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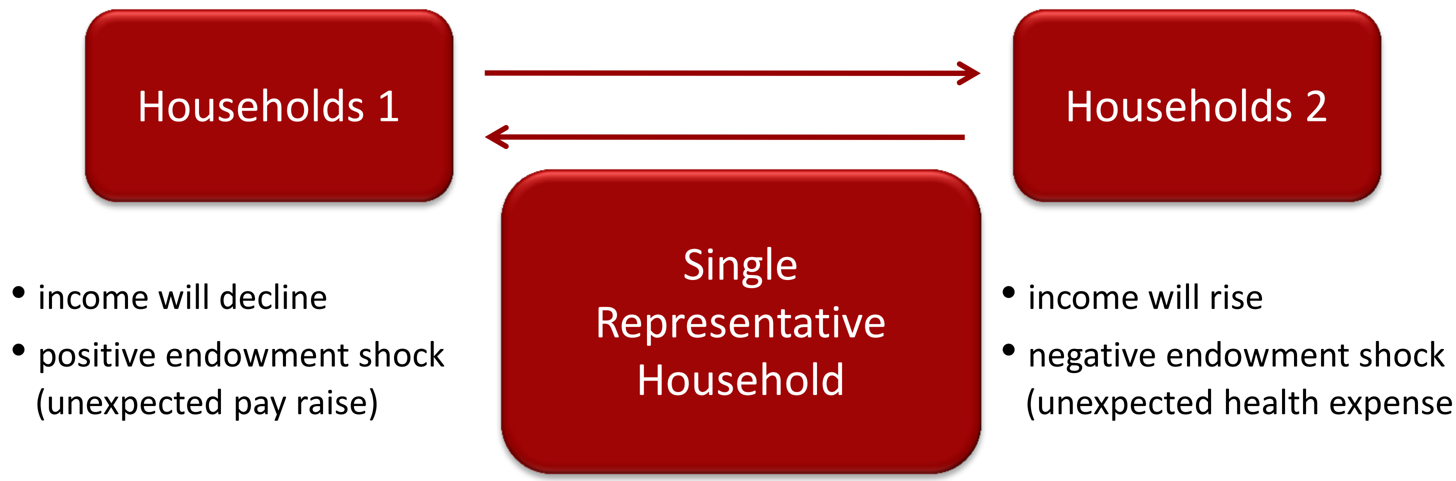
LECTURE 12: “FRICTIONAL FINANCE”

