Optimal Expectations

Markus K. Brunnermeier and Jonathan Parker

Princeton University

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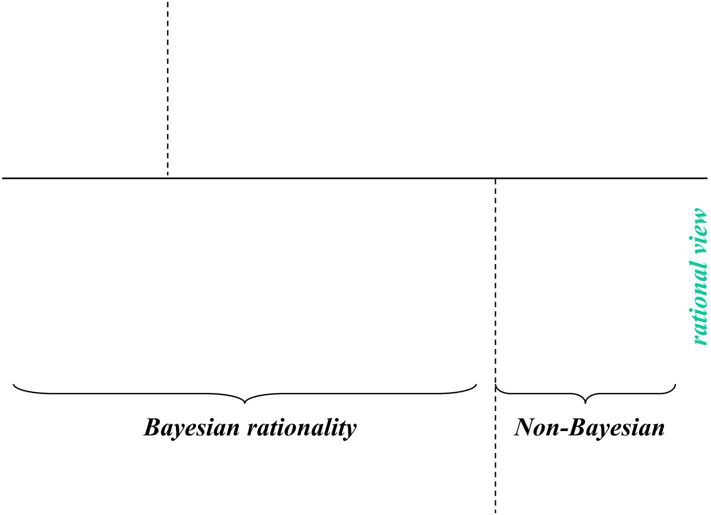
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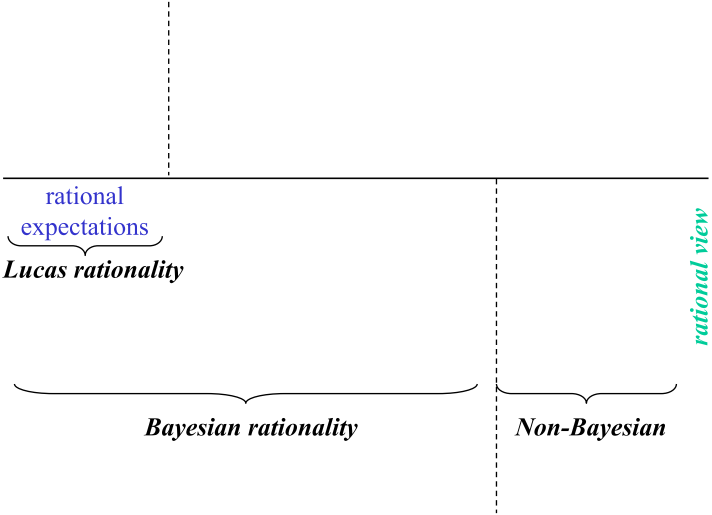
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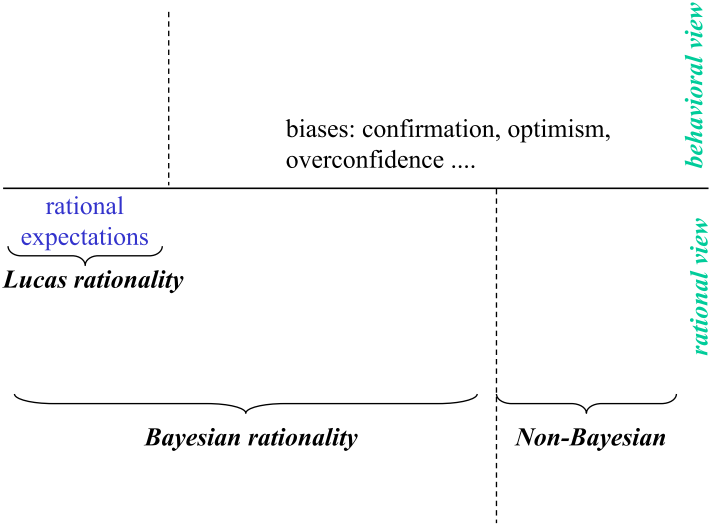
