Macroeconomics with Financial Frictions

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Background reading

- “Macroeconomics with Financial Frictions”
  - Brunnermeier, Eisenbach and Sannikov
    - Proceeding of the Econometric Society World Congress in Shanghai, 2010
- “The I Theory of Money”
  - Brunnermeier and Sannikov
- “The Maturity Rat Race”
  - Brunnermeier & Oehmke

- See www.princeton.edu/~markus

Motivation

- Financial crises occur periodically Kindleberger (1993)
- Financial frictions drive/amplify business cycle
  - Fisher (1933)
  - Keynes (1936)
  - Gurley-Shaw (1955)
  - Minsky (1975)
- Financial sector helps to
    - overcome financing frictions and
    - channels resources

  … but

  - Credit crunch due to
    adverse feedback loops & liquidity spirals
    - Non-linear dynamics

Heterogeneous agents

- Lending-borrowing/insuring since agents are different

- Poor-rich
- Productive
- Less patient
- Less risk averse
- More optimistic

Limited direct lending due to frictions

- Rich-poor
- Less productive
- More patient
- More risk averse
- More pessimistic

- Friction → $p_s MRS_s$ different even after transactions
- **Wealth distribution matters!**
- **Financial sector is not a veil**

Structuring the Macro-literature on Frictions

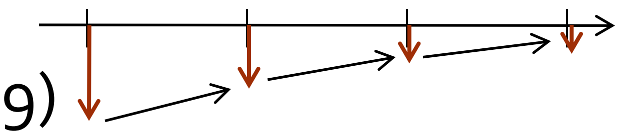
1. Persistence, amplification and instability
   a. Persistence: Carlstrom, Fuerst
   b. Amplification: Bernanke, Gertler, Gilchrist
   c. Instability: Brunnermeier, Sannikov
2. Credit quantity constraints through margins
   a. Credit rationing: Stiglitz, Weiss
   b. Margin spirals : Brunnermeier, Pederson
   c. Endogenous constraints: Geanakoplos
3. Demand for liquid assets & Bubbles – "self insurance"
   a. OLG, Aiyagari, Bewley, Krusell-Smith, Holmstroem Tirole,…
4. Financial intermediaries & Theory of Money

Recurring Theme: Liquidity Mismatch

- Instability of financial system arises from the fragility of liquidity
- Asset side
  - Technological liquidity refers to reversibility of investment
  - Market liquidity refers to price impact of capital sale
- Liability side
  - Funding liquidity refers to maturity structure of debt and sensitivity of margins
- The *liquidity mismatch* between assets and liabilities determines the severity of the amplification effects

Amplification & Instability - Overview

- Bernanke & Gertler (1989), Carlstrom & Fuerst (1997)
  - Perfect (technological) liquidity, but persistence
  - Bad shocks erode net worth, cut back on investments, leading to low productivity & low net worth of in the next period



- Kiyotaki & Moore (1997), BGG (1999)
  - Technological/market illiquidity
  - KM: Leverage bounded by margins; BGG: Verification cost (CSV)
  - Stronger amplification effects through prices (low net worth reduces leveraged institutions' demand for assets, lowering prices and further depressing net worth)
- Brunnermeier & Sannikov (2010)
  - Instability and volatility dynamics, volatility paradox
- Brunnermeier & Pedersen (2009), Geanakoplos
  - Volatility interaction with margins/haircuts (leverage)

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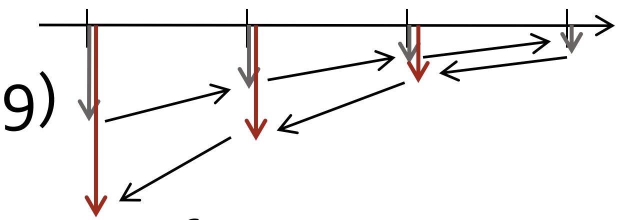

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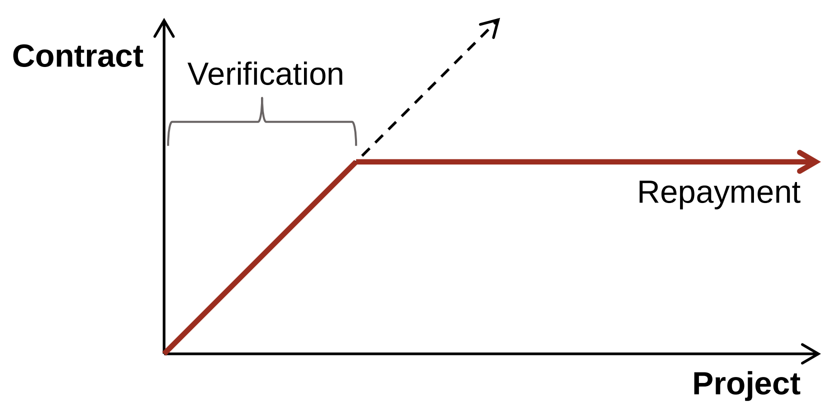
Persistence

- Even in standard real business cycle models, temporary adverse shocks can have long-lasting effects
- Due to feedback effects, persistence is much stronger in models with *financial frictions*
  - Bernanke & Gertler (1989)
  - Carlstrom & Fuerst (1997)
- Negative shocks to net worth exacerbate frictions and lead to lower capital, investment and net worth in future periods

Jump

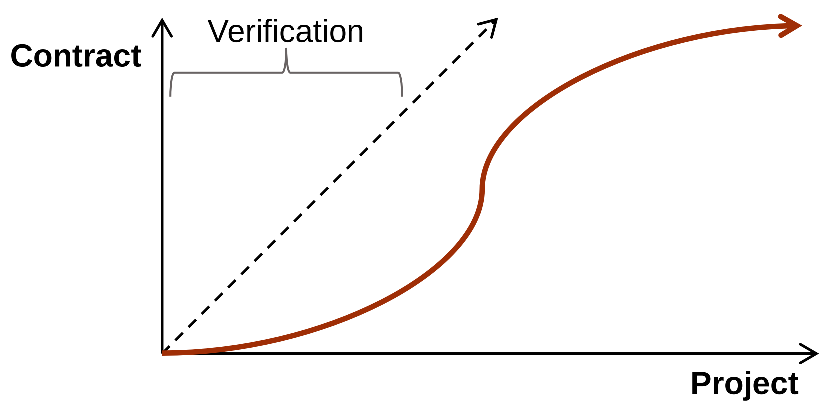
Costly State Verification

- Key friction in previous models is costly state verification, i.e. CSV, a la Townsend (1979)
- Borrowers are subject to an idiosyncratic shock
  - Unobservable to lenders, but can be verified at a cost
- Optimal solution is given by a contract that resembles standard debt



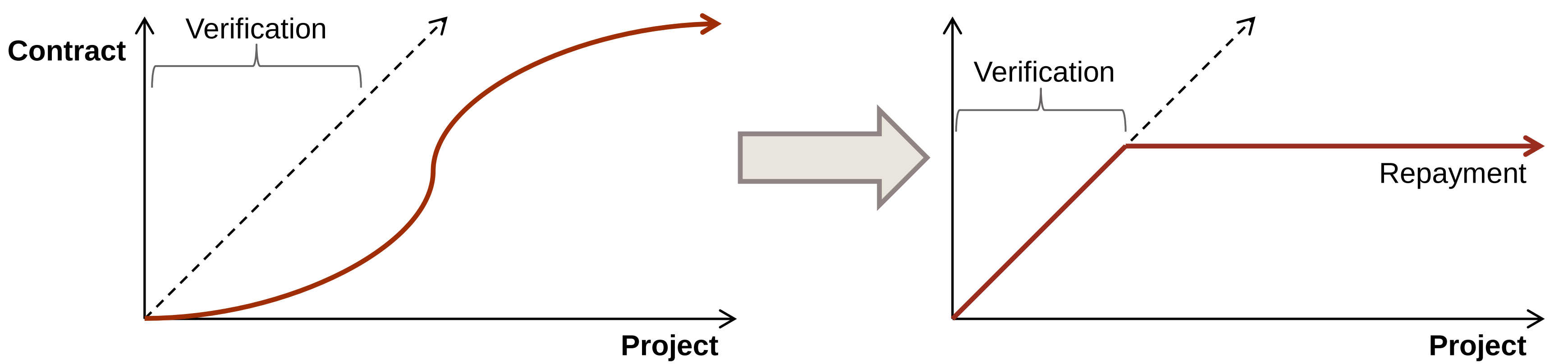
CSV: Contracting

- Competitive market for capital
  - Lender's expected profit is equal to zero
  - Borrower's optimization is equivalent to minimizing expected verification cost
- Financial contract specifies:
  - Debt repayment for each reported outcome
  - Reported outcomes that should be verified



CSV: Optimal Contract

- Incentive compatibility implies that
  - Repayment outside of VR is constant
  - Repayment outside of VR is weakly greater than inside
- Maximizing repayment in VR reduces the size and thus the expected verification cost



Carlstrom & Fuerst

- Output is produced according to $Y_t = A_t f(K_t)$
- Fraction η of entrepreneurs and $1 - \eta$ of households
  - Only entrepreneurs can create new capital from consumption goods
- Individual investment yields ωi_t of capital
  - Shock is given by $\omega \sim G$ with $E[\omega] = 1$
  - This implies consumption goods are converted to capital one-to-one in the *aggregate*
  - *No technological illiquidity!*

CF: Costly State Verification

- Households can verify ω at cost μi_t
  - Optimal contract is debt with audit threshold $\bar{\omega}$
  - Entrepreneur with net worth n_t borrows $i_t - n_t$ and repays $\min\{\omega_t, \bar{\omega}\} \times i_t$
- Auditing threshold is set by HH breakeven condition
  - $\left[\int_0^{\bar{\omega}} (\omega - \mu) dg(\omega) + \left(1 - G(\bar{\omega})\right)\bar{\omega}\right] i_t q_t = i_t - n_t$
  - Here, q_t is the price of capital
- No positive interest (within period borrowing) and no risk premium (no aggregate investment risk)

CF: Supply of Capital

- Entrepreneur's optimization:
  - $\max_{i_t} \int_{\bar{\omega}_t}^{\infty} (\omega - \bar{\omega}_t) dG(\omega)\, i_t q_t$
  - Subject to HH breakeven constraint
- Linear investment rule $i_t = \psi(q_t) n_t$
  - Leverage $\psi(q_t)$ is increasing in q_t
- Aggregate supply of capital is increasing in
  - Price of capital q_t
  - Aggregate net worth N_t

CF: Demand for Capital

- Return to holding capital:
  - $R^k_{t+1} = \frac{A_{t+1} f'(K_{t+1}) + (1-\delta) q_{t+1}}{q_t}$
- Risk averse HH have discount factor $\underline{\beta}$
  - Standard utility maximization
  - Budget constraint:
    $c_t \leq A_t f'(K_t) k_t + q_t[(1-\delta)k_t - k_{t+1}]$
  - Euler equation: $u'(c_t) = \underline{\beta} E_t\left[R^k_{t+1} u'(c_{t+1})\right]$

CF: Demand for Capital

- Risk-neutral entrepreneurs are less patient, $\beta < \underline{\beta}$
  - Euler equation: $1 = \beta E_t\left[R^k_{t+1}\rho(q_t)\right]$
  - Return on internal funds:
    $\rho(q_t) \equiv \int_{\bar{\omega}_t}^{\infty}(\omega - \bar{\omega}_t)dG(\omega)\,\psi(q_t)q_t$
- Aggregate demand for capital is decreasing in q_t

CF: Persistence & Dampening

- Negative shock in period t decreases N_t
  - This increases financial friction and decreases I_t
- Decrease in capital supply leads to
  - Lower capital: K_{t+1}
  - Lower output: Y_{t+1}
  - Lower net worth: N_{t+1}
  - Feedback effects in future periods $t + 2, \ldots$
- Decrease in capital supply also leads to
  - Increased price of capital q_t
  - Dampening effect on propagation of net worth shock

Dynamic Amplification

- Bernanke, Gertler and Gilchrist (1999) introduce *technological illiquidity* in the form of nonlinear adjustment costs to capital
- Negative shock in period t decreases N_t
  - This increases financial friction and decreases I_t
- In contrast to the dampening mechanism present in CF, decrease in capital supply leads to
  - Decreased price of capital due to adjustment costs
  - *Amplification* effect on propagation of net worth shock

Bernanke, Gertler & Gilchrist

- BGG assume separate investment sector
  - This separates entrepreneurs' capital decisions from adjustment costs
- $\Phi(\cdot)$ represents *technological illiquidity*
  - Increasing and concave with $\Phi(0) = 0$
  - $K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta)K_t$
- FOC of investment sector
  - $\max_{I_t}\{q_t K_{t+1} - I_t\} \Rightarrow q_t = \Phi'\left(\frac{I_t}{K_t}\right)^{-1}$

jump to KM97

BGG: Entrepreneurs

- Entrepreneurs alone can hold capital used in production
- At time t, entrepreneurs purchase capital for $t + 1$
  - To purchase k_{t+1}, an entrepreneur borrows $q_t k_{t+1} - n_t$
  - Here, n_t represents entrepreneur net worth
- Assume gross return to capital is given by ωR_{t+1}^k
  - Here $\omega \sim G$ with $E[\omega] = 1$ and ω i.i.d.
  - R_{t+1}^k is the endogenous aggregate equilibrium return

BGG: Costly State Verification

- Entrepreneurs borrow from HH in a CSV framework
- If R_{t+1}^k is deterministic, then threshold satisfies:
  - $\left[(1-\mu)\int_0^{\bar{\omega}} \omega dG(\omega) + \left(1 - G(\bar{\omega})\right)\bar{\omega}\right] R_{t+1}^k q_t k_{t+1} = R_{t+1}(q_t k_{t+1} - n_t)$
  - Here, R_{t+1} is the risk-free rate
- If there is aggregate risk in R_{t+1}^k then BGG argue that entrepreneurs will insure HH against risk
  - This amounts to setting $\bar{\omega}$ as a function of R_{t+1}^k
  - As in CF, HH perfectly diversify against entrepreneur idiosyncratic risk

BGG: Supply of Capital

- Entrepreneurs solve the following problem:
  - $\max_{k_{t+1}} E\left[\int_{\bar{\omega}}^{\infty}(\omega-\bar{\omega})dG(\omega)\, R_{t+1}^{k} q_t k_{t+1}\right]$
  - Subject to HH breakeven condition (state-by-state)
- Optimal leverage is again given by a linear rule
  - $q_t k_{t+1} = \psi\left(\frac{E\left[R_{t+1}^{k}\right]}{R_{t+1}}\right) n_t$
  - In a log-linearized solution, the remaining moments are insignificant
- Aggregate capital supply is increasing in $E\left[R_{t+1}^{k}\right]$ and aggregate net worth N_t

BGG: Demand for Capital

- Return on capital is determined in a general equilibrium framework
  - Gross return to holding a unit of capital
  - $E\left[R_{t+1}^{k}\right] = E\left[\frac{A_{t+1}f'(K_{t+1}) + q_{t+1}(1-\delta) + q_{t+1}\Phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \frac{I_{t+1}}{K_{t+1}}}{q_t}\right]$
- Capital demand is decreasing in expected return $E\left[R_{t+1}^{k}\right]$

BGG: Persistence & Amplification

- Shocks to net worth N_t are persistent
  - They affect capital holdings, and thus N_{t+1}, …
- *Technological illiquidity* introduces amplification effect
  - Decrease in capital leads to reduced price of capital from $q_t = \Phi'\left(\frac{I_t}{K_t}\right)^{-1}$
  - Lower price of capital further decreases net worth

Kiyotaki & Moore 97

- Kiyotaki, Moore (1997) adopt a
  - collateral constraint instead of CSV
  - market illiquidity – second best use of capital
- Durable asset has two roles:
  - Collateral for borrowing
  - Input for production
- Output is produced in two sectors, differ in productivity
- Aggregate capital is fixed, resulting in extreme *technological illiquidity*
  - Investment is completely irreversible

Jump

KM: Amplification

- *Static* amplification occurs because fire-sales of capital from productive sector to less productive sector depress asset prices
  - Importance of *market liquidity* of physical capital
- *Dynamic* amplification occurs because a temporary shock translates into a persistent decline in output and asset prices

KM: Agents

- Two types of infinitely-lived risk neutral agents
- Mass η of productive agents
  - Constant-returns-to-scale production technology yielding $y_{t+1} = ak_t$
  - Discount factor $\beta < 1$
- Mass $1 - \eta$ of unproductive agents
  - Decreasing-returns-to-scale production $y_{t+1} = F(k_t)$
  - Discount factor $\underline{\beta} \in (\beta, 1)$

KM: Frictions

- Since productive agents are less patient, they will want to borrow b_t from unproductive agents
  - However, friction arises in that each productive agent's technology requires *his* individual human capital
  - Productive agents cannot pre-commit human capital
- This results in a collateral constraint $Rb_t \leq q_{t+1}k_t$
  - Productive agent will never repay more than the value of his asset holdings, i.e. collateral

KM: Demand for Assets

- Since there is no uncertainty, a *productive agent* will borrow the maximum quantity and will not consume any of the output
  - Budget constraint: $q_t k_t + b_t \leq (a + q_t) k_{t-1} - R b_{t-1}$
  - Demand for assets: $k_t = \frac{1}{q_t - \frac{q_{t+1}}{R}} [(a + q_t) k_{t-1} - R b_{t-1}]$
- Unproductive agents are not borrowing constrained
  - $R = \underline{\beta}^{-1}$ and asset demand is set by equating margins
  - Demand for assets: $R = \frac{F'(\underline{k}_t) + q_{t+1}}{q_t}$

KM: Equilibrium

- With fixed supply of capital, market clearing requires $\eta K_t + (1-\eta)\underline{K}_t = \bar{K}$
  - This implies $M(K_t) \equiv \frac{1}{R} F'\left(\frac{\bar{K}-\eta K_t}{1-\eta}\right) = q_t - \frac{1}{R} q_{t+1}$
  - Note that $M(\cdot)$ is increasing
- Iterating forward, we obtain: $q_t = \sum_{s=0}^{\infty} \frac{1}{R^s} M(K_{t+s})$

KM: Steady State

- In steady state, productive agents use tradable output a to pay interest on borrowing:
- This implies that steady state price q^* must satisfy:
  - $q^* - \frac{1}{R} q^* = a$
- Further, steady state capital K^* must satisfy:
  - $\frac{1}{R} F'\left(\frac{\bar{K} - \eta K^*}{1-\eta}\right) = a$
  - This reflects inefficiency since marginal products correspond only to *tradable* output

