Clock Games: Theory and Experiments

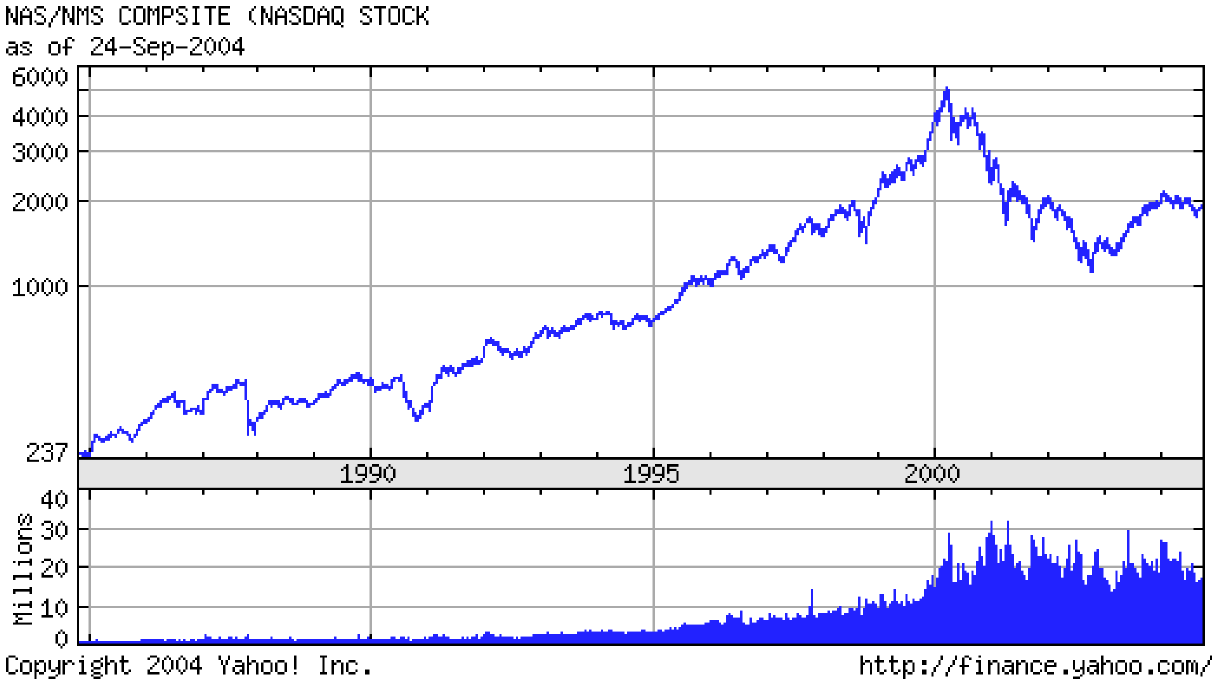
- Markus K. Brunnermeier
  Princeton University
- John Morgan
  UC Berkeley

Timing is crucial - 1

- A firm contemplates a new product introduction for some high tech product
- Waiting reduces the costs of production and thereby increases profits
- However, waiting too long risks entry by a rival
- When should the new product be introduced?

Timing is crucial - 2

- It’s 1 January 2000 and tech stocks are zooming up...



Timing is crucial - 3

- Mobutu has long been in power in (then) Zaire. Should you lead a revolution against him?
- Move too soon, a Mobutu will “deal” with you.
- Move too late the and the power vacuum will be filled.

Timing is crucial - 4

- Common features of many timing problems
  - time has to be *“ripe”*
  - *Congestion* effect: there is only room for *K* players
  - *Waiting* motive: first movers risk more
  - Uncertainty about rivals’ moves
- Examples
  - Currency attacks
  - Debt renegotiation

Timing is crucial - 5

- Differences
  - Few key players – few cohorts – many players
  - Rivals’ moves are difficult/easy to predict
  - Rivals’ moves are observable/unobservable
- Observations
  - Initial delay
  - Sudden onset of action
- Objectives of paper
  - Provide tractable model
  - Experimentally verify predictions given complexity of the game

Some related literature

- Theory on timing games
  - Pre-emption games
  - War of attrition games
  - Recent papers
    - Park & Smith (2003)
    - Morris (1995)
    - AB (2002,2003)
- Experimental literature
  - McKelvey and Palfrey (1992)

Model Setup

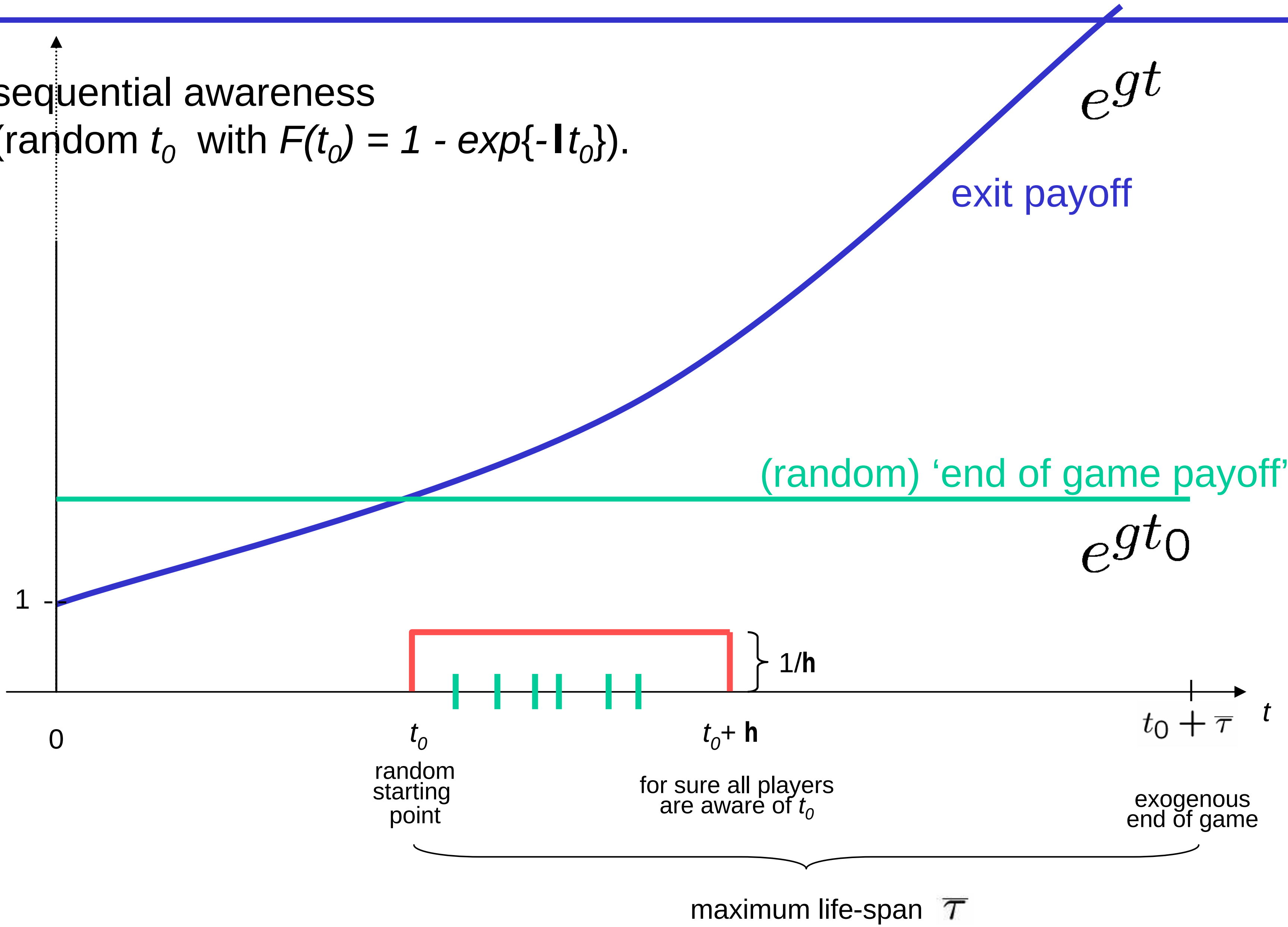
- *I* players each deciding when to "move"
- Players receive private signals about a state relevant variable $\rightarrow$ player *i*'s clock starts at t_i
  - Key tension: When you learn about the change in the state variable, don't know how many others have already learned this.
- Game ends when a critical number, *K*, of the players exit.