Frictionless Finance

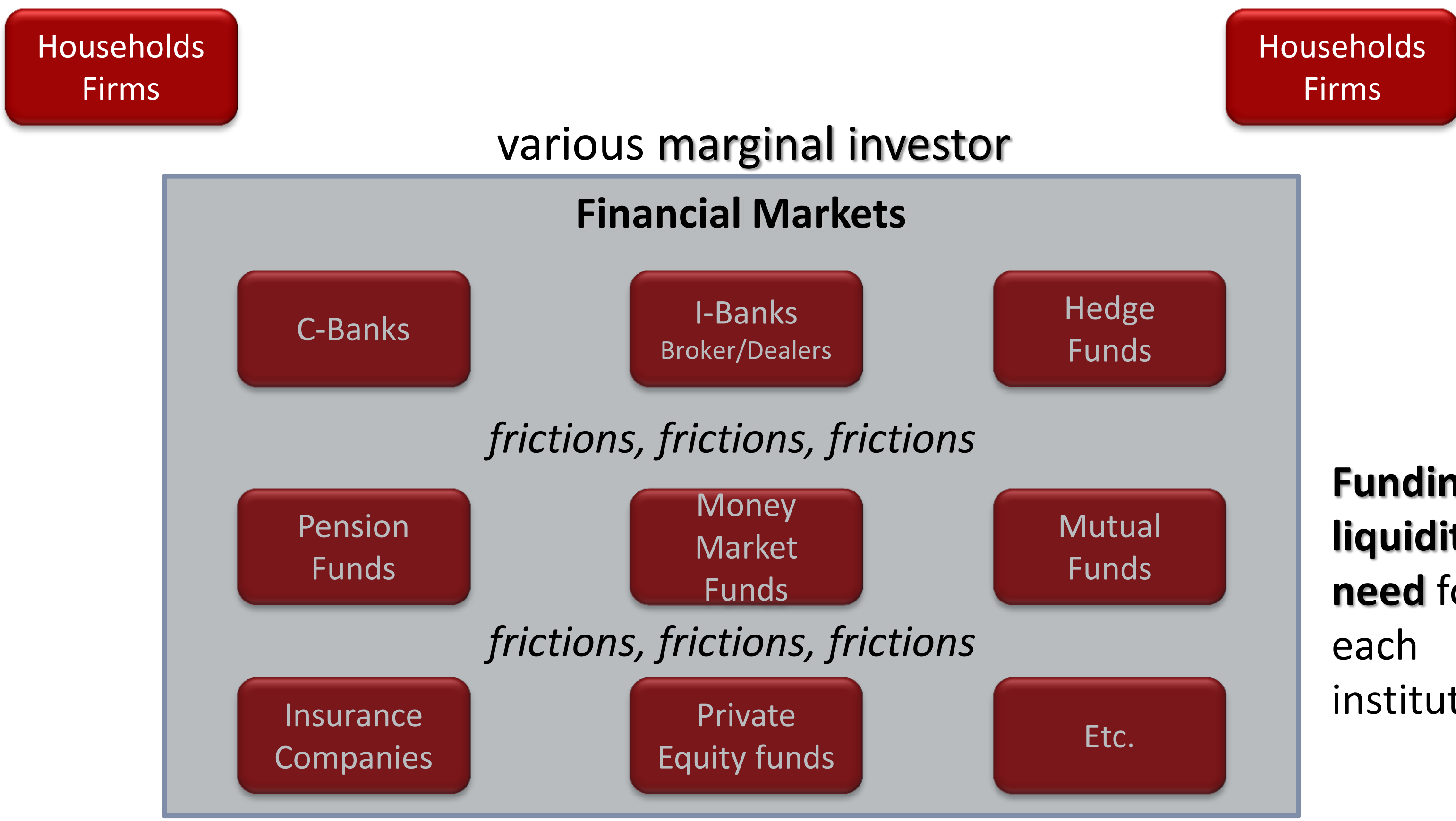
- Endowment Economy



- Aggregate endowment income/shock (no trading)
- Intertemporal Elasticity of Substitution (incentive to smooth consumption)
- Risk aversion

"Frictional Finance"

- Production Economy



Funding liquidity need for each institution

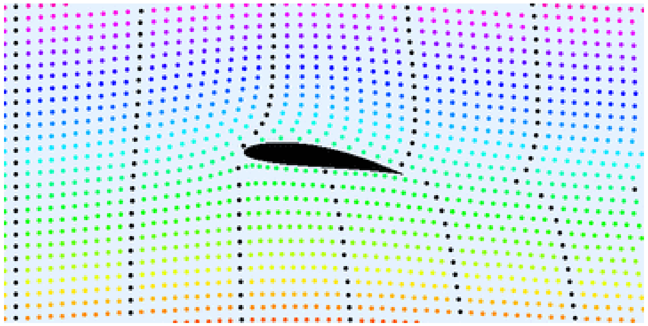
Analogy to Physics

Frictionless

- Physics
  - Gravity (Newton/Einstein)
    - Drop of stone = drop of feather
- Finance
  - Arrow-Debreu Finance
    - Law of one price
- Good benchmark!

Frictions

- Aviation



- Finance
  - Price impact/transaction costs
    Market illiquidity
  - Collateral/margins/haircuts
    Funding liquidity

Relative vs. Absolute Asset Pricing

- Relative asset pricing
  - No (risk-free) arbitrage $\Rightarrow$ Bounds on Prices

- Absolute asset pricing
  - Needed whenever replication strategy is not perfect
  - Additional risk requires a risk premium
    - which needs to be priced!
    - Marginal investor $\neq$ representative agent
      - Wealth/consumption of subgroup with expertise matters!

Forms of Frictions

1. Extra dollar costs
2. Waiting costs due to search frictions
3. quantity constraint $\Rightarrow$ Lagrange multiplier λ

- Market Liquidity
- Funding Liquidity

Leverage and Margins

- Financing a *long position* of $x_t^{j+} > 0$ shares at price $p_t^j = 100$:
  - Borrow \$90 dollar per share;
  - Margin/haircut: $m_t^{j+} = 100 - 90 = 10$ $\Rightarrow$ Capital use: $\$10 x_t^{j+}$
- Financing a *short position* of $x_t^{j-} > 0$ shares:
  - Borrow securities, and lend collateral of \$110 dollar per share
  - Short-sell securities at price of 100
  - Margin/haircut: $m_t^{j-} = 110 - 100 = 10$ $\Rightarrow$ Capital use: $\$10 x_t^{j-}$
- Positions frequently marked to market
  - payment of $x_t^j (p_t^j - p_{t-1}^j)$ plus interest
  - margins potentially adjusted – *more later on this*
- Margins/haircuts must be financed with capital:

$$\sum_j \left(x_t^{j+} m_t^{j+} + x_t^{j-} m_t^{j-} \right) \le W_t \text{ where } x^j = x_t^{j+} - x_t^{j-}$$

with perfect cross-margining $M_t\left(x_t^1, \cdots, x_t^J\right) \le W_t$

Liquidity Concepts

A | L

Funding liquidity

- Can't **roll over** short term debt
- **Margin**-funding is recalled

Liquidity Concepts

A	L
Market liquidity	**Funding liquidity**
• Can only sell assets at **fire-sale prices**	▪ Can't **roll over** short term debt ▪ **Margin**-funding is recalled

Liquidity Concepts

A | L

Market liquidity

- Can only sell assets at **fire-sale prices**

Funding liquidity

- Can't **roll over** short term debt
- **Margin**-funding is recalled

measures	quantity	price	quantity	price
static	Trading volume	Bid-ask	Unsecured vs. collateralize funding	TED spread (term spread)
		VIX Downside correlation	Haircuts/ margins/LTV	
dynamic			Debt maturity to • Asset maturity • Asset market liq	