KM: Productivity Shock

- Log-linearized deviations around steady state:
  - Unexpected one-time shock that reduces production of all agents by factor $1-\Delta$
- Change in assets for given change in asset price:
  - $\hat{K}_t = -\frac{\xi}{1+\xi}\left(\Delta + \frac{R}{R-1}\hat{q}_t\right),\ \hat{K}_{t+s} = \frac{\xi}{1+\xi}\hat{K}_{t+s-1}$
  - $\frac{1}{\xi} = \frac{d \log M(K)}{d \log K}|_{K=K^*}$
- Reduction in assets comes from two shocks:
  - Lost output Δ
  - Capital losses on previous assets $\frac{R}{R-1}\hat{q}_t$

KM: Productivity Shock

- Change in price for given change in assets:
  - Log-linearize the equation $q_t = \sum_{s=0}^{\infty} \frac{1}{R^s} M(K_{t+s})$
  - This provides: $\hat{q}_t = \frac{1}{\xi} \frac{R-1}{R} \sum_{s=0}^{\infty} \frac{1}{R^s} \widehat{K}_{t+s}$
- Combining equations:
  - $\widehat{K}_t = -\left(1 + \frac{1}{(\xi+1)(R-1)}\right)\Delta$
  - $\hat{q}_t = -\frac{1}{\xi}\Delta$

KM: Static vs. Dynamic Amplification

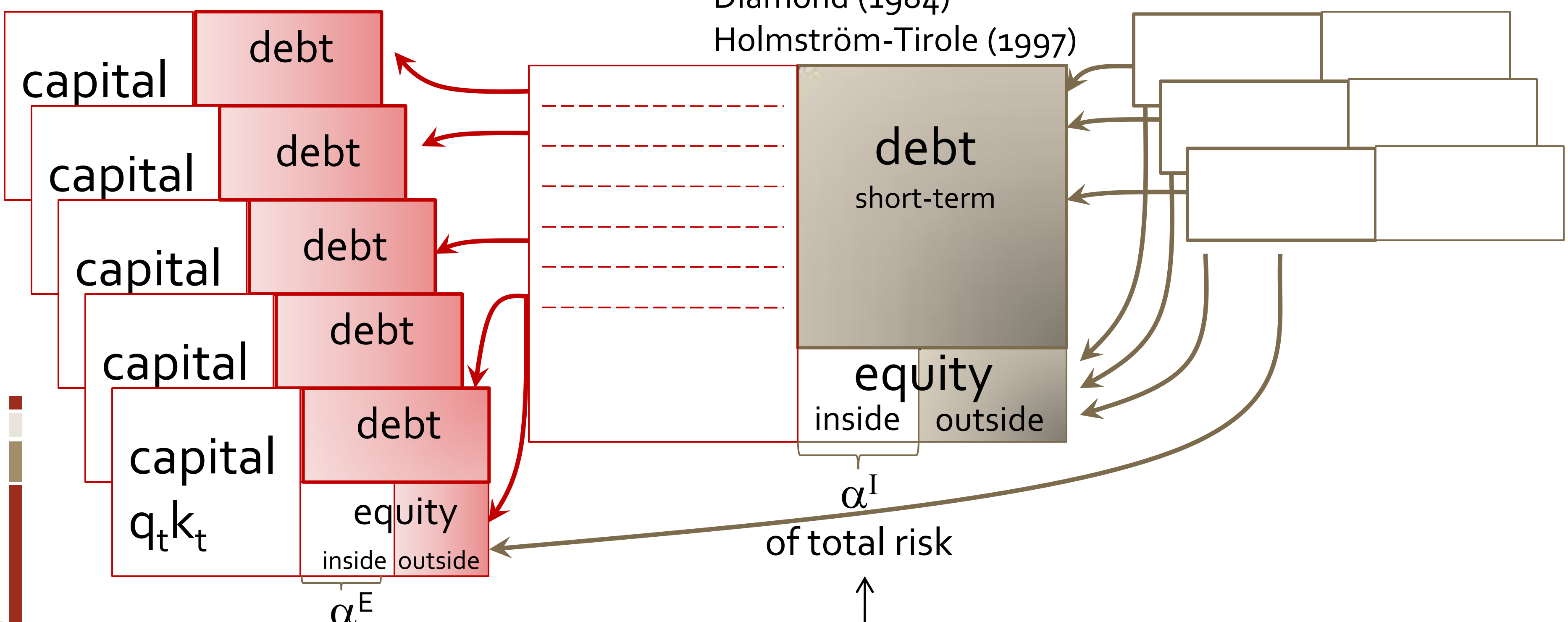
- We can decompose the previous equations into static and dynamic multiplier effects
  - Static effect results from assuming $q_{t+1} = q^*$
- Static multiplier:
  - $\widehat{K}_t^s = -\Delta$
  - $\hat{q}_t^S = -\frac{(R-1)}{R}\frac{1}{\xi}\Delta$
- Dynamic multiplier:
  - $\widehat{K}_t^D = -\frac{1}{(\xi+1)(R-1)}\Delta$
  - $\hat{q}_t^D = -\frac{1}{R}\frac{1}{\xi}\Delta$

BruSan10: Instability & Non-Linear Effects

- Previous papers only considered log-linearized solutions around steady state
- Brunnermeier & Sannikov (2010) build a continuous time model to study full dynamics
  - Show that financial system exhibits inherent instability due to highly non-linear effects
  - These effects are asymmetric and only arise in the downturn
- Agents choose a *capital cushion*
  - Mitigates moderate shocks near steady state
  - High volatility away from steady state

BS: Model overview

- Productive
- Intermediary
  - Monitoring
    Diamond (1984)
    Holmström-Tirole (1997)
- Less productive



incentive for entrepreneur to exert effort

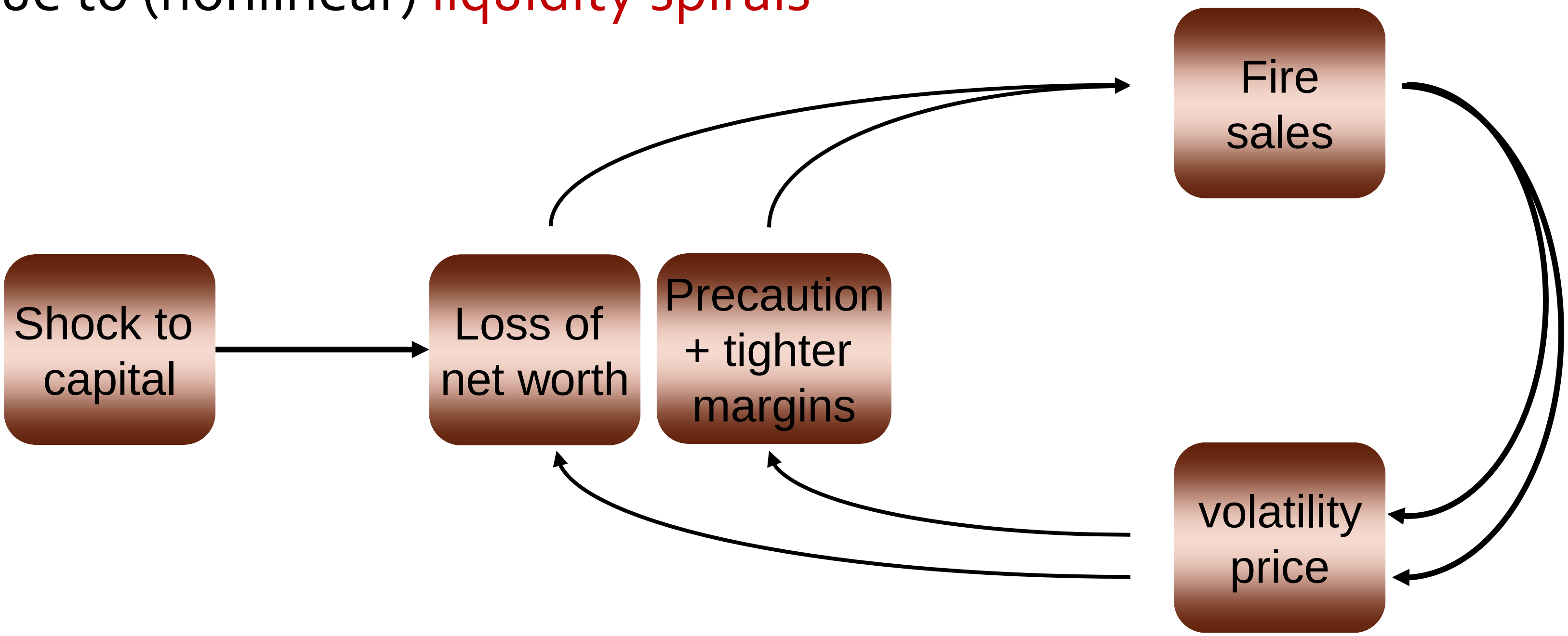
incentive for intermediaries to monitor (have to hold outside equity)

BS: Preview of results

- Full equilibrium dynamics + volatility dynamics
  - Near "steady state"
    - (large) payouts balance profit making
    - intermediaries must be unconstrained and amplification is low
  - Below "steady state"
    - intermediaries constrained, try to preserve capital leading to high amplification and volatility → precaution
- Crises episodes have significant endogenous risk, correlated asset prices, larger spreads and risk premia
- "Volatility paradox"
- SDF is driven by constraint & $c \geq 0$
- Securitization and hedging of idiosyncratic risks can lead to higher leverage, and greater systemic risk

BS: … with volatility dynamics + precaution

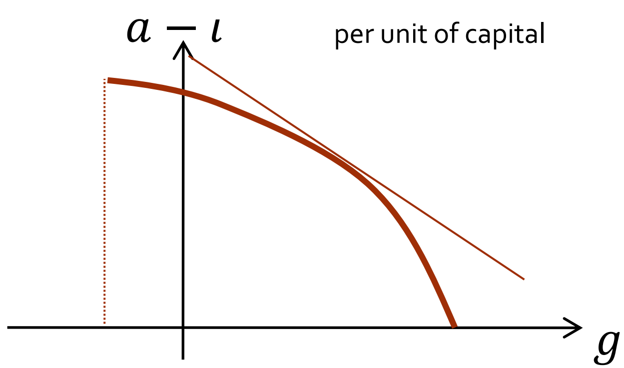
- Unstable dynamics away from steady state due to (nonlinear) liquidity spirals



- Volatility dynamics leads affects size of "safety cushion"
  - Note: log-linearization with zero probability shocks → no safety cushion

BS: Model details

- Output $y_t = a k_t$ (spend for consumption - investment)
- Capital $dk_t = \underbrace{(\Phi(\iota_t) - \delta)}_{=g} k_t dt + \sigma k_t dZ_t$ ← investment rate
- Agents
  - More productive
    - $U = E_0[\int_0^\infty e^{-\rho t} c_t \, dt]$
    - Production frontier



  - Less productive
    - $U = E_0[\int_0^\infty e^{-rt} c_t \, dt]$
    - Production frontier
      - $\underline{\delta} > \delta$
      - $\underline{\iota}_t = 0$
  - Endogenous price process for capital

$$dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$$

$$q_t \geq \underline{q} = \frac{a}{r + \underline{\delta}}$$ if HH limited to buy-hold strategy

BS: Market value of capital/assets $k_t q_t$

- Capital
  - $dk_t = g(\iota)k_t dt + \sigma k_t dZ_t$ "cash flow news" (dividends a_t)
- Price
  - $dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$ "SDF news"

- $k_t q_t$ value dynamics

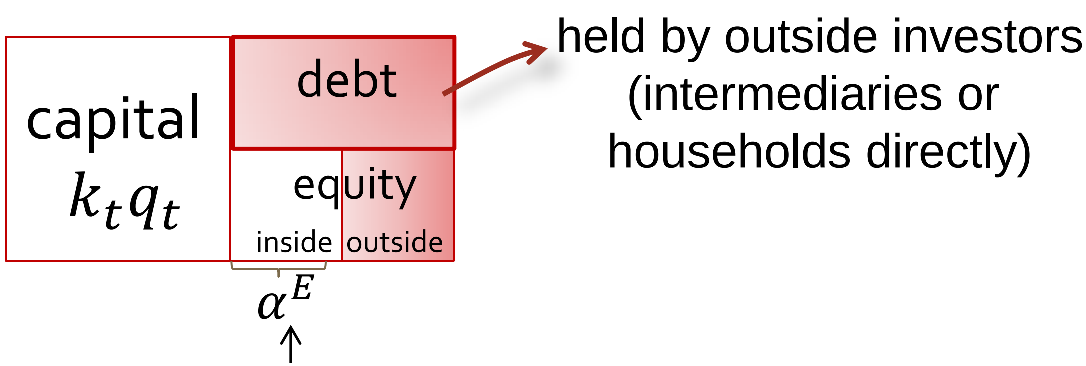
BS: Market value of capital/assets $k_t q_t$

- Capital
  - $dk_t = g(\iota)k_t dt + \sigma k_t dZ_t$ exogenous risk
- Price
  - $dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$ endogenous risk
- $k_t q_t$ value dynamics
  - $d(k_t q_t) =$
    $\left(\Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right)(k_t q_t)dt + \left(\sigma + \sigma_t^q\right)(k_t q_t)dZ_t$

    exogenous endogenous
    risk
  - Ito's Lemma product rule: $d(X_t Y_t) = dX_t Y_t + X_t dY_t + \sigma^X \sigma^Y dt$

BS: Contracting friction

- Focus on contracts in which agents is required to hold sufficient levered equity stake in projects



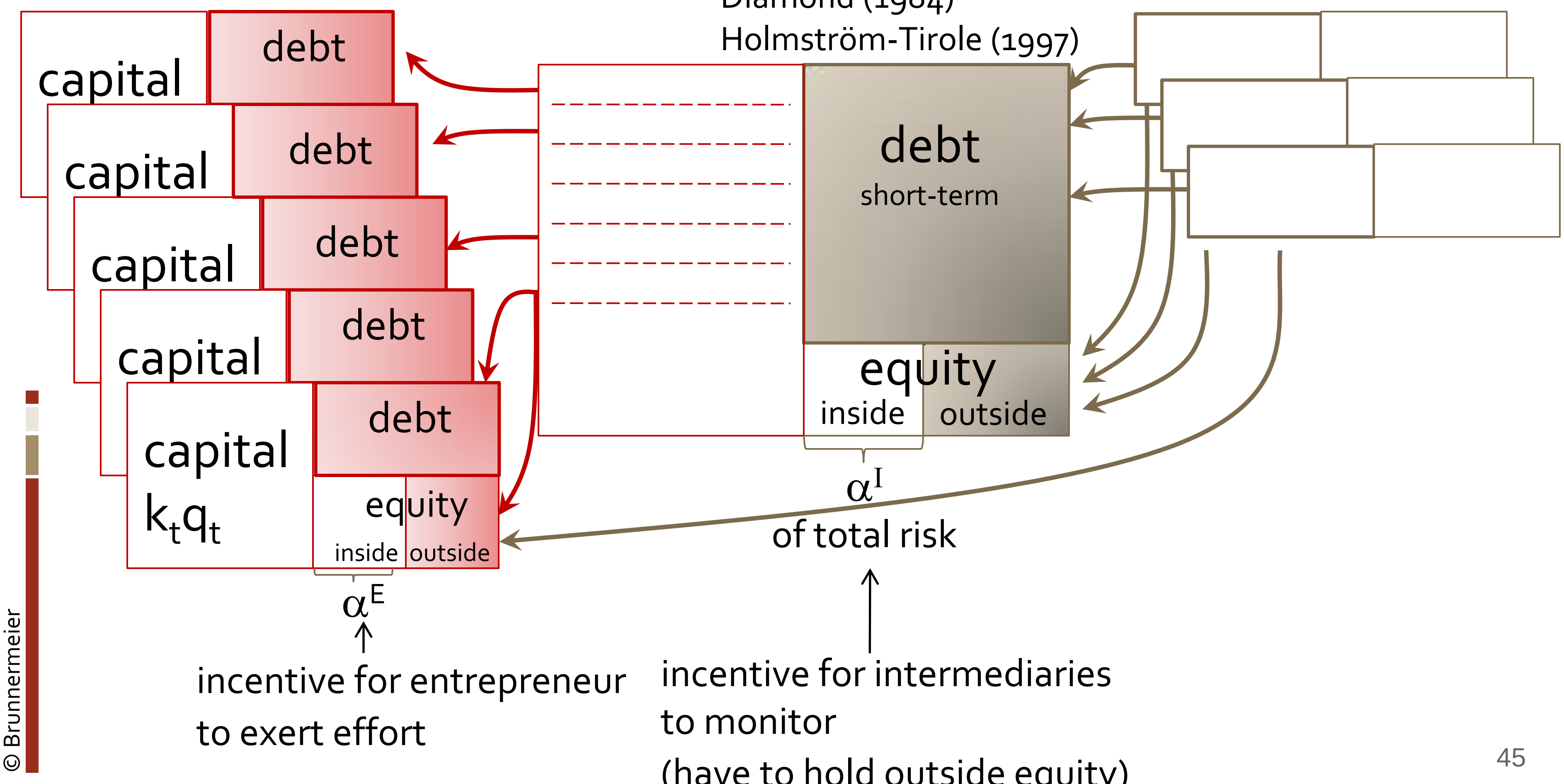
- The more risk entrepreneur wants to unload, the more they have to be monitored (by someone who takes on exposure)

BS: Microfoundation of contracts (extra)

- Agency problem of entrepreneur
  - Increase capital depreciation rate, private benefit b per $1 destroyed
  - Incentive constraint: entrepreneur equity stake $\geq$ b
- Are these contracts optimal? No
  - Entrepreneur reward depends on $k_t q_t$, but q_t is determined by market – why not hedge q_t to get a better performance?
  - Shocks to k_t are common across entrepreneurs, why not hedge those and get first best?
  - In practice markets aggregate information to determine $k_t q_t$, but hard to distinguish between shocks to k_t (cash flow news) and q_t (SDF news)
- Optimal contracts get first-best, but miss important phenomena
- Same as in Kiyotaki & Moore, BGG, He & Krishnamurthy

BS: Interlinked balance sheets

- Productive
- Intermediary
  - Monitoring
    Diamond (1984)
    Holmström-Tirole (1997)
- Less productive