Currency Attack Example

- There are *I* key players in the market
- Each learns about a payoff relevant event: That there have been significant outflows in the reserves of some country.
  - But each is unsure of the timing that others have learned
- Upside from staying in: Enjoy supernormal profits from domestic exchange rate
- Downside: Once there outflows by enough (*K*) key players, a devaluation will occur.

Model setup

- sequential awareness
  (random t_0 with $F(t_0) = 1 - exp\{-\lambda t_0\}$).



Payoff structure

- Payoffs
  - 'exit payoff' (random) for first K players
  - 'end of game payoff' for last I-K players

  - Tie-breaking rule
    if K^{th}, $K+1^{th}$, …, $K+n^{th}$ player exit at the same time $t > t_0$, exiting players receive the exit payoff with equal probability.

introduction

theory

- model setup
- **unobservable actions**
- observable actions
- information clustering

experiment

conclusion

Costs and benefits of delay

- Log Marginal Costs: Hazard rate associated with the end of the game x expected payoff drop from this event
- Log Marginal Benefits: The growth rate from waiting an additional period
- In equilibrium we have the usual MC = MB condition for each of the players.

Delay - unobserved actions …

At $t = t_i + \tau$

$$(1-\Delta h)\underbrace{g e^{g(t_i+\tau)}\Delta}_{\text{benefit of waiting}} = \Delta h \underbrace{E\left[e^{g(t_i+\tau)} - e^{g t_0} \middle| D_\Delta, t_i\right]}_{\text{benefit of exiting}}$$

benefit of waiting

benefit of exiting

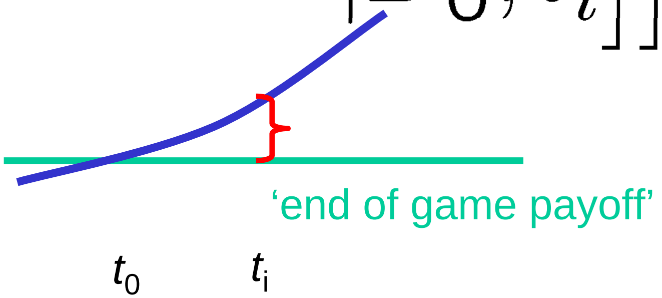
= size of payoff drop

For $\Delta \to 0$

$$g = hE\left[1 - e^{-g(t_i+\tau-t_0)} \middle| D_0, t_i\right]$$

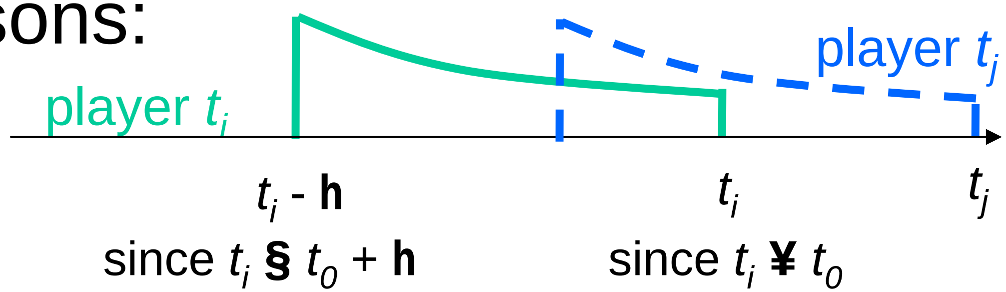
Solving for τ

$$\tau = \frac{1}{g}\left[\ln\frac{h}{h-g} + \ln E\left[e^{-g(t_i-t_0)} \middle| D_0, t_i\right]\right]$$



Equilibrium hazard rate – unobserved actions

- If everybody waits for τ periods, then at $t_i + \tau$
  Prob(payoff drop at $t_i + \tau + \Delta$) =
  =Prob(K^{th} of *others* received signal before $t_i + \Delta$)
- Random for **two** reasons:
  - t_0 is random



  - Timing of K^{th} signal within window of awareness is random

$$\binom{I-1}{1} \pi\left(\Delta|t_0\right) \binom{I-2}{K-1} \left[\Pi\left(\Delta|t_0\right)\right]^{K-1} \left[1-\Pi\left(\Delta|t_0\right)\right]^{I-1-K}$$

- Condition on fact that payoff drop did not occur

… Delay – unobservable actions

Proposition 1: In unique symmetric equilibrium

delay $\tau = \frac{1}{g} \ln \left(\frac{\lambda F(K,I,\eta(\lambda - g))}{g - (g-\lambda) F(K,I,\eta\lambda)} \right)$,

