A Macroeconomic Model with a Financial Sector

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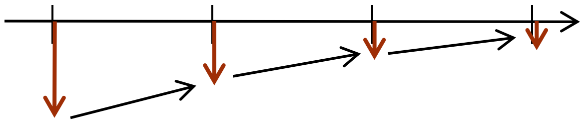
Motivation

- Financial instability
  - Persistence of shocks
  - Amplification
  - Non-linear liquidity spirals - adverse feedback loops
    - Go beyond log-linearization
  - Endogenous risk
  - "Volatility paradox"
- Asset pricing implications
  - Fat tails
  - Endogenous correlation structure

Amplification & Instability - Overview

- Bernanke & Gertler (1989), Carlstrom & Fuerst (1997)
  - Perfect (technological) liquidity, but persistence
  - Bad shocks erode net worth, cut back on investments, leading to low productivity & low net worth of in the next period



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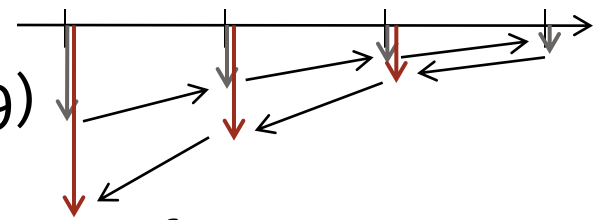

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  - Technological/market illiquidity
  - KM: Leverage bounded by margins; BGG: Verification cost (CSV)
  - Stronger amplification effects through prices (low net worth reduces leveraged institutions' demand for assets, lowering prices and further depressing net worth)

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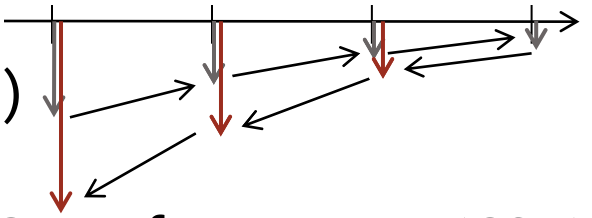

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- Brunnermeier & Sannikov (2010) *- only equity constraint*
  - Instability and volatility dynamics, volatility paradox

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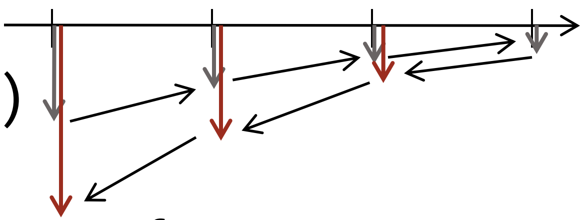

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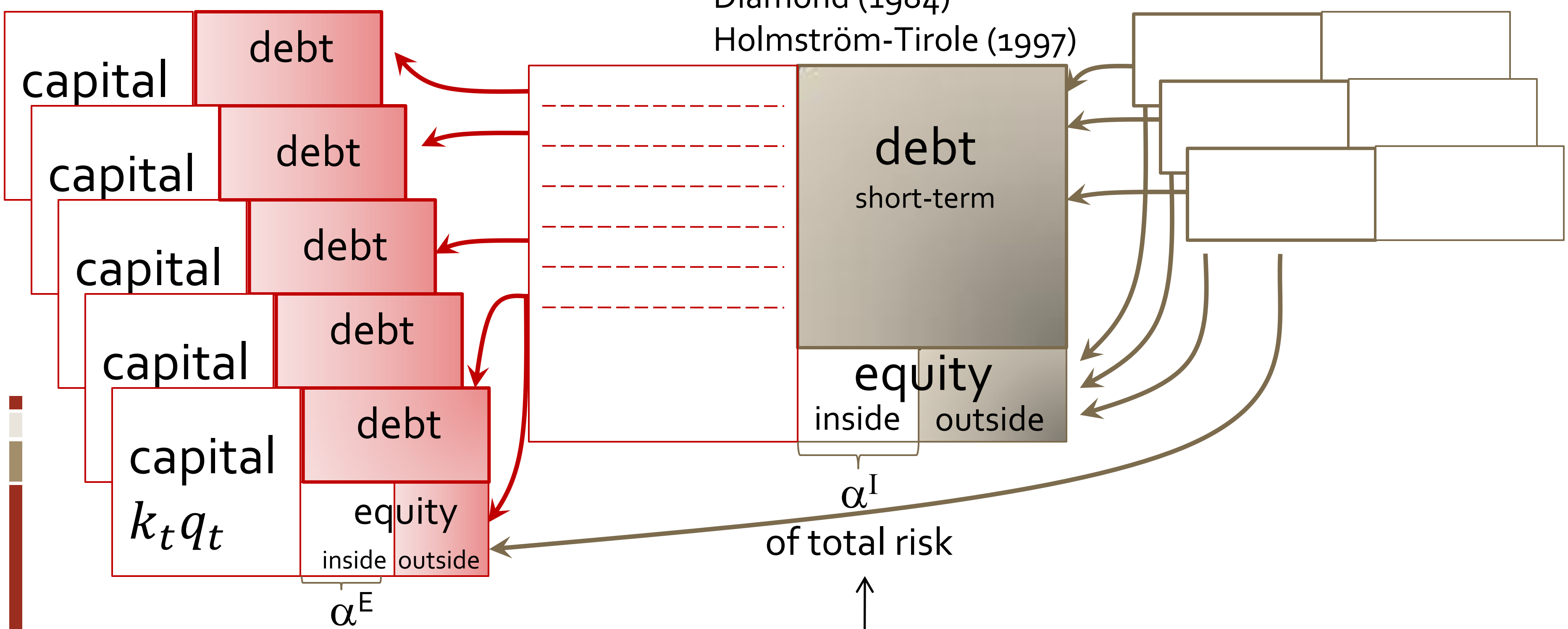
- Brunnermeier & Pedersen (2009), Geanakoplos
  - Volatility interaction with margins/haircuts (leverage) – *debt constraint*

Preview of results

- Full equilibrium dynamics + volatility dynamics
  - Near "steady state"
    - (large) payouts balance profit making
    - intermediaries must be unconstrained and amplification is low
  - Below "steady state"
    - intermediaries constrained, try to preserve capital leading to high amplification and volatility $\longrightarrow$ precaution
- Crises episodes have significant endogenous risk, correlated asset prices, larger spreads and risk premia
- "Volatility paradox"
- SDF is driven by constraint & $c \geq 0$
- Securitization and hedging of idiosyncratic risks can lead to higher leverage, and greater systemic risk

Model setup

- Productive
- Intermediary
  - Monitoring
    Diamond (1984)
    Holmström-Tirole (1997)
- Less productive