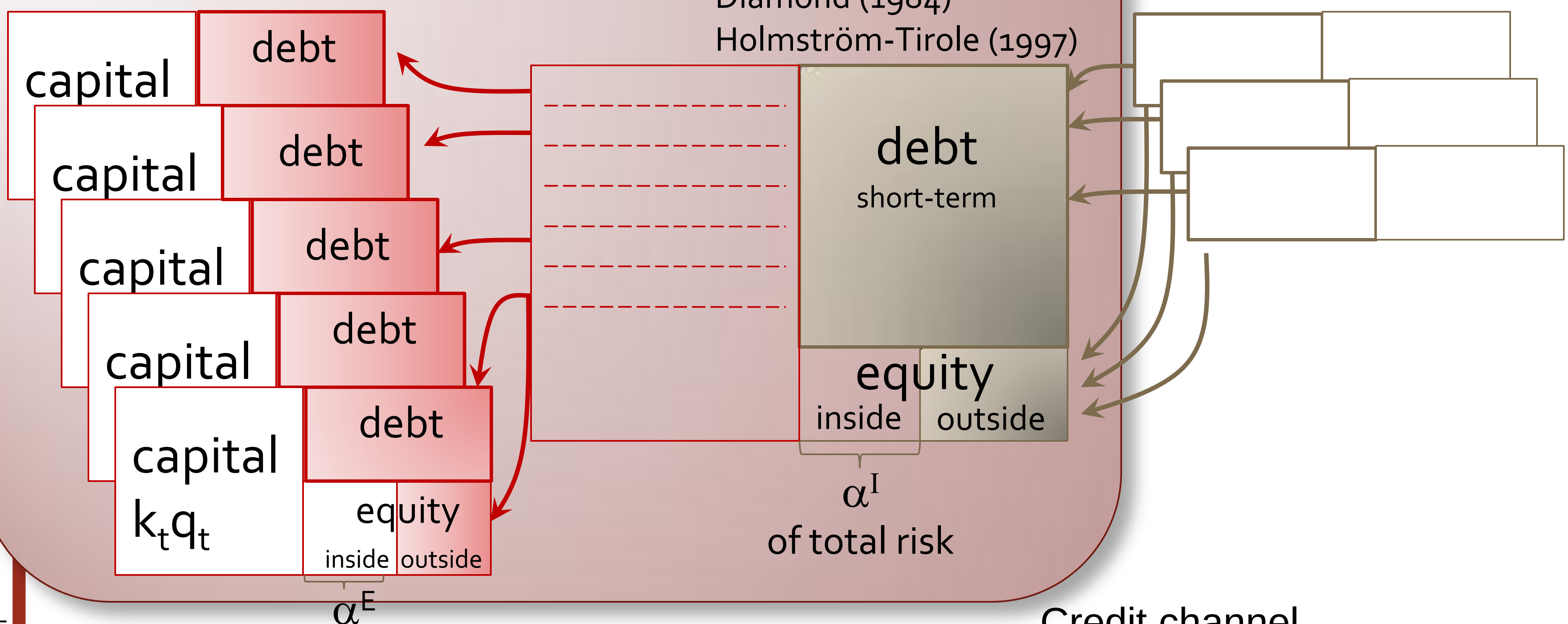
BS: Microfoundation of capital structures

- *Assumption:* value of assets $q_t k_t^i$ is contractable, k_t^i not
- Agency problem of entrepreneur
  - Can take projects w/ NPV<0, private benefit b(*m*)<1 per $1 destroyed
  - *m* is amount of monitoring by intermediary
  - Incentive constraint: $\alpha^E \geq b(m)$, binds in equ. $\Rightarrow \alpha^E(m)$
- Agency problem of intermediary
  - Save monitoring cost c(*m*) per $1 if shirking
  - Incentive constraint: $\alpha^I \geq c(m)$
- Solvency constraint: $n_t \geq 0$ (implied by IC constraints)
- Assume c(*m*) + b(*m*) is a constant for all *m*

  entrepreneurs' & intermediaries' net worth are substitutes
  - Special case: if entrepreneurs' net worth =0, then *m* s.t. b(*m*)=0

BS: Merging productive HH & Intermediaries

- Productive
- Intermediary
  - Monitoring
    Diamond (1984)
    Holmström-Tirole (1997)
- Less productive

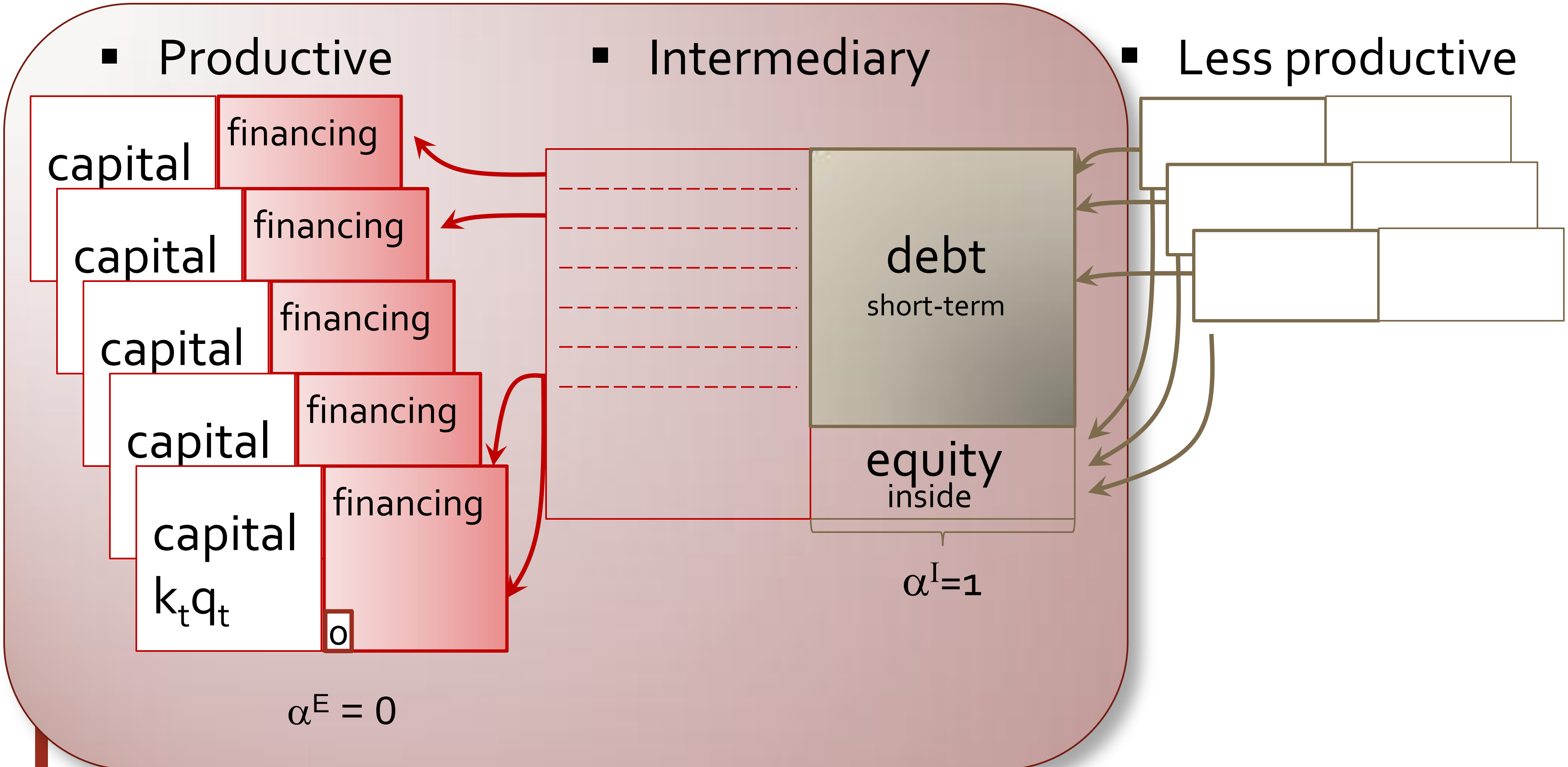


$\alpha := \alpha^E + \alpha^I \geq b(m) + c(m)$

"merged experts"

Credit channel
- Lending channel
- Borrowers' balance sheet channel

BS: Merging productive HH & Intermediaries



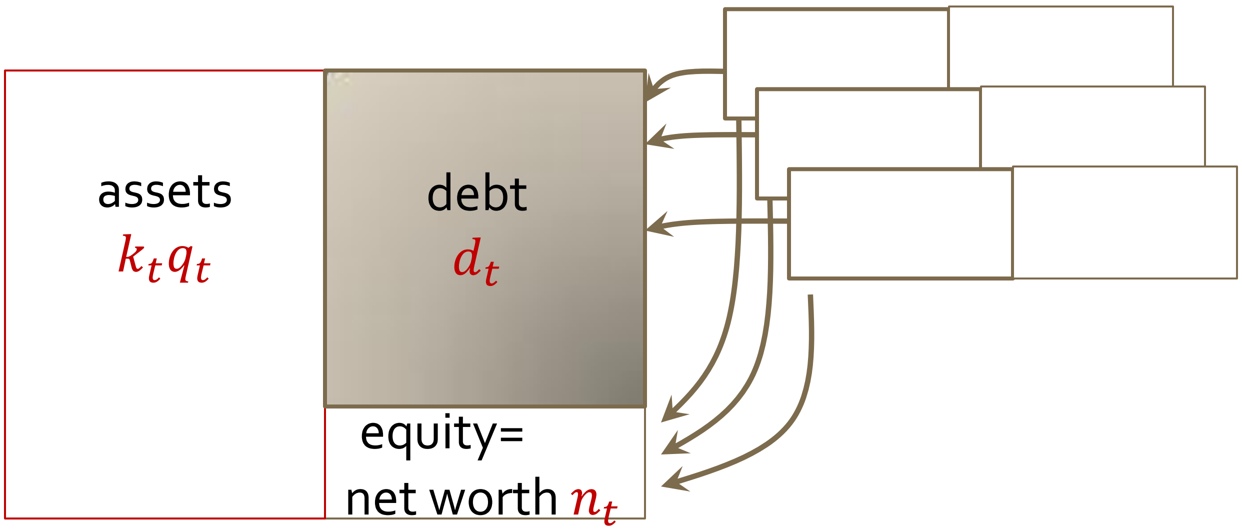
- Productive entrepreneurs have no capital, $\alpha^E = 0$

  Perfect monitoring required, b(m)=0

- Intermediary can't issue outside equity, $\alpha^I = 1$ (appropriate choice of b(m), c(m))

BS: Balance sheet dynamics

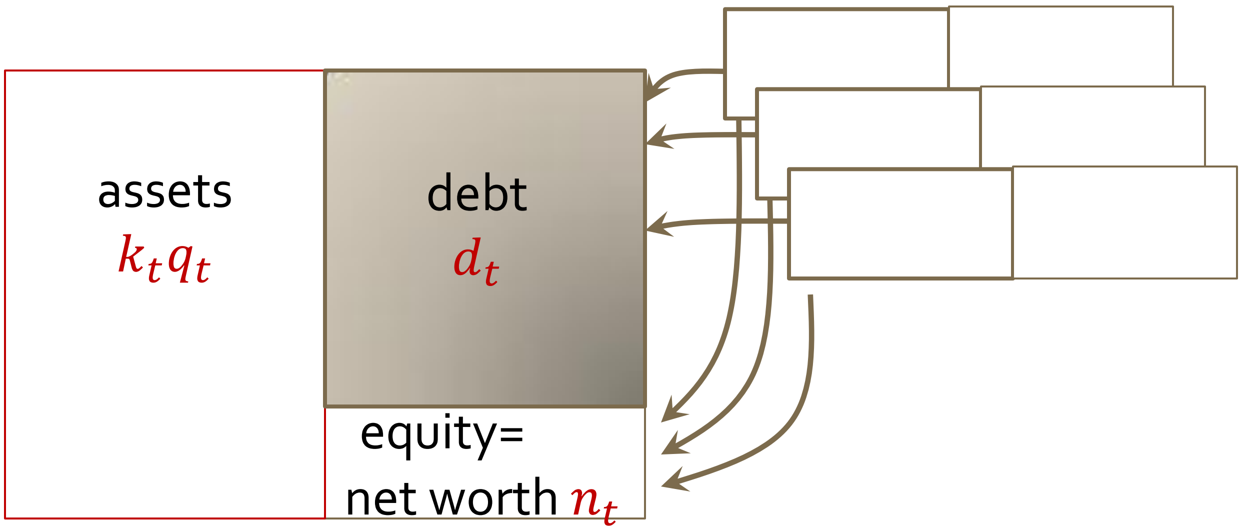
- Productive
- Intermediary
- Less productive



assume $\alpha = 1$ (for today)

BS: Balance sheet dynamics

- Productive
- Intermediary
- Less productive



$$dr_t^k = \left(\frac{a-\iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right)dt + \left(\sigma + \sigma_t^q\right)dZ_t$$

$$dn_t = rn_t dt + (dr^k - rdt)(k_t q_t) - dc_t = \cdots$$

BS: Intuition – main forces at work

- Investment
  - *Scale up*
    - Scalable profitable investment opportunity
    - Higher leverage (borrow at r)
  - *Scale back*
    - Precaution: - don't exploit full (GE) debt capacity – "dry powder"
      - Ultimately, stay away from fire-sales prices
      - Debt can't be rolled over if $d > k_t \underline{q}$ (note, price is depressed)
      - Solvency constraint
- Consumption
  - Consume *early* and borrow $r < \rho$
  - Consume *late* to overcome investment frictions

aggregate leverage!

BS: Definition of equilibrium

- An equilibrium consists of functions that for each history of macro shocks $\{Z_s, s \in [0, t]\}$ specify
  - q_t the price of capital
  - k_t^i, k_t^h capital holdings and
  - dc_t^i, dc_t^h consumption of representative expert and households
  - ι_t rate of internal investment of a representative expert, per unit of capital
  - r_t the risk-free rate
- such that
  - intermediaries and households maximize their utility, given prices q_t as given and
  - markets for capital and consumption goods clear

BS: Solving for equilibrium

1. Households: risk free rate of r_t = households discount rate
   - Makes HH indifferent between consuming and saving, s.t. consumption market clears
   - Required return when their capital >0 $\underbrace{\frac{a}{q_t} - \underline{\delta} + \mu_t^q + \sigma\sigma_t^q}_{expected\ return\ from\ capital} = r$

2. Experts choose $\{k_t, \iota_t, c_t\}$ dynamically to maximize utility

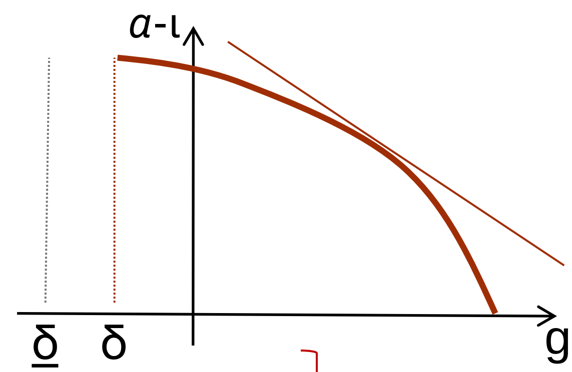
$$\max_{c,\iota,k} E\left[\int_0^\infty e^{-\rho t} dc_t\right] \quad \text{s.t.}$$

$$dn_t = -dc_t + \left(\Phi(i_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right)(k_t q_t)dt$$
$$+\left(\sigma + \sigma_t^q\right)(k_t q_t)dZ_t + [(a - \iota_t)k_t - rd_t]dt$$

$$dn_t \geq 0$$

3. Markets clear: total demand for capital is K_t

BS: Solving for equilibrium



1. Internal investment (static)
2. External investment k_t
   - Given price dynamics $dq_t/q_t = \mu_t^q dt + \sigma_t^q dZ_t$
   - Solvency constraint $n_t \geq 0$
3. When to consume? dc_t

dynamic optimization

- Bellman equation w/ value function $\theta_t n_t$

payoff experts generate from a dollar of net worth by trading undervalued capital

proportional to net worth, atomistic experts have no price impact

$$\rho\theta_t n_t dt = \max_{k_t, dc_t} E\left[dc_t + d(\theta_t n_t)\right]$$

BS: Solving dynamic optimization

- Let value of extra $

$$d\theta_t = \mu_t^\theta \theta_t dt + \sigma_t^\theta \theta_t dZ_t$$

  - recall $dn_t = \ldots.$
- Use Ito's lemma to expand the Bellman equation

$$\rho\theta_t n_t dt = \max_{k_t, dc_t} E[dc_t + d(\theta_t n_t)]$$

  - Risk free: $\underbrace{r}_{risk-free} + \underbrace{\mu_t^\theta}_{E[change\ of\ invest-ment\ opportunities]} = \underbrace{\rho}_{required\ return}$
  - Capital: $\underbrace{\frac{a}{q_t} + g_t + \mu_t^q + \sigma\sigma_t^q - r}_{E[excess\ return\ of\ capital]} = \underbrace{-\sigma_t^\theta(\sigma + \sigma_t^q)}_{capital\ risk\ premium}$
  - $\theta_t \geq 1$, and $dc_t^i > 0$ only when $\theta_t = 1$.
- $e^{-\rho t}\theta_t/\theta_0$ is the experts' stochastic discount factor

BS: Scale invariance

- Model is scale invariant
  - K_t total physical capital
  - N_t total net worth of all experts
- Solve q_t and θ_t as a function of the single state variable
  - $\eta_t = \frac{N_t}{K_t}$

⇒ Mechanic application of Ito's lemma
Pricing equations get transformed into
ordinary differential equations for $q(\eta)$ and $\theta(\eta)$

BS: Solution mechanics – detail slide

- Start with: $dK_t = g(q_t)K_t\,dt + \sigma K_t\,dZ_t$,
  $dN_t = r\,N_t\,dt - dC_t + a\,K_t\,dt - \iota(q_t)\,K_t\,dt +$
  $K_t\,q_t\,[\,(g(q_t) - r + \mu_t^q + \sigma\sigma_t^q)\,dt + (\sigma + \sigma_t^q)\,dZ_t\,]$
- Ito's lemma $\Rightarrow d\eta_t = d(N_t/K_t) = (r - g(q_t) + \sigma^2)\,(\eta_t - q_t)\,dt$
  $+ (a - \iota(q_t)\ + q_t\,\mu_t^q)\,dt + (q_t(\sigma + \sigma_t^q) - \sigma\,\eta_t)\,dZ_t$
- $q_t\,\sigma_t^q = q'(\eta)\,(q_t(\sigma + \sigma_t^q) - \sigma\,\eta_t)$

  $\Rightarrow$

$$\sigma_t^q = \frac{q'(\eta_t)\sigma(q_t - \eta_t)}{q_t(1 - q'(\eta_t))} \text{ and } \sigma_t^\eta = \frac{\sigma(q_t - \eta_t)}{1 - q'(\eta_t)}$$

- $q_t\,\mu_t^q = q'(\eta) * ((r - g(q_t) + \sigma^2)\,(\eta_t - q_t) + a - \iota(q_t)\ + p_t\,\mu_t^q)$
  $+ \frac{1}{2}\,(q_t(\sigma + \sigma_t^q) - \sigma\,\eta_t)^2\,q''(\eta) \Rightarrow$

$$\mu_t^q = \frac{q'(\eta_t)[(r - g(q_t) + \sigma^2)(\eta_t - q_t) + a - \iota(q_t)] + \frac{1}{2}(\sigma_t^\eta)^2 q''(\eta_t)}{q_t(1 - q'(\eta_t))}$$

