., excess demand is decreasing), binding funding constraints along with destabilizing margins (margin effect) or speculators' losses (loss effect) can lead to an increasing demand curve. Further, it is natural to focus on *stable* equilibria in which a small negative (positive) price perturbation leads to excess demand (supply), which, intuitively, "pushes" the price back up (down) to its equilibrium level.

Proposition 3 (Fragility) *There exist $\underline{x}, \underline{\theta}, \underline{a} > 0$ such that*
(i) With informed financiers, the market is fragile at time 1 if speculators' position $|x_0|$ is larger than $\underline{x}$ and of the same sign as the demand shock Z_1.
(ii) With uninformed financiers the market is fragile as in (i) and additionally if the ARCH

parameter θ is larger than $\underline{\theta}$ and the probability, a, of sequential arrival of customers is smaller than $\underline{a}$.

Numerical Example. We illustrate how fragility arises due to destabilizing margins or dealer losses by way of a numerical example. We consider the more interesting (and arguably more realistic) case in which the financiers are uninformed, and we choose parameters as follows.

The fundamental value has ARCH volatility parameters $\underline{\sigma} = 10$ and $\theta = 0.3$, which implies clustering of volatility. The initial price is $p_0 = 130$, the aggregate demand shock of the customers who have arrived at time 1 is $Z_1 = z_0 + z_1 = 30$, and the customers' risk aversion coefficient is $\gamma = 0.05$. The speculators' have an initial position of $x_0 = 0$ and a cash wealth of $W_1 = 900$. Finally, the financier uses a VaR with $\pi = 1\%$ and customers learn their endowment shocks sequentially with probability $a = 1\%$.

Panel A of Figure 2 illustrates how the speculators' demand x_1 and the customers' supply (i.e., the negative of the customers' demand as per Equation (12)) depend on the price p_1 when the fundamental value is $v_1 = 120$ and the speculators' wealth shock is $\eta_1 = 0$. Customers' supply is given by the upward sloping dashed line since, naturally, their supply is greater when the price is higher. Customers supply $Z_1 = 30$ shares, namely the shares that they anticipate receiving at time $t = 3$, when the market is perfectly liquid, $p_1 = v_1 = 120$ (i.e. illiquidity is $|\Lambda_1| = 0$). For lower prices, they supply fewer shares.

The speculators' demand, x_1, must satisfy the margin constraints. It is instructive to consider first the simpler limiting case $a \to 0$ for which the margin requirement is simply $m = \bar{\sigma} + \bar{\theta}|\Delta p_1| = 2.326(10 + 0.3|\Delta p_1|)$. This implies that speculators demand $|x_1| \leq W_1/(\bar{\sigma} + \bar{\theta}|\Delta p_1|)$. Graphically, this means that their demand must be inside the "hyperbolic star" defined by the four (dotted) hyperbolas (that are partially overlaid by a solid demand curve in Figure 2). At the price $p_1 = p_0 = 130$, the margin is smallest and hence the constraint is most relaxed. As p_1 departs from $p_0 = 130$, margins increase and speculators become more constrained — the horizontal distance between two hyperbolas shrinks.

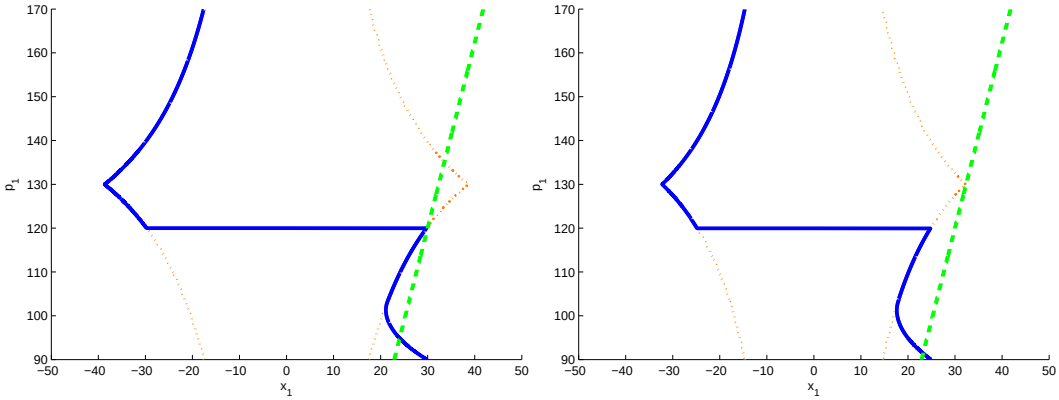

Figure 2: **Speculator Demand and Customer Supply.** This figure illustrates how margins can be destabilizing when financiers are uninformed and the fundamentals have volatility clustering. The solid curve is the speculators' optimal demand for $a = 1\%$. The upward sloping dashed line is the customers' supply, that is, the negative of their demand. In Panel A the speculators experience a zero wealth shock, $\eta_1 = 0$, while in Panel B they face a negative wealth shock of $\eta_1 = -150$, otherwise everything is the same. In Panel A, perfect liquidity $p_1 = v_1 = 120$ is one of two stable equilibria, while in Panel B the unique equilibrium is illiquid.

After establishing the hyperbolic star, it is easy to derive the demand curve for $a \to 0$: For $p_1 = v_1 = 120$, the security's expected return is zero and the speculator is indifferent between all his possible positions on the horizontal line. For price levels $p_1 > v_1$ above this line, the risk neutral speculators want to short-sell the asset, $x_1 < 0$, and their demand is constrained by the upper left side of the star. Similarly, for prices below v_1, speculators buy the asset, $x_1 > 0$, and their demand is limited by the margin constraint. Interestingly, the speculators' demand is upward sloping for prices below 120. As the price declines, the financiers' estimate of fundamental volatility and consequently of margins increase.

We now generalize the analysis to the case where $a > 0$. The margin setting becomes more complicated since uninformed financiers must filter out to what extent the equilibrium price change is caused by a movement in fundamentals Δv_1 and/or an occurrence of a liquidity event with an order imbalance caused by the presence of customers 0 and 1, but not customer 2. Since customers 0 and 1 want to sell ($Z_1 = 30$), a price increase or modest price decline is

most likely due to a change in fundamentals, and hence the margin setting is similar to the case of $a = 0$. This is why speculators' demand curve for prices above 100 almost perfectly overlays the relevant part of the hyperbolic star in Figure 2. However, for a large price drop, say below 100, financiers assign a larger conditional probability that a liquidity event has occurred. Hence, they are willing to set a lower margin (relative to the one implying the hyperbolic star) because they expect the speculator to profit as the price rebounds in period 2 — hence, the cushioning effect discussed above reappears in the extreme here. This explains why the speculators' demand curve is backward bending only in a limited price range and becomes downward sloping for p_1 below roughly 100.[8]

Panel A shows that there are two stable equilibria: a perfect liquidity equilibrium with price $p_1 = v_1 = 120$ and an illiquid equilibrium with a price of about 91 (and an uninteresting unstable equilibrium with p_1 just below 120).

Panel B of Figure 2 shows the same plot as Panel A, but with a negative wealth shock to speculators of $\eta_1 = -150$ instead of $\eta_1 = 0$. In this case, perfect liquidity with $p_1 = v_1$ is no longer an equilibrium since the speculators cannot fund a large enough position. The unique equilibrium is highly illiquid because of the speculators' lower wealth and, importantly, because of endogenously higher margins.

This "disconnect" between the perfect-liquidity equilibrium and the illiquid equilibrium and the resulting fragility is illustrated more directly in Figure 3. Panel A plots the equilibrium price correspondence for different exogenous funding shocks η_1 (with fixed $\Delta v_1 = -10$) and shows that a marginal reduction in funding cannot always lead to a smooth reduction in market liquidity. Rather, there must be a level of funding such that an infinitesimal drop in funding leads to a discontinuous drop in market liquidity.

The dark (blue) line in Figure 3 shows the equilibrium with the highest market liquidity and the light (red) line shows the equilibrium with the lowest market liquidity. We note that

[8]We note that the cushioning effect relies on the financiers' knowledge that the market will become liquid in period $t = 2$. This is not the case in the earlier period 0, though, and, in an earlier version of the paper, we showed that the cushioning effect disappears in a stationary infinite horizon setting in which the "complementary" customers arrive in each period with a constant arrival probability.