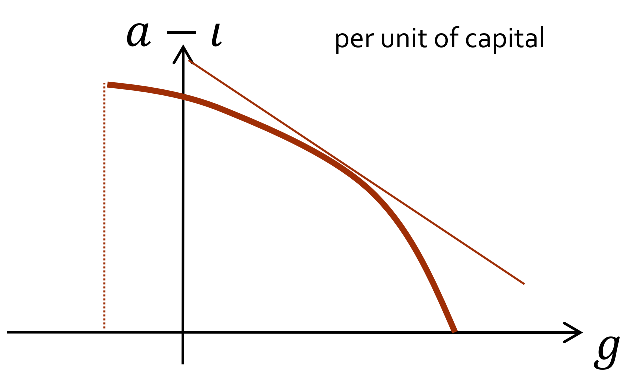


incentive for entrepreneur to exert effort

incentive for intermediaries to monitor
(have to hold outside equity)

Model details

- Output $y_t = a k_t$ (spend for consumption - investment)
- Capital $dk_t = \underbrace{(\Phi(\iota_t) - \delta)}_{=g} k_t dt + \sigma k_t dZ_t$ investment rate
- Agents
  - More productive
    - U =E$_0$[$\int_0^\infty e^{-\rho t}$ c$_t$ dt]
    - Production frontier



  - Less productive
    - U =E$_0$[$\int_0^\infty e^{-rt}$ c$_t$ dt]
    - Production frontier
      - $\underline{\delta} > \delta$
      - $\underline{\iota}_t = 0$
  - Endogenous price process for capital

$$dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$$

$$q_t \geq \underline{q} = \frac{a}{r+\underline{\delta}}$$ if HH limited to buy-hold strategy

Market value of capital/assets $k_t q_t$

- Capital
  - $dk_t = g(\iota)k_t dt + \sigma k_t dZ_t$ "cash flow news" (dividends a_t)
- Price
  - $dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$ "SDF news"

- $k_t q_t$ value dynamics

Market value of capital/assets $k_t q_t$

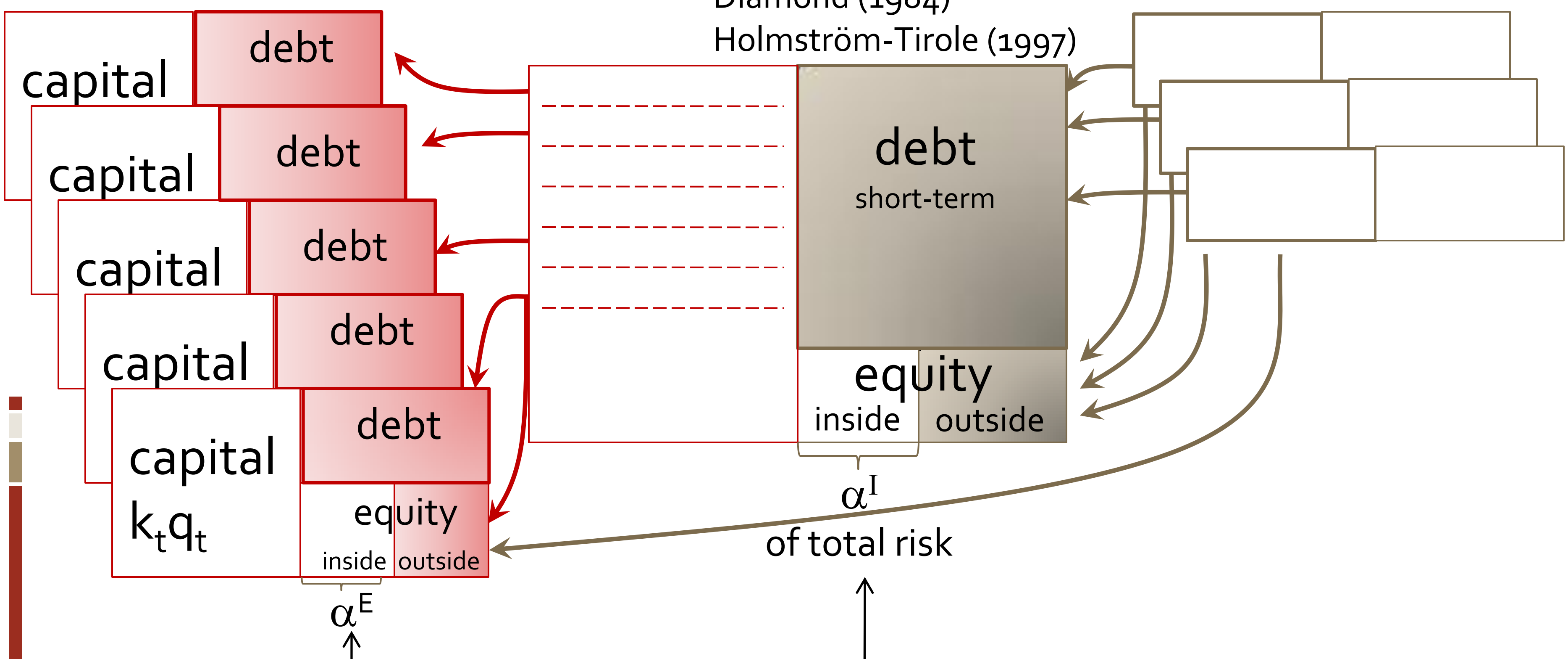
- Capital
  - $dk_t = g(\iota)k_t dt + \sigma k_t dZ_t$ exogenous risk
- Price
  - $dq_t = \mu_t^q q_t dt + \sigma_t^q q_t dZ_t$ endogenous risk
- $k_t q_t$ value dynamics
  - $d(k_t q_t) =$
    $\left(\Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right)(k_t q_t)dt + \left(\sigma + \sigma_t^q\right)(k_t q_t)dZ_t$

    exogenous endogenous

    risk
  - Ito's Lemma product rule: $d(X_t Y_t) = dX_t Y_t + X_t dY_t + \sigma^X \sigma^Y dt$

Interlinked balance sheets

- Productive
- Intermediary
  - Monitoring
    Diamond (1984)
    Holmström-Tirole (1997)
- Less productive



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Merging productive HH & Intermediaries