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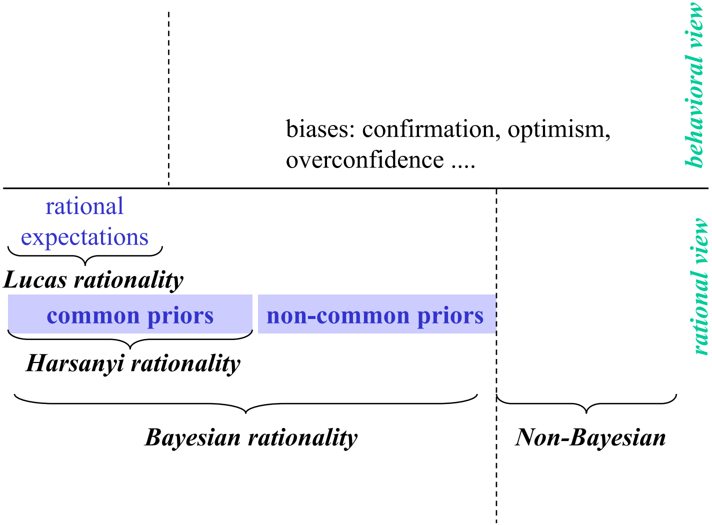
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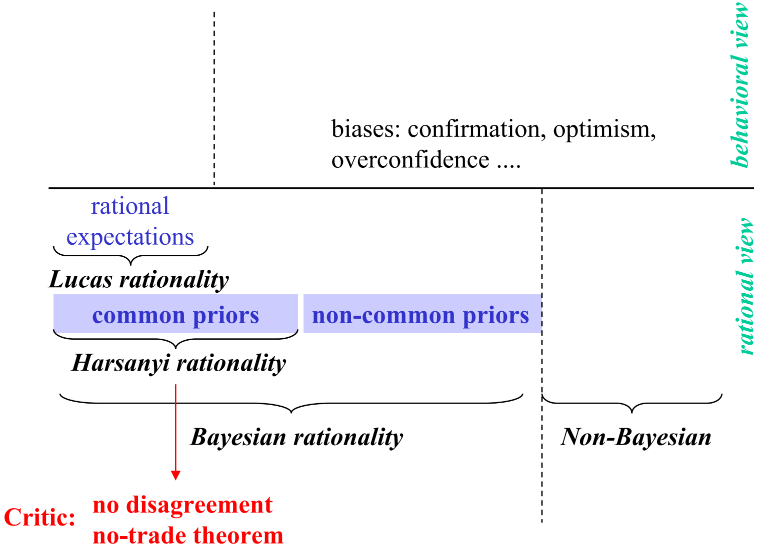
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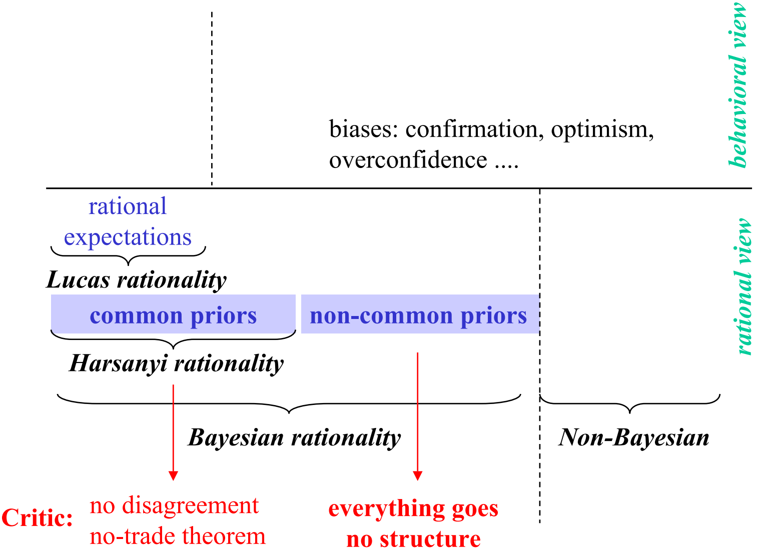
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Overview: Three Main Elements