BS: Solving… - detail slide

$(a-\iota(q_t))/q_t + g(q_t) + \mu_t^q + \sigma\sigma_t^q - r = -\theta'(\eta_t)/\theta(\eta_t)\,\sigma_t^\eta(\sigma+\sigma_t^q)$ and

$$\mu_t^q = \frac{q'(\eta_t)[(r-g(q_t)+\sigma^2)(\eta_t-q_t)+a-\iota(q_t)]+\frac{1}{2}(\sigma_t^\eta)^2 q''(\eta_t)}{q_t(1-q'(\eta_t))} \Rightarrow$$

$$\frac{a-\iota(q_t)}{q_t}+g(q_t)+\frac{q'(\eta_t)[(r-g(q_t)+\sigma^2)(\eta_t-q_t)+a-\iota(q_t)]+\frac{1}{2}(\sigma_t^\eta)^2 q''(\eta_t)}{q_t(1-q'(\eta_t))}$$
$$+\sigma\sigma_t^q - r = -\frac{\theta'(\eta_t)}{\theta(\eta_t)}\sigma_t^\eta(\sigma+\sigma_t^q)$$

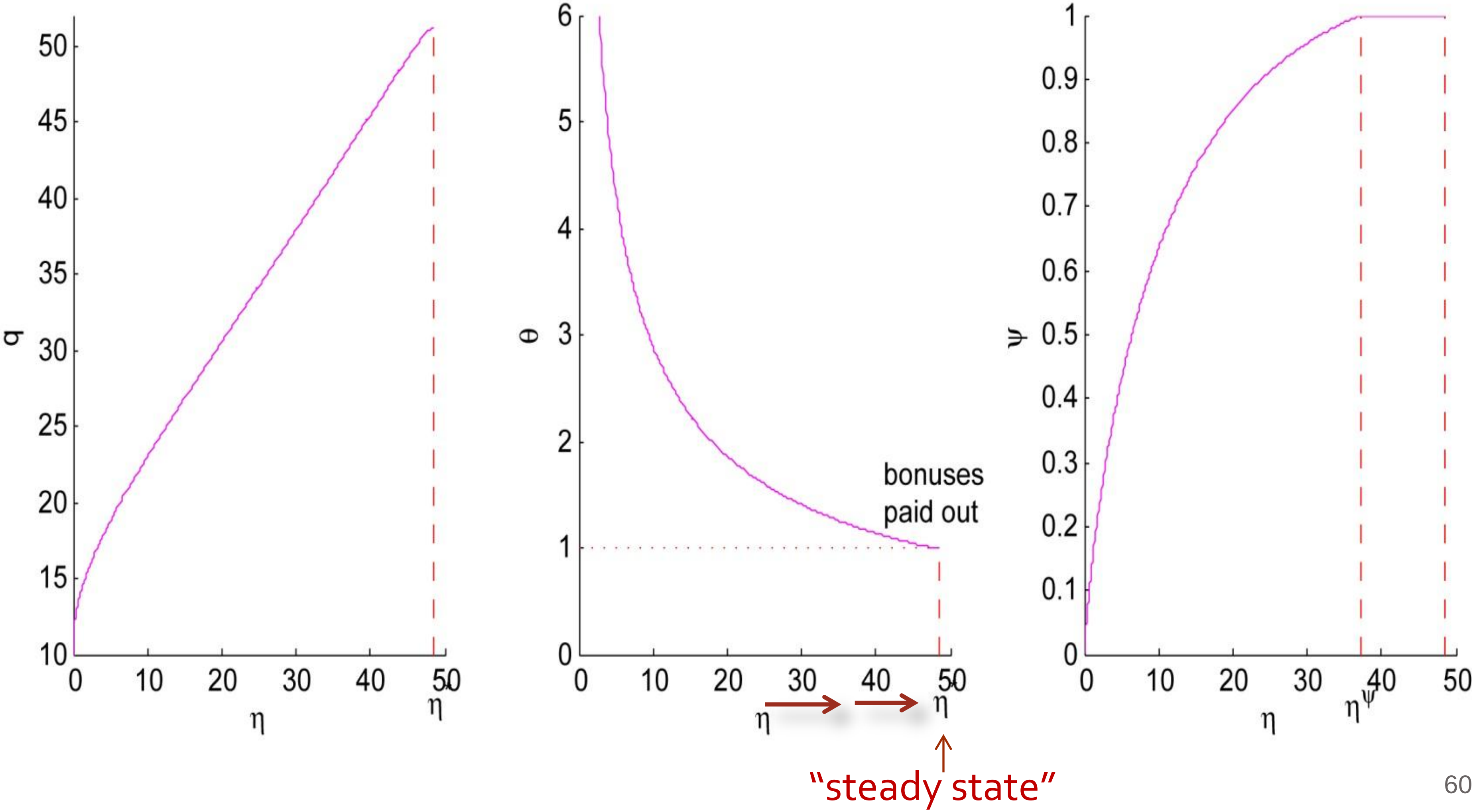
$(\rho\text{-}r)\,\theta(\eta) = \mu_t^\theta \Rightarrow$ where $\sigma_t^q = \frac{q'(\eta_t)\sigma(q_t-\eta_t)}{q_t(1-q'(\eta_t))}$ and $\sigma_t^\eta = \frac{\sigma(q_t-\eta_t)}{1-q'(\eta_t)}$

$$(\rho-r)\theta(\eta) = \theta'(\eta)((r-g(q_t)+\sigma^2)(\eta-q_t)+a-\iota(q_t)+q_t\mu_t^q)+\frac{1}{2}(\sigma_t^\eta)^2\theta''(\eta_t)$$

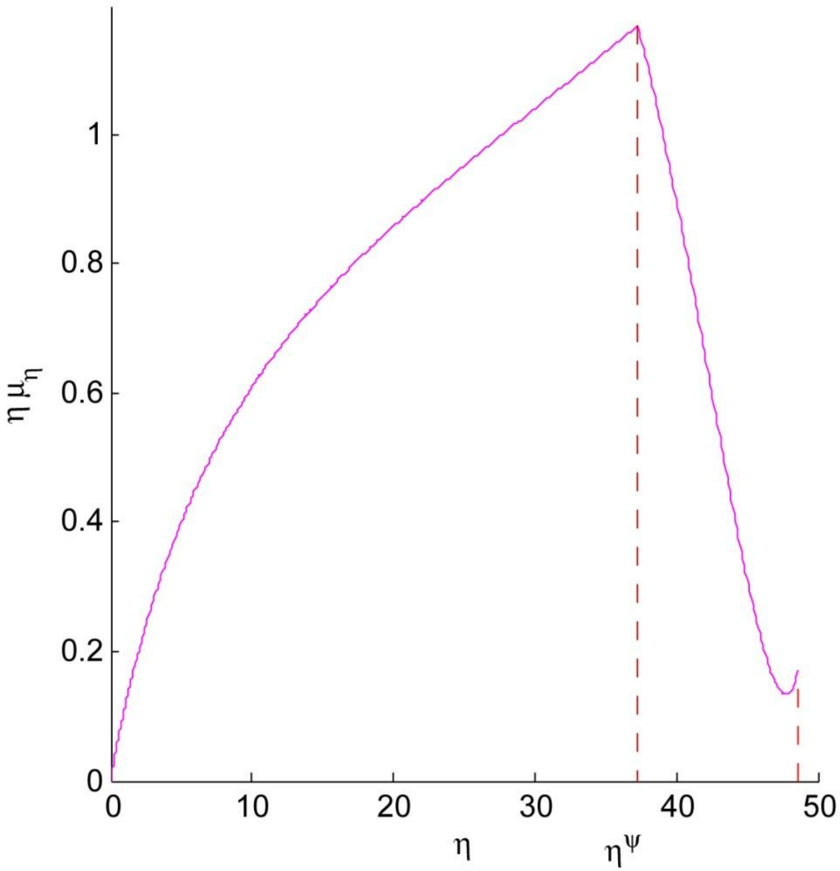
- Boundary conditions: $q(0) = a/(r+\delta^*)$, $q'(\eta^*) = 0$, $\theta(\eta^*) = 1$, $\theta'(\eta^*) = 0$

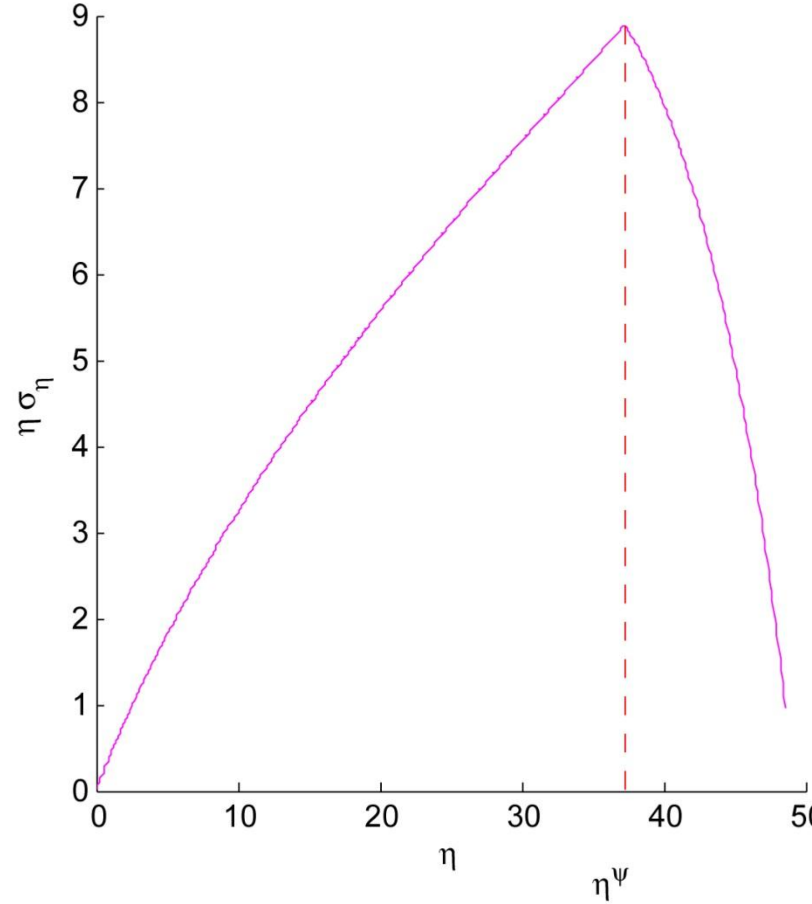
BS: Equilibrium

- Boundary conditions: $q(0) = \underline{q}$, $\theta(0) = \infty$, $\theta(\eta^*) = 1$, $q'(\eta^*) = \theta'(\eta^*) = 0$

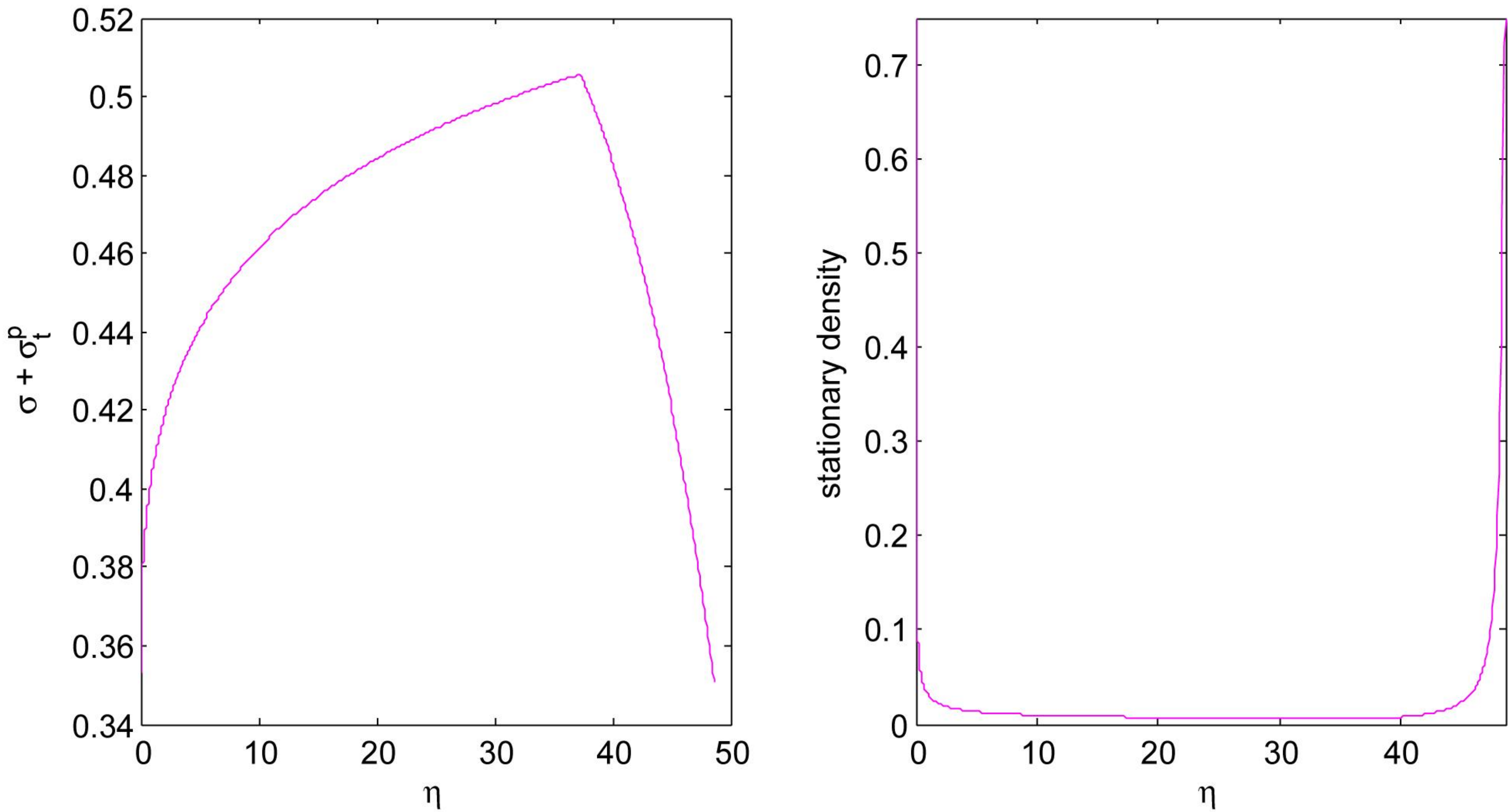


BS: Equilibrium dynamics



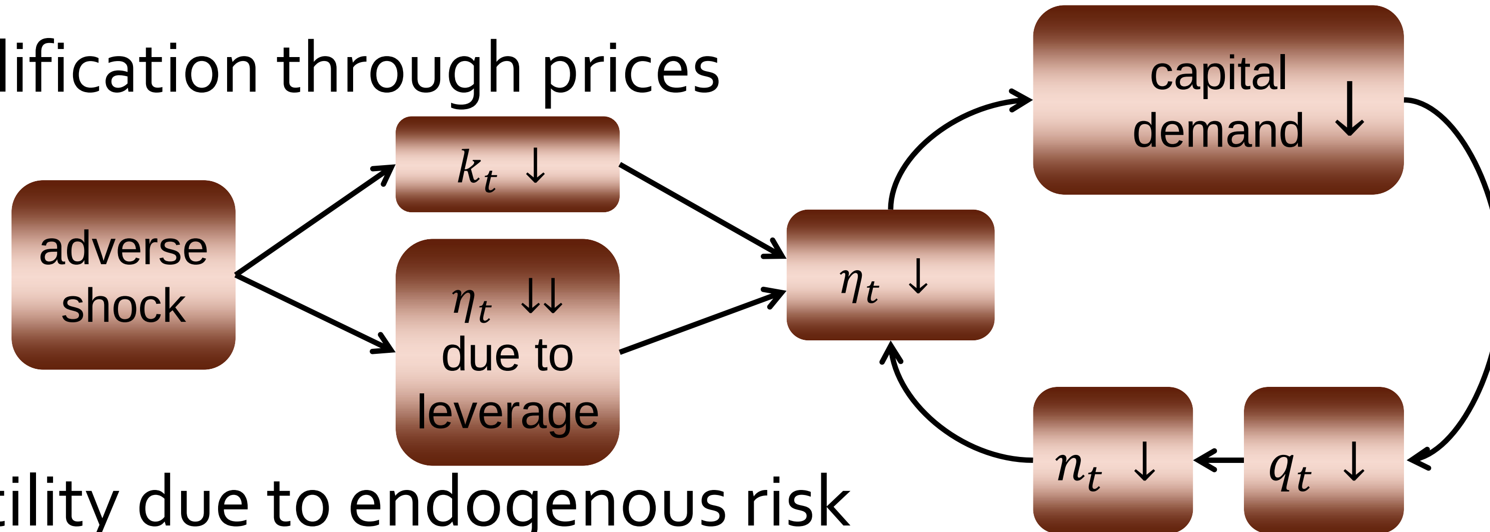


BS: Endogenous risk & "Instability"