- Productive
- Intermediary
  - Monitoring
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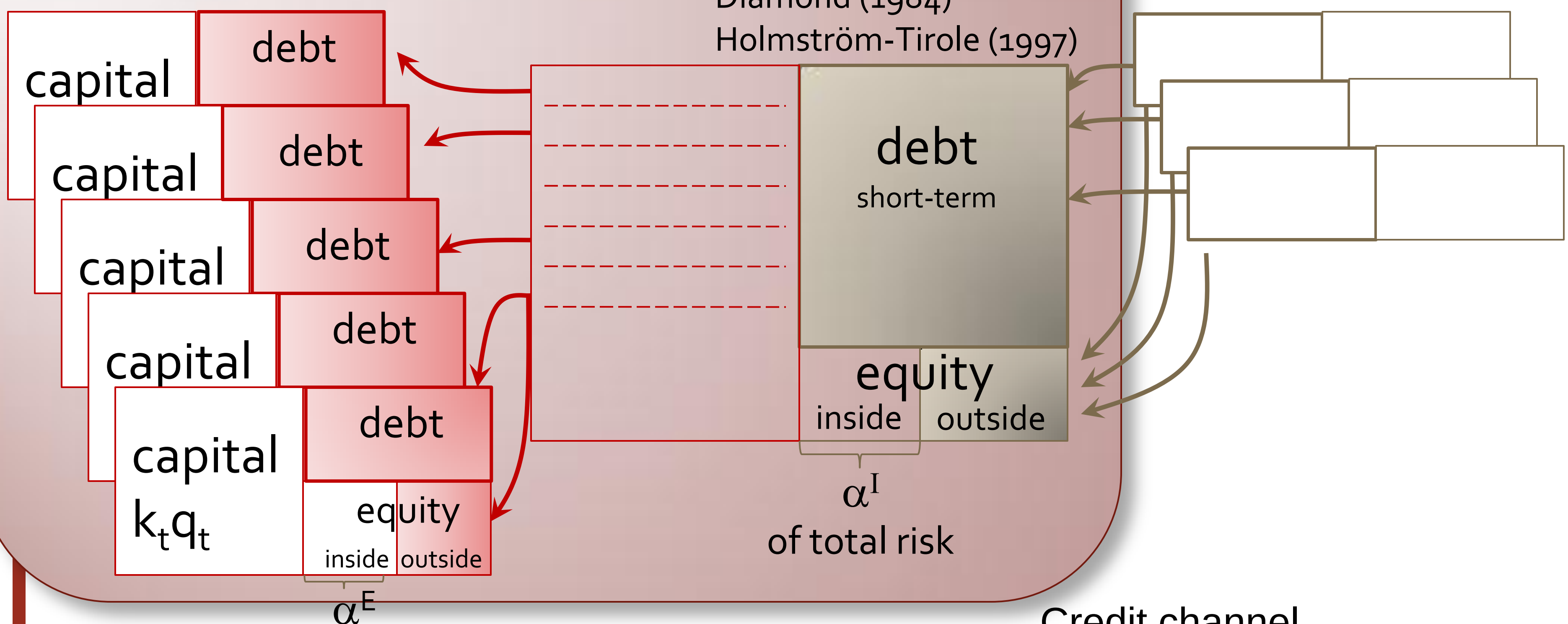
- Less productive



$\alpha := \alpha^E + \alpha^I \geq$ b(m) + c(m)

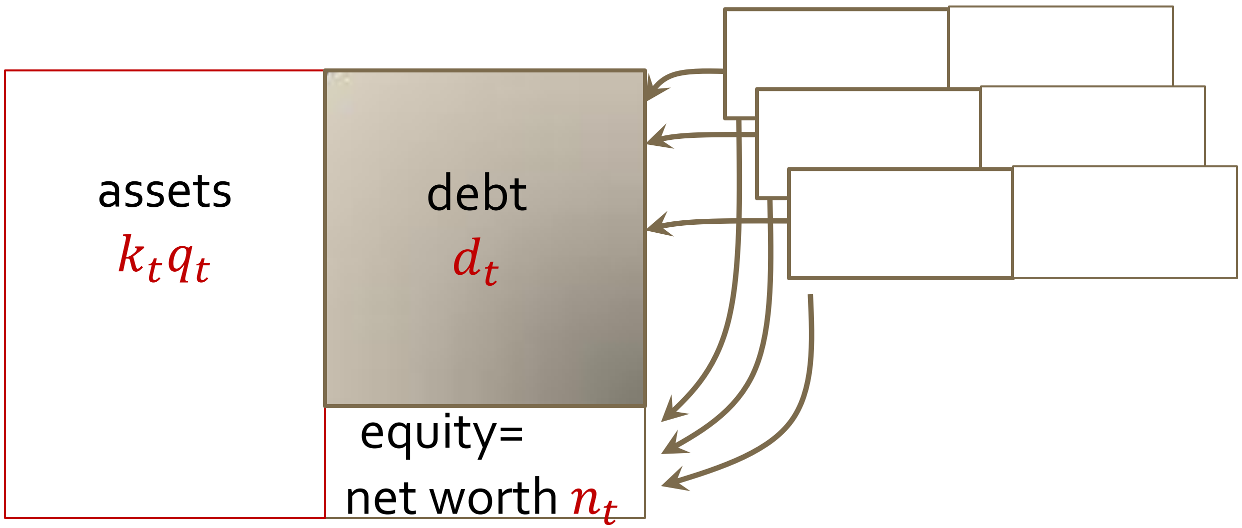
"merged experts"

Credit channel

- Lending channel
- Borrowers' balance sheet channel

Balance sheet dynamics

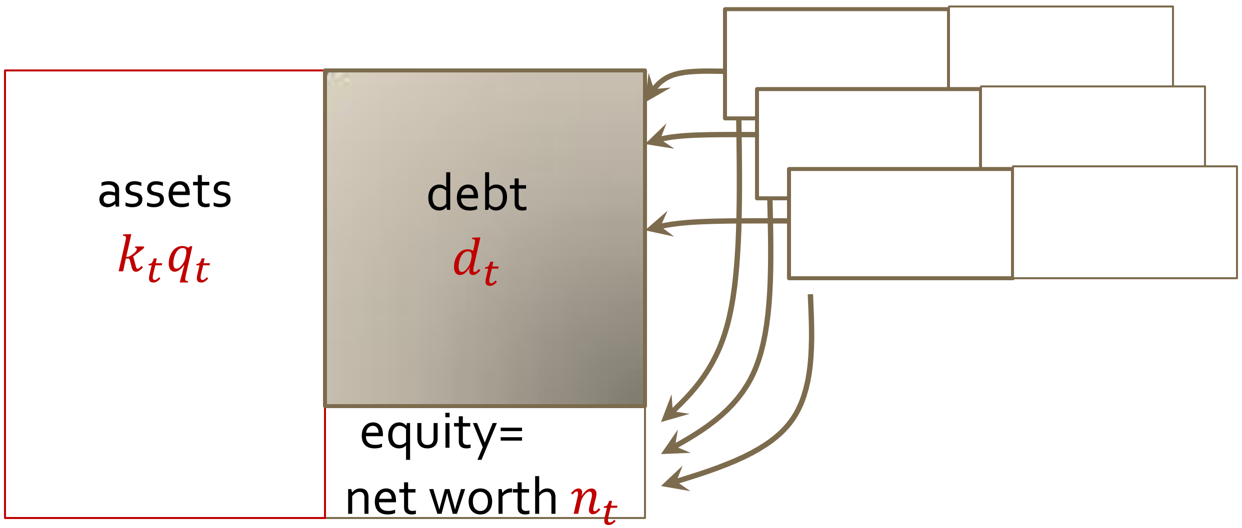
- Productive
- Intermediary
- Less productive



assume $\alpha = 1$ (for today)