1. **Felicity at t:** $\hat{E}_t [U(c_1, ..., c_T)]$
   - Agents care about utility flow today and
   - expected utility flows in the future

   $\Rightarrow$ happier if more optimistic
2. **No split personality**
   - Distorted beliefs distort actions

   $\Rightarrow$ better outcomes if more rational
3. **Optimal beliefs balance these forces**
   - Beliefs maximize well-being $\frac{1}{T} E \left[\sum_{t=1}^{T} \hat{E}_t [U(c_1, ..., c_T)] \right]$

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Outline

The General Framework

Actions: At each t agent chooses c_t to maximize felicity$_t$ given subjective beliefs $\hat{\pi}\left(s_t|\underline{s}_{t-1}\right)$, and resource constraints.

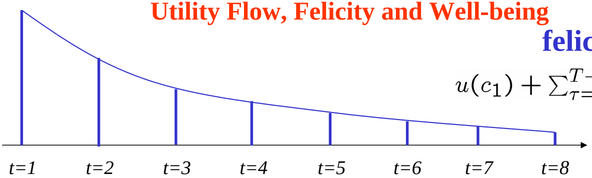
Felicity at t: $\hat{E}_t[U(c_1, ..., c_T)]$

with time-separable exponential discounting equals

$$\underbrace{\sum_{\tau=1}^{t-1} \beta^\tau u\left(c_\tau\right)}_{\text{'memory' utility}} + \beta^t u\left(c_t\right) + \underbrace{\hat{E}_t\left[\sum_{\tau=t+1}^{T} \beta^\tau u\left(c_\tau\right)\right]}_{\text{'expected' utility}}$$

Note: βs for past consumption could be replaced with δ.

Utility Flow, Felicity and Well-being



felicity at *t* = 1

$$u(c_1) + \sum_{\tau=1}^{T-t} \beta^{\tau} u(c_{1+\tau})$$

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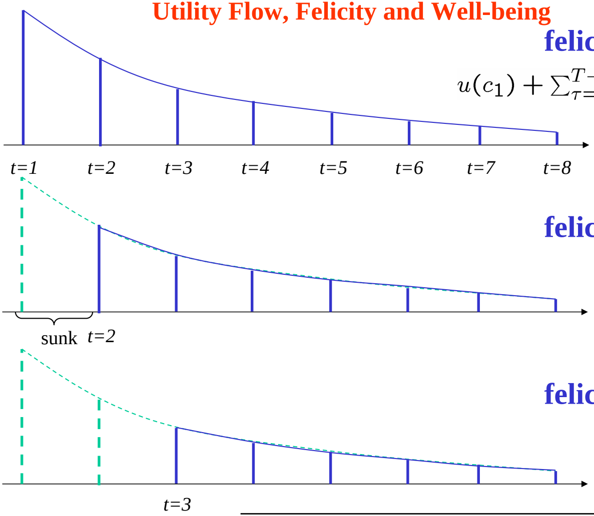

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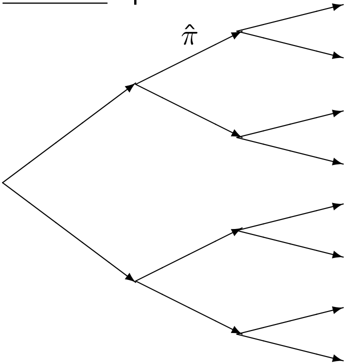
felicity at *t* = 2

felicity at *t* = 3

Well-being

Beliefs: At $t = 0$ optimal beliefs are $\hat{\pi}^{OE}\left(s_t|\underline{s}_{t-1}\right)$



that maximize

Well-being: $\mathcal{W} = \frac{1}{T} E\left[\sum_{t=1}^{T} \hat{E}_t\left[U(\cdot)\right]\right]$

subject to:

- agent behavior given these beliefs
- $\hat{\pi}^{OE}\left(s_t|\underline{s}_{t-1}\right)$ are probabilities
- $\hat{\pi}^{OE}\left(s_t|\underline{s}_{t-1}\right) = 0$ if $\pi\left(s_t|\underline{s}_{t-1}\right) = 0$

Two-period **Example** with Consumption at $t = 2$

	$t = 1$	$t = 2$
felicity in period 1		$\beta \hat{E}[u(c_2)]$
felicity in period 2		$\beta u(c_2)$

Actions maximize felicity: $\beta \hat{E}[u(c_2)]$

Beliefs maximize well-being: $\mathcal{W} = \frac{1}{2}\beta \hat{E}[u(c_2)] + \frac{1}{2}\beta E[u(c_2)]$