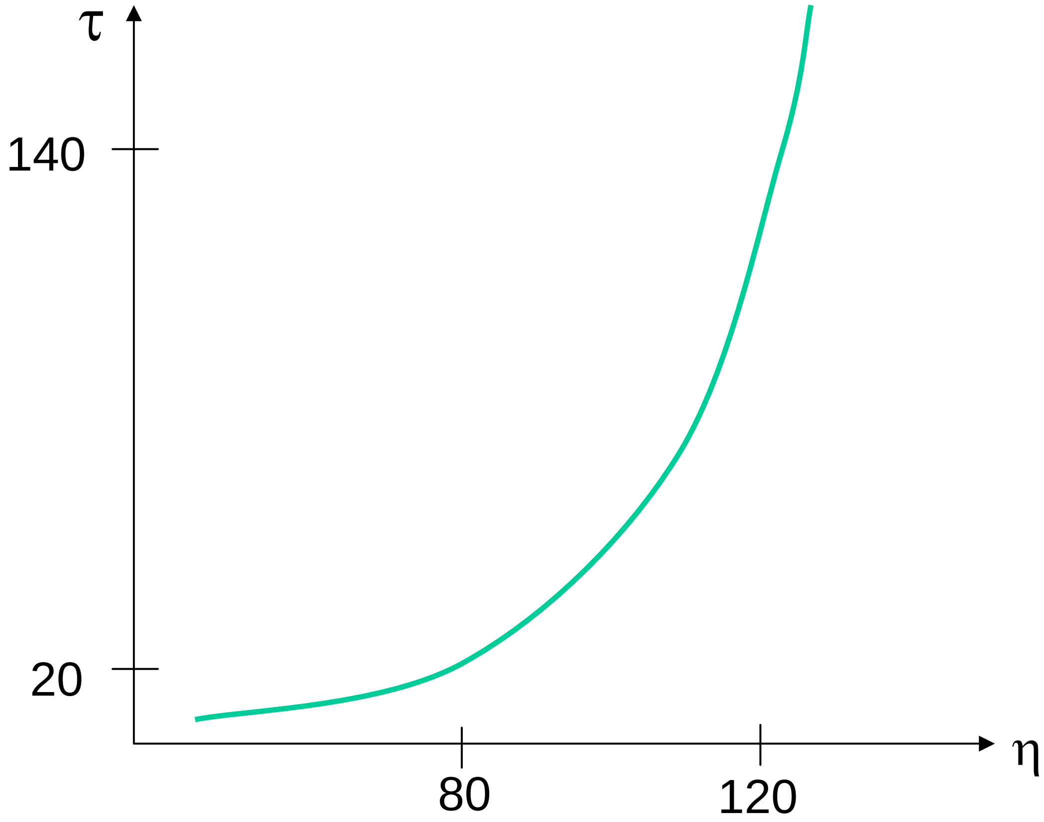
where $F(a, b, x)$ is a Kummer function.

Integral representation

$$\frac{\Gamma(b)}{\Gamma(b-a)\Gamma(a)} \int_0^1 e^{xz} z^{a-1} (1-z)^{b-a-1} dz$$

Delay increases with window of awareness

Proposition 2: Delay increases with window of η.



Why?

Makes it more difficult to predict moves of others.

Herding – observable actions

Proposition 3: All players exit after observing the first player exiting (if first player exits in equilibrium after receiving the signal).

Intuition: backwards induction

Delay of first player – observable action

$$g = h_1 \frac{I-K}{I-1} E\left[1 - e^{-g(t_i+\tau-t_0)} | D_0, t_i\right]$$

h_1 = hazard rate of the first player exiting

$\frac{I-K}{I-1}$ = probability of *not* receiving the high exit payoff when herding after first

$$\tau_1 = \frac{1}{g}\left[\ln \frac{h_1}{h_1 - g\frac{I-1}{I-K}} + \ln E\left[e^{-g(t_i-t_0)} | D_0, t_i\right]\right]$$

Simplifies to a ratio of Kummer functions

introduction

theory

model setup

unobservable actions

observable actions

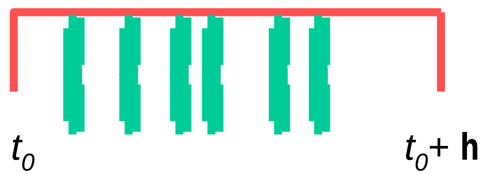
Information clustering

experiment

conclusion

Information Clustering

- Our model - CC model - AB model



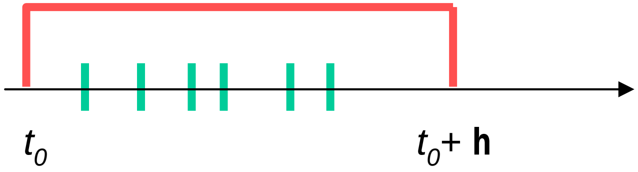
I players

continuum of players
in *I* groups

continuum of players
no information clustering

Comparison to AB (unobservable)

- Proposition 6: $\tau > \tau_{AB}$.
  - 2 effects:
    - Individual player carries more weight (focus of CC-model)
    - Synchronization is more complicated
      - In AB: hazard rate > prob. of being "Kth" player conditional on knowing to "Kth" player, t_i knows that next player exits an instant later with probability 1 and causes payoff drop.
      - In BM: hazard rate > prob. of being "Kth"
        ***** prob. K+1th follows in next instant.



- Proposition 7: Fix $K=\kappa I$. As $I \to \infty$, $\tau \to \tau_{AB}$.
  - (Kummer functions converge to exponentials.)

Comparison with AB (observable)

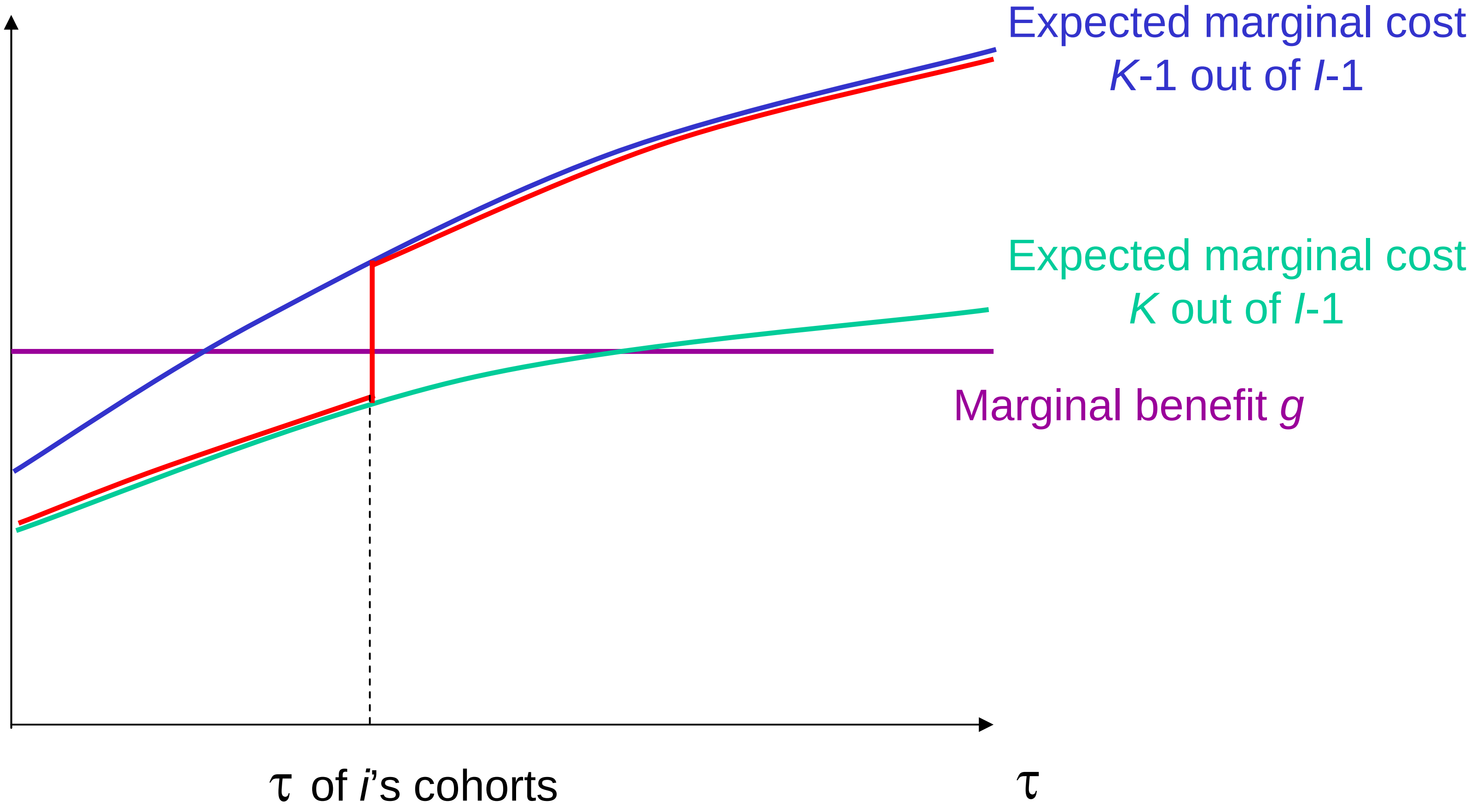
- Proposition 9: $\tau_{1,AB} = 0$.
  - Intuition
    - If at $t_i + \tau_{AB,1}$ payoff hadn't occurred it will occur with prob. one in next instant (i.e. hazard rate=∞)
    - Hazard rate is continuous
    - For any $\tau_{AB,1}>0$ player i has incentive to exit earlier.
    - Hence, $\tau_{AB,1}=0$.