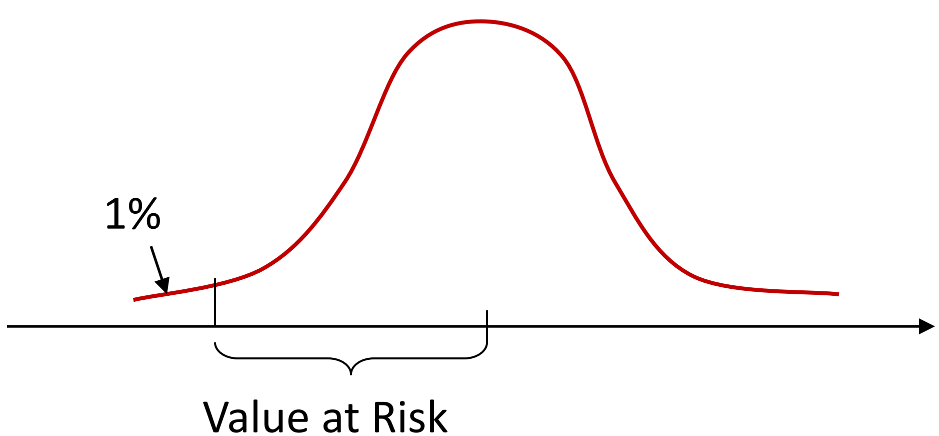
Market and Funding Liquidity

- Step 1:
  - exogenous margin requirement (funding liquidity)
  - Derive modified CAPM

- Step 2:
  - Endogenous margins as a function of future volatility
  - Liquidity spirals
    (adverse feedback loops between market and funding liquidity)

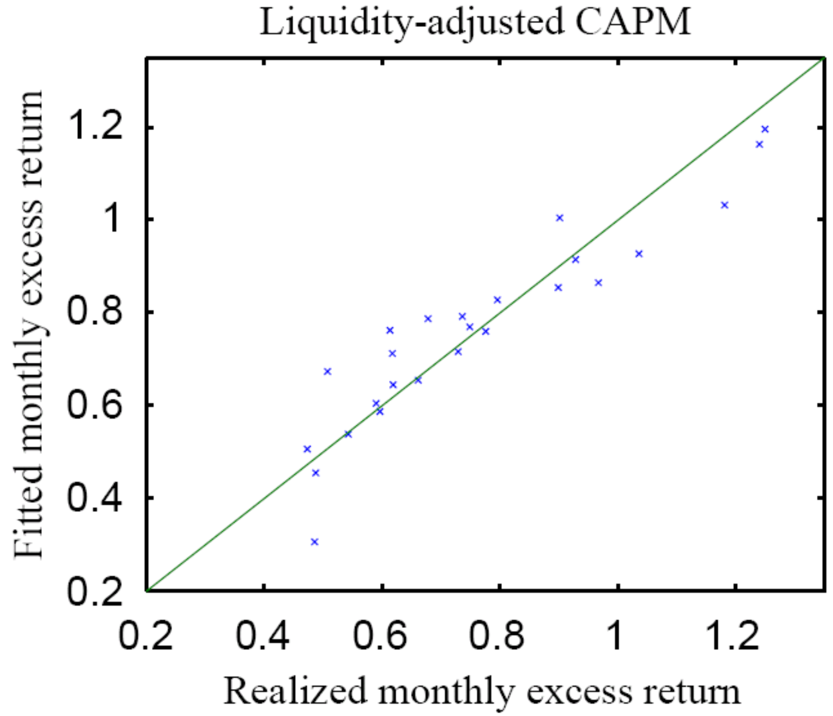
Margins – Value at Risk (VaR)

- How are margins set by brokers/exchanges?
  - Value at Risk: $\Pr[-(p_{t+1} - p_t) \geq m] = 1\%$



Exogenous Margin constraints

- Cross section of stocks better explained by liquidity-adjusted CAPM



Model of Funding Liquidity

- Overlapping generations economy:

$$\max_x x'\left(E_t[P_{t+1}] - \left(1+r^f\right)P_t\right) - \frac{\gamma^i}{2} x'\Sigma_\mathrm{t} x$$

  - Portfolio constraint: $\sum_j m_t^j \left|x^j\right| P_t^j \leq W_t^i$
  - Lagrange multiplier ψ

- Solution given by FOC

$$x^i = \frac{1}{\gamma^i}\Sigma_t^{-1}\left[E_t[P_{t+1}] - \left(1+r^f\right)P_t - \psi_t D(m_t) P_t\right]$$

$D(\cdot)$ makes a vector into a diagonal matrix

Equilibrium

- Competitive equilibrium with net supply $\bar{x}$

$$\sum_i x^i = \bar{x}$$

- Equilibrium price, with $\frac{1}{\gamma} \equiv \sum_i \frac{1}{\gamma^i}$ and $\phi = \frac{\gamma}{\gamma^b}$

$$P_t = D\left(1 + r^f + \psi_t \phi m_t\right)^{-1} [E_t[P_{t+1}] - \gamma \Sigma_t \bar{x}]$$