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Discussion

1. Subjective probabilities are chosen once and forever
   - Bayes' Rule (LIE) holds,
   - Can be interpreted as choice of priors
2. If beliefs are objective, wellbeing = felicity
   - Only incentive to distort beliefs is anticipatory utility gain
3. Rational expectations are optimal *only if*
   - anticipatory utility does enter felicities or
   - anticipatory utility does not enter well-being $\mathcal{W}$.
4. Different memory discounting in felicity
   - Paper's results hold qualitatively for any memory discounting
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6. Why optimal expectations?
   - It is optimal: "as if" interpretation
   - Parents/Upbringing affects (prior) beliefs
   - Neuroscientific "story":

     prefrontal cortex exerts effort to reduce overoptimism

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7. Payoff: biases are endogenous
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   "That the chance of loss is frequently undervalued, ..."
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Applications

- Portfolio choice
  - ⇒ *preference for skewed returns*
- General equilibrium
  - ⇒ *endogenous heterogenous prior beliefs*
  - ⇒ *equity premium puzzle versus long shot phenomena*
- Consumption-savings problem with stochastic income
  - ⇒ *optimism and overconfidence in future income*
  - ⇒ *consumption profiles concave due to "news"*
  - ⇒ *choose incomplete consumption insurance*
- Optimal timing of a single task
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Portfolio Choice

- Setup
  1. Two period problem:
     invest in period 1, consume in period 2
  2. Two assets:
     a risk-free asset, return R; a risky asset, return $R + Z$
  3. Uncertainty:
     $S > 2$ states, $\pi_s > 0$ for $s = 1$ to S,
     $Z_s < Z_{s+1}$, $Z_1 < 0 < Z_S$
  4. $c \geq 0$ in all states

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Portfolio Choice

Stage 2: Agent $\max_\alpha \beta \sum_{s=1}^{S} \hat{\pi}_s u(R+\alpha Z_s)$

$$\text{FOC:} \quad 0 = \sum_{s=1}^{S} \hat{\pi}_s u'(R+\alpha Z_s) Z_s \qquad \Rightarrow \alpha^*(\hat{\pi})$$

Stage 1: Choose $\hat{\pi}_s$ to maximize well-being

$$\underbrace{\frac{1}{2}\beta \sum_{s=1}^{S} \hat{\pi}_s u(R+\alpha^* Z_s)}_{\text{felicity at } t=1} + \frac{1}{2}\beta \underbrace{\sum_{s=1}^{S} \pi_s u(R+\alpha^* Z_s)}_{\text{'average' utility at } t=2}$$

$$\text{FOC:} \quad \underbrace{\frac{\beta}{2}(u_S - u_{s'})}_{\text{benefits of anticipation}} = \underbrace{\frac{\beta}{2}\sum_{s=1}^{S} \pi_s u'(R+\alpha^* Z_s) Z_s \frac{d\alpha^*}{d\hat{\pi}_{s'}}}_{\text{costs of changed behavior}}$$

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Proposition Excess risk taking due to optimism

(i) Agents are optimistic about states with high portfolio payou

if $\alpha^* > 0$, $\sum_{s=1}^{S} (\hat{\pi}_s - \pi_s)\, u'(R + \alpha^* Z_s)\, Z_s > 0$;

if $\alpha^* < 0$, $\sum_{s=1}^{S} (\hat{\pi}_s - \pi_s)\, u'(R + \alpha^* Z_s)\, Z_s < 0$.

(ii) Agents go even more long (short) than agent with RE **or** in the opposite direction

if $E[Z] > 0$, then $\alpha^* > \alpha^{RE} > 0$ or $\alpha^* < 0$;

if $E[Z] < 0$, then $\alpha^* < \alpha^{RE} < 0$ or $\alpha^* > 0$;