Balance sheet dynamics

- Productive
- Intermediary
- Less productive



$$dr_t^k = \left(\frac{a-\iota_t}{q_t} + \Phi(\iota_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right)dt + \left(\sigma + \sigma_t^q\right)dZ_t$$

$$dn_t = rn_t dt + (dr^k - r dt)(k_t q_t) - dc_t = \cdots$$

Intuition – main forces at work

- Investment
  - *Scale up*
    - Scalable profitable investment opportunity
    - Higher leverage (borrow at r)
  - *Scale back*
    - Precaution: - don't exploit full (GE) debt capacity – "dry powder"
      - Ultimately, stay away from fire-sales prices
      - Debt can't be rolled over if $d > k_t \underline{q}$ (note, price is depressed)
      - Solvency constraint
- Consumption
  - Consume *early* and borrow $r < \rho$
  - Consume *late* to overcome investment frictions

aggregate leverage!

Definition of equilibrium

- An equilibrium consists of functions that for each history of macro shocks $\{Z_s, s \in [0, t]\}$ specify
  - q_t the price of capital
  - k_t^i, k_t^h capital holdings and
  - dc_t^i, dc_t^h consumption of representative expert and households
  - ι_t rate of internal investment of a representative expert, per unit of capital
  - r_t the risk-free rate
- such that
  - intermediaries and households maximize their utility, given prices q_t as given and
  - markets for capital and consumption goods clear

Solving for equilibrium

1. Households: risk free rate of r_t = households discount rate
   - Makes HH indifferent between consuming and saving, s.t. consumption market clears
   - Required return when their capital >0 $\underbrace{\frac{a}{q_t} - \underline{\delta} + \mu_t^q + \sigma\sigma_t^q}_{expected\ return\ from\ capital} = r$

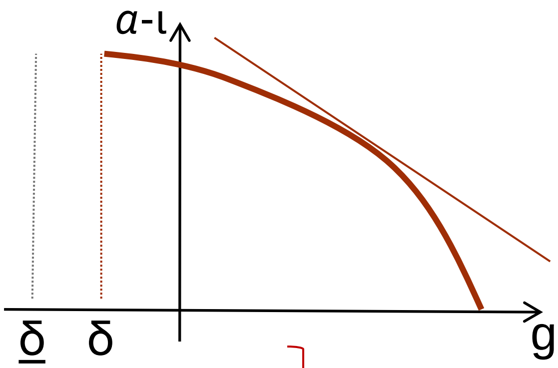
2. Experts choose $\{k_t, \iota_t, c_t\}$ dynamically to maximize utility

$$\max_{c,\iota,k} E\left[\int_0^\infty e^{-\rho t} dc_t\right] \quad \text{s.t.}$$

$$\begin{aligned} dn_t \; &= \; -dc_t + \left(\Phi(i_t) - \delta + \mu_t^q + \sigma\sigma_t^q\right)(k_t q_t)dt \\ &\quad +\left(\sigma + \sigma_t^q\right)(k_t q_t)dZ_t + [(a - \iota_t)k_t - rd_t]dt \\ dn_t &\geq 0 \end{aligned}$$

3. Markets clear: total demand for capital is K_t

Solving for equilibrium



1. Internal investment (static)
2. External investment k_t
   - Given price dynamics $dq_t/q_t = \mu_t^q dt + \sigma_t^q dZ_t$
   - Solvency constraint $n_t \geq 0$
3. When to consume? dc_t

dynamic optimization

- Bellman equation w/ value function $\theta_t n_t$

payoff experts generate from a dollar of net worth by trading undervalued capital

proportional to net worth, atomistic experts have no price impact

$$\rho\theta_t n_t dt = \max_{k_t, dc_t} E\left[dc_t + d(\theta_t n_t)\right]$$

Solving dynamic optimization

- Let value of extra $

$$d\theta_t = \mu_t^\theta \theta_t dt + \sigma_t^\theta \theta_t dZ_t$$

  - recall $dn_t = \ldots.$

- Use Ito's lemma to expand the Bellman equation

$$\rho\theta_t n_t dt = \max_{k_t, dc_t} E[dc_t + d(\theta_t n_t)]$$

  - Risk free: $\underbrace{r}_{risk-free} + \underbrace{\mu_t^\theta}_{E[change\ of\ investment\ opportunities]} = \underbrace{\rho}_{required\ return}$
  - Capital: $\underbrace{\frac{a}{q_t} + g_t + \mu_t^q + \sigma\sigma_t^q - r}_{E[excess\ return\ of\ capital]} = \underbrace{-\sigma_t^\theta(\sigma + \sigma_t^q)}_{capital\ risk\ premium}$
  - $\theta_t \geq 1$, and $dc_t^i > 0$ only when $\theta_t = 1$.