Margin CAPM

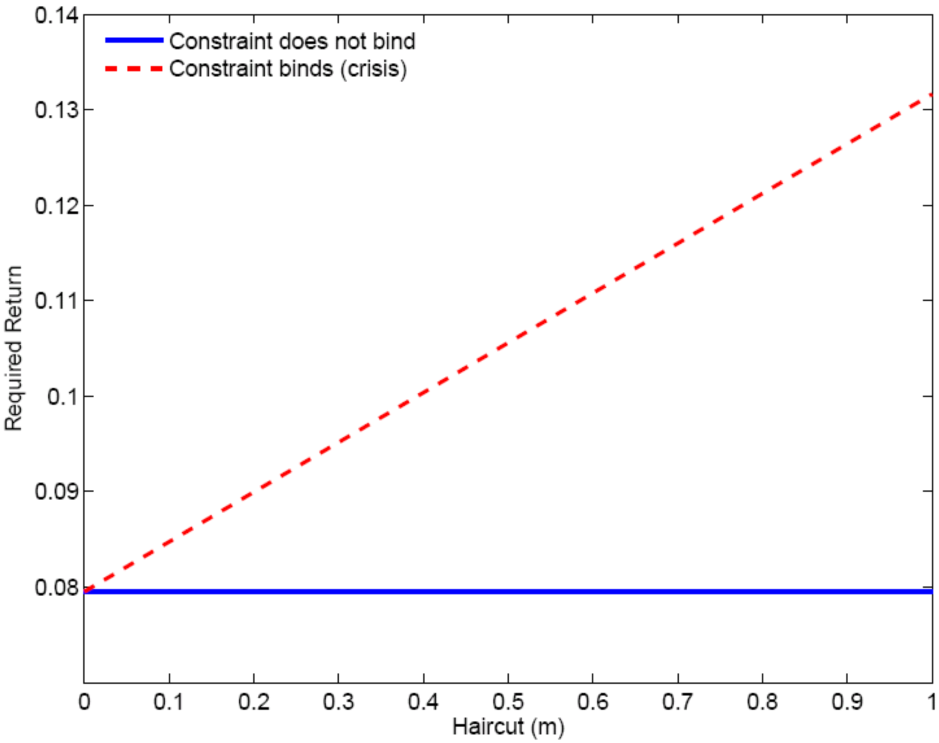
- Equilibrium required return

$$E_t[r_{t+1}^s] = r^f + \beta_t^s \lambda_t + \psi_t \phi_t m_t^s$$

  – Lagrange multiplier ψ_t
  – Fraction of constrained agents ϕ_t
  – Margin requirement m_t
  – Risk premium $\lambda_t = E_t[r_{t+1}^M] - r^f - \psi_t$

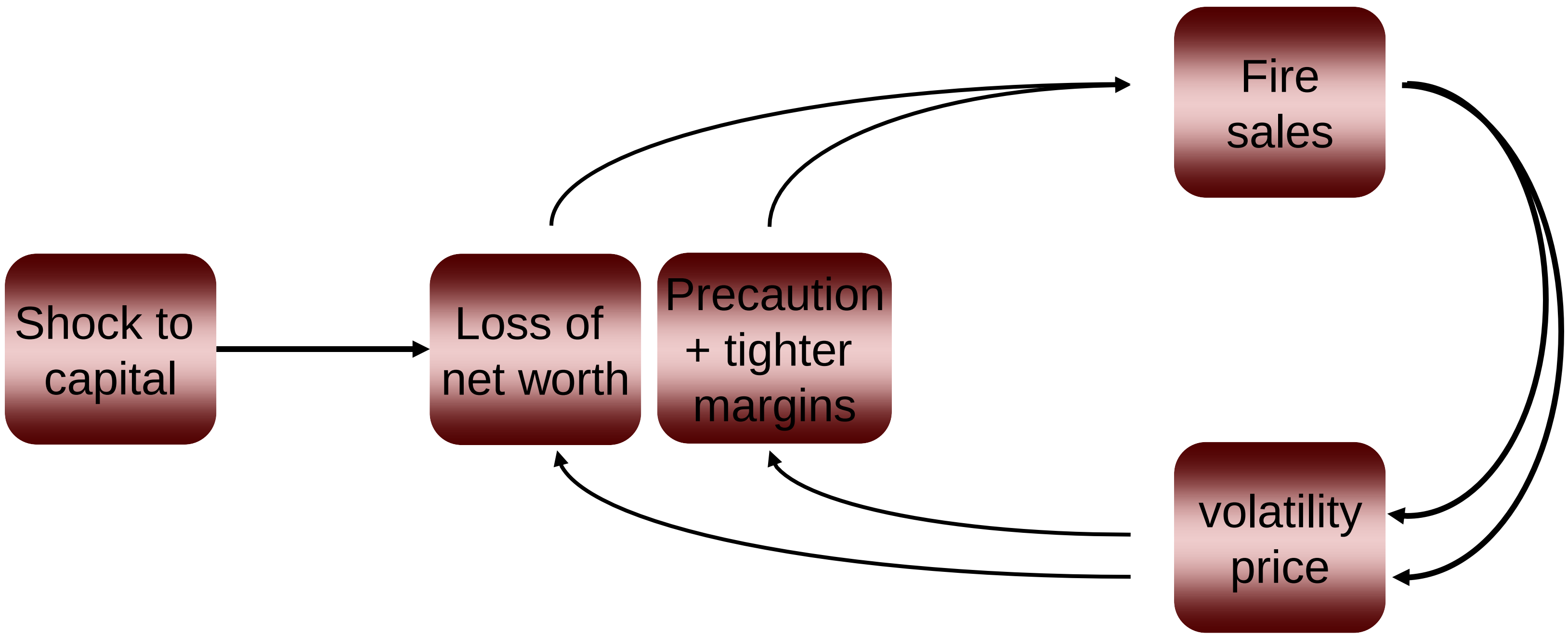
Margin CAPM

- For high margin securities, required return increases when balance sheets are constrained