Preference for Skewed Returns

- Empirical Phenomena:
  - Horse race long shots: Golec and Tamarkin (1998)
  - Lottery demand: Garrett and Sobel (1999)
  - Security design? Swedish lottery bonds, PS-Lotteriesparen
- Setup
  - 2 states with payoffs: $Z_1 < 0 < Z_2$,
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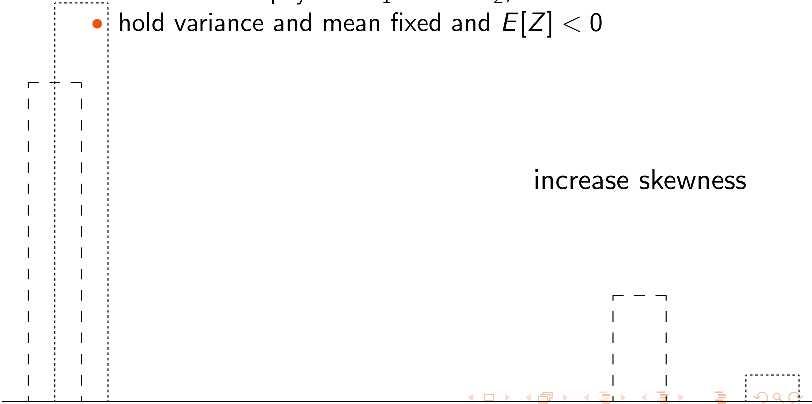

Proposition Skewness

An agent with an unbounded utility function holds some of the asset even though its mean payoff is negative if the payoff is sufficiently skewed.

- Remark:
  - Agent goes long for large π_1 even though $E[Z] < 0$, since
    - there is not much room to short and distort beliefs
    - shorting becomes very risky

General Equilibrium

- Empirical Phenomena:
  - betting & gambling
  - high trading volume (stock and FX market)
  - home bias
  - $\Leftarrow$ **endogenous** *heterogenous prior beliefs*?
  - negatively skewed: equity premium puzzle
  - positively skewed: IPO underperformance, long-shots
- Setup:
  The portfolio choice problem with
  - A continuum of agents with identical endowments
  - A fixed supply of ‘bonds’ with normalization $R = 1$
  - The risky asset in zero net supply: $1 + Z_s = \frac{1+\varepsilon_s}{P_e}$

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Proposition Hetereogeneous Priors

For $S > 2$ agents split into two groups with different beliefs

(i) Optimists with $\hat{E}^i \left[Z^{OE} \right] > 0$ and $\alpha^{OE,i} > 0 = \alpha^{RE}$

(ii) Pessimists with $\hat{E}^j \left[Z^{OE} \right] < 0$ and $\alpha^{OE,j} < 0$

both groups trade against each other and $\{\hat{\pi}^i\} \neq \{\pi\} \neq \{\hat{\pi}^j\}$.

- **Example**
  - $u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$ with $\gamma = 3$,
  - $\pi_1 = 0.25$, $\pi_2 = 0.75$,
  - $\varepsilon_1 = -0.6$, $\varepsilon_2 = 0.2$ so $P^{RE} = 1$.

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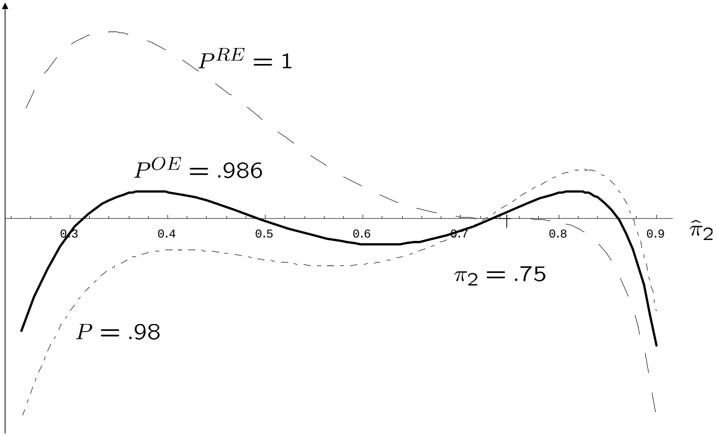

Figure: Wellbeing as a function of subjective beliefs, $\hat{\pi}_2$

In this example, as we vary the economic environment, beliefs change . . .

$P^{OE} > P^{RE} = 1$ if payoff is positively skewed (long-shots, IPO)
$P^{OE} < P^{RE} = 1$ if payoff is negatively skewed (stock market).

Conjecture

For multi-asset case with positive net supply:

- Heterogeneity in beliefs is less pronounced.
- Agents invest in different skewed assets (forgo diversification benefits to hold skewed assets.)