- $e^{-\rho t}\theta_t/\theta_0$ is the experts' stochastic discount factor

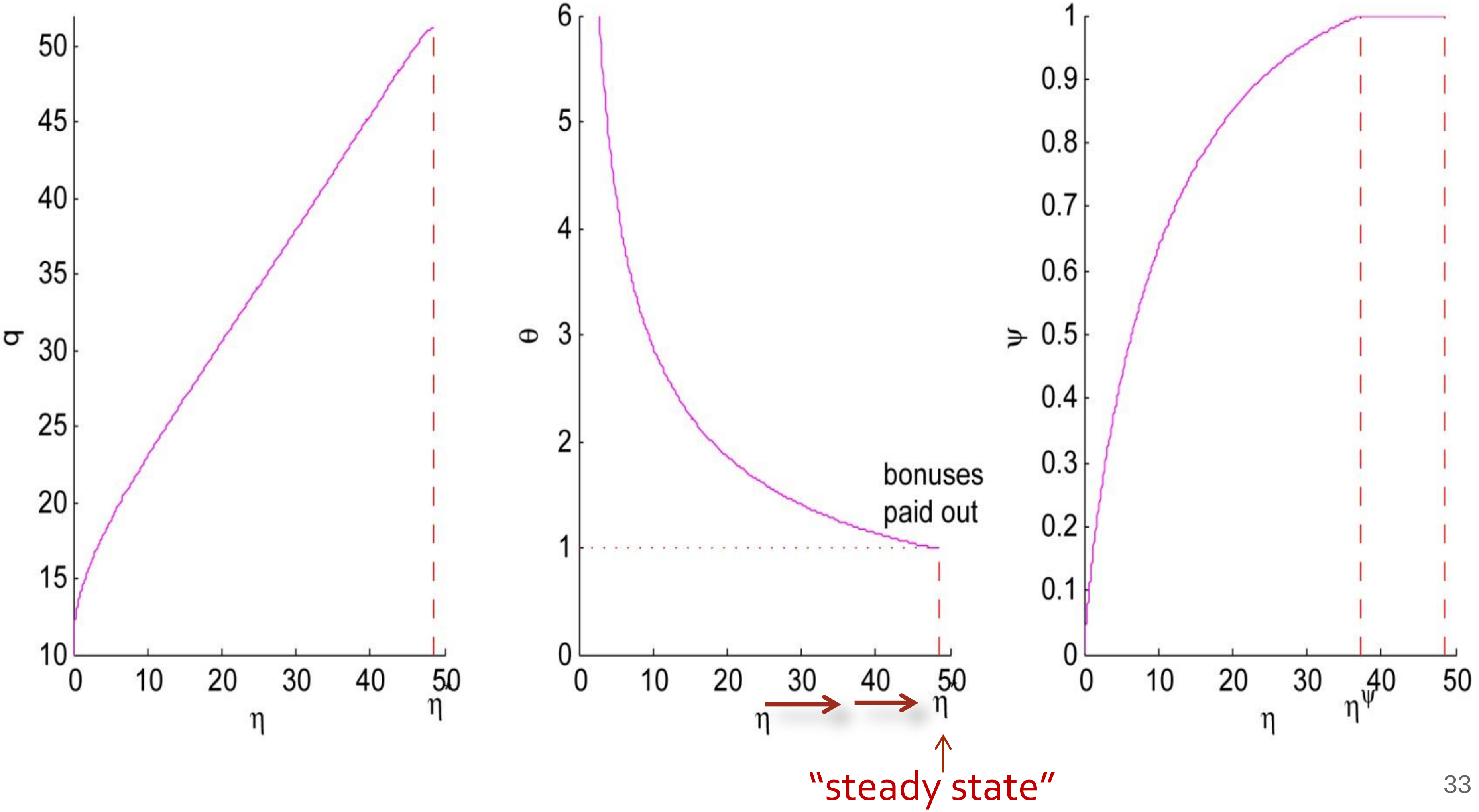
Scale invariance

- Model is scale invariant
  - K_t total physical capital
  - N_t total net worth of all experts
- Solve q_t and θ_t as a function of the single state variable
  - $\eta_t = \frac{N_t}{K_t}$

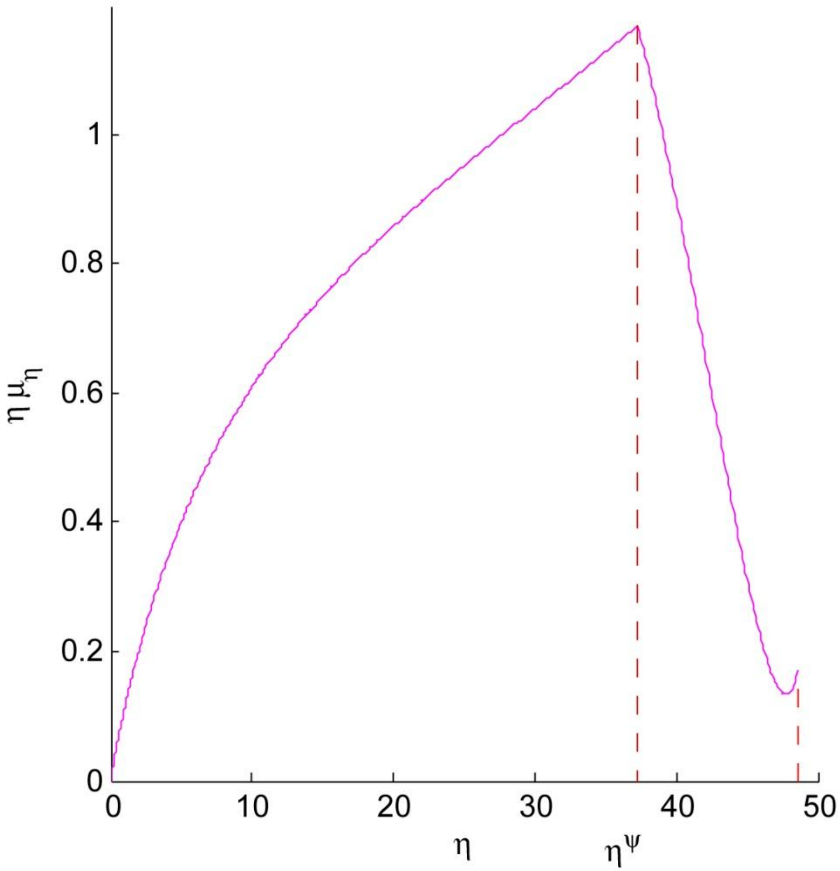
➡ Mechanic application of Ito's lemma
Pricing equations get transformed into
ordinary differential equations for $q(\eta)$ and $\theta(\eta)$