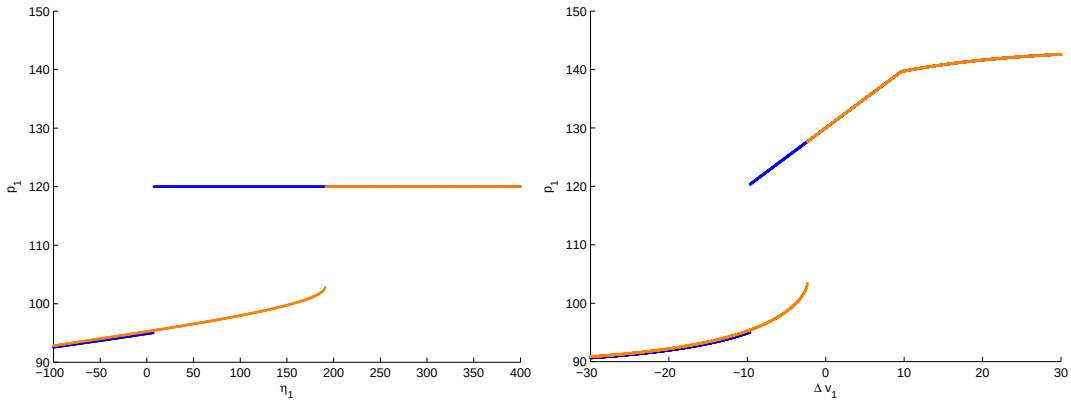

Figure 3: **Fragility due to Destabilizing Margins.** The figure shows the equilibrium price as a function of the speculators' wealth shock η_1 (Panel A) and of fundamental shocks Δv_1 (Panel B). This is drawn for the equilibrium with the highest market liquidity (light/ red line) and the equilibrium with the lowest market liquidity (dark/ blue line). The margins are destabilizing since financiers are uninformed and fundamentals exhibit volatility clustering. The equilibrium prices are discontinuous, which reflects *fragility* in liquidity since a small shock can lead to a disproportionately large price effect.

the financiers' filtering problem and, hence, the margin function depend on the equilibrium selection. Since the margin affects the speculators' trades, the equilibrium selection affects the equilibrium outcome everywhere — prices are slightly affected even outside the η region (v_1 region) with fragility.

Panel B of Figure 3 plots the equilibrium price correspondence for different realizations of the fundamental shock Δv_1 (with fixed $\eta_1 = 0$) and shows the same form of discontinuity for adverse fundamental shocks to v_1. The discontinuity with respect to Δv_1 is most easily understood in conjunction with Panel A of Figure 2. As Δv_1 falls, the horizontal line of speculator demand shifts downward, and the customer supply line moves downward. As a result, the perfect liquidity equilibrium vanishes. Panel B also reveals the interesting asymmetry that negative fundamental shocks lead to larger price movements than corresponding positive shocks (for $Z_1 := z_0 + z_1 > 0$). This asymmetry arises even without a loss effect since $x_0 = 0$.

Fragility can also arise because of shocks to customer demand or volatility. Indeed, the market can also be suddenly pushed into an illiquid equilibrium with high margins due to an

increase in demand and an increase in volatility. Paradoxically, a marginally larger demand for liquidity by customers can lead to a drastic reduction of liquidity supply by the speculators when it pushes the equilibrium over the edge.

While the example above has speculators with zero initial positions, $x_0 = 0$, it is also interesting to consider $x_0 > 0$. In this case, lower prices lead to losses for the speculators, and graphically this means that the constraints in the "hyperbolic star" tighten (i.e., the gap between the hyperbolas narrows) at low prices. Because of this "loss effect," the discontinuous price drop associated with the illiquid equilibrium is even larger.

In summary, this example shows how destabilizing margins and dealer losses give rise to a discontinuity in prices which can help to explain the sudden market liquidity dry-ups observed in many markets. For example, Russia's default in 1998 was in itself only a trivial wealth shock relative to global arbitrage capital. Nevertheless, it had a large effect on liquidity in global financial markets, consistent with our fragility result that a small wealth shock can push the equilibrium over the edge.

Liquidity Spirals. To further emphasize the importance of speculators' funding liquidity, we now show how it can make market liquidity highly sensitive to shocks. We identify two amplification mechanisms: a "margin spiral" due to increasing margins as speculator financing worsens, and a "loss spiral" due to escalating speculator losses. Figure 4 illustrates these "liquidity spirals": A shock to speculator capital ($\eta_1 < 0$) forces speculators to provide less market liquidity, which increases the price impact of the customer demand pressure. With uninformed financiers and ARCH effects the resulting price swing increases financiers' estimate of the fundamental volatility and, hence, increases the margin, thereby worsening speculator funding problems even further, and so on, leading to a "margin spiral." Similarly, increased market illiquidity can lead to losses on speculators' existing positions, worsening their funding problem and so on, leading to a "loss spiral." Mathematically, the spirals can be expressed as follows:

Proposition 4

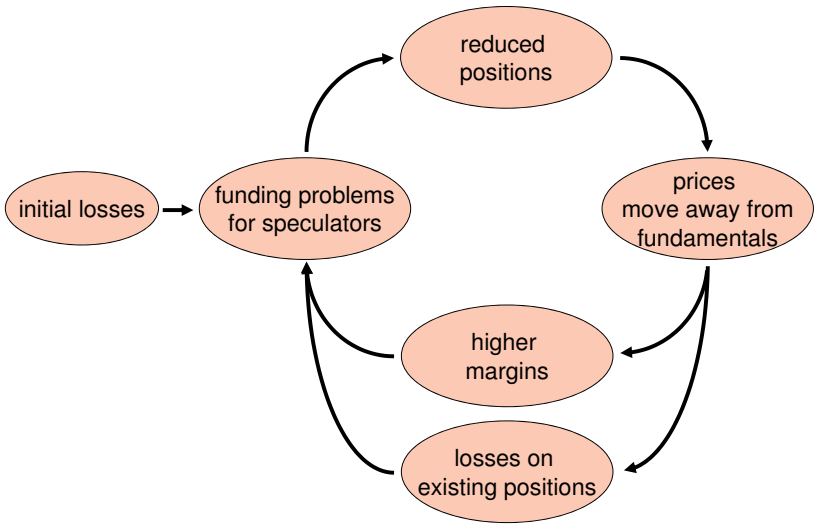

Figure 4: **Liquidity Spirals**

(i) *If speculators' capital constraint is slack then the price* p_1 *is equal to* v_1 *and insensitive to local changes in speculator wealth.*

(ii) (Liquidity Spirals) *In a stable illiquid equilibrium with selling pressure from customers,* $Z_1, x_1 > 0$*, the price sensitivity to speculator wealth shocks* η_1 *is*

$$\frac{\partial p_1}{\partial \eta_1} = \frac{1}{\frac{2}{\gamma(\sigma_2)^2} m_1^+ + \frac{\partial m_1^+}{\partial p_1} x_1 - x_0} \tag{23}$$

and with buying pressure from customers, $Z_1, x_1 < 0$*,*

$$\frac{\partial p_1}{\partial \eta_1} = \frac{-1}{\frac{2}{\gamma(\sigma_2)^2} m_1^- + \frac{\partial m_1^-}{\partial p_1} x_1 + x_0}. \tag{24}$$

A margin spiral arises if $\frac{\partial m_1^+}{\partial p_1} < 0$ *or* $\frac{\partial m_1^-}{\partial p_1} > 0$*, which happens with positive probability if speculators are uninformed and* a *is small enough. A loss spiral arises if speculators' previous position is in the opposite direction as the demand pressure* $x_0 Z_1 > 0$*.*

This proposition is intuitive. Imagine first what happens if speculators face a wealth shock of \$1, margins are constant, and speculators have no inventory $x_0 = 0$. In this case, the speculator must reduce his position by $1/m_1$. Since the slope of each of the two customer demand curves is[9] $1/(\gamma\,(\sigma_2)^2)$, we get a total price effect of $1/(\frac{2}{\gamma(\sigma_2)^2}m_1)$.