BS: Endogenous Risk through Amplification

- Amplification through prices



- Volatility due to endogenous risk

$$\sigma_t^q = \frac{q'(\eta_t)\sigma(q_t - \eta_t)}{1 - q'(\eta_t)}.$$

← amplification

- Key to amplification is $q'(\eta)$
  - Depends how constrained experts are

BS: Dynamics near and away from SS

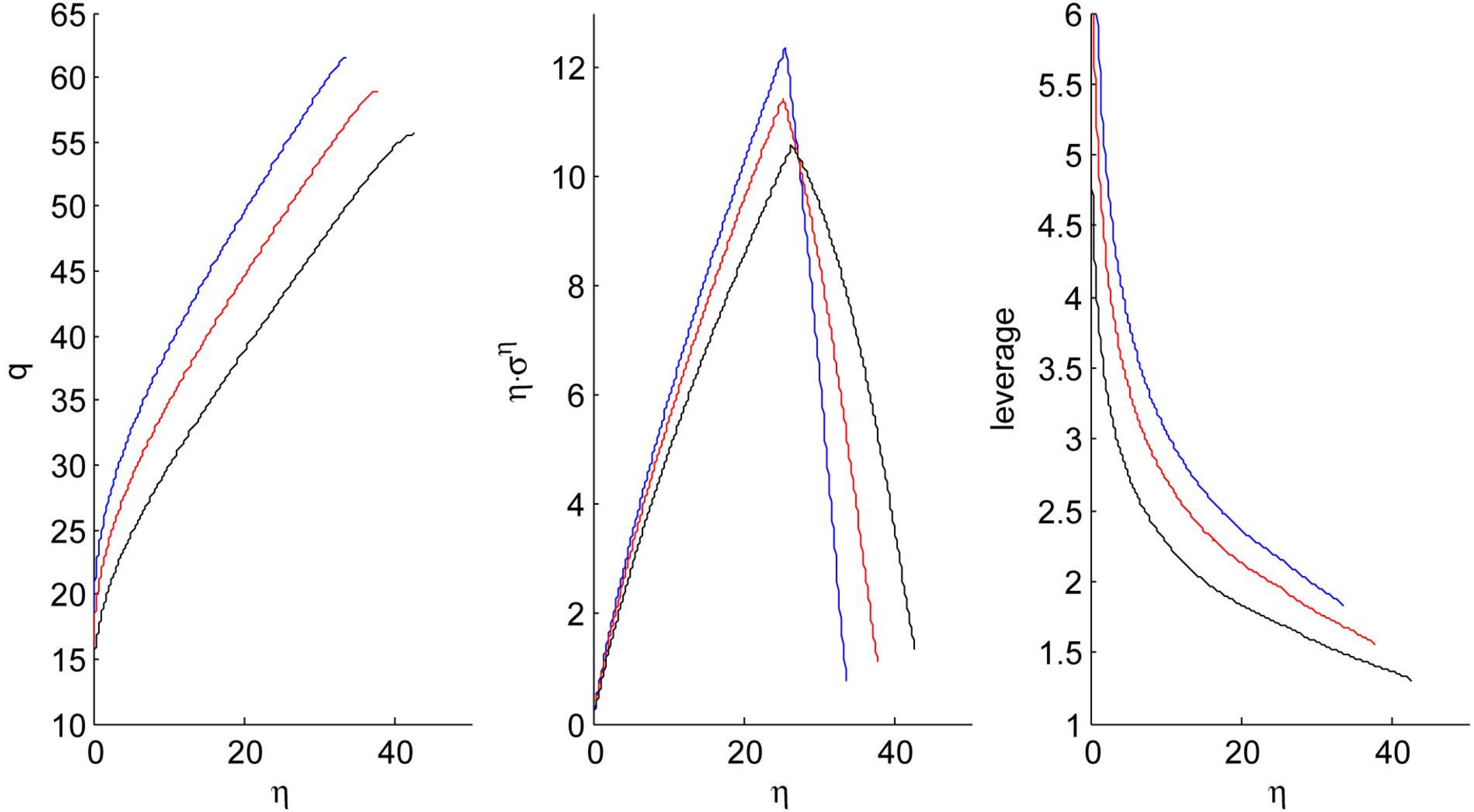
- Intermediaries choose payouts endogenously
  - Exogenous exit rate in BGG/KM
  - Payouts occur when intermediaries are least constrained

$$q'(\eta^*) = 0$$

- Steady state: experts unconstrained
  - Bad shock leads to lower payout rather than lower capital demand
  - $q'(\eta^*) = 0, \sigma_t^q(\eta^*) = 0$
- Below steady state: experts constrained
  - Negative shock leads to lower demand
  - $q'(\eta^*)$ is high, strong amplification, $\sigma_t^q(\eta^*)$ is high
  - … but when η is close to 0,
    $q \approx \underline{q}(\eta_t), q'(\eta)$ and $\sigma_t^q(\eta^*)$ is low

Note difference to BGG/KM

BS: "Volatility Paradox" … σ (.025 ,.05 ,.1)



- As σ decreases, η^* goes down, $q(\eta^*)$ goes up, $\sigma^\eta(\eta^*)$ may go up, max σ^η goes up

BS: Ext1: asset pricing (cross section)

- **Capital:** Correlation increases with σ^q
  - Extend model to many types i of capital

$$\frac{dk_t^i}{k_t^i} = \left(\Phi(\iota_t^i) - \delta\right)dt + \sigma dZ_t + \sigma' dz_t^i$$

aggregate shock (under σdZ_t) — uncorrelated shock (under $\sigma' dz_t^i$)

  - Experts hold diversified portfolios
    - Equilibrium looks as before, (all types of capital have same price) but
    - Volatility of $q_t k_t$ is $\sigma + \sigma' + \sigma^q$
    - Endogenous risk is perfectly correlated, exogenous risk not
    - For uncorrelated z^i and z^j
      correlation $(q_t^i k_t^i, q_t^j k_t^j)$ is $(\sigma + \sigma^q)/(\sigma + \sigma' + \sigma^q)$
      which is increasing in σ^q

BS: Ext1: asset pricing (cross section)

- **Outside equity:**
  - Negative skewness
  - Excess volatility
  - Pricing kernel: e^{-rt}
    - Needs risk aversion!

- **Derivatives:**
  - Volatility smirk (Bates 2000)
  - More pronounced for index options (Driessen et al. 2009)

BS: Ext2: Idiosyncratic jump losses

$$dk_t^i = gk_t^i dt + \sigma k_t^i dZ_t + k_t^i dJ_t^i$$

- J_t^i is an idiosyncratic compensated Poisson loss process, recovery distribution F and intensity $\lambda(\sigma_t^q)$
- $q_t k_t^i$ drops below debt d_t, costly state verification

- Time-varying interest rate spread
- Allows for direct comparison with BGG

BS: Ext. 2: Idiosyncratic losses

$$dk_t^i = gk_t^i dt + \sigma k_t^i dZ_t + k_t^i dJ_t^i$$

- J_t^i is an idiosyncratic compensated Poisson loss process, recovery distribution F and intensity $\lambda(\sigma_t^q)$
- $q_t k_t^i$ drops below debt d_t, costly state verification
- Debt holders' loss rate $\lambda(\sigma^p) v \int_0^{\frac{d}{v}} (\frac{d}{v} - x) dF(x)$
- Verification cost rate

$$\lambda(\sigma^p) v \underbrace{\int_0^{\frac{d}{v}} cx dF(x)}_{C(\frac{d}{v})}$$

- Leverage bounded not only by precautionary motive, but also by the cost of borrowing

Asset	Liabilities
$v_t = k_t q_t$	$d_t = k_t q_t - n_t$
	n_t

BS: Ext2: Equilibrium

- Experts borrowing rate $> r$
  - Compensates for verification cost
- Rate depends on leverage, price volatility
- $d\eta_t$ = diffusion process (without jumps) because losses cancel out in aggregate

BS: Ext3: Securitization

- Experts can contract on shocks Z_t and dJ_t^i directly among each other, zero contracting costs
- In principle, good thing (avoid verification costs)
- Equilibrium
  - experts fully hedge idiosyncratic risks
  - experts hold their share (do not hedge) aggregate risk Z_t, market price of risk depends on $\sigma_t^\theta(\sigma + \sigma_t^q)$
  - with securitization experts lever up more (as a function of η_t) and bonus payments occur "sooner"
  - financial system becomes less stable
  - risk taking is endogenous (Arrow 1971, Obstfeld 1994)

BS: Conclusion

- Incorporate financial sector in macromodel
  - Higher growth
  - Exhibits instability
    - similar to existing models (BGG, KM) in term of persistence/amplification, but
    - non-linear liquidity spirals (away from steady state) lead to instability
- Risk taking is endogenous
  - "Volatility paradox:" Lower exogenous risk leads to greater leverage and may lead to higher endogenous risk
  - Correlation of assets increases in crisis
  - With idiosyncratic jumps: countercyclical credit spreads
  - Securitization helps share idiosyncratic risk, but leads to more endogenous risk taking and amplifies systemic risk
- Welfare: (Pecuniary) Externalities
  - excessive exposure to crises events

Overview

- Persistence
- Dynamic Amplification
  - Technological illiquidity BGG
  - Market illiquidity KM97
- Instability, Volatility Dynamics, Volatility Paradox
- Volatility and Credit Rationing/Margins/Leverage
- Demand for Liquid Assets

Credit Rationing

- Credit rationing refers to a failure of market clearing in credit
  - In particular, an excess demand for credit that fails to increase market interest rate
- Stiglitz, Weiss (1981) show how asymmetric information on risk can lead to credit rationing

Stiglitz, Weiss

- Entrepreneurs borrow from competitive lenders at interest rate r
  - Risky investment projects with $R \sim G(\cdot \,|\sigma_i)$
  - Mean preserving spreads, so heterogeneity is only in risk
- Assume entrepreneur borrows B
- Entrepreneur's payoff is convex in R
  - $\pi_e(R, r) = \max\{R - (1 + r)B, 0\}$
- Lender's payoff is concave in R
  - $\pi_l(R, r) = \min\{R, (1 + r)B\}$

SW: Adverse Selection

- Due to convexity, entrepreneur's expected payoff is increasing in riskiness σ_i
  - Only entrepreneurs with sufficiently risky projects will apply for loans, i.e. $\sigma_i \geq \sigma^*$
- Zero-profit condition: $\int \pi_e(R, r) dG(R|\sigma^*) = 0$
  - This determines cutoff σ^*
  - Note that σ^* is increasing in r
- Lender's payoff is not monotonic in r
  - Ex-post payoff is increasing in r
  - Higher cutoff σ^* leads to riskier selection of borrowers

SW: Credit Rationing

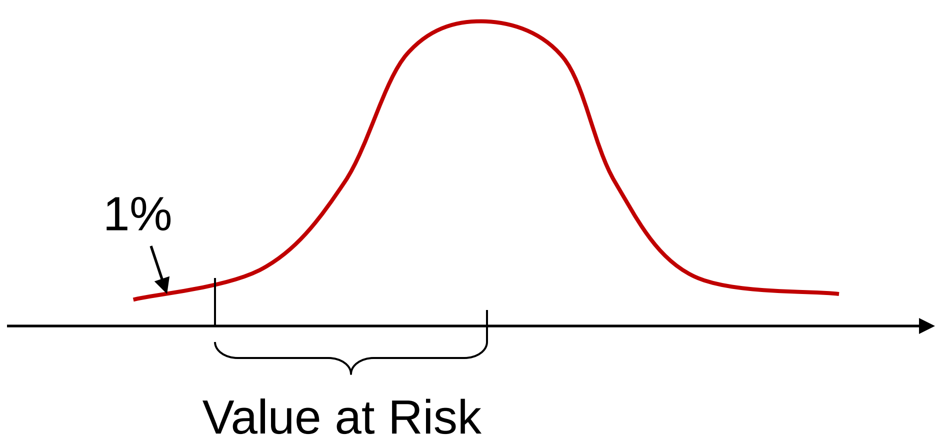
- Lenders will only lend at the profit maximizing interest rate r
- Excess demand for funds from borrowers will not increase the market rate
  - There exist entrepreneurs who would like to borrow, willing to pay a rate higher than the prevailing one
- Adverse selection leads to failure of credit markets

Brunnermeier-Pedersen: Margin Spiral

- For collateralized lending, debt constraints are directly linked to the volatility of collateral
  - Constraints are more binding in volatile environments
  - Feedback effect between volatility and constraints
- These margin spirals force agents to delever in times of crisis
  - Collateral runs
  - Multiple equilibria

BP: Margins – Value at Risk (VaR)

- Margins give incentive to hold well diversified portfolio
- How are margins set by brokers/exchanges?
  - **Value at Risk**: $\Pr(-(p_{t+1} - p_t) \geq m) = 1\%$



BP: Leverage and Margins

- Financing a *long position* of $x^{j+}_t > 0$ shares at price $p^j_t = 100$:
  - Borrow \$90\$ dollar per share;
  - Margin/haircut: $m^{j+}_t = 100 - 90 = 10$
  - Capital use: \$10 x^{j+}_t
- Financing a *short position* of $x^{j-}_t > 0$ shares:
  - Borrow securities, and lend collateral of 110 dollar per share
  - Short-sell securities at price of 100
  - Margin/haircut: $m^{j-}_t = 110 - 100 = 10$
  - Capital use: \$10 x^{j-}_t
- Positions frequently marked to market
  - payment of $x^j_t (p^j_t - p^j_{t-1})$ plus interest
  - margins potentially adjusted – *more later on this*
- Margins/haircuts must be financed with capital:

$$\sum_j (x^{j+}_t m^{j+}_t + x^{j-}_t m^{j-}_t) \leq W_t \text{, where } x^j = x_t^{j+} - x_t^{j-}$$

$$\text{with perfect cross-margining: } M_t (x_t^1, \ldots, x_t^J) \leq W_t$$

BP: Liquidity Concepts (recall)

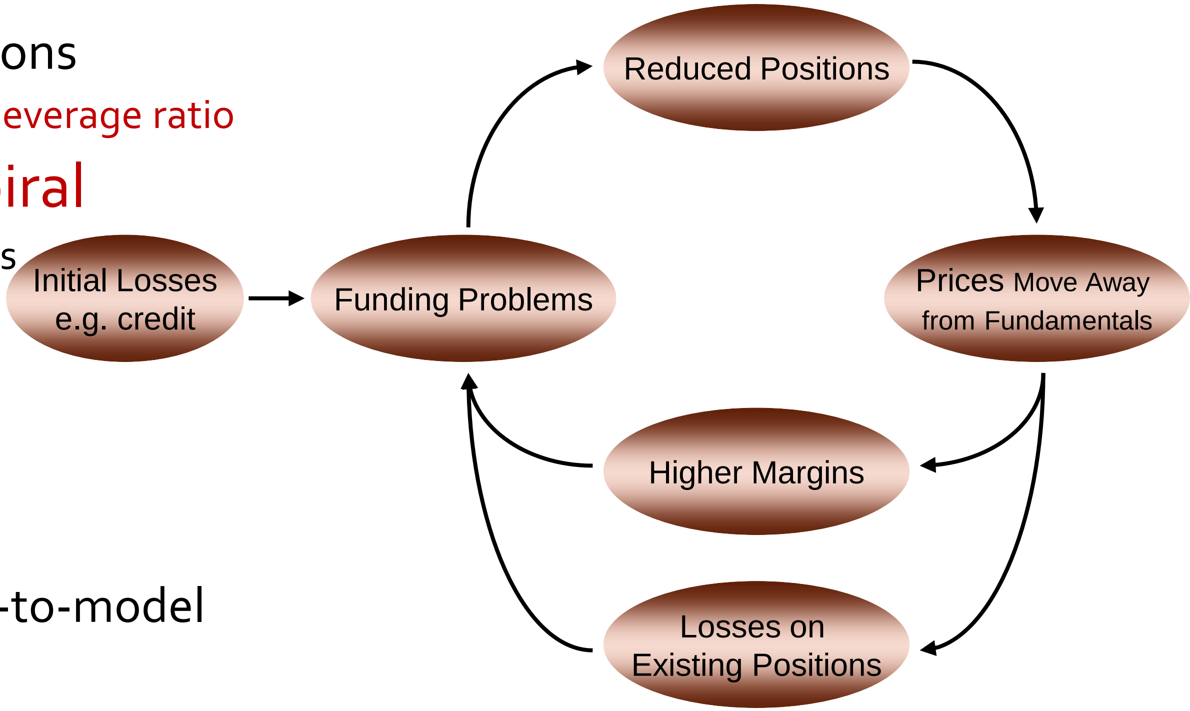
A	L
	Funding liquidity ▪ Can't **roll over** short term debt ▪ **Margin**-funding is recalled

BP: Liquidity Concepts (recall)

A	L
Market liquidity	**Funding liquidity**
▪ Can only sell assets at **fire-sale prices**	▪ Can't **roll over** short term debt ▪ **Margin**-funding is recalled

BP: Liquidity Spirals

- Borrowers' balance sheet
  - Loss spiral – like in BGG/KM
    - Net wealth > α x
      for asym. info reasons
    - constant or increasing leverage ratio
  - Margin/haircut spiral
    - Higher margins/haircuts
    - No rollover
    - redemptions
    - forces to delever



- Mark-to-market vs. mark-to-model
  - worsens loss spiral
  - improves margin spiral

Source: Brunnermeier & Pedersen (2009)

• Both spirals reinforce each other

BP: Margin Spirals - Intuition

1. Volatility of collateral increases
   - Permanent price shock is accompanied by higher future volatility (e.g. ARCH)
     - Realization how difficult it is to value structured products
   - Value-at-Risk shoots up
   - Margins/haircuts increase = collateral value declines
   - Funding liquidity dries up
   - Note: all "expert buyers" are hit at the same time, SV 92
2. Adverse selection of collateral
   - As margins/ABCP rate increase, selection of collateral worsens
   - SIVs sell-off high quality assets first (empirical evidence)
   - Remaining collateral is of worse quality

BP: Model Setup

- Time: t=0,1,2
- One asset with final asset payoff v (later: assets j=1,…,J)
- Market illiquidity measure: $\Lambda_t = |E_t(v) - p_t|$

  (deviation from "fair value" due to selling/buying pressure)

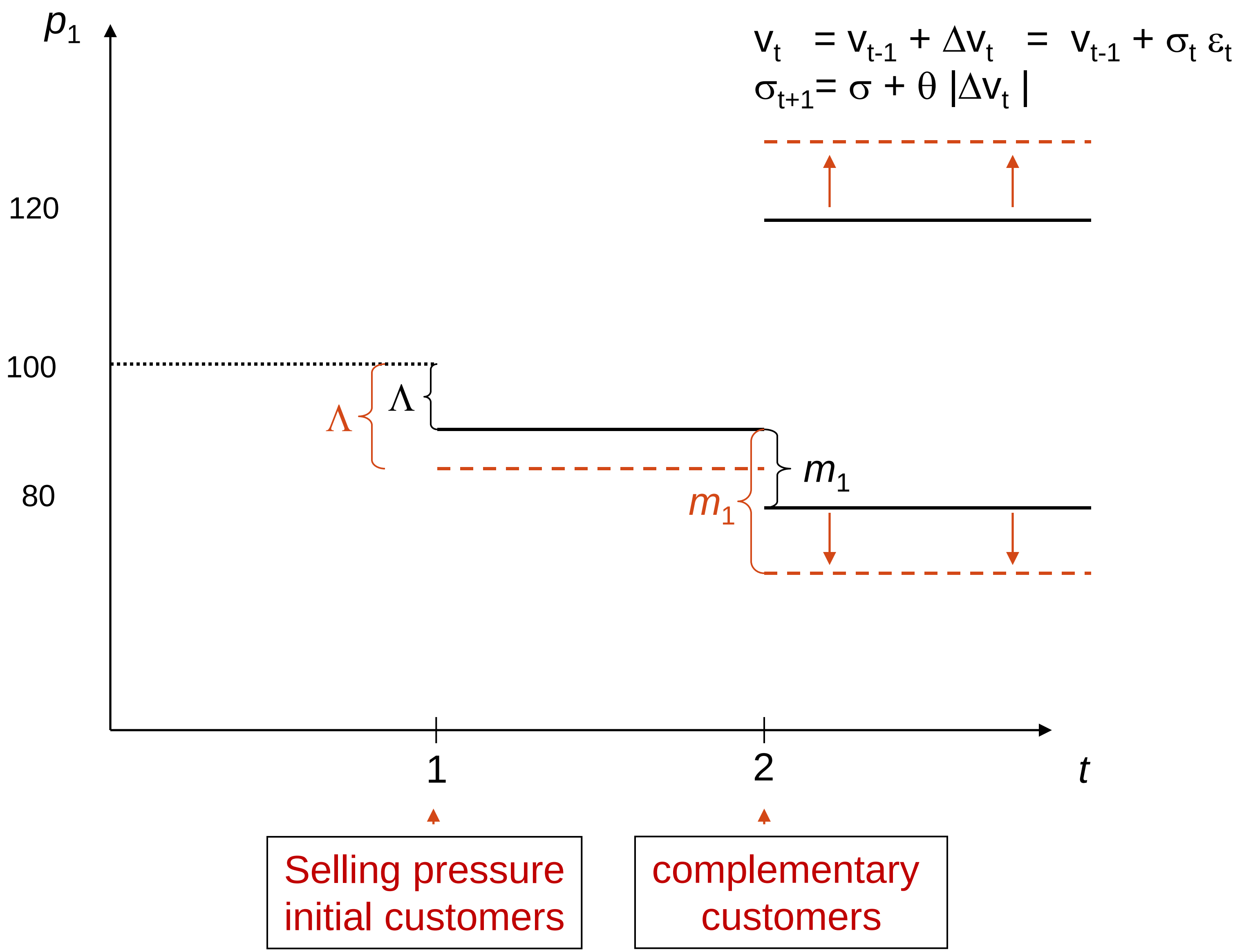
- Agents
  - Initial customers with supply $S(z, E_t[v]-p_t)$ at t=1,2
  - Complementary customers' demand $D(z, E_2[v]-p_2)$ at t=2
  - Risk-neutral dealers provide *immediacy* and
    - face capital constraint
  - $xm(\sigma, \Lambda) \leq W(\Lambda) \quad := \quad \max\{0, \underbrace{B}_{\text{cash}} + \underbrace{x_0(E_1[v]-\Lambda)}_{\text{“price” of stock holding}}\}$

BP: Financiers' Margin Setting

- Margins are set based on Value-at-Risk
- Financiers do not know whether price move is due to
  - *Likely*, movement in fundamental
  - *Rare*, Selling/buying pressure by customers who suffered asynchronous endowment shocks.