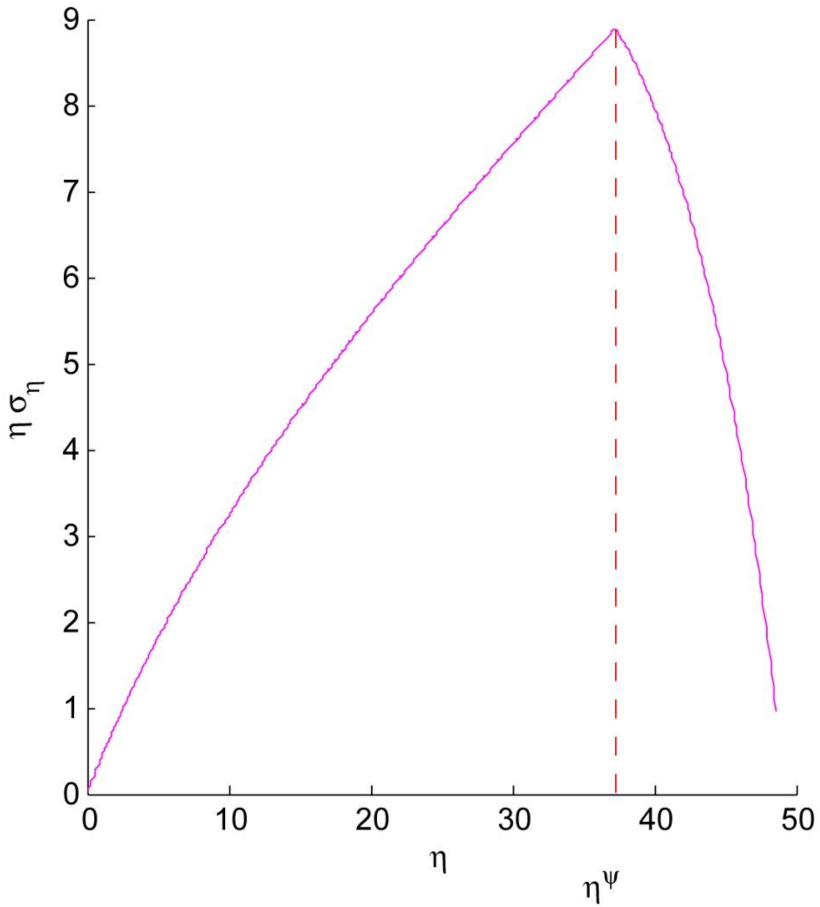
Equilibrium

- Boundary conditions: $q(0) = \underline{q}$, $\theta(0)= \infty$, $\theta(\eta^*) = 1$, $q'(\eta^*)= \theta'(\eta^*) = 0$