Endogenous Margin Constraint

- Loss and Margin Spiral



Brunnermeier-Pedersen Model

- Time: $t = 0,1,2$
- One asset with final asset payoff v (later: assets j=1,…,J)
- Market illiquidity measure: $\Lambda_t = |E_t[v] - p_t|$

  (deviation from "fair value" due to selling/buying pressure)
- Agents
  - Initial customers with supply $S(z, E_t[v] - p_t)$ at $t = 1,2$
  - Complementary customers' demand $D(z, E_2[v] - p_2)$ at $t = 2$
  - Risk-neutral dealers provide *immediacy* and
    - face capital constraint

$$x_m(\sigma, \Lambda) \leq W(\Lambda) := \max\left\{0, \underbrace{B}_{\text{cash}} + x_0 \underbrace{(E_1[v] - \Lambda)}_{\text{"price" of stock holding}}\right\}$$

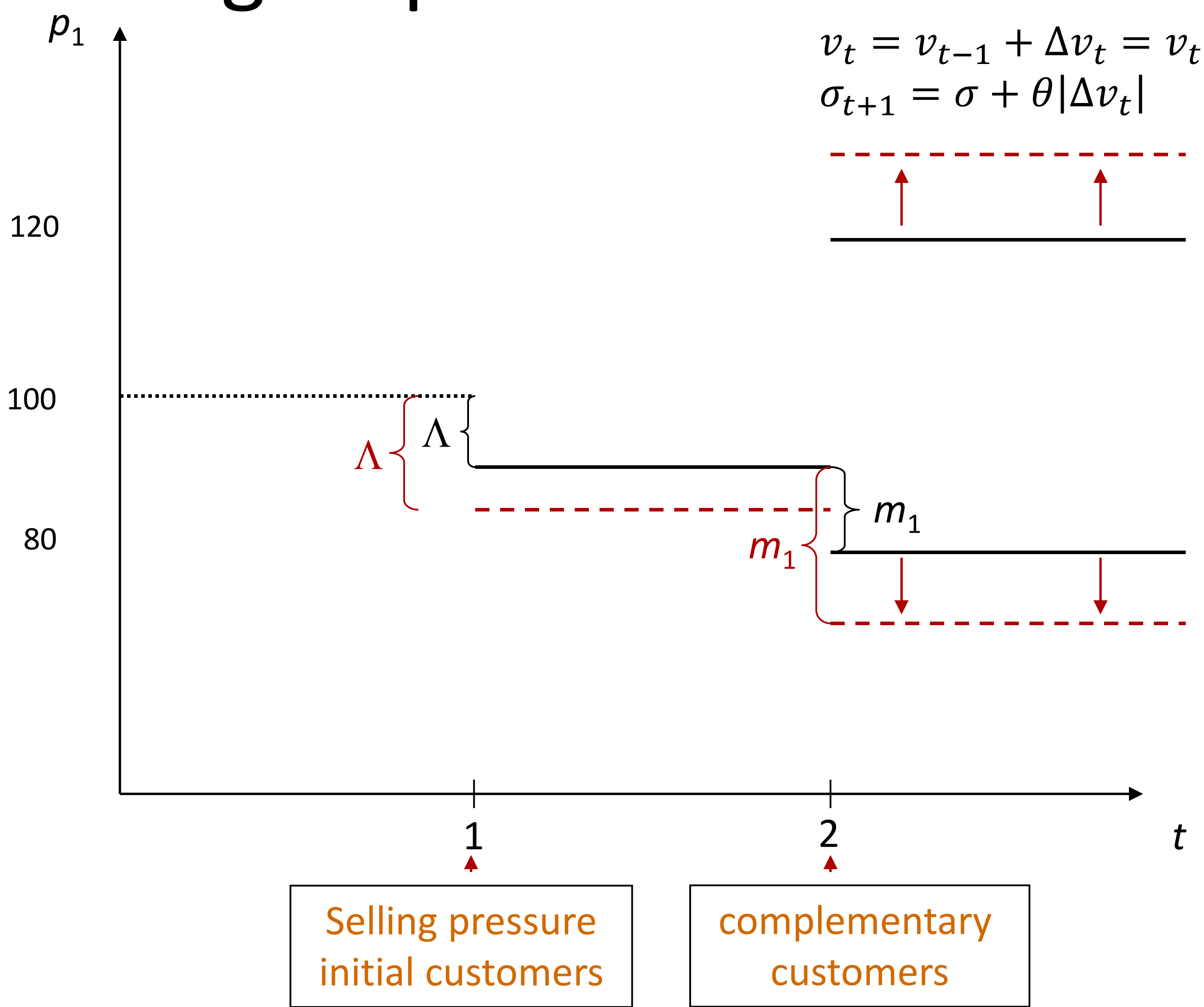
Financiers' margin setting

- Margins are set based on Value-at-Risk
- Financiers do not know whether price move is due to
  - Likely, movement in fundamental
  - Rare, Selling/buying pressure by customers who suffered asynchronous endowment shocks.

$$m_1^{j+} = \Phi^{-1}(1-\pi)\sigma_2 = \bar{\sigma} + \bar{\theta}|\Delta p_1| = m_1^{j-}$$

Margin Spiral – Increased Vol.

$$v_t = v_{t-1} + \Delta v_t = v_{t-1} + \sigma_t \varepsilon_t$$
$$\sigma_{t+1} = \sigma + \theta |\Delta v_t|$$