The two additional terms in the denominator imply amplification or dampening effects due to changes in the margin requirement and to profit/losses on the speculators' existing positions. To see that, recall that for any $k > 0$ and l with $|l| < k$, it holds that $\frac{1}{k-l} = \frac{1}{k} + \frac{l}{k^2} + \frac{l^2}{k^3} + ...$; so with $k = \frac{2}{\gamma(\sigma_2)^2}m_1$ and $l = -\frac{\partial m_1^{\pm}}{\partial p_1}x_1 \pm x_0$, each term in this infinite series corresponds to one loop around the circle in Figure 4. The total effect of the changing margin and speculators' positions amplifies the effect if $l > 0$. Intuitively, if e.g. $Z_1 > 0$, then customer selling pressure is pushing down the price, and $\frac{\partial m_1^+}{\partial p_1} < 0$ means that as prices go down, margins increase, making speculators' funding tighter and thus destabilizing the system. Similarly, when customers are buying, $\frac{\partial m_1^-}{\partial p_1} > 0$ implies that increasing prices leads to increased margins, making it harder for speculators to short-sell, thus destabilizing the system. The system is also destabilized if speculators lose money on their previous position as prices move away from fundamentals similar to e.g., Shleifer and Vishny (1997).

Interestingly, the total effect of a margin spiral together with a loss spiral is greater than the sum of their separate effects. This can be seen mathematically by using simple convexity arguments, and it can be seen intuitively from the flow diagram of Figure 4.

Note that spirals can also be "started" by shocks to liquidity demand Z_1, fundamentals v_1, or volatility. It is straightforward to compute the price sensitivity with respect to such shocks. They are just multiples of $\frac{\partial p_1}{\partial \eta_1}$. For instance, a fundamental shock affects the price both because of its direct effect on the final payoff and because of its effect on customers' estimate of future volatility — and both of these effects are amplified by the liquidity spirals.

Our analysis sheds some new light on the 1987 stock market crash, complementing the standard culprit, portfolio insurance trading. In the 1987 stock market crash, numerous market makers hit (or violated) their funding constraint:

[9] See Equation (12).

"By the end of trading on October 19, [1987] thirteen [NYSE specialist] units had no buying power"

— SEC (1988), page 4-58

Several of these firms managed to reduce their positions and continue their operations. Others did not. For instance, Tompane was so illiquid that it was taken over by Merrill Lynch Specialists and Beauchamp was taken over by Spear, Leeds & Kellogg (Beauchamp's clearing broker). Also, market makers outside the NYSE experienced funding troubles: the Amex market makers Damm Frank and Santangelo were taken over; at least 12 OTC market makers ceased operations; and several trading firms went bankrupt.

These funding problems were due to (i) reductions in capital arising from trading losses and defaults on unsecured customer debt, (ii) an increased funding need stemming from increased inventory and, (iii) increased margins. One New York City bank, for instance, increased margins/haircuts from 20% to 25% for certain borrowers, and another bank increased margins from 25% to 30% for all specialists (SEC (1988) page 5-27 and 5-28). Other banks reduced the funding period by making intra-day margin calls, and at least two banks made intra-day margin calls based on assumed 15% and 25% losses, thus effectively increasing the haircut by 15% and 25%. Also, some broker-dealers experienced a reduction in their line of credit and – as Figure 1 shows – margins at the futures exchanges also drastically increased (SEC (1988) and Wigmore (1998)).

In summary, our results on fragility and liquidity spirals imply that during "bad" times, small changes in underlying funding conditions (or liquidity demand) can lead to sharp reductions in liquidity. The 1987 crash exhibited several of the predicted features, namely capital constrained dealers, increased margins, and increased illiquidity.

4 Commonality and Flight to Quality (Time 1)

We now turn to the cross-sectional implications of illiquidity. Since speculators are risk-neutral, they optimally invest all their capital in securities that have the greatest expected

profit $|\Lambda^j|$ per capital use, i.e., per dollar margin m^j, as expressed in Equation (14). That equation also introduces the shadow cost of capital ϕ_1 as the marginal value of an extra dollar. The speculators' shadow cost of capital ϕ_1 captures well the notion of funding liquidity: a high ϕ means that the available funding — from capital W_1 and from collateralized financing with margins m_1^j — is low relative to the needed funding, which depends on the investment opportunities deriving from demand shocks z^j.

The market liquidity of all assets depend on the speculators' funding liquidity, especially for high-margin assets, and this has several interesting implications:

Proposition 5 *There exists $c > 0$ such that, for $\theta^j < c$ for all j and either informed financiers or uninformed with $a < c$, we have:*

(i) (Commonality of Market Liquidity) *The market illiquidities $|\Lambda|$ of any two securities, k and l, co-move,*

$$\mathrm{Cov}_0\left(|\Lambda_1^k|, |\Lambda_1^l|\right) \geq 0\,, \tag{25}$$

and market illiquidity co-moves with funding illiquidity as measured by speculators' shadow cost of capital ϕ_1

$$\mathrm{Cov}_0\left[|\Lambda_1^k|, \phi_1\right] \geq 0\,. \tag{26}$$

(ii) (Commonality of Fragility) *Jumps in market liquidity occur simultaneously for all assets for which speculators are marginal investors.*

(iii) (Quality=Liquidity) *If asset l has lower fundamental volatility than asset k, $\underline{\sigma}^l < \underline{\sigma}^k$, then l also has lower market illiquidity,*

$$|\Lambda_1^l| \leq |\Lambda_1^k| \tag{27}$$

if $x_1^k \neq 0$ or $|Z_1^k| \geq |Z_1^l|$.

(iv) (Flight to Quality) *The market liquidity differential between high- and low-fundamental-volatility securities is bigger when speculator funding is tight, that is, $\underline{\sigma}^l < \underline{\sigma}^k$ implies that*

$|\Lambda_1^k|$ *increases more with a negative wealth shock to the speculator,*

$$\frac{\partial|\Lambda_1^l|}{\partial(-\eta_1)} \leq \frac{\partial|\Lambda_1^k|}{\partial(-\eta_1)}\,, \tag{28}$$

if $x_1^k \neq 0$ *or* $|Z_1^k| \geq |Z_1^l|$. *Hence, if* $x_1^k \neq 0$ *or* $|Z_1^k| \geq |Z_1^l|$ *a.s., then*

$$\mathrm{Cov}_0(|\Lambda_1^l|, \phi_1) \leq \mathrm{Cov}_0(|\Lambda_1^k|, \phi_1)\,. \tag{29}$$

Numerical Example, continued. To illustrate these cross-sectional predictions, we extend the numerical example of Section 3 to two securities. The two securities only differ in their long-run fundamental volatility: $\bar{\sigma}^1 = 7.5$ and $\bar{\sigma}^2 = 10$. The other parameters are as before, except that we double W_1 to 1800 since the speculators now trade two securities, the financiers remain uninformed, and we focus on the simpler limited case with $a \to 0$.

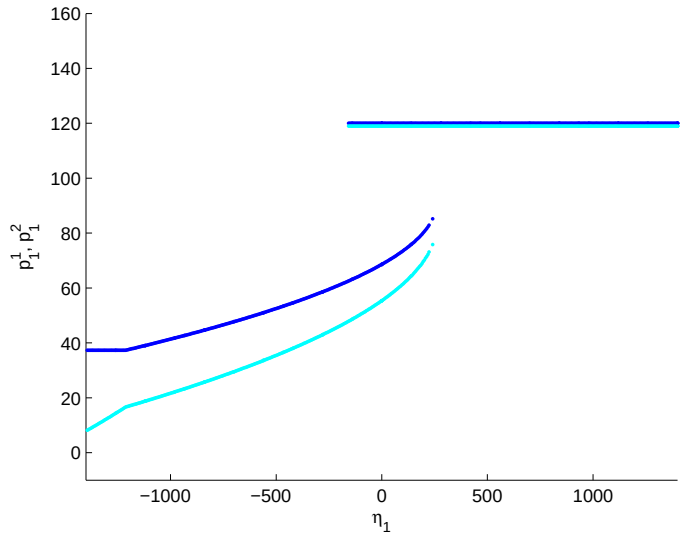

Figure 5: **Flight to Quality and Commonality in Liquidity.** The figure plots the prices p_1^j of assets 1 and 2 as functions of speculators' funding shocks η_1. Asset 1 (darker curve) has lower long-run fundamental risk than asset 2 (lighter curve), $\bar{\sigma}^1 = 7.5 < 10 = \bar{\sigma}^2$.

Figure 5 depicts the assets' equilibrium prices for different values of the funding shock η_1.

First note that as speculator funding tightens and our funding illiquidity measure ϕ_1 rises, the market illiquidity measure $|\Lambda_1^j|$ rises for both assets. Hence, for random η_1, we see our commonality in liquidity result $\text{Cov}_0\left[|\Lambda_1^k|, |\Lambda_1^l|\right] > 0$.