- Corollary: $\tau_1 > \tau_{1,AB} = 0$.
  - Same reasoning as in case with unobservable actions.

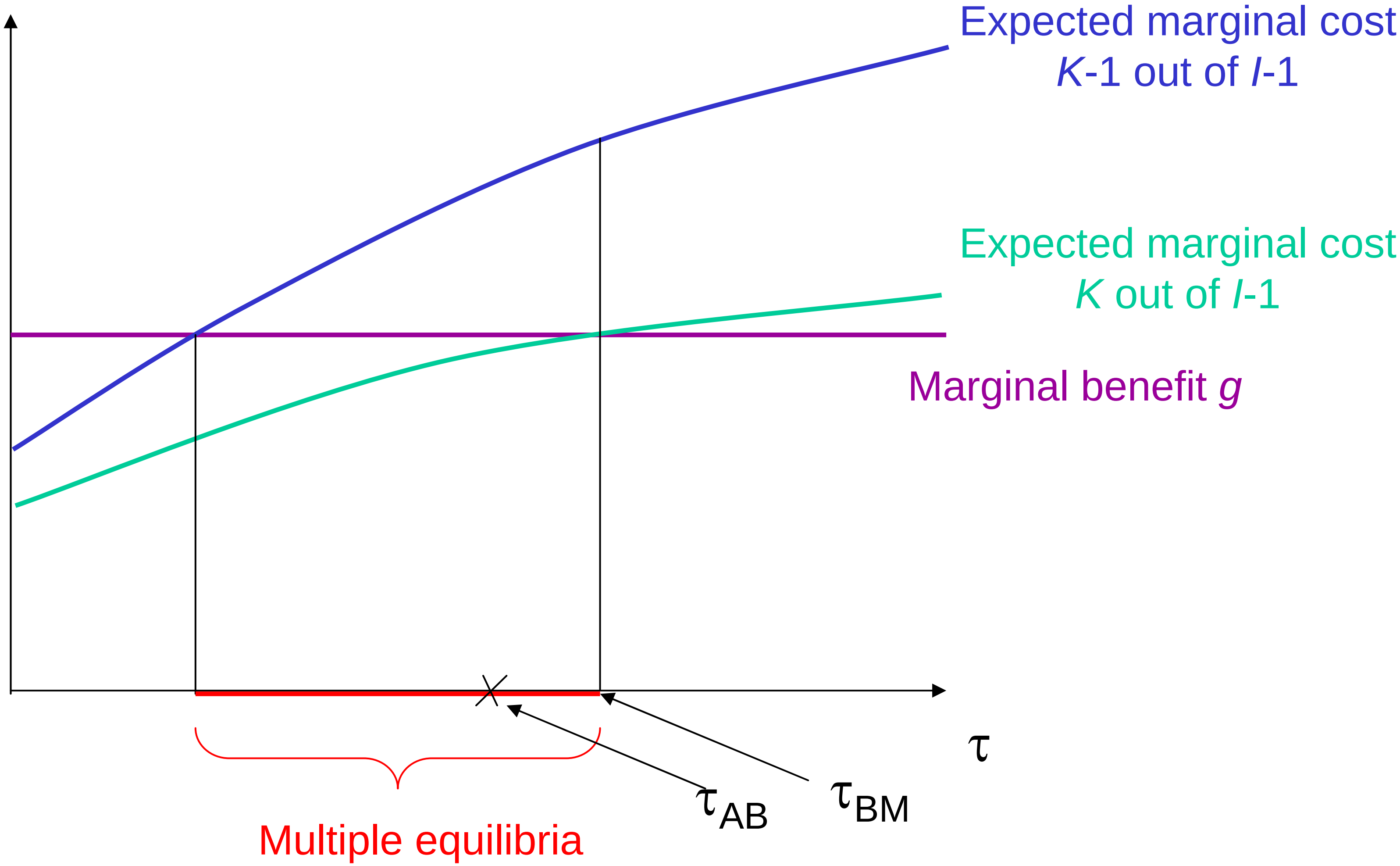
Isolating information clustering (CC-model)

- Continuum of players, but I cohorts
- Difference:
  player i knows that his cohort exits at $t_i + \tau$
  - Before $t_i+\tau$: drop if K^{th} cohort out of (I-1) exits
  - After $t_i+\tau$: drop if (K-1)th out of (I-1) exits
    (since own cohort exited)
  - At $t_i+\tau$: drop occurs with strictly positive prob.

Isolating information clustering (CC-model)



Isolating information clustering (CC-model)



Reasoning for τ_1 in case with observable actions is analogous.

Key conclusions

- Information clustering creates an additional force for equilibrium delay in the unobservable case
- Information clustering is *necessary* to get equilibrium delay in the observable case
- What matters for equilibrium delay?
  - Transparency
  - Synchronicity
  - Clustering