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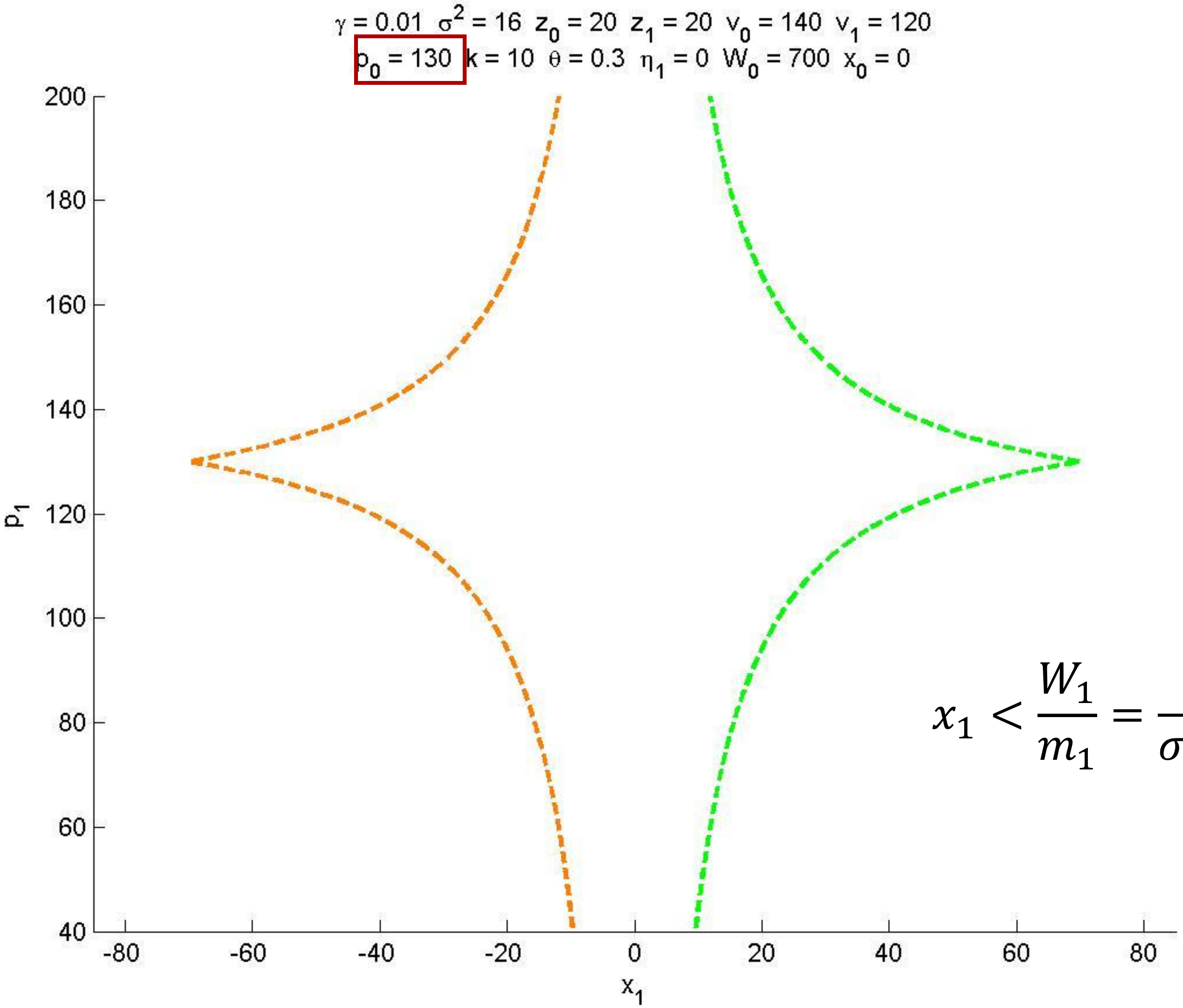
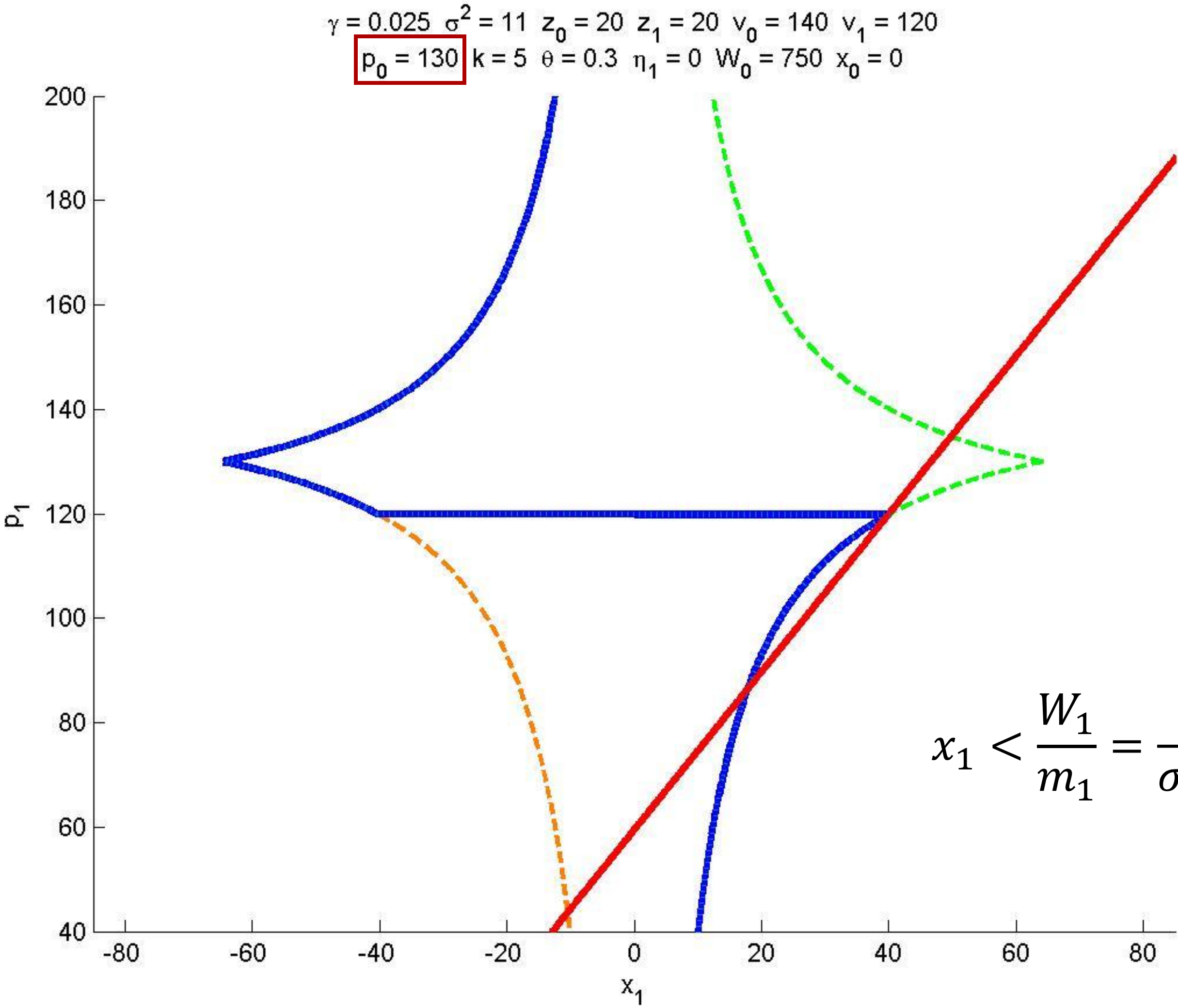