Complicates Aggregation:
Representative agent has different preference structure from individual (possibly identical) investors.

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Consumption & Savings

- Empirical Phenomena:
  - households **expect** upward sloping consumption profile (Barsky et al. (1997))
  - **actual** average consumption growth is non-positive and profiles are concave (Gourinchas & Parker (2002))
- Setup:
  - Finite-lived agent, quadratic utility $u(c_t) = ac_t - \frac{1}{2}bc_t^2$,
  - one risk-free asset, $R\beta = 1$,
  - i.i.d. income:

Objective prob.: y_t independent over time $\Pi\left(y_t | \underline{y}_{t-1}\right)$
$d\Pi\left(y_t\right) > 0$ for all $y \in \left[\underline{y}, \bar{y}\right]$.

Subjective prob.: $\hat{\Pi}\left(y_t | \underline{y}_{t-1}\right) \geq 0$ for all $y \in \left[\underline{y}, \bar{y}\right]$

Optimal Consumption

Euler equation:

$$c_t\left(A_t, \underline{y}_t\right) = \hat{E}\left[c_{t+1}\left(A_{t+1}, \underline{y}_{t+1}\right)|\underline{y}_t\right]$$

Consumption rule:

$$c_t^*\left(\underline{y}_t\right) = \frac{1-R^{-1}}{1-R^{-(T-t)}}\left(A_t + y_t + \sum_{\tau=1}^{T-t} R^{-\tau}\hat{E}\left[y_{t+\tau}|\underline{y}_t\right]\right)$$

Note: c_t^* depends only on $\hat{E}\left[y_{t+\tau}|\underline{y}_t\right]$ (not higher moments)

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Optimal Beliefs

So $\Rightarrow$ Variance only lowers anticipatory utility, but does not affect c

$\Rightarrow$ OE exhibit no uncertainty for quadratic utility.

Therefore

$$\hat{E}\left[u\left(c^*_{t+\tau}\right)|\underline{y}_t\right] = u\left(\hat{E}\left[c^*_{t+\tau}|\underline{y}_t\right]\right)$$

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Certainty + Euler equation ⇒ wellbeing simplifies to

$$\frac{1}{T}\sum_{t=1}^{T}\psi_t E\left[u\left(c_t^*\left(\underline{y}_t\right)\right)\right]$$

and FOC implies an actual consumption path of

$$c_t^*\left(\underline{y}_t\right)=\frac{a}{b}-\frac{\psi_{t+\tau}}{\psi_t}R^\tau\left(\frac{a}{b}-E\left[c_{t+\tau}^*\left(\underline{y}_{t+\tau}\right)|\underline{y}_t\right]\right)$$

where $\psi_t=\beta^{t-1}\left(1+\sum_{\tau=1}^{T-t}\left(\beta^\tau+(\beta\delta)^\tau\right)\right)$

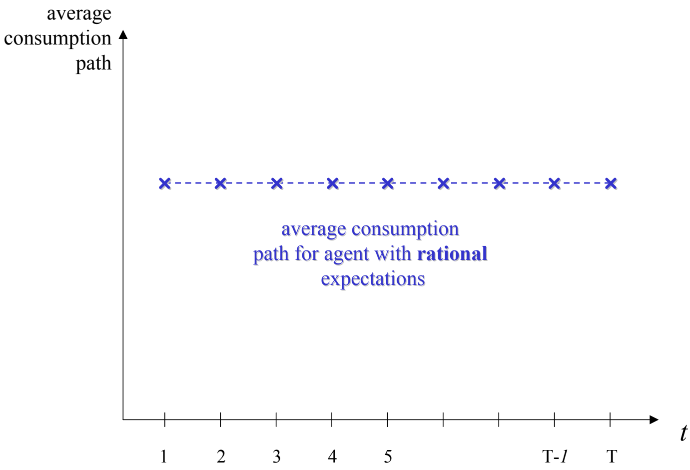
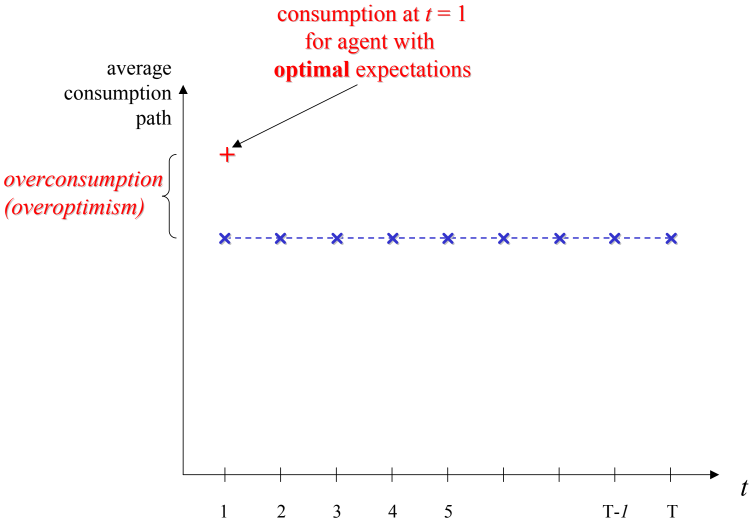