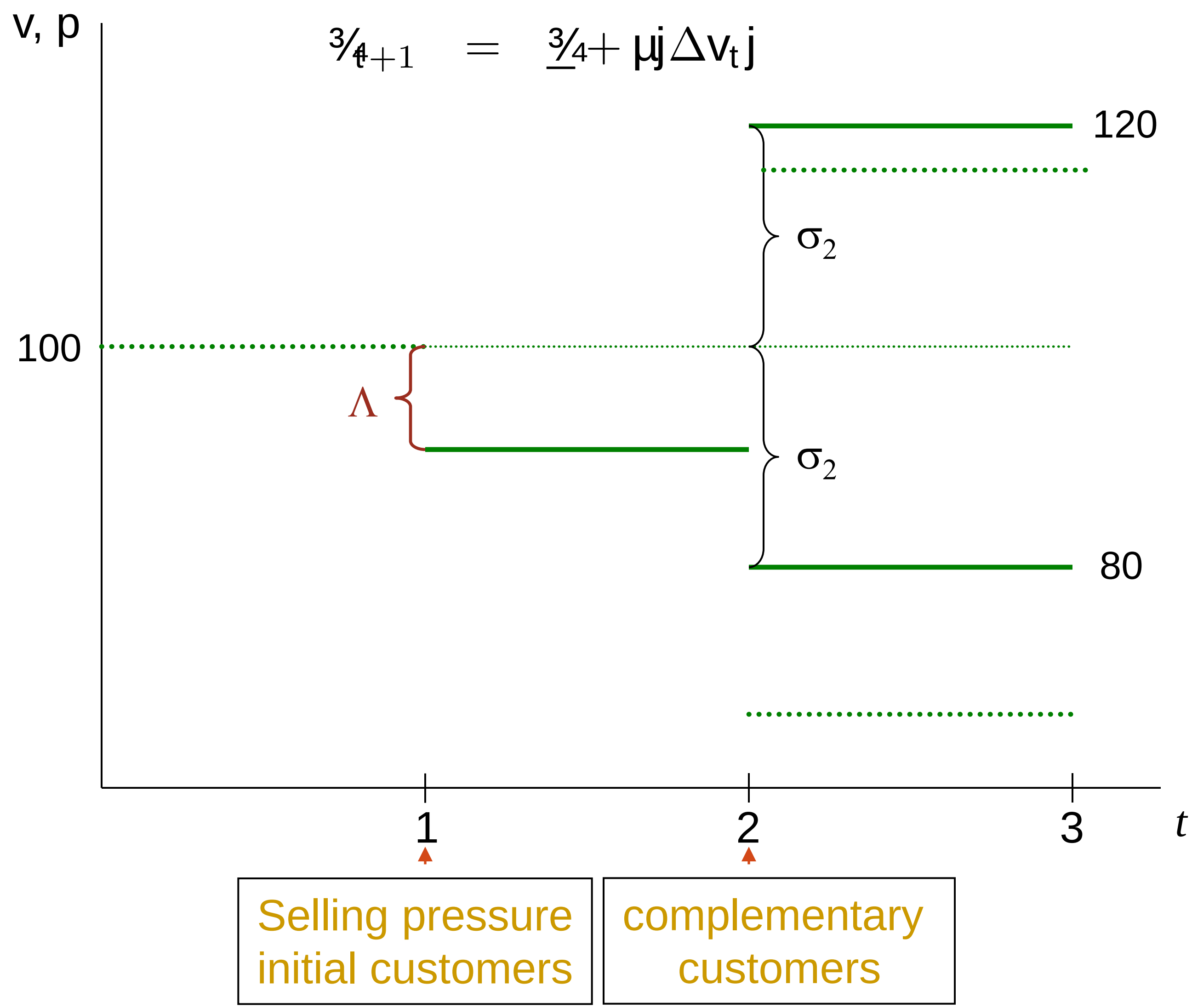
$$m_1^{j+} = \Phi^{-1}(1-\pi)\sigma_2 = \sigma + \bar{\theta}|\Delta p_1| = m_1^{j-}$$

BP: Margin Spiral – Increased Vol.

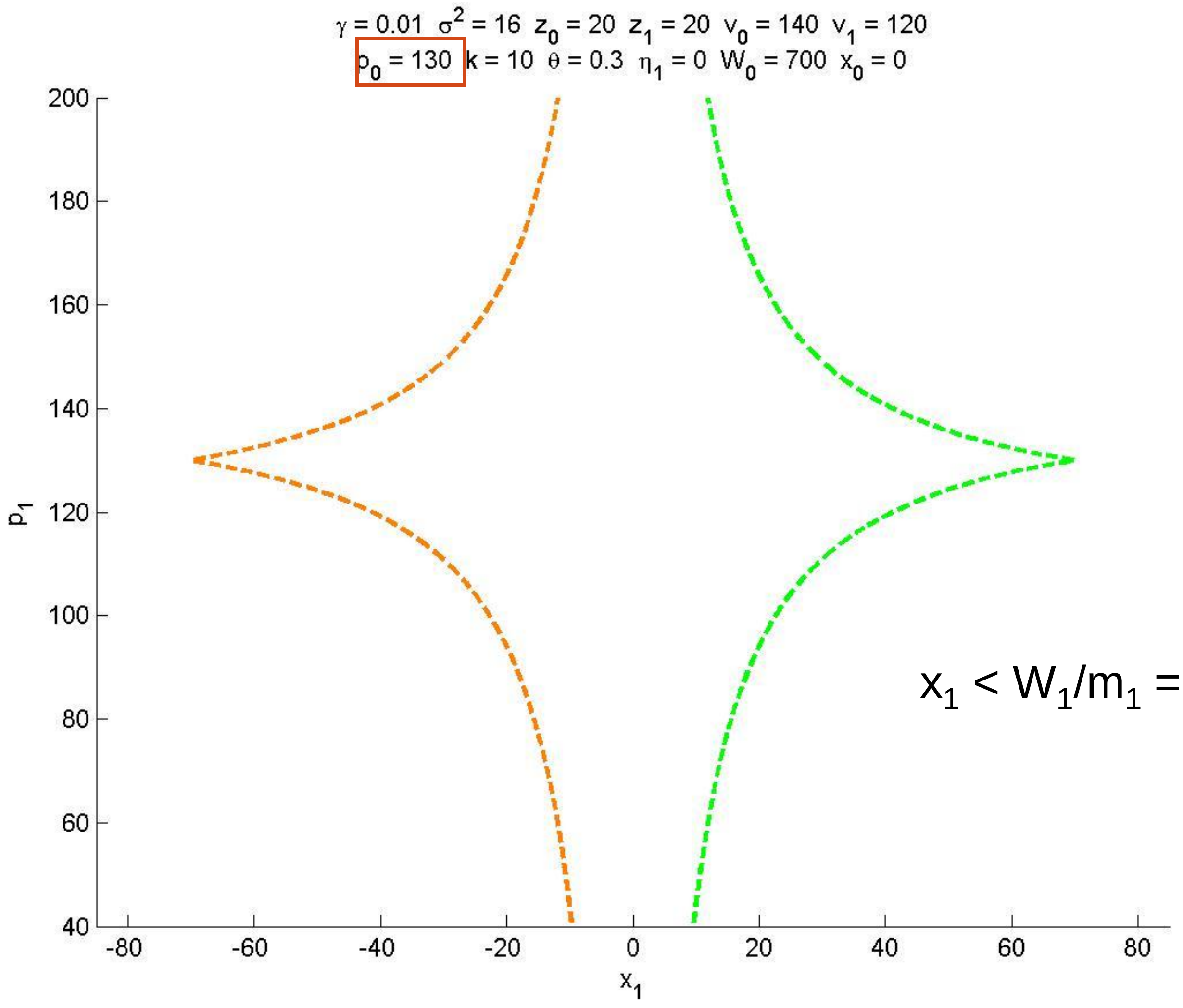


BP: Model Setup in a Figure

$$v_t = v_{t-1} + \Delta v_t = v_{t-1} + ¾_t$$
$$¾_{t+1} = \underline{¾} + µ|\Delta v_t|$$



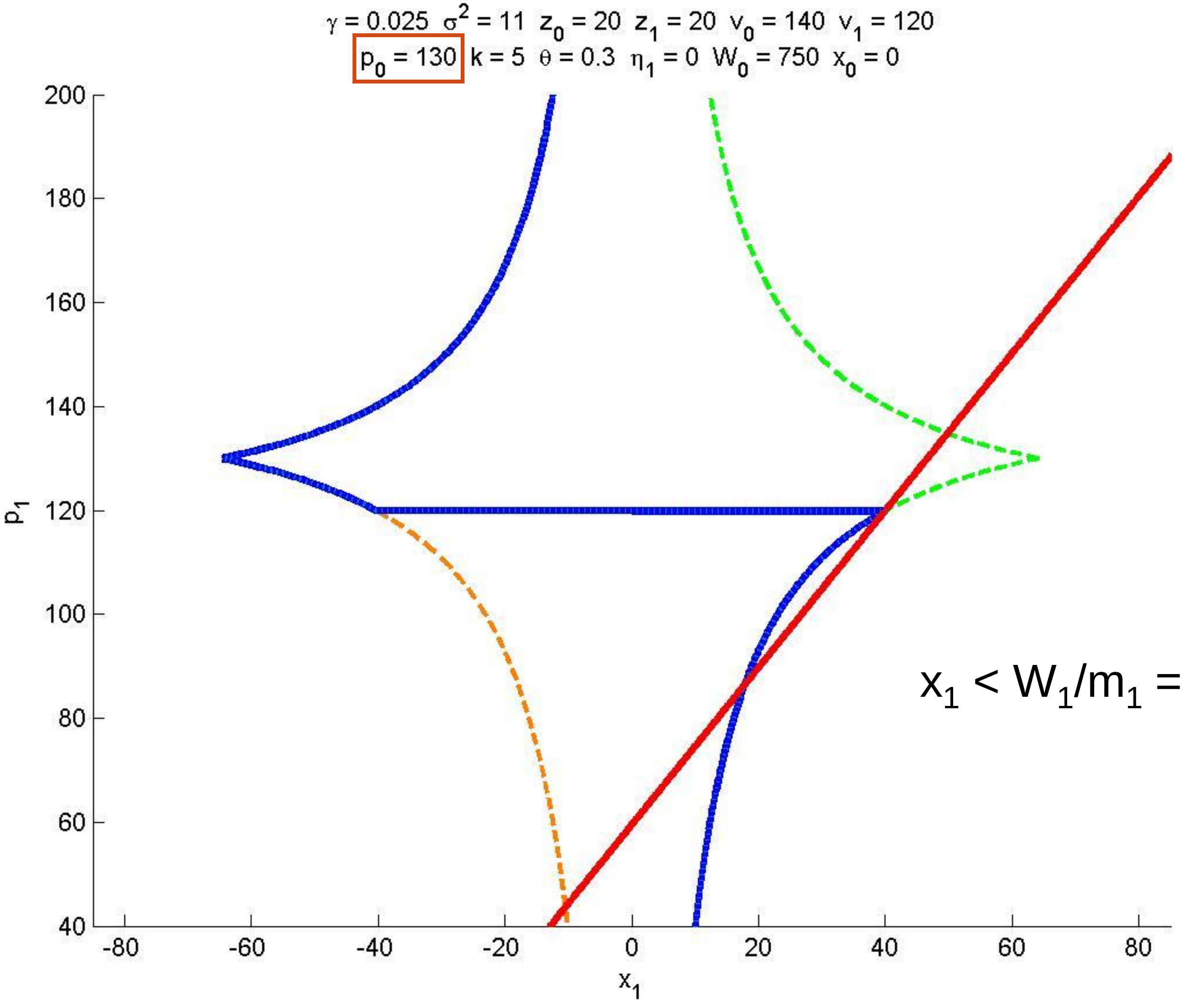
1. Margin Spiral – Increased Vol.



customers' supply

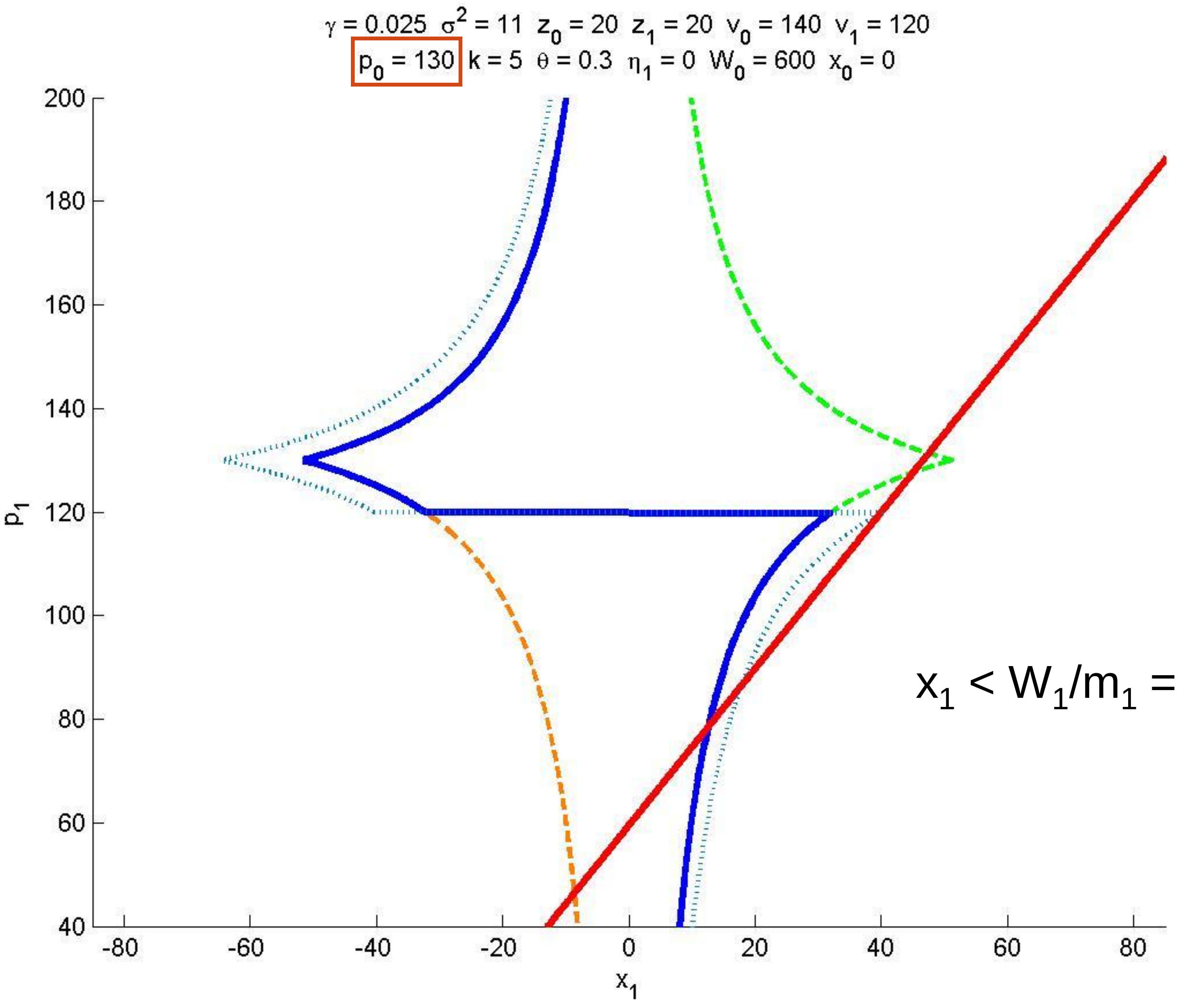
$x_1 < W_1/m_1 = W_1/(\sigma + \bar{\theta}|\Delta p_1|)$