customers' supply

$$x_1 < \frac{W_1}{m_1} = \frac{W_1}{\sigma + \theta|\Delta \mathrm{p}_1|}$$

Margin Spiral – Increased Vol.



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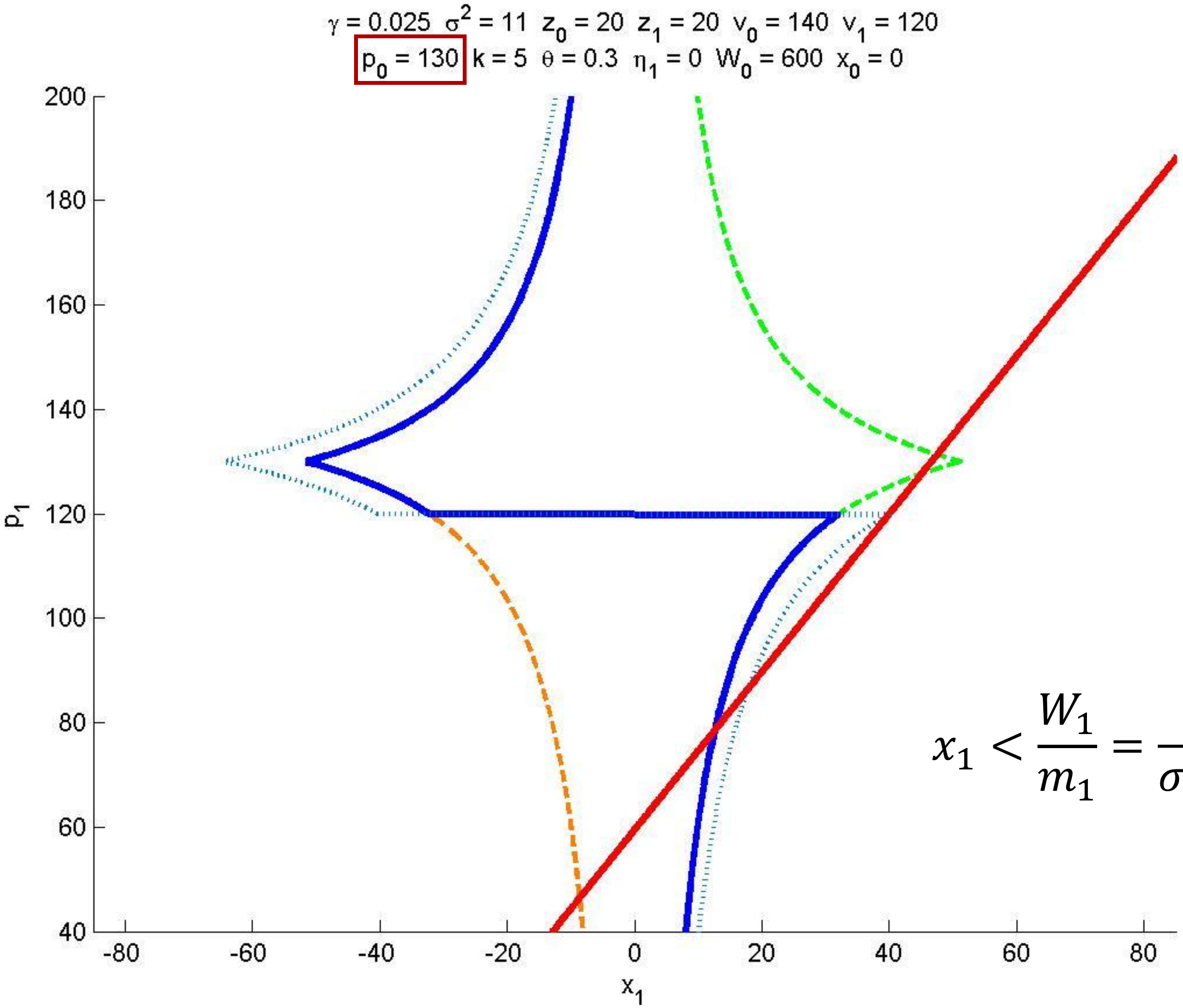