introduction

theory

experiment

design and procedures

measures – delay & herding

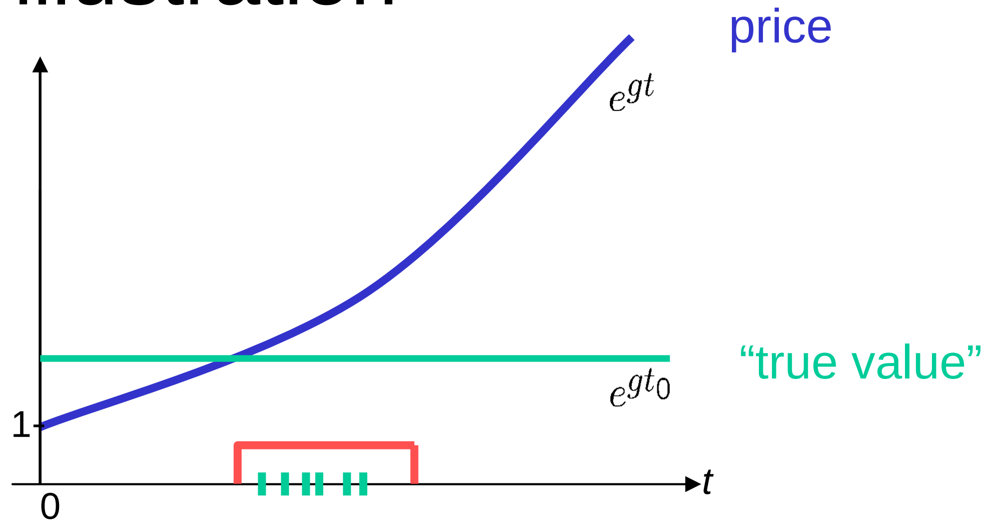
results

further insights

conclusion

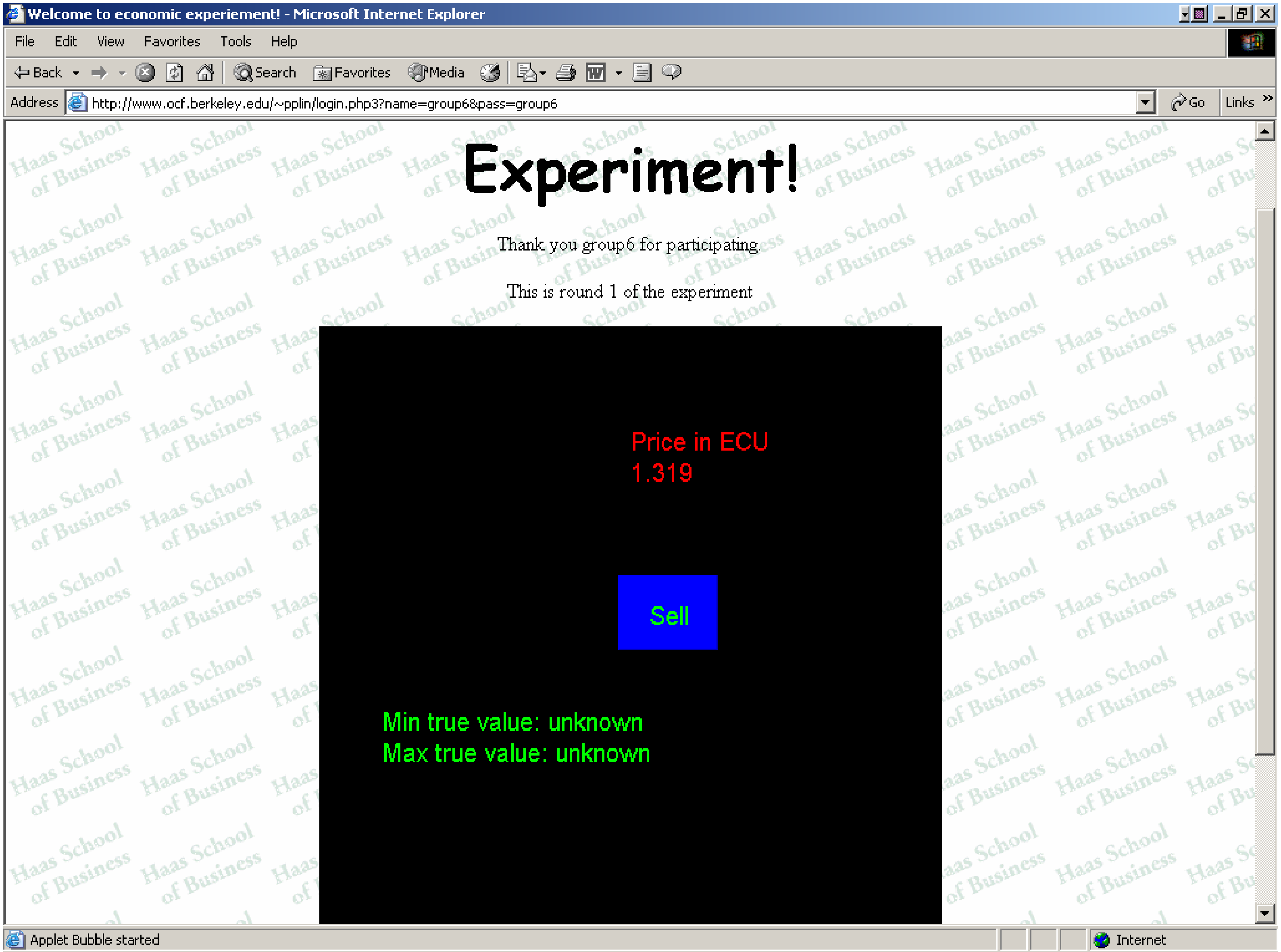
Experimental design - 1

- Stock market illustration

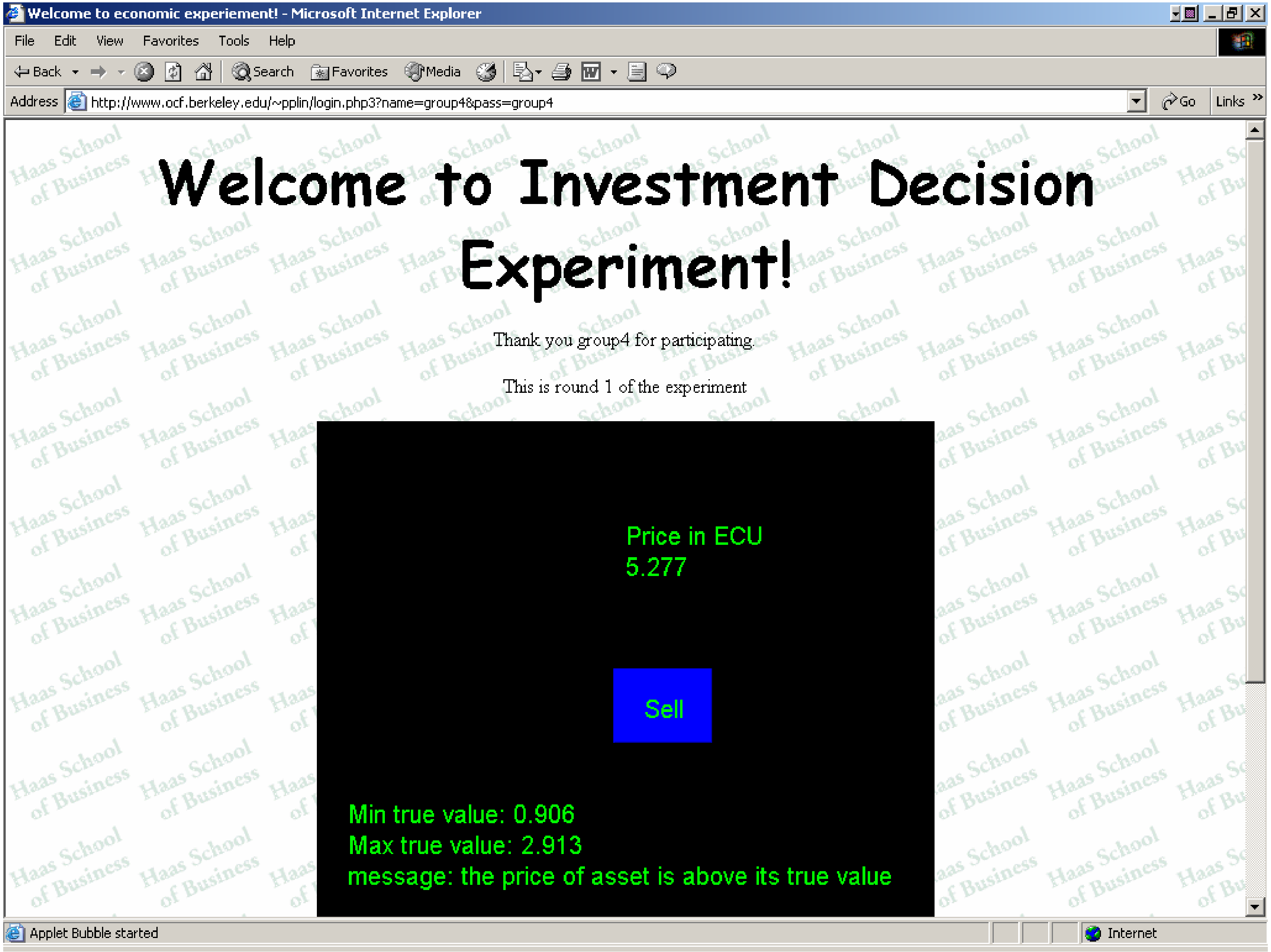


- g=2%, λ=1%, (½ second)
- 2 parallel rounds (randomly matched)
- 6 players per round
- First 3 sell at exit price e^{gt}, others e^{gt_0}

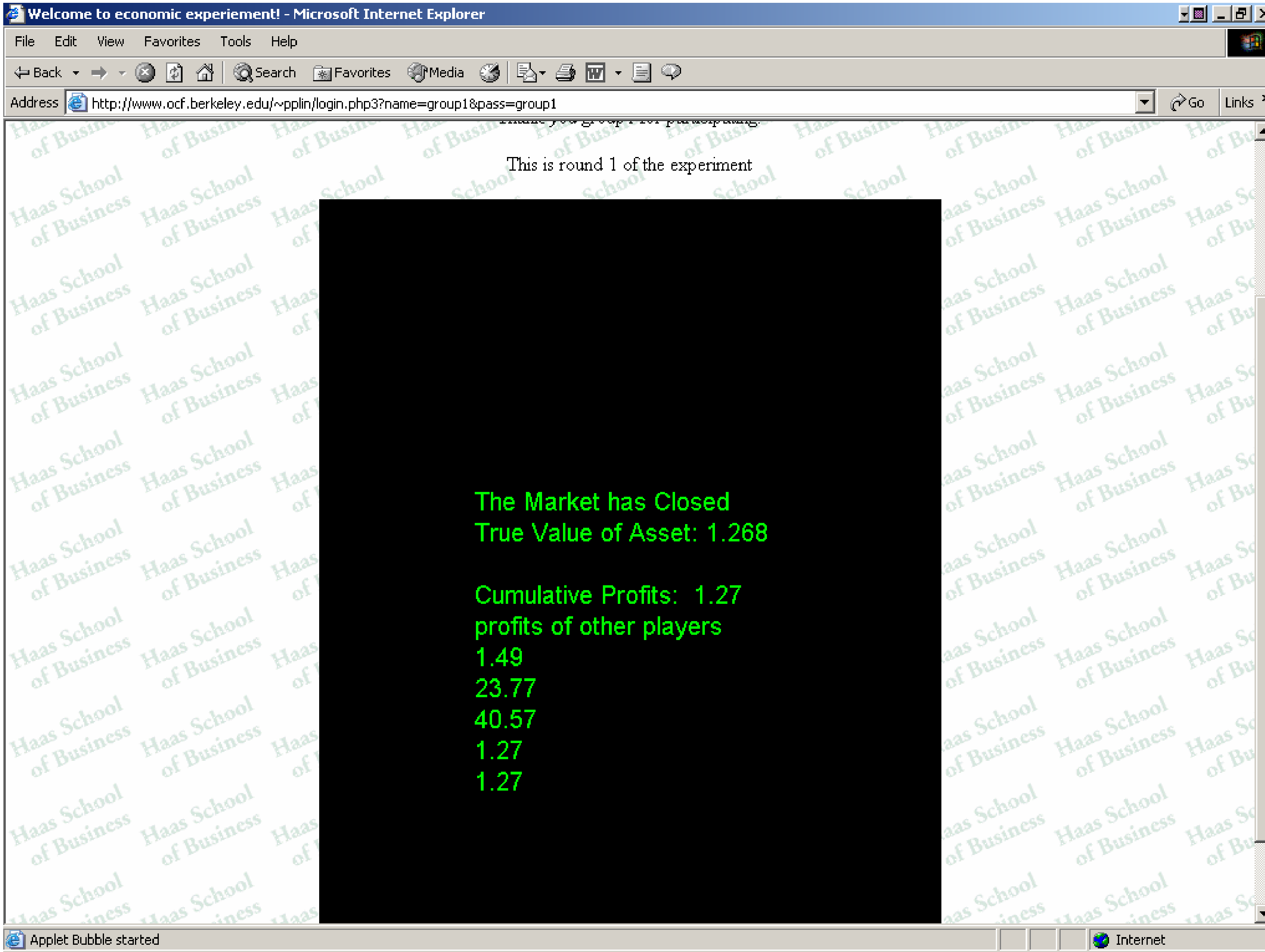
Experimental design - 2



Experimental design - 3



Experimental design - 4



Experimental design - 5

- Average payout: ECU 30.32 = $15.16
- Hovering to avoid coordination via mouse-click
- "Learning by doing:" focus on periods 20,…,45
- Obvious mistakes: sale within 10 periods (5 sec.)
- 16 Sessions
- Treatments:

	unobservable	observable
$\eta=50$	*Compressed*	X
$\eta=90$	*Baseline*	*Observable*

Measures

- Delay measures
  - delay $t_{\text{exit } i} - t_i$
  - bubble length $t_{\text{exit } [K]} - t_0$
    - Notice censoring!
- Herding measure
  - $\text{GAP}^{2,1}$ $t_{\text{exit } [2]} - t_{\text{exit } [1]}$
  - $\text{GAP}^{3,2}$ $t_{\text{exit } [3]} - t_{\text{exit } [2]}$