The "commonality in fragility" cannot directly be seen from Figure 5, but it is suggestive that both assets have the same range of η_1 with two equilibrium prices p_1^j. The intuition for this result is the following. Whenever funding is unconstrained, there is perfect market liquidity provision for all assets. If funding is constrained, then it cannot be the case that speculators provide perfect liquidity for one asset but not for the other, since they would always have an incentive to shift funds towards the asset with non-perfect market liquidity. Hence, market illiquidity jumps for both assets at exactly the same funding level.

Our result relating fundamental volatility to market liquidity ("Quality=Liquidity") is reflected in p_1^2 being below p_1^1 for any given funding level. Hence, the high-fundamental-volatility asset 2 is always less liquid than the low-fundamental-volatility asset 1.

The graph also illustrates our result on "flight to quality." To see this, consider the two securities' relative price sensitivity with respect to η_1. For large wealth shocks, market liquidity is perfect for both assets, i.e. $p_1^1 = p_1^2 = v_1^1 = v_1^2 = 120$, so in this high range of funding, market liquidity is insensitive to marginal changes in funding. For sufficiently small funding levels, $\eta_1 < -157$, market illiquidity of both assets increases as η_1 drops since speculators must take smaller stakes in both assets. Importantly, as funding decreases, p_1^2 decreases more steeply than p_1^1, that is, asset 2 is more sensitive to funding declines than asset 1. This is because speculators cut back more on the "funding-intensive" asset 2 with its high margin requirement. The speculators want to maximize their profit per dollar margin, $|\Lambda^j|/m^j$ and therefore $|\Lambda^2|$ must be higher than $|\Lambda^1|$ to compensate speculators for using more capital for margin.

Both price functions exhibit a kink around $\eta = -1210$, because, for sufficiently low funding levels, speculators put all their capital into asset 2. This is because the customers are more eager to sell the more volatile asset 2, leading to more attractive prices for the speculators.

5 Liquidity Risk (Time 0)

We now turn attention to the initial time period $t = 0$ and demonstrate that (i) funding liquidity risk matters even before margin requirements actually bind, (ii) the pricing kernel depends on future funding liquidity, ϕ_{t+1}, (iii) the conditional distribution of prices p_1 is skewed due to the funding constraint (inducing fat tails ex-ante), and (iv) margins m_0 and illiquidity Λ_0 can be positively related due to liquidity risk even if financiers are informed.

The speculators' trading activity at time 0 naturally depends on their expectations about the next period and, in particular, the time 1 illiquidity described in detail above. Further, speculators risk having negative wealth W_1 at time 1, in which case they have utility $\varphi_t W_t$. If speculators have no dis-utility associated with negative wealth levels ($\varphi_t = 0$), then they go to their limit already at time 0 and the analysis is similar to time 1 (see appendix).

We focus on the more realistic case in which the speculators have dis-utility in connection with $W_1 < 0$ and, therefore, choose not trade to their constraint at time $t = 0$ when their wealth is large enough. To understand this, note that while most firms legally have limited liability, the capital W_t in our model refers to pledgable capital allocated to trading. For instance, Lehman Brothers' 2001 Annual Report (page 46) states:

> "The following must be funded with cash capital: Secured funding 'haircuts,' to reflect the estimated value of cash that would be advanced to the Company by counterparties against available inventory, Fixed assets and goodwill, [and] Operational cash ... "

Hence, if Lehman suffers a large loss on its pledgable capital such that $W_t < 0$, then it incurs monetary costs that must be covered with its unpledgable capital like operational cash (which could also hurt Lehman's other businesses). In addition the firm incurs non-monetary cost, like loss in reputation and in goodwill, that reduces its ability to exploit future profitable investment opportunities. To capture these effects, we let a speculator's utility be $\phi_1 W_1$, where ϕ_1 is given by the right-hand side of (14) both for positive and negative values of W_1. With this assumption, equilibrium prices at time $t = 0$ are such that the speculators do not

trade to their constraint at time $t = 0$ when their wealth is large enough. In fact, this is the weakest assumption that curbs the speculators' risk taking since it makes their objective function linear. Higher "bankruptcy costs" would lead to more cautious trading at time 0 and qualitatively similar results.[10]

If the speculator is not constrained at time $t = 0$, then the first-order condition for his position in security j is $E_0[\phi_1(p_1^j - p_0^j)] = 0$. (We leave the case of a constrained time-0 speculator for the appendix.) Consequently, the funding liquidity ϕ_1 determines the pricing kernel $\pi_1/E_0[\phi_1]$ for the cross section of securities:

$$p_0^j = \frac{E_0[\phi_1 p_1^j]}{E_0[\phi_1]} = E_0[p_1^j] + \frac{Cov_0[\phi_1, p_1^j]}{E_0[\phi_1]}. \tag{30}$$

Equation (30) shows that the price at time 0 is the expected time-1 price — which already depends on the liquidity shortage at time 1 — further adjusted for liquidity risk in the form of a covariance term. The liquidity risk term is intuitive: The time-0 price is lower if the covariance is negative, that is, if the security has a low payoff during future funding liquidity crises when ϕ_1 is high.

An illustration of the importance of funding-liquidity management is the "LTCM crisis." The hedge fund Long Term Capital Management (LTCM) had been aware of funding liquidity risk. Indeed, they estimated that in times of severe stress, haircuts on AAA-rated commercial mortgages would increase from 2% to 10%, and similarly for other securities (HBS Case N9-200-007(A)). In response to this, LTCM had negotiated long-term financing with margins fixed for several weeks on many of their collateralized loans. Other firms with similar strategies, however, experienced increased margins. Due to an escalating liquidity spiral, LTCM

[10] We note that risk aversion also limits speculators' trading in the real world. Our model based on margin constraints differs from one driven purely by risk aversion in several ways. E.g. an adverse shock that lowers speculator wealth at $t = 1$ creates a profitable investment opportunity which one might think partially offsets the loss — a natural "dynamic hedge." Because of this dynamic hedge, in a model driven by risk-aversion speculators (with a relative risk aversion coefficient larger than one) increase their $t = 0$ hedging demand, which in turn, lowers illiquidity in $t = 0$. However, exactly the opposite occurs in a setting with capital constraints. Capital constraints prevent speculators from taking advantage of investment opportunities in $t = 1$ so they cannot exploit this "dynamic hedge." Hence, speculators are reluctant to trade away the illiquidity in $t = 0$.

could ultimately not fund its positions in spite of its numerous measures to control funding risk, and it was taken over by 14 banks in September 1998. Another recent example is the funding problems of the hedge fund Amaranth in September 2006, which reportedly ended with losses in excess of $6 billion.

Numerical Example, continued. To better understand funding liquidity risk, we return to our numerical example with one security, $\eta_1 = 0$ and $a \to 0$. We first consider the setting with uninformed financiers and later turn to the case with informed financiers.

Figure 6 depicts the price p_0 and expected time-1 price $E_0[p_1]$ for different initial wealth levels, W_0, for which the speculators' funding constraint is not binding in $t = 0$. The figure shows that even though the speculators are unconstrained at time 0, market liquidity provision is limited with prices below the fundamental value of $E_0[v] = 130$. The price is below the fundamental for two reasons: First, the expected time-1 price is below the fundamental value because of the risk that speculators cannot accommodate the customer selling pressure at that time. Second, p_0 is even below $E_0[p_1]$, since speculators face liquidity risk: Holding the security leads to losses in the states of nature when speculators are constrained and investment opportunities are good, implying that $\text{Cov}[\phi_1, p_1] < 0$. The additional compensation for liquidity risk is $\frac{Cov_0[\phi_1, p_1^j]}{E_0[\phi_1]}$ as seen in Equation (30), which is the difference between the solid line p_1 and the dashed $E_0[p_1]$.[11]

The funding constraint not only affects the price level, it also introduces skewness in the p_1-distribution conditional on the sign of the demand pressure. For $Z_1 > 0$, speculators

[11] Our numerical analysis shows that $\text{Cov}[\phi_1, p_1]$ is non-positive over the whole range of (non-binding) funding levels W_0. The correlation is non-positive since low v_1 realizations typically lead to large price drops that widen the gap $v_1 - p_1$. This increases $\phi_1 = 1 + \frac{v_1 - p_1}{m_1}$, despite the mitigating effect that an increase in $m_1 = \bar{\sigma} + \bar{\theta}|\Delta p_1|$ causes. As a function of W_0 the correlation coefficient is U-shaped and below $-.8$ for $W_0 \in [1000, 2500]$. The U-shaped pattern in speculators wealth is best understood by considering extreme funding levels: for extremely large speculator wealth, the price approaches the fundamental value, and for zero speculator wealth the speculators become unimportant because they don't have the means to move the price. For sufficiently low wealth levels, a more subtle mechanism is at work that explains why the correlation becomes less negative as the speculator wealth declines. High realization of v_1 lead to a less sharp increase in price ($\Delta v_1 > \Delta p_1$), since the price increase is held back by higher margins. This, in turn, increases ϕ_1 when p_1 increases - a positive relationship. While this second mechanism is is at work for high realizations of v_1, ex-ante it is still dominated by the effect driven by low realizations of v_1, and hence $\text{Cov}[\phi_1, p_1]$ is non-positive for all W_0.