customers' supply

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Multiple Assets

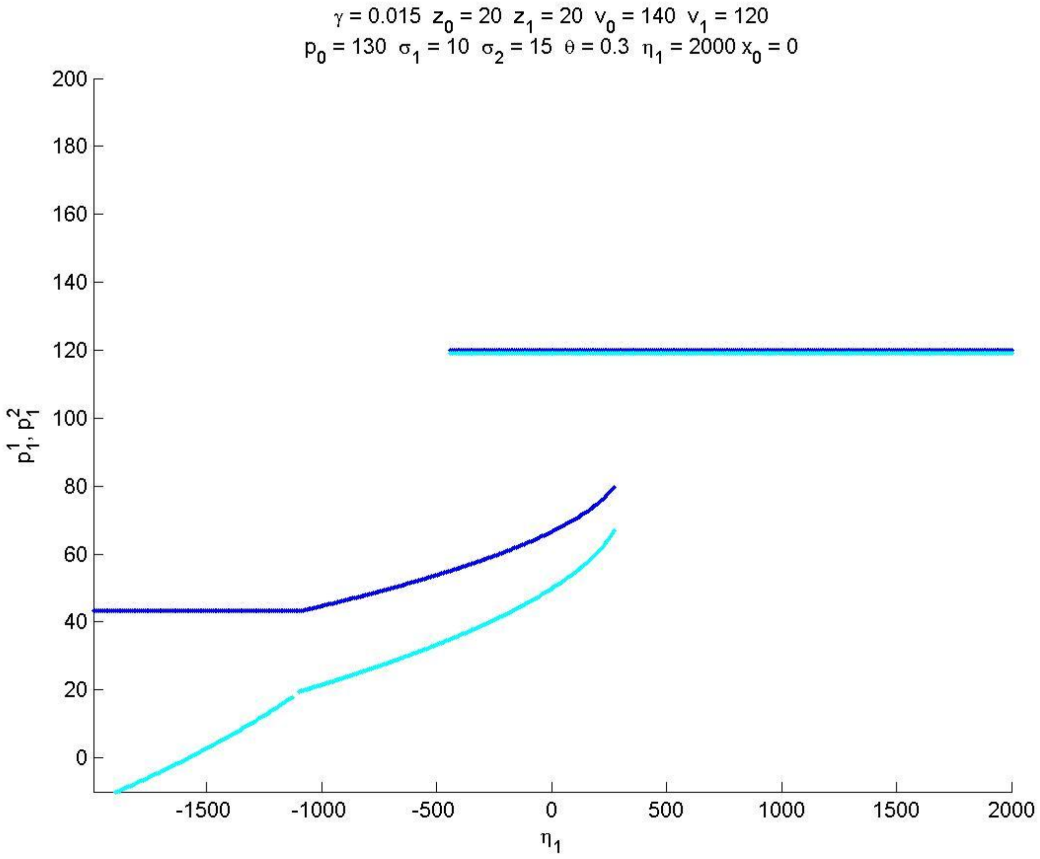
- Dealer maximizes expected profit per capital use
  - Expected profit $E_1\left[v^j\right] - p^j = \Lambda^j$
  - Capital use m^j
- Dealers
  - Invest only in securities with highest ratio $\frac{\Lambda^j}{m^j}$
- Hence, illiquidity/margin ratio $\frac{\Lambda^j}{m^j}$ is constant

Commonality & Flight to Quality

- Commonality
  - Since funding liquidity is driving common factor
- Flight to Quality
  - Quality=Liquidity
    Assets with lower fund vol. have better liquidity
  - Flight
    liquidity differential widens when funding liquidity becomes tight

Flight to Quality

m^2=Volatility of Security2 = 2 > 1 = Volatility of Security1=m^1



Summary

- Financial Frictions matter
- Relative asset pricing ⇒ bounds on asset prices
- Marginal investor matters
  - Can vary from asset to asset (depending on expertise)
- Funding Liquidity constraint
  ⇒ Lagrange multiplier modifies CAPM
- Market Liquidity is impaired
  ⇒ feeds back to Funding Liquidity (collateral)