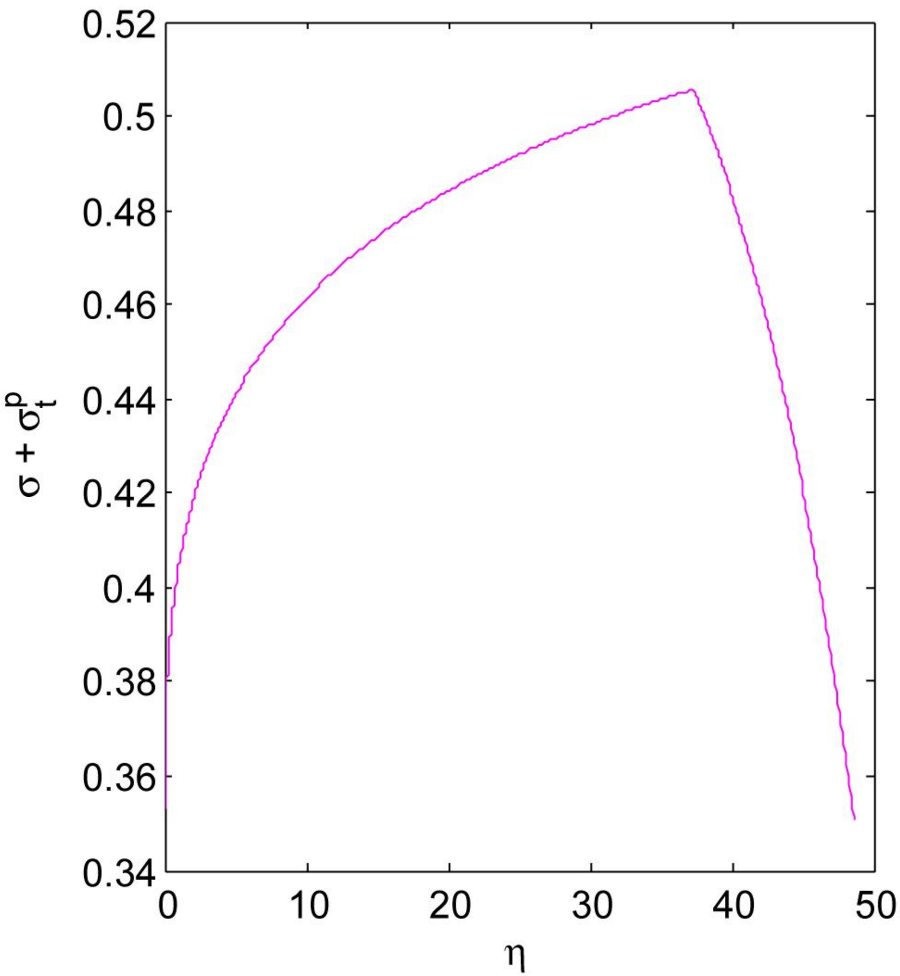


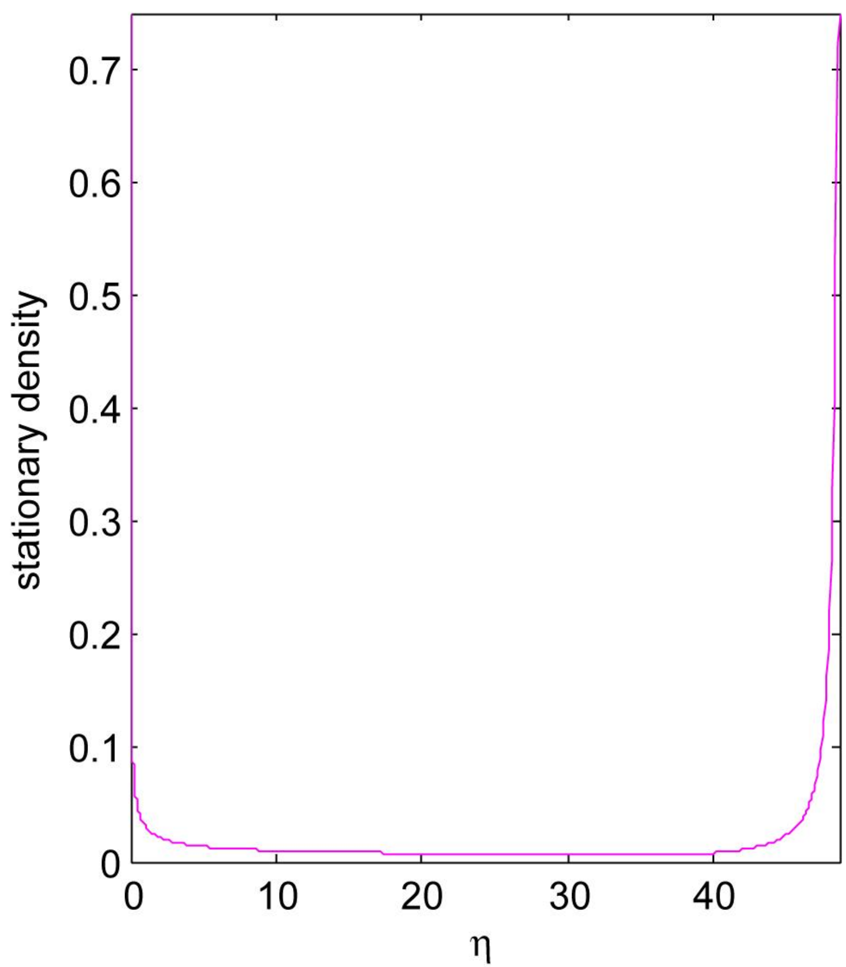
Equilibrium dynamics





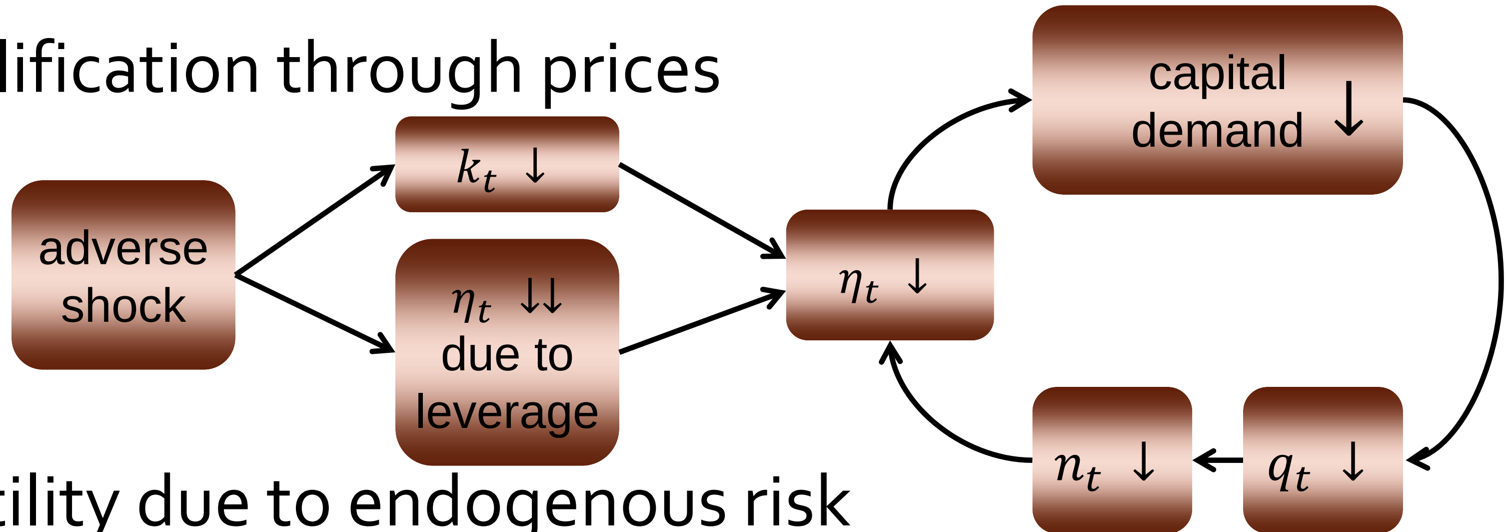
Endogenous risk & “Instability”





Endogenous Risk through Amplification

- Amplification through prices



- Volatility due to endogenous risk

$$\sigma_t^q = \frac{q'(\eta_t)\sigma(q_t - \eta_t)}{1 - q'(\eta_t)}.$$

amplification

- Key to amplification is $q'(\eta)$
  - Depends how constrained experts are

Dynamics near and away from SS

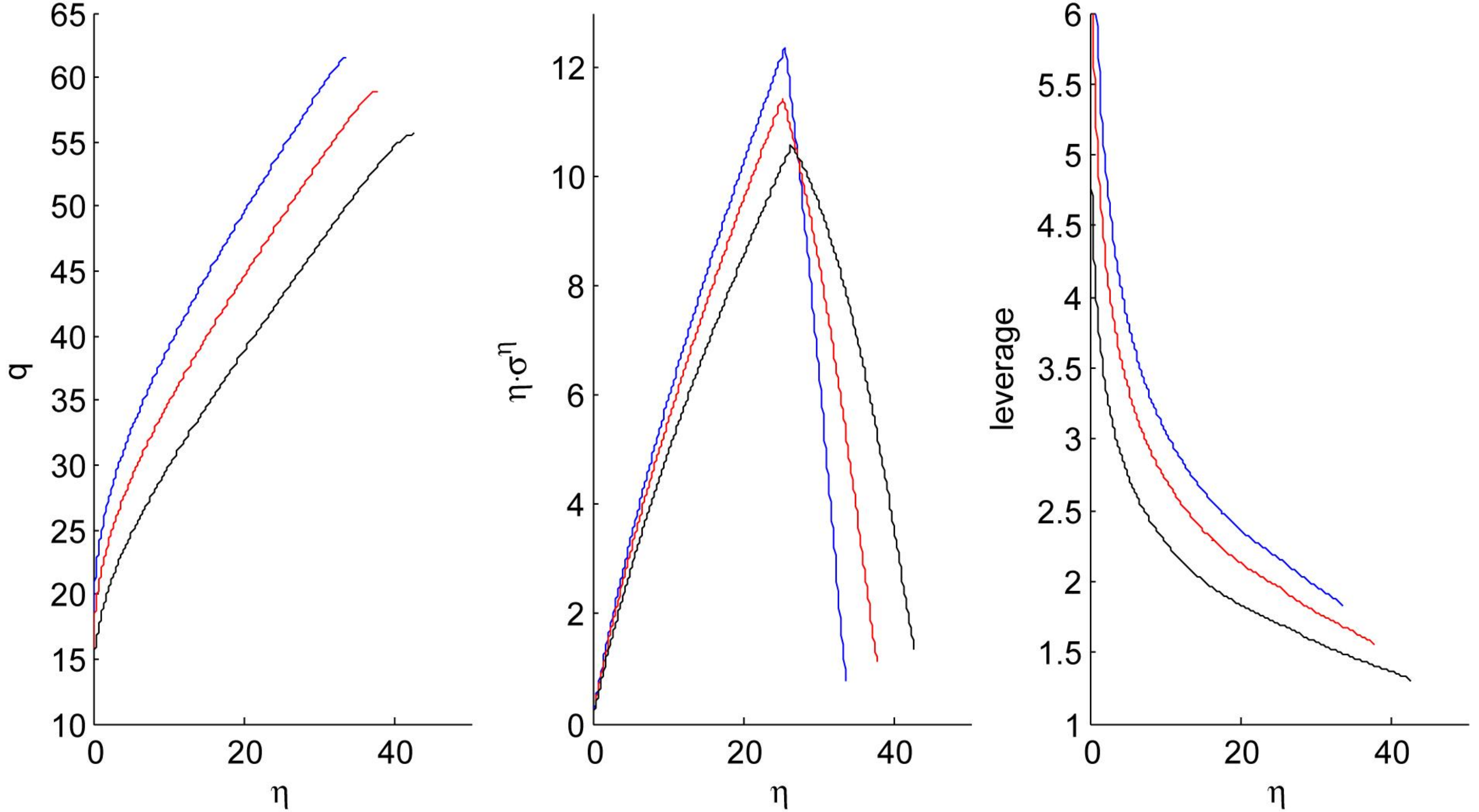
- Intermediaries choose payouts endogenously
  - Exogenous exit rate in BGG/KM
  - Payouts occur when intermediaries are least constrained