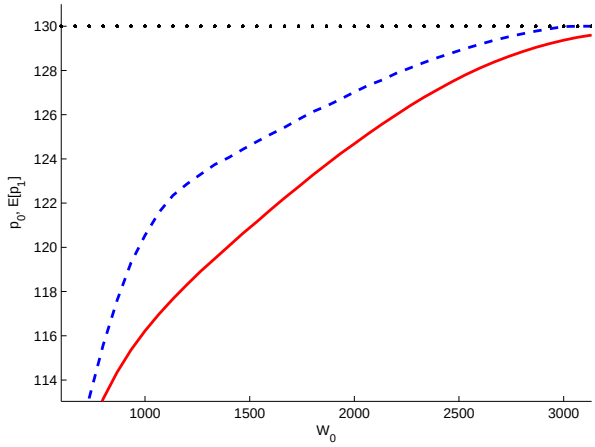

Figure 6: **Illiquidity at Time 0.** This graph shows the price p_0 at time 0 (solid line), the expected time-1 price $E_0[p_1]$ (dashed line), and the fundamental value $E_0[v] = 130$ (dotted line) for different levels of speculator funding W_0. The price p_0 is below the fundamental value due to illiquidity, in particular, because of customer selling pressure and the risk that speculators will hit their capital constraints at time 1, even though speculators are not constrained at time 0 for the depicted wealth levels.

take long positions and, consequently, negative v_1-shocks lead to capital losses with resulting liquidity spirals. This amplification triggers a sharper price drop than the corresponding price increase for positive v_1-shocks. Figure 7 shows this negative skewness for different funding levels W_0. The effect is not monotone — zero dealer wealth implies no skewness, for instance.

For negative realizations of Z_1, customers want to buy (not sell as above), and funding constraints induce a positive skewness in the p_1-distribution. The speculator's return is negatively skewed, as above, since it is still its losses that are amplified. This is consistent with the casual evidence that hedge fund return indexes are negatively skewed. It also suggests that from an ex-ante point of view, i.e., prior to the realization of Z_1, funding constraints lead to higher kurtosis of the price distribution (fat tails).

Finally, we can also show numerically that unlike at time $t = 1$, margins can be positively related to illiquidity at time 0, even when financiers are fully informed.[12] This is because of the liquidity risk between time 0 and time 1. To see this, note that if we reduce the

[12]The simulation results are available upon request from the authors.

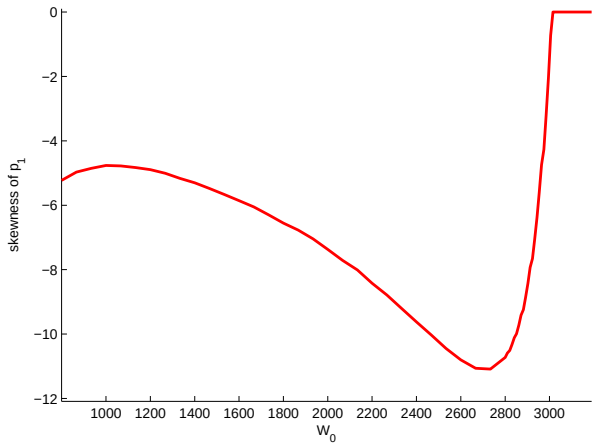

Figure 7: **Conditional Price Skewness.** The figure shows the conditional skewness of p_1 for different funding levels W_0. While the funding constraint is not binding at time 0, it can become binding at time 1, leading to large price drops due to liquidity spirals. Price increases are not amplified, and this asymmetry results in skewness.

speculators' initial wealth W_0, then the market becomes less liquid in the sense that the price is further from the fundamental value. At the same time, the equilibrium price in $t = 1$ is more volatile and thus equilibrium margins at time 0 can actually increase.

6 New Testable Predictions

Our analysis provides a theoretical framework that delivers a unified explanation for a host of stylized empirical facts. Our analysis further suggests a novel line of empirical work that tests the model at a deeper level — namely its prediction that speculator funding is a driving force underlying these market liquidity effects.

First, it would be of interest to empirically study the determinants of margin requirements, e.g. using data from futures markets or from prime brokers. Our model suggests that both fundamental volatility and liquidity-driven volatility affect margins (Propositions 1 and 2). Empirically, fundamental volatility can be captured using price changes over a longer time period, while the sum of fundamental and liquidity-based volatility can be captured by

short-term price changes as in the literature on variance ratios (see, e.g. Campbell, Lo, and MacKinlay (1997)). Our model predicts that, in markets where it is harder for financiers to be informed, margins depend on the total fundamental and liquidity-based volatility. In particular, in times of liquidity crises, margins increase in such markets, and, more generally, margins should co-move with illiquidity in the time series and in the cross section.[13]

Second, our model suggests that an exogenous shock to speculator capital should lead to a reduction in market liquidity (Proposition 4). Hence, a clean test of the model would be to identify exogenous capital shocks such as an unconnected decision to close down a trading desk, a merger leading to reduced total trading capital, or a loss in one market unrelated to the fundamentals of another market, and then study the market liquidity and margin around such events.

Third, the model implies that the effect of speculator capital on market liquidity is highly non-linear: a marginal change in capital has a small effect when speculators are far from their constraints, but a large effect when speculators are close to their constraints — illiquidity can suddenly jump (Propositions 3 and 4).

Fourth, the model suggests that a cause of the commonality in liquidity is that the speculators' shadow cost of capital is a driving state variable. Hence, a measure of speculator capital tightness should help explain the empirical co-movement of market liquidity. Further our result "commonality of fragility" suggests that especially sharp liquidity reductions occur simultaneously across several assets (Proposition 5(i)–(ii)).

Fifth, the model predicts that the sensitivity of margins and market liquidity to speculator capital is larger for securities that are risky and illiquid on average. Hence, the model suggests that a shock to speculator capital would lead to a reduction in market liquidity through a spiral effect that is stronger for illiquid securities (Proposition 5(iv)).

Sixth, speculators are predicted to have negatively skewed returns since, when they hit their constraints, they make significant losses because of the endogenous liquidity spirals,

[13] One must be cautious with the interpretation of the empirical results related to changes in Regulation T since this regulation may not affect speculators but affects the demanders of liquidity, namely the customers.

and, in contrast, their gains are not amplified when prices return to fundamentals. This leads to conditional skewness and unconditional kurtosis of security prices (Section 5).

7 Conclusion

By linking funding and market liquidity this paper provides a unified framework that explains the following stylized facts: (i) Liquidity suddenly dries up; we argue that fragility in liquidity is in part due to destabilizing margins which arise when financiers are imperfectly informed the fundamentals follow an ARCH process. (ii) Market liquidity and fragility co-moves across assets since changes in funding conditions affects speculators' market liquidity provision of all assets. (iii) Market liquidity is correlated with volatility, since trading more volatile assets requires higher margin payments and speculators provide market liquidity across assets such that illiquidity per capital use, i.e. illiquidity per dollar margin, is constant. (iv) Flight to quality phenomena arise in our framework since when funding becomes scarce speculators cut back on the market liquidity provision especially for capital intensive, i.e. high margin, assets. (v) Market liquidity moves with the market since funding conditions do. In addition to explaining these stylized facts, the model also makes a number of more specific, testable predictions that could inspire further empirical research on margins.

Our analysis also suggests policy implications for central banks. Central banks can help mitigate market liquidity problems by controlling funding liquidity. If a central bank is better than the typical financiers of speculators at distinguishing liquidity shocks from fundamental shocks, then the central bank can convey this information and urge financiers to relax their funding requirements — as the Federal Reserve Bank of New York did during the 1987 stock market crash. Central banks can also improve market liquidity by boosting speculator funding conditions during a liquidity crisis, or by simply stating the intention to provide extra funding during times of crisis, which would loosen margin requirements immediately as financiers' worst-case scenarios improve.