Figure: Consumption Path



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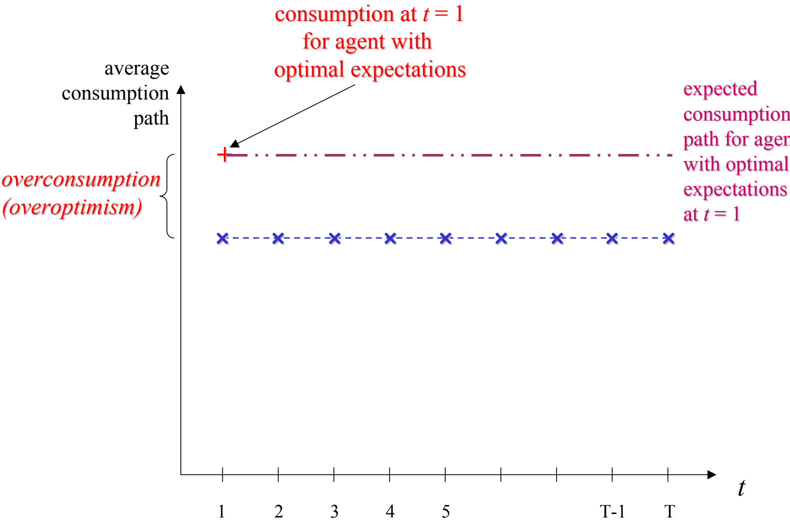

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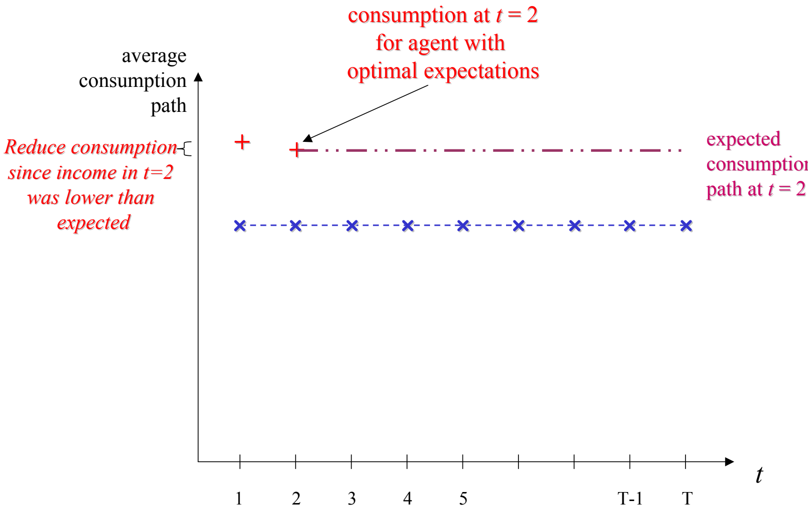