$$q'(\eta^*) = 0$$

- Steady state: experts unconstrained
  - Bad shock leads to lower payout rather than lower capital demand
  - $q'(\eta^*) = 0, \sigma_t^q(\eta^*) = 0$
- Below steady state: experts constrained
  - Negative shock leads to lower demand
  - $q'(\eta^*)$ is high, strong amplification, $\sigma_t^q(\eta^*)$ is high
  - ... but when η is close to 0,
    $q \approx \underline{q}(\eta_t), q'(\eta)$ and $\sigma_t^q(\eta^*)$ is low

Note difference to BGG/KM

"Volatility Paradox" … σ (.025 ,.05 ,.1)



- As σ decreases, η^* goes down, $q(\eta^*)$ goes up, $\sigma^\eta(\eta^*)$ may go up, max σ^η goes up

Ext1: asset pricing (cross section)

- **Capital:** Correlation increases with σ^q
  - Extend model to many types i of capital

$$\frac{dk_t^i}{k_t^i} = \left(\Phi\left(\iota_t^i\right) - \delta\right)dt + \underbrace{\sigma dZ_t}_{\text{aggregate shock}} + \underbrace{\sigma' dz_t^i}_{\text{uncorrelated shock}}$$

  - Experts hold diversified portfolios
    - Equilibrium looks as before, (all types of capital have same price) but
    - Volatility of $q_t k_t$ is $\sigma + \sigma' + \sigma^q$
    - Endogenous risk is perfectly correlated, exogenous risk not
    - For uncorrelated z^i and z^j

      correlation $(q_t^i k_t^i, q_t^j k_t^j)$ is $(\sigma + \sigma^q)/(\sigma + \sigma' + \sigma^q)$

      which is increasing in σ^q

Ext1: asset pricing (cross section)

- **Outside equity:**
  - Negative skewness
  - Excess volatility
  - Pricing kernel: e^{-rt}
    - Needs risk aversion!

- **Derivatives:**
  - Volatility smirk (Bates 2000)
  - More pronounced for index options (Driessen et al. 2009)

Ext2: Idiosyncratic jump losses

$$dk_t^i = gk_t^i dt + \sigma k_t^i dZ_t + k_t^i dJ_t^i$$

- J_t^i is an idiosyncratic compensated Poisson loss process, recovery distribution F and intensity $\lambda(\sigma_t^q)$
- $q_t k_t^i$ drops below debt d_t, costly state verification

- Time-varying interest rate spread
- Allows for direct comparison with BGG

Ext. 2: Idiosyncratic losses

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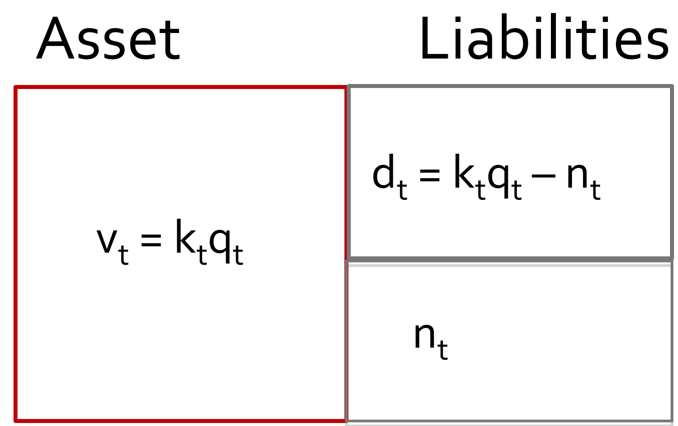
- $q_t k_t^i$ drops below debt d_t, costly state verification
- Debt holders' loss rate $\lambda(\sigma^p)v\int_0^{\frac{d}{v}}(\frac{d}{v}-x)dF(x)$
- Verification cost rate

$$\lambda(\sigma^p)v\underbrace{\int_0^{\frac{d}{v}} cx\,dF(x)}_{C(\frac{d}{v})}$$

- Leverage bounded not only by precautionary motive, but also by the cost of borrowing



Ext2: Equilibrium

- Experts borrowing rate $> r$
  - Compensates for verification cost
- Rate depends on leverage, price volatility
- $d\eta_t$ = diffusion process (without jumps) because losses cancel out in aggregate

Ext3: Securitization

- Experts can contract on shocks Z_t and dJ_t^i directly among each other, zero contracting costs
- In principle, good thing (avoid verification costs)
- Equilibrium
  - experts fully hedge idiosyncratic risks
  - experts hold their share (do not hedge) aggregate risk Z_t, market price of risk depends on $\sigma_t^\theta(\sigma + \sigma_t^q)$
  - with securitization experts lever up more (as a function of η_t) and bonus payments occur "sooner"
  - financial system becomes less stable
  - risk taking is endogenous (Arrow 1971, Obstfeld 1994)

Conclusion

- Incorporate financial sector in macromodel
  - Higher growth
  - Exhibits instability
    - similar to existing models (BGG, KM) in term of persistence/amplification, but
    - non-linear liquidity spirals (away from steady state) lead to instability
- Risk taking is endogenous
  - "Volatility paradox:" Lower exogenous risk leads to greater leverage and may lead to higher endogenous risk
  - Correlation of assets increases in crisis
  - With idiosyncratic jumps: countercyclical credit spreads
  - Securitization helps share idiosyncratic risk, but leads to more endogenous risk taking and amplifies systemic risk
- Welfare: (Pecuniary) Externalities
  - excessive exposure to crises events

Thank you! ☺