A Appendix: Real World Margin Constraints

A central element of our paper is the capital constraints that the main providers of market liquidity face. To set the stage for our model, we review these institutional features for securities firms such as hedge funds, banks' proprietary trading desks, and market makers. (Readers mainly interested in our theory can skip to Section 1.)

A.1 Funding Requirements for Hedge Funds

We first consider the funding issues faced by hedge funds since they have relatively simple balance sheets and face little regulation. A hedge fund's capital consists of its equity capital supplied by the partners, and possible long-term debt financing that can be relied upon during a potential funding crisis. Since a hedge fund is a partnership, the equity is not locked into the firm indefinitely, as in a corporation. The investors (that is, the partners) can withdraw their capital at certain times, but — to ensure funding — the withdrawal is subject to initial lock-up periods and general redemption notice periods before specific redemption dates (typically at least a month, often several months or even years). A hedge fund usually does not issue long-term unsecured bonds, but some (large) hedge funds manage to obtain debt financing in the form of medium-term bank loans or in the form of a guaranteed line of credit.[14] Recently, some hedge funds have even raised capital by issuing bonds or permanent equity (e.g., see The Economist 1/27/2007, page 75).

The main source of leverage for hedge funds is collateralized borrowing financed by the hedge fund's prime broker(s). The prime brokerage business is opaque since the terms of the financing are subject to negotiation and are hidden to outsiders. We describe stylized financing terms and, later, we discuss caveats.

If a hedge fund buys at time t a long position of $x_t^j > 0$ shares of a security j at price p_t^j, it has to come up with $x_t^j p_t^j$ dollars. The security can, however, be used as collateral for a

[14] A line of credit may have a "material adverse change" clause or other covenants subject to discretionary interpretation of the lender. Such covenants imply that the line of credit may not be a reliable source of funding during a crisis.

new loan of, say, l_t^j dollars. The difference between the price of the security and the collateral value is denoted as the margin requirement $m_t^{j+} = p_t^j - l_t^j$. Hence, this position uses $x_t^j m_t^{j+}$ dollars of the fund's capital. The collateralized funding implies that the capital use depends on margins, not notional amounts.

The margins on fixed income securities and over-the-counter (OTC) derivatives are set through a negotiation between the hedge fund and the prime broker that finances the trade. The margins are typically set so as to make the loan almost risk free for the broker, that is, such that it covers the largest possible adverse price move with a certain degree of confidence (i.e., it covers the Value-at-Risk).[15]

If the hedge fund wants to sell short a security, $x^j < 0$, then the fund asks one of its brokers to locate a security that can be borrowed, and then the fund sells the borrowed security. Duffie, Gârleanu, and Pedersen (2002) describe in detail the institutional arrangements of shorting. The broker keeps the proceeds of the short sale to be able to repurchase the security if the hedge fund fails and, additionally, requires that the hedge fund posts a margin m_t^{j-} that covers the largest possible adverse price move with a certain degree of confidence.

In the US, margins on equities are subject to Regulation T, which stipulates that non-brokers/dealers must have an initial margin (downpayment) of 50% of the market value of the underlying stock, both for new long and short positions. Hedge funds can, however, circumvent Regulation T by, for instance, organizing the transaction as a total return swap, which is a derivative that is functionally equivalent to buying the stock.

The margin on exchange traded futures (or options) is set by the exchange. The principle for setting the margin for futures or options is the same as that described above. The margin is set so as to make the exchange almost immune to the default risk of the counterparty, and hence riskier contracts have larger margins.

A hedge fund must finance all of its positions, that is, the sum of all the margin require-

[15] An explicit equation for the margin is given by (6) in Section 1. Often brokers also take into account the delay between the time a failure by the hedge fund is noticed, and the time the security is actually sold. Hence, the margin of a one-day collateralized loan depends on the estimated risk of holding the asset over a time period that is often set as five to ten days.

ments on long and short positions cannot exceed the hedge fund's capital. In our model, this is captured by the key Equation (4) in Section 1.

At the end of the financing period, time $t+1$, the position is "marked-to-market," which means that the hedge fund receives any gains (or pays any losses) that have occurred between t and $t+1$, that is, the fund receives $x_t^j(p_{t+1}^j - p_t^j)$ and pays interest on the loan at the funding rate. If the trade is kept on, the broker keeps the margin to protect against losses going forward from time $t+1$. The margin can be adjusted if the risk of the collateral has changed, unless the counterparties have contractually fixed the margin for a certain period. Stock exchanges and self-regulatory organizations (e.g., NASD) also impose maintenance/continuation margins for existing stock positions. For example, the NYSE and the NASD require that investors maintain a minimum margin of 25% for long stock positions and 30% for short stock positions.

Instead of posting risk-free assets (cash), a hedge fund can also post other risky assets, say asset k, to cover its margin on position, say x^j. However, in this case a "haircut," h_t^k, is subtracted from asset k's market value to account for the riskiness of the collateral. The funding constraint becomes $x_t^j m_t^j \leq W_t - x_t^k h_t^k$. Moving the haircut term to the left-hand side reveals that the haircut is equivalent to a margin, since the hedge fund could alternatively have used the risky security to raise cash and then used this cash to cover the margins for asset j. We therefore use the terms margins and haircuts interchangeably.

We have described how funding constraints work when margins and haircuts are set separately for each security position. It is, however, sometimes possible to "cross-margin", i.e. to jointly finance several trades that are part of the same strategy. This leads to a lower total margin if the risks of the various positions are partially offsetting. For instance, much of the interest rate risk is eliminated in a "spread trade" with a long position in one bond and a short position in a similar bond. Hence, the margin/haircut of a jointly financed spread trade is smaller than the sum of the margins of the long and short bonds. For a strategy that is financed jointly, we can reinterpret security j as such a strategy. Prime brokers compete by,

among other things, offering low margins and haircuts — a key consideration for hedge funds — which means that it is becoming increasingly easy to finance more and more strategies jointly. In the extreme, one can imagine a joint financing of a hedge fund's total position such that the "portfolio margin" would be equal to the maximum portfolio loss with a certain confidence level. Currently, it is often not practical to jointly finance a large portfolio. This is because a large hedge fund finances its trades using several brokers; both a hedge fund and a broker can consist of several legal entities (possibly located in different jurisdictions); certain trades need separate margins paid to exchanges (e.g., futures and options) or to other counterparties of the prime broker (e.g., securities lenders); prime brokers may not have sufficiently sophisticated models to evaluate the diversification benefits (e.g., because they do not have enough data on the historical performance of newer products such as CDOs); and because of other practical difficulties in providing joint financing. Further, if the margin requirement relies on assumed stress scenarios in which the securities are perfectly correlated (e.g., due to predatory trading, as in Brunnermeier and Pedersen (2005)), then the portfolio margin constraint coincides with position-by-position margins.

A.2 Funding Requirements for Banks

A bank's capital consists of equity capital plus its long-term borrowings (including credit lines secured from commercial banks, alone or in syndicates), reduced by assets that cannot be readily employed (e.g., goodwill, intangible assets, property, equipment, and capital needed for daily operations), and further reduced by uncollateralized loans extended by the bank to others (see e.g., Goldman Sachs' 2003 Annual Report). Banks also raise money using short-term uncollateralized loans such as commercial paper and promissory notes, and, in the case of commercial banks, demand deposits. These sources of financing cannot, however, be relied on in times of funding crisis since lenders may be unwilling to continue lending, and therefore this short-term funding is often not included in measures of capital.

The financing of a bank's trading activity is largely based on *collateralized* borrowing.

Banks can finance long positions using collateralized borrowing from corporations, other banks, insurance companies, and the Federal Reserve Bank, and can borrow securities to shortsell from, for instance, mutual funds and pension funds. These transactions typically require margins which must be financed by the bank's capital as captured by the funding constraint (4).

The financing of a bank's proprietary trading is more complicated than that of a hedge fund, however. For instance, banks may negotiate zero margins with certain counterparties, and banks can often sell short shares held in-house, that is, held in a customer's margin account (in "street name") such that the bank does not need to use capital to borrow the shares externally. Further, a bank receives margins when financing hedge funds (i.e., the margin is negative from the point of view of the bank). However, often the bank wants to pass on the trade to an exchange or another counterparty and hence has to pay a margin to the exchange. In spite of these caveats, we believe that in times of stress, banks face margin requirements and are ultimately subject to a funding constraint in the spirit of (4). For instance, Goldman Sachs' 2003 Annual Report (page 62) states that it seeks to maintain net capital in excess of total margins and haircuts that it would face in periods of market stress plus the total draws on unfunded commitments at such times. In addition, Goldman Sachs recognizes that it may not have access to short-term borrowing during a crisis, that margins and haircuts may increase during such a crisis, and that counterparties may withdraw funds at such times.