1. Margin Spiral – Increased Vol.



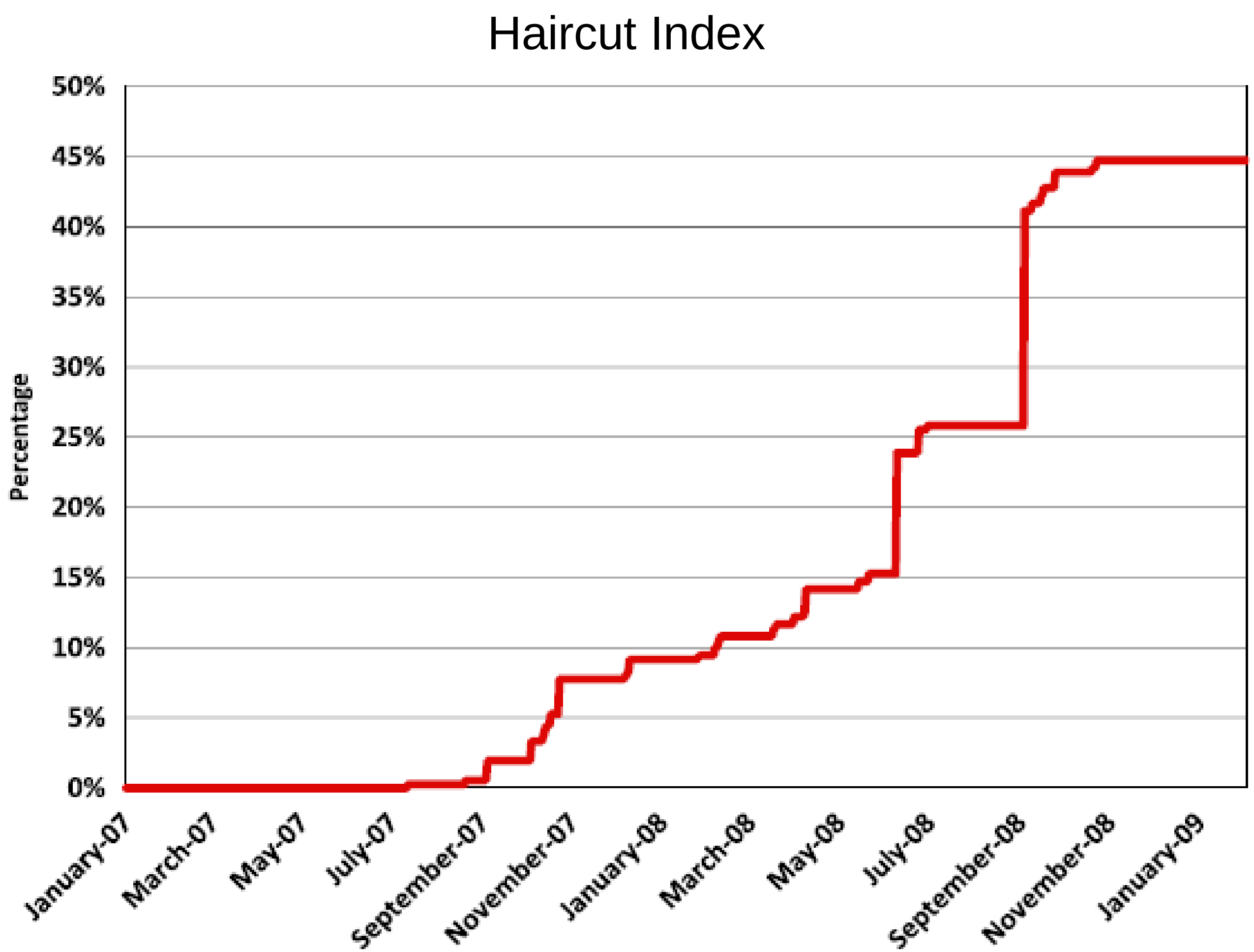
$x_1 < W_1/m_1 = W_1/(\sigma + \bar{\theta}|\Delta p_1|)$

1. Margin Spiral – Increased Vol.



$x_1 < W_1/m_1 = W_1/(\sigma + \bar{\theta}|\Delta p_1|)$

Data Gorton and Metrick (2011)



"The Run on Repo"

Copeland, Martin, Walker (2011)

Margins **very stable** in tri-party repo r

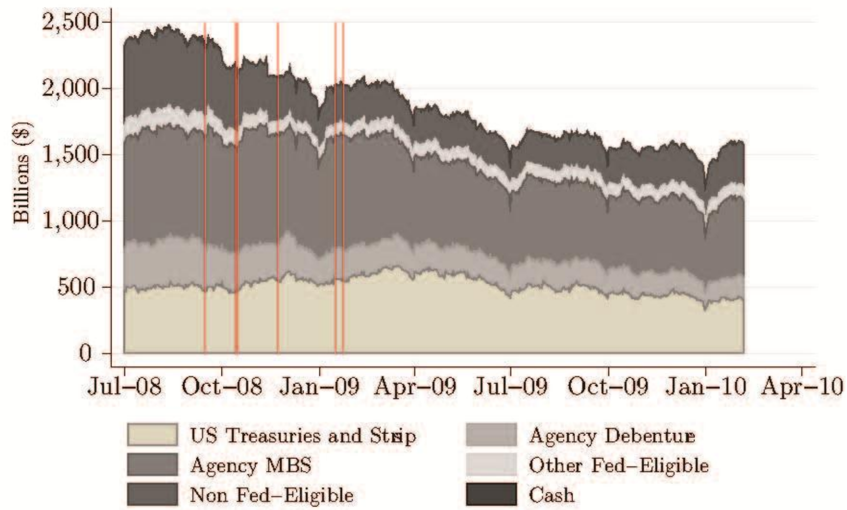
- contrasts with Gorton and Metri
- **no general run on certain type:**
  - http://www.ny.frb.org/research/s

Run (non-renewed financing) only on

- Bear Stearns (anecdotally)
- Lehman (in the data)

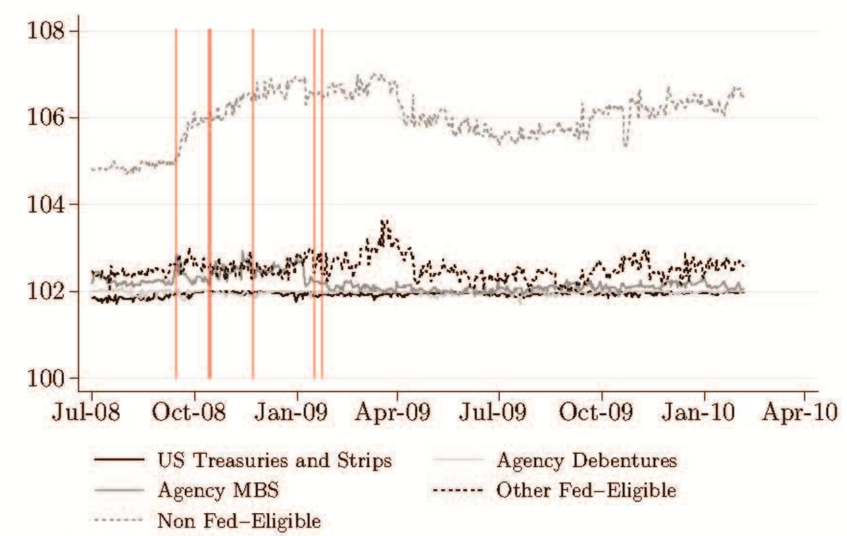
Like 100% haircut…
(**counterparty specific!**)

Figure 6: Stacked Graph of Collateral



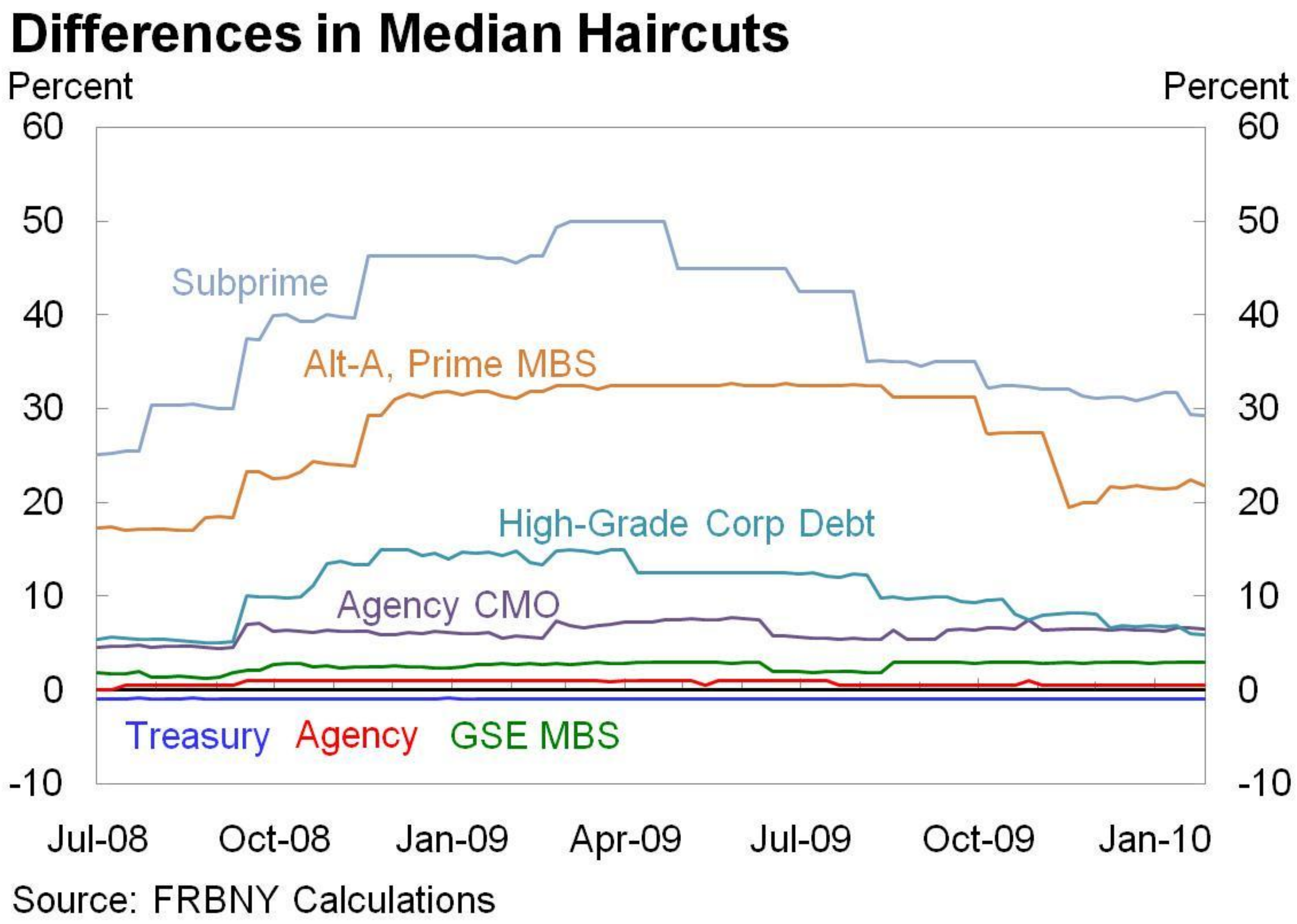
Note: July 17, 2008 excluded because no data was available for BNYM on that date.
Red lines correspond to important market events. From left to right: 9/15/08 (Lehman), 10/14/08 (9 banks receive aid), 10/16/08 (UBS), 11/23/08 (Citi), 1/16/09 (B of A), 1/24/09 (Citi).

Figure 7: Median Haircuts by Asset Type



Note: Red lines correspond to important market events. From left to right: 9/15/08 (Lehman), 10/14/08 (9 banks receive aid), 10/16/08 (UBS), 11/23/08 (Citi), 1/16/09 (B of A), 1/24/09 (Citi).

Bilateral and Tri-party Haircuts?



Tri-party Repo Haircuts April 2011

Asset Group	Cash Investor Margins Levels		
	10th Percentile	Median	90th Percentile
ABS Investment Grade	2.0%	5.0%	10.0%
ABS Non Investment Grade	2.0%	5.5%	8.0%
Agency CMOs	2.0%	3.0%	5.0%
Agency Debentures & Strips	2.0%	2.0%	3.0%
Agency MBS	2.0%	2.0%	5.0%
CMO Private Label Investment Grade	3.0%	5.0%	10.0%
CMO Private Label Non Investment Grade	2.0%	5.0%	8.0%
Corporates Investment Grade	2.0%	5.0%	8.0%
Corporates Non Investment Grade	2.0%	8.0%	11.2%
Equities	5.0%	8.0%	15.0%
Money Market	2.0%	5.0%	5.0%
US Treasuries excluding Strips	1.1%	2.0%	2.0%
US Treasuries Strips	2.0%	2.0%	2.0%

- This is triparty repo by different asset classes
- Reported by FRBNY

http://www.newyorkfed.org/tripartyrepo/margin_data.html

Overview

- Persistence
- Dynamic Amplification
  - Technological illiquidity BGG
  - Market illiquidity KM97
- Instability, Volatility Dynamics, Volatility Paradox
- Volatility and Credit Rationing/Margins/Leverage
- Demand for Liquid Assets
- Financial Intermediation

Demand for Liquid Assets

- *Technological and market illiquidity* create time amplification and instability
  - Fire-sales lead to time varying price of capital
  - Liquidity spirals emerge when price volatility interacts with debt constraints
- Focus on demand for liquid instruments
  - No amplification effects, i.e. reversible investment and constant price of capital q
    - Borrowing constraint = collateral constraint
  - Introduce idiosyncratic risk, aggregate risk, and finally amplification

Outline

- Deterministic Fluctuations
  - Overlapping generations
  - Completing markets with liquid asset
- Idiosyncratic Risk
  - Precautionary savings
  - Constrained efficiency
- Aggregate Risk
  - Bounded rationality
- Amplification Revisited

Overlapping Generations

- Samuelson (1958) considers an infinite-horizon economy with two-period lived overlapping agents
  - Population growth rate n
- Preferences given by $u(c_t^t, c_{t+1}^t)$
  - Pareto optimal allocation satisfies $\frac{u_1}{u_2} = 1 + n$
- OLG economies have multiple equilibria that can be Pareto ranked

OLG: Multiple Equilibria

- Assume $u(c_t^t, c_{t+1}^t) = \log c_t^t + \beta \log c_{t+1}^t$
  - Endowment $y_t^t = e, y_{t+1}^t = 1 - e$
- Assume complete markets and interest rate r
- Agent's FOC implies that $\frac{c_{t+1}^t}{\beta c_t^t} = 1 + r$
  - For $r = n$, this corresponds to the *Pareto solution*
  - For $r = \frac{1-e}{\beta e} - 1$, agents will consume their endowment
- Autarky solution is clearly *Pareto inferior*

OLG: Completion with Durable Asset

- Autarky solution is the **unique** equilibrium implemented in a sequential exchange economy
  - Young agents cannot transfer wealth to next period
- A durable asset provides a <u>store of value</u>
  - Effective store of value reflects *market liquidity*
  - Pareto solution can be attained as a competitive equilibrium in which the price level grows at same rate as the population, i.e. $b_{t+1} = (1+n)b_t$
  - Old agents trade durable asset for young agents' consumption goods

OLG: Production

- Diamond (1965) introduces a capital good and production
  - Constant-returns-to-scale production $Y_t = F(K_t, L_t)$
- Optimal level of capital is given by the *golden rule*, i.e. $f'(k^*) = n$
  - Here, lowercase letters signify **per capita** values
- Individual (and firm) optimization implies that
  - $\frac{u_1}{u_2} = 1 + r = 1 + f'(k)$
  - It is possible that $r < n \Rightarrow k > k^* \Rightarrow$ Pareto inefficient

OLG: Production & Efficiency

- Diamond recommends issuing government debt at interest rate r
- Tirole (1985) introduces a rational bubble asset trading at price b_t
  - $b_{t+1} = \frac{1+r_{t+1}}{1+n} b_t$
- Both solutions *crowd out* investment and increase r
  - If baseline economy is inefficient, then an appropriately chosen debt issuance or bubble size can achieve Pareto optimum with $r = n$

OLG: Crowding Out vs. Crowding In

- Depending on the framework, government debt and presence of bubbles can have two opposite effects
  - Crowding out refers to the decreased investment to increase in supply of capital
  - Crowding in refers to increased investment due to improved risk transfer
- Woodford (1990) explores both of these effects

OLG: Woodford 1

- Consider a model with two types of agents
  - Per capita production $f(k)$
  - Alternating endowments $\bar{e} > \underline{e} > 0$
  - No borrowing
- Stationary solution
  - High endowment agents are *unconstrained*, consuming $\bar{c}$ and saving part of endowment
  - Low endowment agents are *constrained*, consuming $\underline{c} \leq \bar{c}$ and depleting savings

OLG: Crowding Out

- Euler equations
  - Unconstrained: $u'(\bar{c}) = \beta(1+r)u'(\underline{c})$
  - Constrained: $u'(\underline{c}) \geq \beta(1+r)u'(\bar{c})$
- Interest rate is lower than discount rate
  - $f'(k) - 1 = r \leq \beta^{-1} - 1 \equiv \rho \Rightarrow$ Pareto inefficient
- Increasing debt provides *market liquidity*
  - This increases interest rate and reduces capital stock
  - With $r = \rho \Rightarrow \underline{c} = \bar{c}$ (full insurance)

OLG: Woodford 2

- Assume agents now have alternating *opportunities* (instead of endowments)
  - Unproductive agents can only hold government debt
  - Productive agents can hold debt *and* capital
- Stationary solution
  - Unproductive agents are *unconstrained*, consuming $\bar{c}$ and saving part of endowment (as debt)
  - Productive agents are *constrained*, consuming $\underline{c} \leq \bar{c}$ and investing savings and part of endowment in capital

OLG: Crowding In

- Euler equations
  - Unconstrained: $u'(\bar{c}) = \beta(1+r)u'(\underline{c})$
  - Constrained: $u'(\underline{c}) = \beta f'(k) u'(\bar{c})$
  - Interest rate satisfies $1 + r \leq f'(k)$
- Increasing debt provides *market liquidity*
  - This increases r and k since $\beta(1+r) = \frac{1}{\beta f'(k)}$
  - Transfer from unproductive periods to productive periods
  - Increase debt until both agents are unconstrained

Precautionary Savings

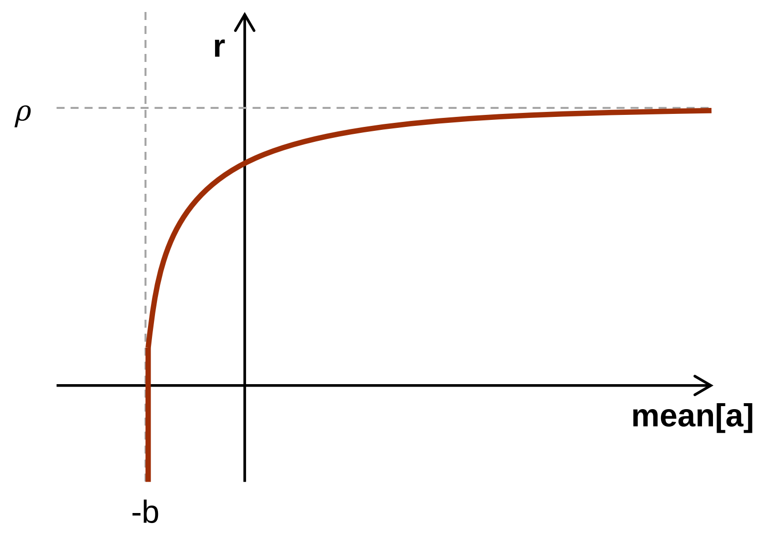
- Consumption smoothing implies that agents will save in high income states and borrow in low income states
  - If markets are incomplete, agents may not be able to efficiently transfer consumption between these outcomes
- Additional precautionary savings motive arises when agents cannot insure against uncertainty
  - Shape of utility function u'''
  - Borrowing constraint $a_t \geq -b$