introduction

theory

experiment

design and procedure

measures – delay & herding

results

further insights

conclusion

Theory Predictions

Table 1: Theory Predictions

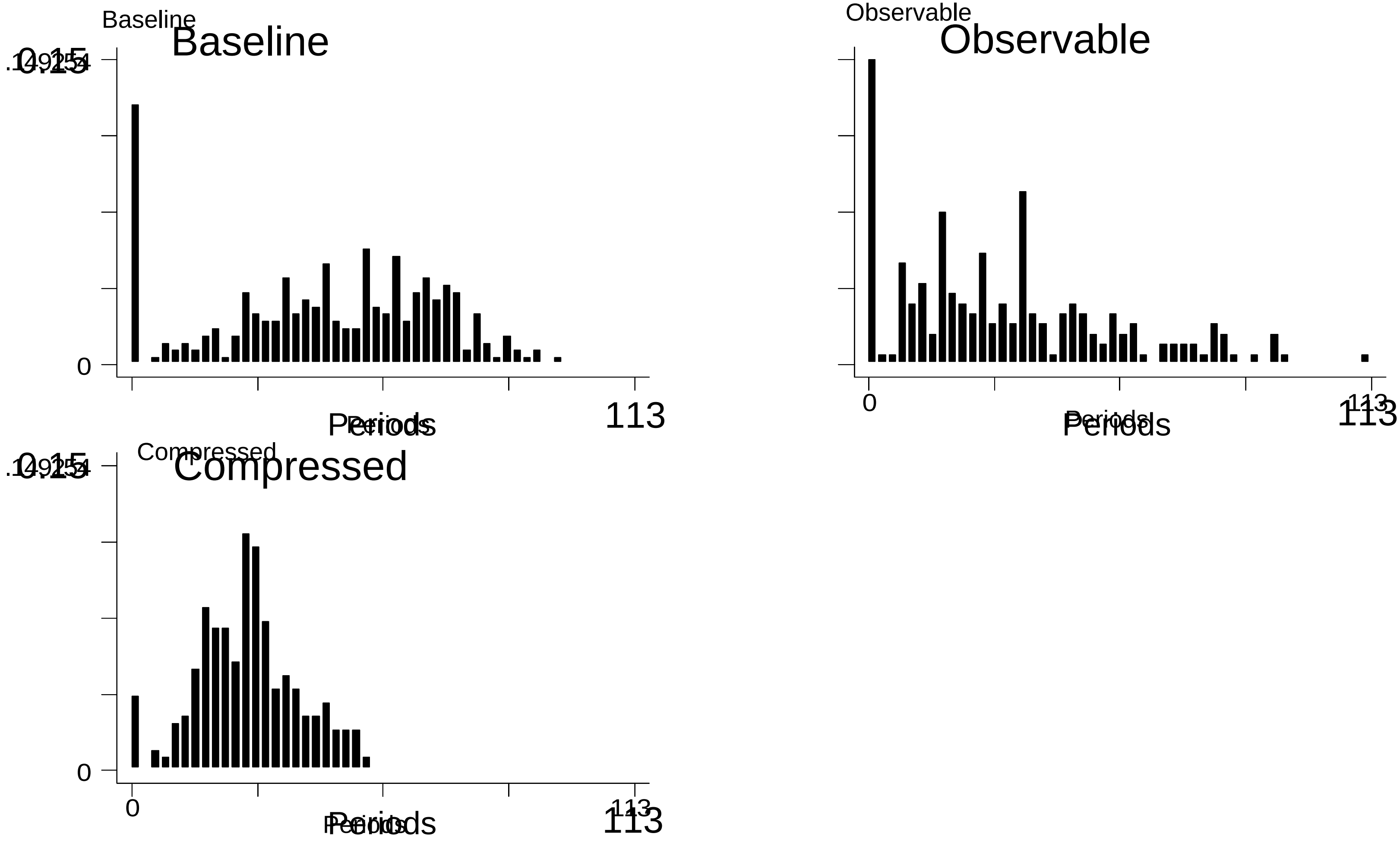
	Treatment		
	Baseline	Compressed	Observable
Bubble Length	62	26	26
Delay Length	23	5	13*
Gap Length	13	7	1

*For the Observable treatment, Delay is only meaningful for first seller

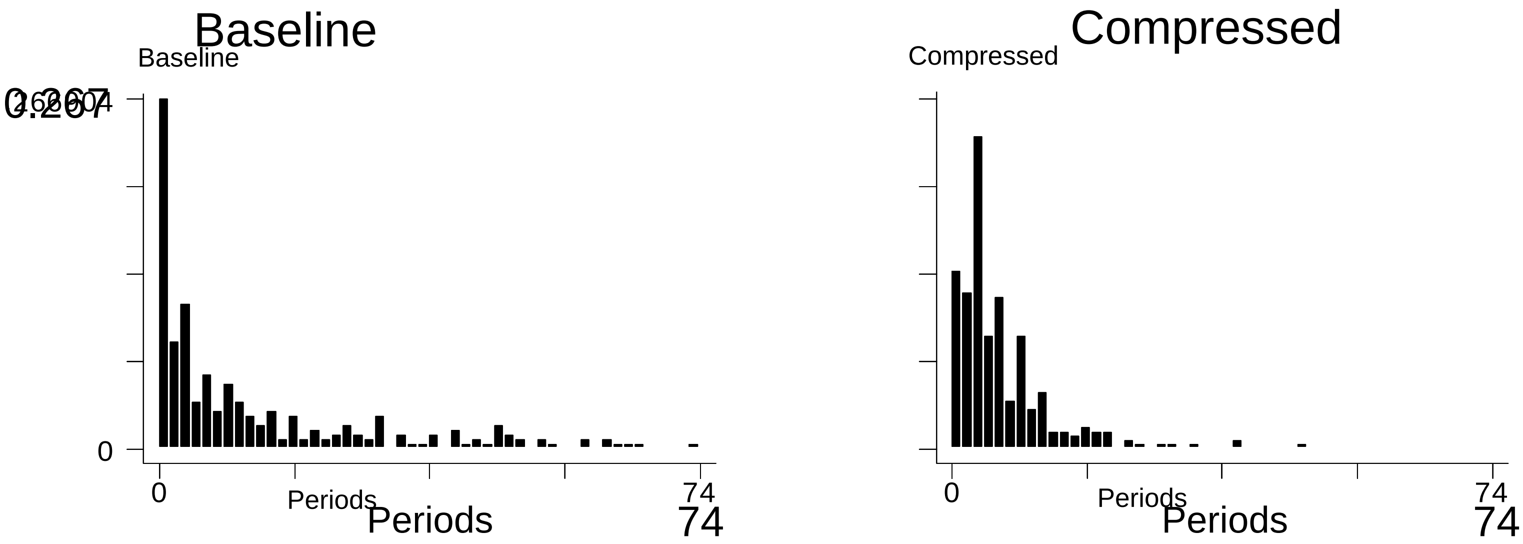
Descriptive Statistic

	Treatment		
	Baseline	Compressed	Observable
Number of Sessions	6	6	4
Bubble Length	44.00	26.46	31.00
	(25.00)	(11.31)	(24.55)
Delay Length			
Seller 1	7.26	3.97	7.06
	(9.60)	(4.25)	(12.72)
Seller 2	10.18	5.34	
	(13.68)	(5.39)	
Seller 3	12.67	6.66	
	(16.14)	(6.79)	
Gap Length			
Between 1st & 2nd seller	23.27	18.59	9.48
	(27.63)	(26.37)	(25.81)
Between 2nd & 3rd seller	15.22	8.68	1.85
	(15.87)	(9.51)	(1.49)