Banks must also satisfy certain regulatory requirements. Commercial banks are subject to the Basel accord, supervised by the Federal Reserve system for US banks. In short, the Basel accord of 1988 requires that a bank's "eligible capital" exceeds 8% of the "risk-weighted asset holdings," which is the sum of each asset holding multiplied by its risk weight. The risk weight is 0% for cash and government securities, 50% for mortgage-backed loans, and 100% for all other assets. The requirement posed by the 1988 Basel accord corresponds to Equation (4) with margins of 0%, 4%, and 8%, respectively. In 1996, the accord was

amended, allowing banks to measure market risk using an internal model based on portfolio VaRs rather than using standardized risk weights.

US broker-speculators, including banks acting as such, are subject to the Securities and Exchange Commission's (SEC)'s "net capital rule" (SEC Rule 15c3-1). This rule stipulates, among other things, that a broker must have a minimum "net capital," which is defined as equity capital plus approved subordinate liabilities minus "securities haircuts" and operational charges. The haircuts are set as security-dependent percentages of the market value. The standard rule requires that the net capital exceeds at least $6\frac{2}{3}\%$ (15:1 leverage) of aggregate indebtedness (broker's total money liabilities) or alternatively 2% of aggregate debit items arising from customer transactions. This constraint is similar in spirit to (4).[16] As of August 20, 2004, SEC amended the net capital rule for Consolidated Supervised Entities (CSE)'s such that CSE's may, under certain circumstances, use their internal risk models to determine whether they fulfill their capital requirement (SEC Release No. 34-49830).

A.3 Funding Requirements for Market Makers

There are various types of market-making firms. Some are small partnerships, whereas others are parts of large investment banks. The small firms are financed in a similar way to hedge funds in that they rely primarily on collateralized financing; the funding of banks was described in Section A.2.

Certain market makers, such as NYSE specialists, have an obligation to make a market and a binding funding constraint means that they cannot fulfill this requirement. Hence, avoiding the funding constraint is especially crucial for such market makers.

Market makers are in principle subject to the SEC's net capital rule (described in Section A.2), but this rule has special exceptions for market makers. Hence, market makers' main regulatory requirements are those imposed by the exchange on which they operate. These constraints are often similar in spirit to (4).

[16] Let L be the lower of $6\frac{2}{3}\%$ of total indebtedness or 2% of debit items and h^j the haircut for security j; then the rule requires that $L \leq W - \sum_j h^j x^j$, that is, $\sum_j h^j x^j \leq W - L$.

B Appendix: Proofs

Proof of Propositions 1 and 2. These results follow from the calculations in the text. ■

Proof of Proposition 3. We prove the proposition for $Z_1 > 0$, implying $p_1 \leq v_1$ and $x_1 \geq 0$. The complementary case is analogous. To see how the equilibrium depends on the exogenous shocks, we first combine the equilibrium condition $x_1 = -\sum_{k=0}^{1} y_1^k$ with the speculator funding constraint to get

$$m_1^+ \left(Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1) \right) \leq b_0 + p_1 x_0 + \eta_1 \tag{31}$$

that is,

$$G(p_1) := m_1^+ \left(Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1) \right) - p_1 x_0 - b_0 \leq \eta_1 \tag{32}$$

For η_1 large enough, this inequality is satisfied for $p_1 = v_1$, that is, it is a stable equilibrium that the market is perfectly liquid. For η_1 low enough, the inequality is violated for $p_1 = \frac{2v_1}{\gamma(\sigma_2)^2} - Z_1$, that is, it is an equilibrium that the speculator is in default. We are interested in intermediate values of η_1. If the left-hand side G of (32) is increasing in p_1 then p_1 is a continuously increasing function of η_1, implying no fragility with respect to η_1.

Fragility arises if G can be decreasing in p_1. Intuitively, this expression measures speculator funding needs at the equilibrium position, and fragility arises if the funding need is greater when prices are lower, that is, further from fundamentals. (This can be shown to be equivalent to a non-monotonic excess demand function.)

When the financier is informed, the left-hand side G of (32) is

$$(\bar{\sigma} + \bar{\theta}|\Delta v_1| + p_1 - v_1) \left(Z_1 + \frac{2}{\gamma(\sigma_2)^2}(p_1 - v_1) \right) - p_1 x_0 - b_0 \tag{33}$$

The first product is a product of two positive increasing functions of p_1, but the second term,

$-p_0x_0$, is decreasing in p_1 if $x_0 > 0$. Since the first term does not depend on x_0, there exists $\underline{x}$ such that, for $x_0 > \underline{x}$, the whole expression is decreasing.

When the financier is uninformed, we first show that there is fragility for $a = 0$. In this case, the left-hand side of (32) is

$$G^0(p_1) := (\bar{\sigma} + \bar{\theta}|\Delta p_1|)\left(Z_1 + \frac{2}{\gamma(\sigma_2)^2}(p_1 - v_1)\right) - p_1x_0 - b_0 \tag{34}$$

When $p_1 < p_0$, $\bar{\theta}|\Delta p_1| = \theta(p_0 - p_1)$ decreases in p_1 and, if $\bar{\theta}$ is large enough, this can make the entire expression decreasing. (Since $\bar{\theta}$ is proportional to θ, this clearly translates directly to θ.) Also, the expression is decreasing if x_0 is large enough.

Finally, on any compact set of prices, the margin function converges uniformly to (22) as a approaches 0. Hence, G converges uniformly to G^0. Since the limit function G^0 has a decreasing part, choose $p_1^a < p_1^b$ such that $\varepsilon := G^0(p_1^a) - G^0(p_1^b) > 0$. By uniform convergence, choose $\underline{a} > 0$ such that for $a < \underline{a}$, G differs from G^0 by at most $\varepsilon/3$. Then we have

$$\begin{aligned} G(p_1^a) - G(p_1^b) &= G^0(p_1^a) - G^0(p_1^b) + [G(p_1^a) - G^0(p_1^a)] - [G(p_1^b) - G^0(p_1^b)] && (35)\\ &\geq \varepsilon - \frac{\varepsilon}{3} - \frac{\varepsilon}{3} = \frac{\varepsilon}{3} > 0 && (36) \end{aligned}$$

which proves that G has a decreasing part.

It can be shown that the price cannot be chosen continuously in η_1 when the left-hand side of (32) can be decreasing. ■

Proof of Proposition 4. When the funding constraint binds, we use the implicit function theorem to compute the derivatives. With y_1 is given by (12), the equilibrium condition $x_1 = -y_1$, the fact that the speculator funding constraint binds in an illiquid equilibrium, and that $v_1 - p_1 > 0$ when $Z_1 > 0$, we have

$$m_1^+\left(Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1)\right) = b_0 + p_1x_0 + \eta_1\,. \tag{37}$$

We differentiate this expression to get

$$\frac{\partial m_1^+}{\partial p_1}\frac{\partial p_1}{\partial \eta_1}\left(Z_1 - \frac{2}{\gamma(\sigma_2)^2}(v_1 - p_1)\right) + m_1^+ \frac{2}{\gamma(\sigma_2)^2}\frac{\partial p_1}{\partial \eta_1} = \frac{\partial p_1}{\partial \eta_1}x_0 + 1\,, \tag{38}$$

which leads to Equation (23) after rearranging. The case of $Z_1 < 0$ (i.e., Equation (24)) is analogous.

Finally, spiral effects happen if one of the last two terms in the denominator of the right-hand side of Equations (23) and (24) is negative. (The total value of the denominator is positive by definition of a stable equilibrium.) When the speculator is informed, $\frac{\partial m_1^+}{\partial p_1} = 1$ and $\frac{\partial m_1^-}{\partial p_1} = -1$ using Proposition 1. Hence, in this case margins are stabilizing.