PCS: Prudence

- Utility maximization $E_0[\sum_{t=0}^{\infty} \beta^t u(c_t)]$
  - Budget constraint: $c_t + a_{t+1} = e_t + (1+r)a_t$
  - Standard Euler equation: $u'(c_t) = \beta(1+r)E_t[u'(c_{t+1})]$
- If $u''' > 0$, then Jensen's inequality implies:
  - $\frac{1}{\beta(1+r)} = \frac{E_t[u'(c_{t+1})]}{u'(c_t)} > \frac{u'(E_t[c_{t+1}])}{u'(c_t)}$
  - Marginal value is greater due to uncertainty in c_{t+1}
  - Difference is attributed to *precautionary savings*
- <u>Prudence</u> refers to curvature of u', i.e. $P = -\frac{u'''}{u''}$

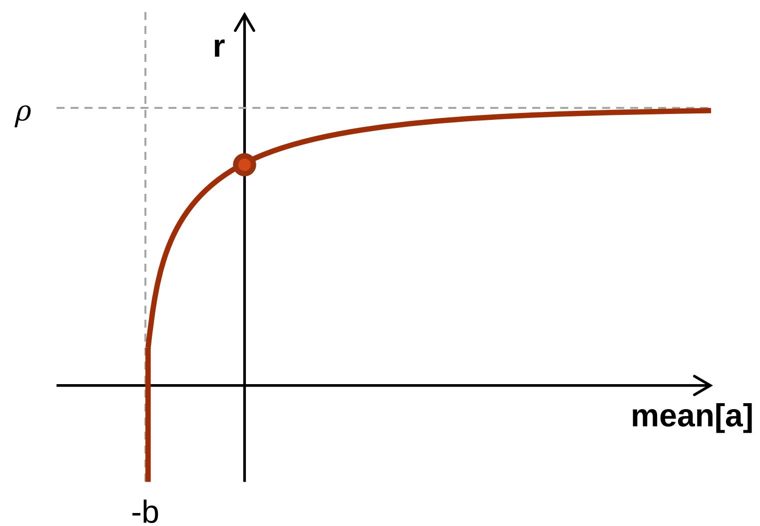
Idiosyncratic Risk

- With *incomplete markets* and *borrowing constraints*, agents engage in precautionary savings in the presence of idiosyncratic income shocks
- Following Bewley (1977), mean asset holdings $E[a]$ result from individual optimization



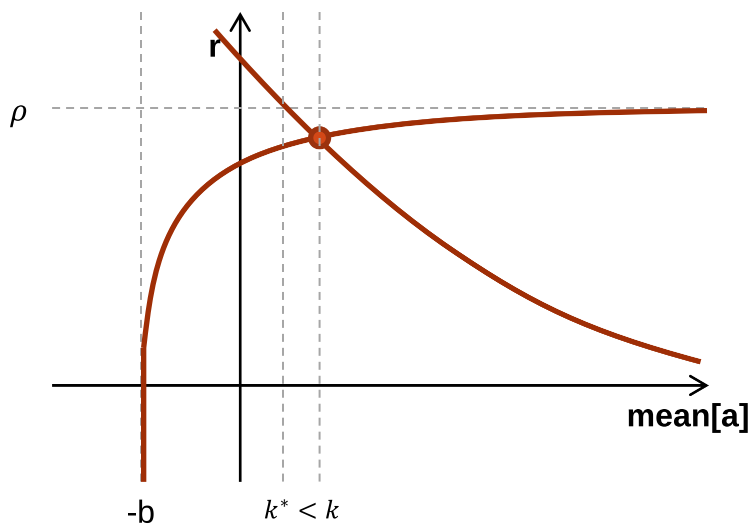
IR: Exchange Economy

- In an exchange economy, aggregate supply of assets must be zero
  - Huggett (1993)
- Equilibrium interest rate always satisfies $r < \rho$



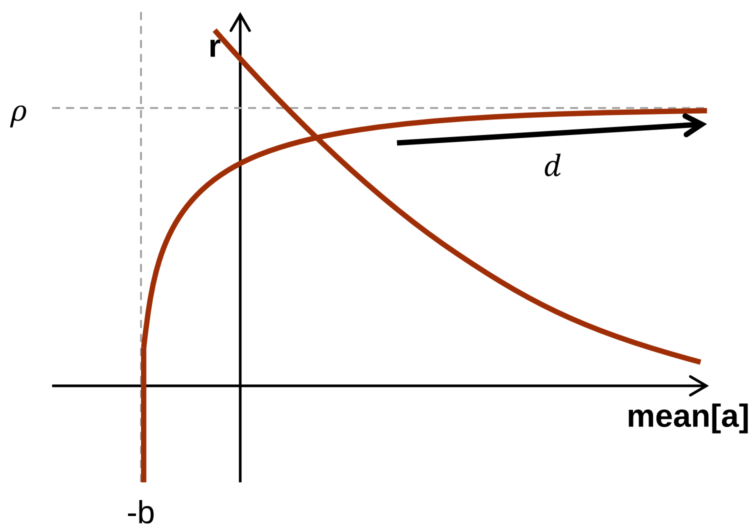
IR: Production Economy

- Aiyagari (1994) combines the previous setup with standard production function $F(K, L)$
  - Constant aggregate labor L
- Demand for capital is given by $f'(k) - \delta = r$
  - Efficient level of capital $f'(k^*) - \delta = \rho \Rightarrow k^* < k$



IR: Production Economy

- Aiyagari (1995) shows that a tax on capital earnings can address this efficiency problem
  - This decreases the net interest rate received by agents
- Government debt does not work "perfectly"
  - No finite amount of government debt will achieve $r = \rho$



Constrained Inefficiency

- Bewley-Aiyagari economies result in competitive allocations that are not only Pareto inefficient, but are also *constrained* Pareto inefficient
  - Social planner can achieve a Pareto superior outcome even facing same market incompleteness
- This result can be attributed to *pecuniary externalities*
  - In competitive equilibrium, agents take prices as given whereas a social planner can induce wealth transfers by affecting relative prices
  - Stiglitz (1982), Geanakoplos-Polemarcharkis (1986)

CI: Aiyagari Economy

- Davila, Hong, Krusell, Rios-Rull (2005) consider welfare increasing changes in Aiyagari setting
- Higher level capital leads to higher wages and lower interest rates
  - Higher wage amplifies contemporaneous effect of labor endowment shock
  - Lower interest rate dampens impact of endowment shock in following periods

CI: Amplification

- Two period setting with $t \in \{0,1\}$
  - Initial wealth y
  - Labor endowment $e \in \{e_1, e_2\}$ (i.i.d)
  - Aggregate labor: $L = \pi e_1 + (1-\pi)e_2$
  - Production function $f(K, L)$
- Agent consumption plan given by $\{c_0, c_1, c_2\}$
  - $c_i \leq e_i w + K(1+r)$
  - $\frac{dU}{dK} = \{-u'(c_0) + \beta(1+r)[\pi u'(c_1) + (1-\pi)u'(c_2)]\} + \beta[\pi u'(c_1)K + (1-\pi)u'(c_2)K]\frac{dr}{dK} + \beta[\pi u'(c_1)e_1 + (1-\pi)u'(c_2)e_2]\frac{dw}{dK}$

CI: Amplification

- The first expression is zero from agent's FOC
  - Agents take prices as given, i.e. assume $\frac{dw}{dK} = \frac{dr}{dK} = 0$
- In a competitive equilibrium $\frac{dr}{dK} = f_{KK}$ and $\frac{dw}{dK} = f_{KL}$
  - f linearly homogeneous implies $Kf_{KK} + Lf_{KL} = 0$
- This provides:
  - $\frac{dU}{dK} = \beta\pi(1-\pi)\frac{Kf_{KK}}{L}(u'(c_1) - u'(c_2))(e_2 - e_1) < 0$
  - Reducing level of capital improves ex-ante utility

CI: Dampening

- Consider addition of third period $t = 2$
  - Same labor endowment $e \in \{e_1, e_2\}$
- Effect of change in capital level at $t = 1$ depends on realization of labor endowment
  - $\Delta = \beta\pi(1-\pi)\frac{Kf_{KK}}{L}(u'(c_1) - u'(c_2))(e_2 - e_1) < 0$
  - $\frac{dU_i}{dK} = \beta\left[\Delta + \beta\left(\pi u'(c_{i1})\right) + (1-\pi)u'(c_{i2})\right)(K_i - K)f_{KK}]$
- Second term is positive if and only if $K_i < K$
  - Increasing capital more desirable for low endowment agents and less desirable for high endowment agents

Aggregate Risk

- Krusell, Smith (1998) introduce aggregate risk into the Aiyagari framework
  - Aggregate productivity shock that follows a Markov process z_t and $Y_t = z_t F(K_t, L_t)$
- Aggregate capital stock determines equilibrium prices r_t, w_t
  - However, the evolution of aggregate stock is affected by the **distribution** of wealth since poor agents may have a much higher propensity to save
  - Tracking whole distribution is practically impossible

AR: Bounded Rationality

- Krusell, Smith assume agents are boundedly rational and approximate the distribution of capital by a finite set of moments M
  - Regression R^2 is relatively high even if $\#M = 1$
- This result is strongly dependent on low risk aversion and low persistence of labor shocks
  - Weak precautionary savings motive except for poorest agents
  - Since wealth-weighted averages are relevant, this has a negligible effect on aggregate quantities

Amplification Revisited

- Investment possibility shocks
  - Production possibilities: Scheinkman & Weiss (1986)
  - Investment possibilities: Kiyotaki & Moore (2008)
- Interim liquidity shocks
  - Exogenous shock: Holmstrom & Tirole (1998)
  - Endogenous shock: Shleifer & Vishny (1997)
- Preference shocks
  - No aggregate risk: Diamond & Dybvig (1984)
  - Aggregate risk: Allen & Gale (1994)

Scheinkman & Weiss

- Two types of agents with perfectly negatively correlated idiosyncratic shocks
  - No aggregate risk, but key difference is that labor supply is now elastic
- Productivity reflects *technological liquidity*
  - Productivity switches according to a Poisson process
  - Productive agents can produce consumption goods
- No capital in the economy
  - Can only save by holding money (fixed supply)
  - Productive agents exchange consumption goods for money from unproductive agents

SW: Aggregate Dynamics

- Aggregate fluctuations due to elastic labor supply
- Price level is determined in equilibrium
  - As productive agents accumulate money, wealth effect induces lower labor supply
  - Aggregate output declines and price level increases
- Effect of changes in money supply depends on distribution of money between agent types
  - Increase in money supply will reduce (increase) aggregate output when productive agents hold less (more) than half the money supply, i.e. when output is high (low)

Kiyotaki & Moore 08

- Two types of agents, entrepreneurs and households
  - Entrepreneurs can invest,
    but only when they have an investment opportunity
  - Opportunities correspond to *technological liquidity*
- Investment opportunities arrive i.i.d. and cannot be insured against
  - Entrepreneur can invest with probability π
- Agents can hold equity or fiat money

KM: Financing

- Entrepreneurs have access to 3 sources of capital
  - New equity claims, but a fraction $1-\theta$ must be held by entrepreneur for at least one period
  - Existing equity claims, but only a fraction ϕ_t of these can be sold right away
  - Money holdings, with no frictions
- Capital frictions represent *illiquidity*

KM: Entrepreneurs

- Budget constraint:
  - $c_t + i_t + q_t(n_{t+1} - i_t) + p_t(m_{t+1} - m_t) = r_t n_t + q_t(1-\delta)n_t$
  - Equity holdings net of investment $n_{t+1} - i_t$
  - Price of equity/capital q_t can be greater than 1 due to limited investment opportunities
- Liquidity constraint:
  - $n_{t+1} \geq (1-\theta)i_t + (1-\phi_t)(1-\delta)n_t$
  - Limits on selling new and existing equity place lower bound on future equity holdings

KM: Investment Opportunity

- For low θ, ϕ_t, liquidity constraints are binding and money has value
- An entrepreneur with an investment opportunity will spend all of his money holding
  - Budget constraint can be rewritten as $c_t^i + q_t^R n_{t+1}^i = r_t n_t + (\phi_t q_t + (1-\phi_t) q_t^R)(1-\delta) n_t + p_t m_t$
  - Replacement cost of capital: $q_t^R \equiv \frac{1-\theta q_t}{1-\theta}$
  - Can create new equity holdings at cost $q_t^R < q_t$, but this reduces value of remaining unsold holdings

KM: No Investment Opportunity

- Entrepreneur without investment opportunity decides on allocation between equity (depends on opportunity at $t+1$) and money
  - Return to money: $R^m_{t+1} \equiv \frac{p_{t+1}}{p_t}$
  - No opportunity: $R^s_{t+1} \equiv \frac{r_{t+1}+q_{t+1}(1-\delta)}{q_t}$
  - Opportunity: $R^i_{t+1} \equiv \frac{r_{t+1}+\left(\phi_{t+1}q_{t+1}+(1-\phi_{t+1})q^R_{t+1}(1-\delta)\right)}{q_t}$

KM: Logarithmic Utility

- Under logarithmic utility, entrepreneurs will consume $1-\beta$ fraction of wealth
- Around steady-state, aggregate level of capital is smaller than in first-best economy, i.e. $K_{t+1} < K^*$
  - Expected return on capital $E_t[f'(K_{t+1}) - \delta] > \rho$
- Conditional liquidity premium arises since $E_t[R^m_{t+1}] < E_t[R^s_{t+1}] < 1 + \rho$
  - Unconditional liquidity premium may also arise (but is smaller) since $E_t\left[R^i_{t+1}\right] < E_t[R^m_{t+1}]$

KM: Real Effects

- Negative shocks to *market liquidity* ϕ_t of equity have aggregate effects
  - Shift away from equity into money
  - Drop in price q_t and increase in p_t
  - Decrease in investment and capital
- Shock to financing conditions feeds back to real economy as a reduction in output
  - KM find that government can counteract effects by buying equity and issuing new money (upward pressure on q_t and downward pressure on p_t)

Holmstrom & Tirole 98

- Three period model with $t \in \{0,1,2\}$
- Entrepreneurs with initial wealth A
  - Investment of I returns RI in $t = 2$ with probability p
  - Interim funding requirement ρI at $t = 1$ with $\rho \sim G$
  - Extreme *technological illiquidity*, as investment is worthless if interim funding is not provided
- Moral hazard problem
  - Efficiency requires $\rho \leq \rho_1 \equiv pR \Rightarrow$ continuation
  - Only $\rho \leq \rho_0 < \rho_1$ of funding can be raised due to manager's private benefit, i.e. principal-agent conflict

HT: Optimal Contracting

- Optimal contract specifies:
  - Investment size I
  - Continuation cutoff $\hat{\rho}$
  - Division of returns contingent on realized ρ
- Entrepreneurs maximize expected surplus, i.e.
  - $\max_{I,\hat{\rho}} \left\{ I \int_0^{\hat{\rho}} (\rho_1 - \rho) dG(\rho) - I \right\}$
- Households can only be promised ρ_0 at $t = 1$
  - Breakeven condition: $I \int_0^{\hat{\rho}} (\rho_0 - \rho) dG(\rho) = I - A$
- Solution provides cutoff $\hat{\rho} \in [\rho_0, \rho_1]$

HT: General Equilibrium

- Without a storage technology, liquidity must come from financial claims on real assets
  - *Market liquidity* of claims becomes crucial
- If there is no aggregate uncertainty, the optimal contract can be implemented:
  - Sell equity
  - Hold part of market portfolio
  - Any surplus is paid to shareholders as dividends

HT: Aggregate Risk

- With aggregate risk, optimal contract may not be implementable
  - Market liquidity of equity is affected by aggregate state
- Consider perfectly correlated projects
  - Liquidity is low when it is needed (bad aggregate state)
  - Liquidity is high when it is not needed (good state)
- This introduces a role for government to provide a store of wealth

Shleifer & Vishny 97

- Fund managers choose how aggressively to exploit an arbitrage opportunity
- Mispricing can widen in interim period
  - Investors question investment and withdraw funds
  - Managers must unwind position when mispricing is largest, i.e. most profitable
  - Low *market liquidity* due to limited knowledge of opportunity
- Fund managers predict this effect, and thus limit arbitrage activity
  - Keep buffer of liquid assets to fund withdrawals

Diamond & Dybvig 83

- Three period model with $t \in \{0,1,2\}$
- Continuum of ex-ante identical agents
  - Endowment of 1 in $t = 0$
  - Idiosyncratic preference shock, i.e. probability λ that agent consumes in $t = 1$ and probability $1 - \lambda$ that agent consumes in $t = 2$
- Preference shock is not observable to outsiders
  - Not insurable, i.e. incomplete markets

DD: Investment

- Good can be stored without cost
  - Payoff of 1 in any period
- Long term investment project
  - Payoff of $R > 1$ in $t = 2$
  - Salvage value of $r \leq 1$ if liquidated early in $t = 1$
  - Market for claims to long-term project at price p
- Trade-off between return and *liquidity*
  - Investment is subject to *technological illiquidity*, i.e. $r \leq 1$
  - Market liquidity is represented by interim price p

DD: Consumption

- Investing x induces contingent consumption plan:
  - $c_1 = px + (1 - x)$
  - $c_2 = Rx + \frac{R(1-x)}{p}$
- In equilibrium, we require $p = 1$
  - If $p < 1$, then agents would store the asset and purchase project at $t = 1$
  - If $p > 1$, then agents would invest and sell project at $t = 1$

DD: Optimality

- With interim markets, any investment plan leads to $c_1 = 1, c_2 = R$
  - If $r < 1$, fraction $1 - \lambda$ of aggregate wealth must be invested in project (market clearing)
  - Since $p > r$, then asset's *market liquidity* is greater than its *technological liquidity*
- This outcome is clearly superior to autarky, with $c_1' = r, c_2' = R$ or $c_1'' = c_2'' = 1$

Allen & Gale

- AG extend DD framework by adding aggregate risk
  - Here, $\lambda = \lambda_H$ with probability π and $\lambda = \lambda_L < \lambda_H$ with probability $1 - \pi$
- Agents observe realization of aggregate state and idiosyncratic preference shock at $t = 1$
  - After resolution of uncertainty, agents can trade claims to long-term project at $p_s \in \{p_H, p_L\}$
  - Asset's *market liquidity* will vary across states
- For simplicity, assume $r = 0$

AG: Prices

- Market clearing requires $p_s \leq R$
  - Late consumers stored goods: $(1-\lambda_s)(1-x)$
  - Early consumers invested goods: $\lambda_s x$
- Cash-in-the-market pricing
  - $p_s = \min\left\{R, \frac{(1-\lambda_s)(1-x)}{\lambda_s x}\right\}$
  - This implies that $p_H \leq p_L$, i.e. *market liquidity* is weaker when there are a large proportion of early consumers
- Despite deterministic project payoffs, there is volatility in prices

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Gross Shadow Banking and Commercial Banking Liabilities