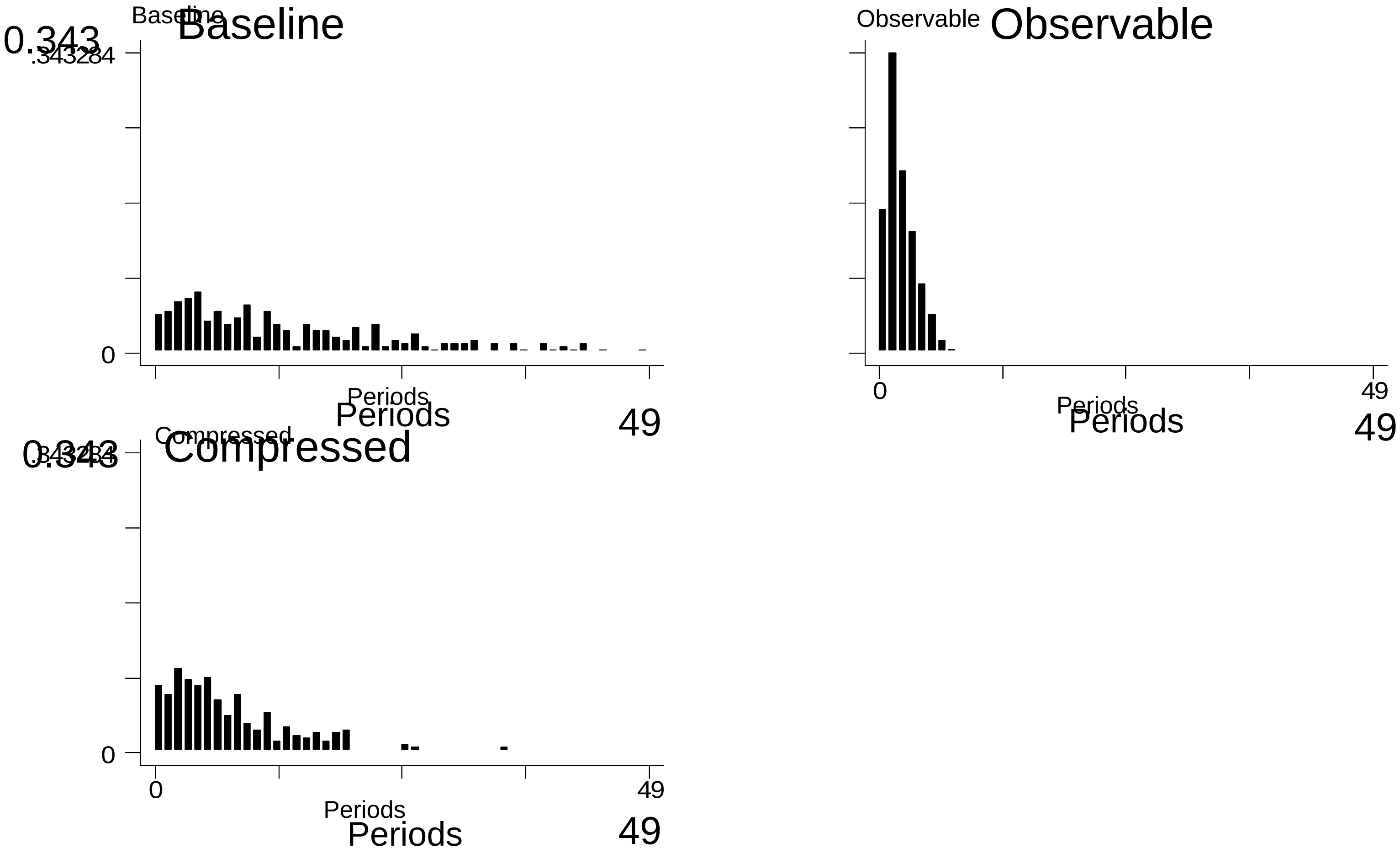
Histograms – Bubble Length



Histogram - Third Player’s Delay



Histogram - Gap Length



Results – Session Level Analysis

- Prediction 1: Bubble Length
  - Baseline > Compressed 5 %
- Prediction 2: Delay
  - Player 1: Baseline > Compressed 5 %
  - Player 2: 5 %
  - Player 3: 1 %
  - Player 1: Baseline > Observable failed to reject =
- Prediction 3: GAP
  - GAP21: Baseline > Observable 5 %
  - GAP32: 5 %

Results: Delay – Individual Level Analysis

	Robust-Cluster-OLS			Tobit	
	Seller 1	Seller 2	Seller 3	Baseline	Compressed
Constant	11.512	15.643	19.597	18.635	9.036
	(12.48)**	(11.33)**	(11.17)**	(35.09)**	(39.32)**
Compressed	−3.471	−5.165	−6.697		
	(4.50)**	(4.60)**	(4.91)**		
Observable	−0.654				
	(0.52)				
t_0	−0.054	−0.069	−0.087		
	(10.39)**	(8.89)**	(9.41)**		
Round Fixed Effects	Yes	Yes	Yes	Yes	Yes
Observations	738	584	583	1681	1788
R-squared	0.23	0.26	0.28		

Results: Herding – Individual Level Analysis

	All Treatments		Observable Treatment Only	
	GAP21	GAP32	GAP21	GAP32
Constant	14.24	11.431	3.278	1.733
	(7.55)**	(12.18)**	(1.57)	(10.35)**
Compressed	–3.858	–6.182		
	(–1.48)	(6.52)**		
Observable	–12.83	–13.028		
	(4.67)**	(15.45)**		
t_0	0.112	0.047	0.084	0.002
	(5.66)**	(5.02)**	(2.48)*	(0.87)
Round Fixed Effects	Yes	Yes	Yes	Yes
Observations	785	786	201	201
R-squared	0.15	0.29	0.25	0.25

introduction

theory

experiment

- design and procedure
- measures – delay & herding
- results
- **further insights**

conclusion

t_0–effect & exiting before t_i

- t_0-effect
  - Risk aversion - stakes are higher for large t_0
  - Difference in risk aversion among players
    - Delay of first seller < Delay of third seller
    - Effect becomes larger for large t_0
  - Misperception of constant arrival rate
  - Waiting for a fixed (absolute) price increase
- Exiting before t_i
  - mistakes
  - Worries that others suffer t_0-effect (risk aversion)
  - Effect is larger in Baseline since bubble is larger

Probit of Non-Delay

	Baseline and Compressed Only		Seller 1 Only
t_0	0.017	+0.3 %	0.017
	(17.44)**		(13.46)**
Compressed	−0.435	-7.1 %	−0.223
	(2.90)**		(1.20)
Observable			−0.03
			(0.18)
Constant	−2.474		−1.373
	(12.85)**		(3.34)**
Round Fixed Effects	Yes		Yes
Observations	2259		738

base rate 8.8 %

Conclusion

- Many timing games have in common
  - Time has to be “ripe”
  - Congestion effect
  - Costly to be pioneer
  - Uncertainty about others moves
- Theoretical predictions of clock games:
  - Delay increases with
    - number of key players
    - uncertainty about others moves
  - Herding/sudden onset if moves are observable
    - Initial delay for first player decreases with number of players
- Experiment
  - Comparative static/Treatment effects are confirmed
  - Delay and herding less strong (in terms of levels)
  - Additional insights: t_0-effect, …