If the speculators are uninformed and a approaches 0, then using Proposition 2 we find that $\frac{\partial m_1^+}{\partial p_1} = \frac{\partial m_1^+}{\partial \Lambda_1}$ approaches $-\bar{\theta} < 0$ for $v_1 - v_0 + \Lambda_1 - \Lambda_0 < 0$ and $\frac{\partial m_1^-}{\partial p_1} = \frac{\partial m_1^-}{\partial \Lambda_1}$ approaches $\bar{\theta} > 0$ for $v_1 - v_0 + \Lambda_1 - \Lambda_0 > 0$. This means that there is a margin spiral with positive probability. The case of a loss spiral is immediately seen to depend on the sign on x_0. ■

Proof of Proposition 5. We first consider the equation that characterizes a constrained equilibrium. When there is selling pressure from customers, $Z_1^j > 0$, it holds that

$$|\Lambda_1^j| = -\Lambda_1^j = v_1^j - p_1^j = \min\{\, \phi_1 m_1^{j+}\,,\, \frac{\gamma(\sigma_2^j)^2}{2} Z_1^j \,\}\,, \tag{39}$$

and if customers are buying, $Z_1^j < 0$, we have

$$|\Lambda_1^j| = \Lambda_1^j = p_1^j - v_1^j = \min\{\, \phi_1 m_1^{j-}\,,\, \frac{\gamma(\sigma_2^j)^2}{2}(-Z_1^j) \,\}\,. \tag{40}$$

We insert the equilibrium condition $x_1^j = -\sum_k y_1^{j,k}$ and Equation (12) for $y_1^{j,k}$ into the

speculators' funding condition to get

$$\sum_{Z_1^j > \frac{2\phi_1 m_1^{j+}}{\gamma(\sigma_2^j)^2}} m_1^{j+}\left(Z_1^j - \frac{2\phi_1 m_1^{j+}}{\gamma(\sigma_2^j)^2}\right) + \sum_{-Z_1^j > \frac{2\phi_1 m_1^{j-}}{\gamma(\sigma_2^j)^2}} m_1^{j-}\left(-Z_1^j - \frac{2\phi_1 m_1^{j-}}{\gamma(\sigma_2^j)^2}\right) = \sum_j x_0^j p_1^j + b_0 + \eta_1 \tag{41}$$

where the margins are evaluated at the prices solving (39)–(40). When ϕ_1 approaches infinity, the left-hand side of (41) becomes zero, and when ϕ_1 approaches zero, the left-hand side approaches the capital needed to make the market perfectly liquid. As in the case of one security, there can be multiple equilibria and fragility (Proposition 3). On a stable equilibrium branch, ϕ_1 increases as η_1 decreases.

Of course, the equilibrium shadow cost of capital ϕ_1 is random since $\eta_1, \Delta v_1^1, \ldots, \Delta v_1^J$ are random. To see the commonality in liquidity, we note that $|\Lambda^j|$ is increasing in ϕ_1 for each $j = k, l$. To see this, consider first the case $Z_1^j > 0$. When the financiers are uninformed, $a = 0$, and $\theta^j = 0$, then $m_1^{j+} = \bar{\sigma}^k$, and, therefore, Equation (39) shows directly that $|\Lambda_1^j|$ increases in ϕ_1 (since the minimum of increasing functions is increasing). When financiers are informed and $\theta^j = 0$ then $m_1^{j+} = \bar{\sigma}^k + \Lambda_1^j$, and, therefore, Equation (39) can be solved to be $|\Lambda_1^j| = \min\{\frac{\phi_1}{1+\phi_1}\bar{\sigma}^j, \frac{\gamma(\sigma_2^k)^2}{2} Z_1^k\}$, which increases in ϕ_1. Similarly, Equation (40) shows that $|\Lambda^j|$ is increasing in ϕ_1 when $Z_1^j < 0$.

Now, since $|\Lambda^j|$ is increasing in ϕ_1 and does not depend on other state variables under these conditions, $\mathrm{Cov}\left(|\Lambda^k(\phi)|, |\Lambda^l(\phi)|\right) > 0$ because any two functions which are both increasing in the same random variable are positively correlated (the proof of this is similar to that of Lemma 1 below). Since $|\Lambda^j|$ is bounded, we can use dominated convergence to establish the existence of $c > 0$ such that part (i) of the proposition applies for any $\theta^j, a < c$.

To see part (ii) of the proposition, note that, for all j, $|\Lambda^j|$ is a continuous function of ϕ_1, which is locally insensitive to ϕ_1 if and only if the speculator is not marginal on security j (i.e., if the second term in Equation (39) or (40) attains the minimum). Hence, $|\Lambda^j|$ jumps if and only if ϕ_1 jumps.

To see part (iii), we write illiquidity using Equations (39)–(40) as

$$|\Lambda_1^j| = \min\{\, \phi_1 m_1^{j,sign(Z_1^j)} \,,\, \frac{\gamma(\sigma_2^j)^2}{2}|Z_1^j|\,\}\,. \tag{42}$$

Hence, using the expression for the margin, if the financier is uninformed and $\theta^j = a = 0$, then

$$|\Lambda_1^j| = \min\{\, \phi_1 \bar{\sigma}_1^j \,,\, \frac{\gamma(\sigma_2^j)^2}{2}|Z_1^j|\,\} \tag{43}$$

and, if the financier is informed and $\theta^j = 0$, then

$$|\Lambda_1^j| = \min\{\, \frac{\phi_1}{1+\phi_1}\,\bar{\sigma}_1^j \,,\, \frac{\gamma(\sigma_2^j)^2}{2}|Z_1^j|\,\}\,. \tag{44}$$

In the case of an uninformed financier as in (43), we see that, if $x_1^k \neq 0$,

$$|\Lambda_1^k| = \phi_1 \bar{\sigma}_1^k > \phi_1 \bar{\sigma}_1^l \geq |\Lambda_1^l| \tag{45}$$

and, if $|Z_1^k| \geq |Z_1^l|$,

$$|\Lambda_1^k| = \min\{\, \phi_1 \bar{\sigma}_1^k \,,\, \frac{\gamma(\sigma_2^k)^2}{2}|Z_1^k|\,\} > \min\{\, \phi_1 \bar{\sigma}_1^l \,,\, \frac{\gamma(\sigma_2^l)^2}{2}|Z_1^l|\,\} = |\Lambda_1^l|\,. \tag{46}$$

Since Λ^k and Λ^l converge to these values as θ^j, a approach zero, we can choose c so that inequality holds for θ^j, a below c. With an informed financier, it is seen that $|\Lambda_1^k| \geq |\Lambda_1^l|$ using similar arguments.

For part (iv) of the proposition, we use that

$$\frac{\partial|\Lambda_1^j|}{\partial(-\eta_1)} = \frac{\partial|\Lambda_1^j|}{\partial\phi_1}\frac{\partial\phi_1}{\partial(-\eta_1)} \tag{47}$$

Further, $\frac{\partial\phi_1}{\partial(-\eta_1)} \geq 0$ and, from Equations (43)–(44), we see that $\frac{\partial|\Lambda_1^k|}{\partial\phi_1} \geq \frac{\partial|\Lambda_1^l|}{\partial\phi_1}$. The result that $\mathrm{Cov}(\Lambda^k, \phi) \geq \mathrm{Cov}(\Lambda^l, \phi)$ now follows from Lemma 1 below. ■

Lemma 1 *Let X be a random variable and g_i, $i = 1, 2$, be weakly increasing functions X,*

where g_1 has a larger derivative than g_2, that is, $g_1'(x) \geq g_2'(x)$ for all x and $g_1'(x) > g_2'(x)$ on a set with non-zero measure. Then,

$$\text{Cov}[X, g_1(X)] > Cov[X, g_2(X)] \tag{48}$$

Proof. For $i = 1, 2$ we have

$$\begin{aligned} \text{Cov}[X, g_i(X)] &= E\left[(X - E[X])g_i(X)\right] &(49)\\ &= E\left[(X - E[X])(\int_{E[X]}^{X} g_i'(y)dy)\right]. &(50) \end{aligned}$$

The latter expression is a product of two terms that always have the same sign. Hence, this is higher if g_i' is larger. ■

Liquidity Risk (Time 0). Section 5 focuses on the case of speculators who are unconstrained at $t = 0$. When a speculator's problem is linear and he is constrained at time 0, then he invests only in securities with the highest expected profit per capital use, where profit is calculated using the pricing kernel $\phi_1^i / E_0[\phi_1^i]$. In this case, his time-0 shadow cost of capital is

$$\phi_0^i = E_0[\phi_1^i]\left(1 + \max_j \left\{ \frac{E_0[\frac{\phi_1^i}{E_0[\phi_1^i]} p_1^j] - p_0^j}{m_0^+}, -\frac{E_0[\frac{\phi_1^i}{E_0[\phi_1^i]} p_1^j] - p_0^j}{m_0^-} \right\}\right) \tag{51}$$

■

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