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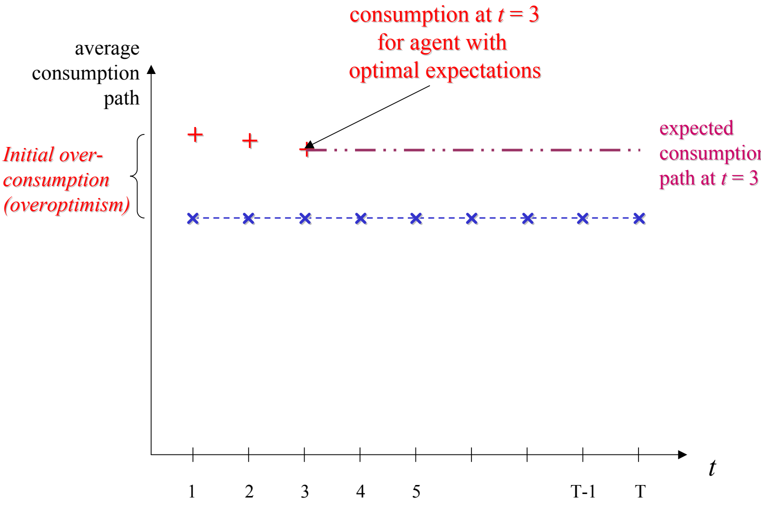

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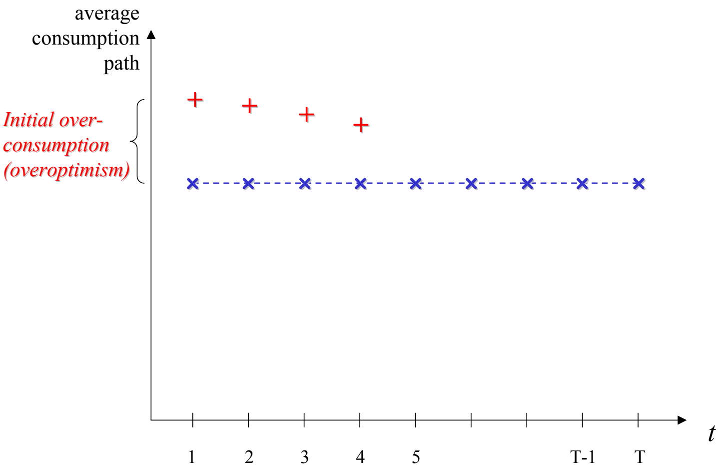

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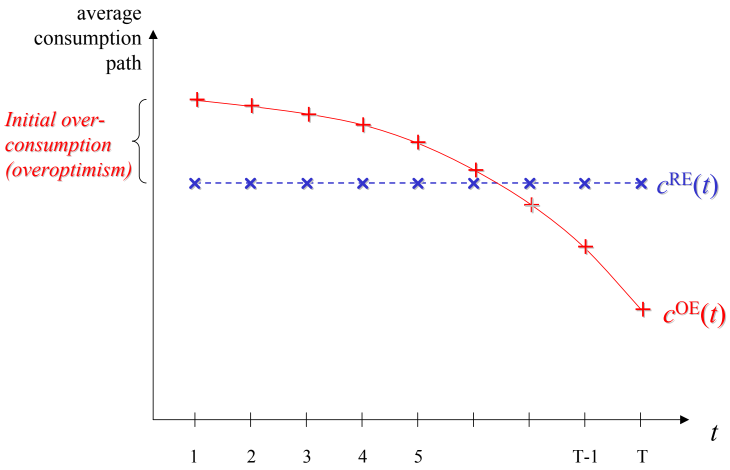

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Proposition Undersaving

For all $t < T$

(i) $\hat{E}\left[\sum_{\tau=0}^{T-t-1} R^{-\tau} y_{t+1+\tau} | \underline{y}_t\right] > E\left[\hat{E}\left[\sum_{\tau=0}^{T-t-1} R^{-\tau} y_{t+1+\tau} | \underline{y}\right.\right.$

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(iv) as $T \to \infty$, $c_t^*\left(\underline{y}_t\right) \to c_t^{RE}\left(\underline{y}_t\right)$

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Conclusion

- Rational expectations are sub-optimal:
  - Agents with rational beliefs makes the ex post best decisions
  - but agents that care about the future can be happier with some optimism
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- Optimal expectations is a structural model of non-rational beliefs
  - beliefs are most distorted when decision errors are small
  - beliefs are most distorted when "dream" benefits are largest
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